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APPLICABILITY OF FUZZY LOGIC TO THE DEFINITION OF THE BUCKLING LENGTH OF STEEL PLANE FRAMES

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Abstract. This paper presents a fuzzy-logic-based approach to the definition of buckling length evoked by the uncertainty of joint stiffness and boundary conditions for the member support. The example demonstrates a lucid and specific application of fuzzy sets in modelling uncertainties in design. To carry out analysis, the extension principle in the form of α-cuts was used. Buckling lengths were analysed utilizing stability solution according to the second-order theory. The beam finite element method with the shape functions of sin and sinh was utilized in the analysis taking into account valuable information on the uncertainties of input data.

Keywords: steel, concrete, structure, design, reliability, random, fuzzy logic, imperfection.

1. Introduction

Two basic conventional methods are generally utilized for analyzing the load-carrying capacity of steel structures by EUROCODES, i.e. stability solution with buckling length and the geometric non-linear solution. Stability solution is still employed by designer engineers and can be expected it will not be rejected from the field of practical applications for some years (Kala, Omishore 2006).

In complex structures with numerous load case combinations, it is not common practice to perform stability calculation for all loading cases; another source of uncertainty is determining joint stiffness and their combinations.

The buckling length (parameter) of a member of a structural system is therefore frequently chosen for all loading cases as one value by the designer's expertise. Buckling length cannot be statistically evaluated as it is a typical vague characteristic of the solution that can be mathematically modelled utilizing fuzzy sets (Omishore, Kala 2006).

2. Support Joint Stiffness as a Fuzzy Number

The vagueness of mathematical modelling rests in the emulation of the function of the studied object by another object called the model. EUROCODE 3 lists a number of methods for the solution of the carrying capacity of the frames in Fig. 1: analysis of buckling length, geometric non-linear solutions, a combination of the fore mentioned methods and simplified procedures according to the first-order theory.

The determination of the internal forces and design of members utilizing the geometric non-linear solution is limited by a correct determination of initial geometric imperfections for each structure. Even though EUROCODE 3 lists the initial geometric imperfections for the basic frame cases, imperfections in atypical structures are not listed. The geometric non-linear solution is frequently utilized in scientific workplaces for the analysis of 'well-defined' structures (Kala 2007a, 2007b, 2007c; Koteš, Vičan 2005). On the contrary, stability solution is more general and provides a solution for all types of structures that frequently occur in buildings. The structure presented in Fig. 1 is one of many structures of nuclear power plants Temelín (ČR) and Mochovce (SR). The majority of the structures of NPP are of an atypical shape. This, due to the above-mentioned reasons, eliminates the utilization of the geometric non-linear solution and predetermines the application of stability solution with buckling length.

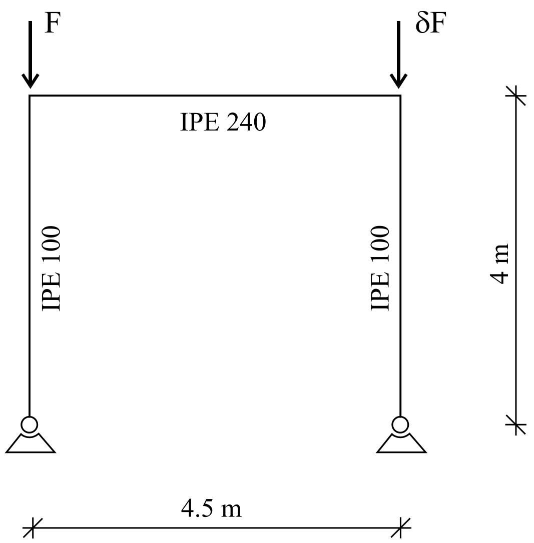

Fig. 1. The geometry of a steel plane frame

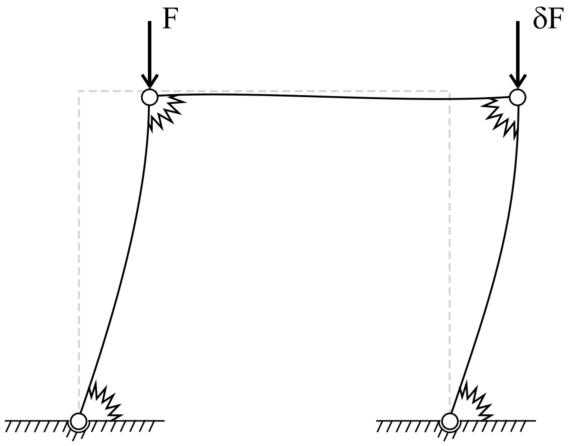

Fig. 2. Buckling a frame with a semi-rigid joint

One of the frequent sources of error in determining buckling length is in an erroneous assignment of boundary conditions for the member support. The fuzzy number of the rotational stiffness of the semi-rigid joint of support is given by the unfamiliarity of the locations of welding the column base into the anchor base plate. The fuzzy number in Fig. 3 was defined from the imprecise measurements of firm EGV, s.r.o. They experimentally obtained two rotational stiffness values of the anchor base plate: 14.7 kNm/rad and 230 kNm/rad. Information on the locations of the anchorage of plates and on the manner by which stiffness was experimentally obtained was not provided by the firm. From realization experience, a more commonly realized variant was 14.7 kNm/rad. Conditions for welding the place of the anchorage of stiffness made 230 kNm/rad and was not often fulfilled; a horizontal deviation of the locations of welding the columns of the structure meant a decrease in rotational stiffness. The evaluation of the truth degree of stiffness realization is depicted in Fig. 3.

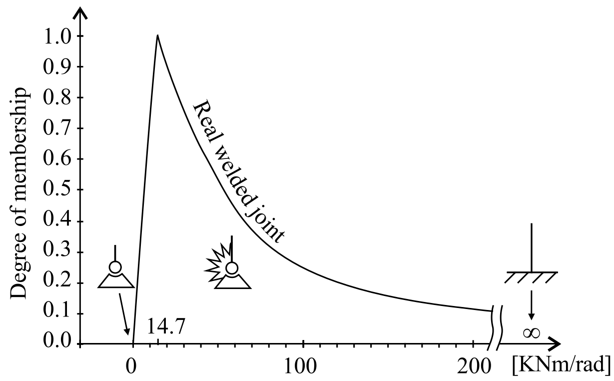

Fig. 3. Stiffness of support joints

3. Beam-to-Column Joint Stiffness as a Fuzzy Number

Another factor on which the correct determination of buckling length is dependent is the stiffness of joint connections, see Fig. 4. Gross errors in the realization of the joint connections of the frame knees were elicited from inspection. The number of bolts did not correspond to design and in some cases, bolt connection was replaced by the poorly performed welded connections.

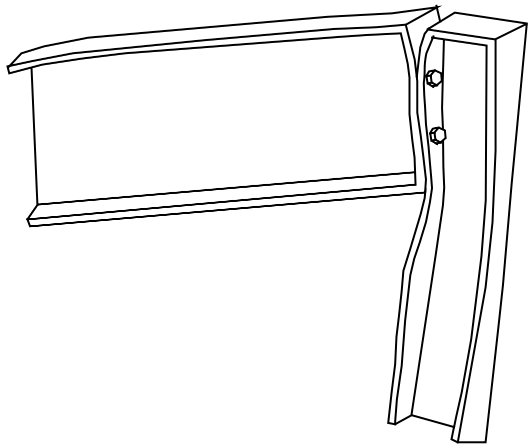

Fig. 4. An example of a real semi-rigid joint

The fuzzy number of the stiffness of the frame knee joints (see Fig. 5) was defined with the aid of stiffness obtained according to EUROCODE 3 for different variants of solutions according to software IDA-NEXIS. Support for the fuzzy number of joint stiffness was defined as the interval ⟨0; ∞⟩, see Fig. 5. The stiffness of zero corresponds to a hinge and that of ∞ to a perfectly rigid joint. Both of these possibilities are extreme and unrealistic cases, i.e. they were allocated for the degree of zero membership.

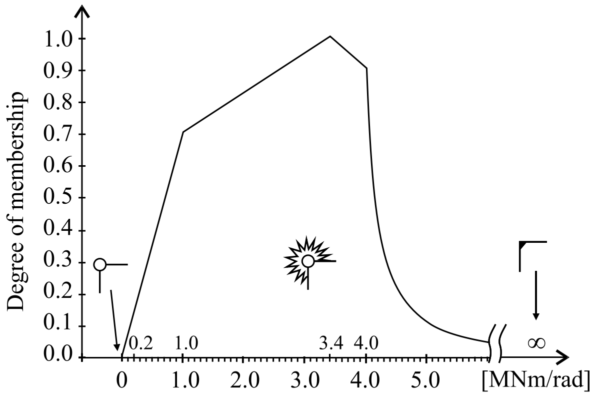

Fig. 5. Stiffness of semi-rigid joints

The stiffness of zero (hinge) or infinity (clamped end) frequently idealizes real joint stiffness in computational models. Maximal stiffness that could be in reality secured by the structural design is the stiffness of approximately 4 MNm/rad, see Fig. 6.

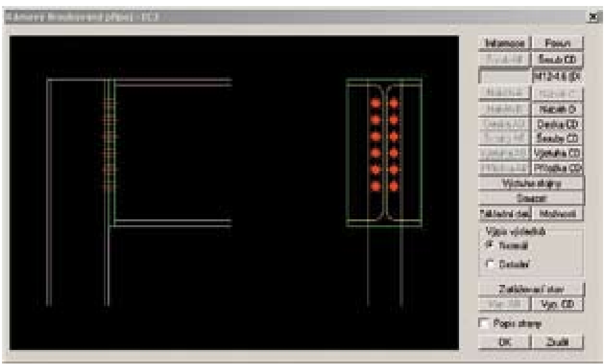

Fig. 6. Semi-rigid joint stiffness modelled

The ideal hinge is the boundary member of the set. In real joint connections, joints with two bolts approach zero stiffness, see Fig. 7. The realization of such joint would be a gross structural error. However, it is necessary to incorporate such type of realization into the set of achievements occurring due to different reasons. Joint stiffness holds value 0.2 MNm/rad. The membership degree of such stiffness was estimated as 0.14, see Fig. 5. The analysis of joint stiffness was carried out utilizing the programme IDA-NEXIS.

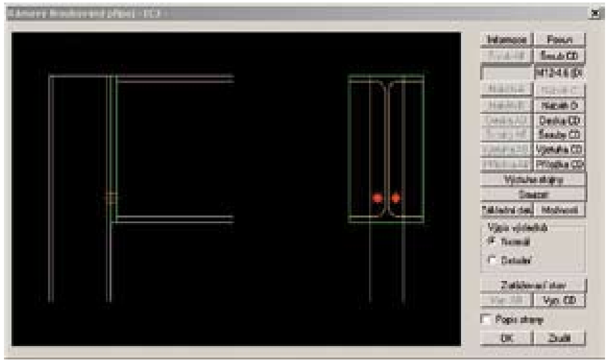

Fig. 7. Semi-rigid joint stiffness modelled

A set of the variants of other values exists between the boundary values. The behaviour of the membership function was estimated from the obtained results. Linear segments were used to substitute behaviour because we did not have further refining information at our disposal.

The membership function increases steeply in the interval ⟨0; 1⟩ MNm/rad. The most frequent occurrences of stiffness realization are assumed in the interval of rotational stiffness of ⟨1; 4⟩ MNm/rad which corresponds to the chosen membership function. Stiffness over 4 MNm/rad does not correspond to any commonly utilized structural design for this structural detail. It could pertain to additionally welded joints or any other structural amendments by which the gross error was solved during joint realization.

It could be objected that the fuzzy set in Fig. 5 should be discrete because the set of bolts and plates has a finite number of members. Due to the fact that there exist a high number of combinations (including welds) to realize them and that there is a deviation from nominal values, joint stiffness may be assumed as a positive real number. A subjective evaluation of the models of membership functions displayed in Fig. 3 and 5 may differ. Generally, it would be very valuable to draw considering the experiences of designers and the analysis of the damage and wreckage of structures.

4. Conclusions

The aim of the study was to analyse the influence of the uncertainty of joint stiffness on the uncertainty of the buckling lengths of both columns. Buckling lengths were analysed utilizing stability solution according to the second-order theory. The beam finite element method with shape functions sin and sinh was utilized in the conducted analysis. The plane frame is single-variably incrementally loaded until the determinant of the tangential stiffness matrix reaches zero. The attained critical load value determines the axial forces in members. The buckling length of the column is calculated as the length of a simply supported beam under the action of the Euler critical load (with bending the stiffness of a corresponding column) that looses stability and buckles.

The fuzzy analyses of buckling lengths were evaluated for the parameter $\delta \in \{1.0, 0.8, 0.6, 0.4, 0.2, 0\}$ according to the general extension principle (Dubois 1980). In the fuzzy sets theory, basic arithmetic operations with fuzzy numbers are developed (addition, subtraction, multiplication, division) (Dubois 1980). The result of the operation is a fuzzy number, to which a membership function appertains. For computational procedures, simplifying assumptions and realization examples are possible, see (Ferracuti *et al.* 2005; Möller *et al.* 2005; Möller, Reuter 2007; Wagenknecht *et al.* 1999; Štemberk, Kalafutová 2008; Tanyildizi 2007; Unal et al. 2007; Kala 2004, 2005, 2007d, 2007e, 2007f, 2007g, 2008; Kala, Omishore 2005). The solution is based on the so-called response function by which the computational model is approximated. Then, fuzzy arithmetic with this substitutive function is realized.

The example illustrates the application of fuzzy sets to the analysis of uncertainty during the design of steel structures. Even though a number of publications concerning this field are available, specialized textbooks for application fields are absent. Complex design models practically compromise fuzzy analysis due to a large number of necessary simulation runs.

The fuzzy sets of buckling lengths are illustrated in Figs 8 and 9. In the case that we want to examine if the frame satisfies the ultimate limit state according to EUROCODE 3, it is necessary to defuzzify the fuzzy set of buckling length, see COG, MOM methods (Dubois 1980). Defuzified buckling length is a singleton by the aid of which the buckling coefficients and load-carrying capacity of the frame according to EUROCODE 3 can be evaluated. Valuable information on the uncertainties of input data was taken into account in the analysis containing a number of other valuable data. The membership functions are distinctively sharp functions, see Figs 8 and 9.

In the case that the right column is unloaded, the core of the buckling length of the left column shifts to the left and the core of the buckling length of the right column shifts to the right. The support of the buckling length of the left column widens with decreasing δ while that of the right column narrows down.

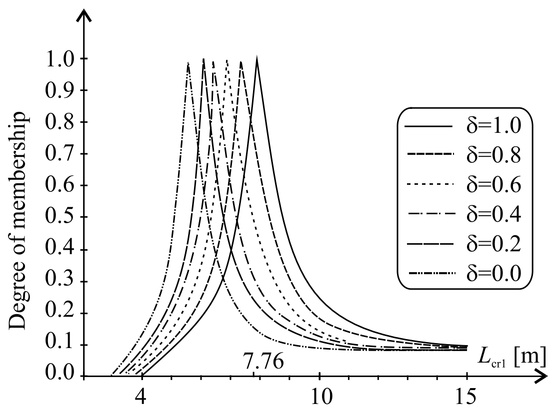

Fig. 8. Fuzzy buckling length of the left column

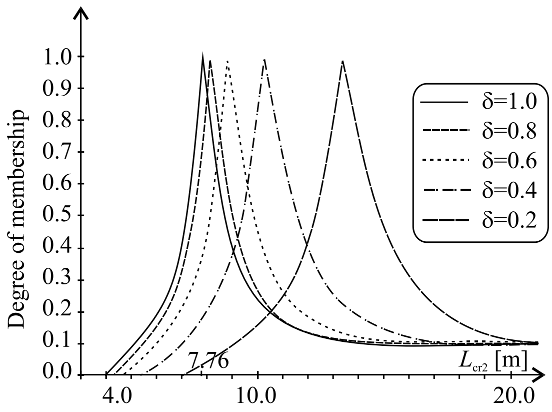

Fig. 9. Fuzzy buckling length of the right column

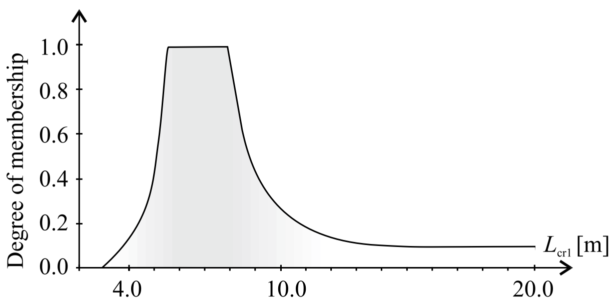

Fig. 10. Fuzzy buckling length of the left column

The fuzzy sets depicted in Fig. 8 and Fig. 9 present fundamental fuzzy outputs that do not make provision for the degree of the membership of parameter δ. In the event that information on the parameter of loading column δ is not available, i.e. each $\delta \in \langle 0;1 \rangle$ is assigned to the degree of membership 1.0, the fuzzy number of buckling length can be obtained utilizing the theoretic operation of union, see Fig. 10 and Fig. 11.

Parameter δ may be considered as a fuzzy number, for which each is assigned a degree of membership, e.g. according to Fig. 12. The outputs of fuzzy buckling lengths obtained with regard to fuzzy number δ from Fig. 12 are depicted in Figs. 13 and 14.

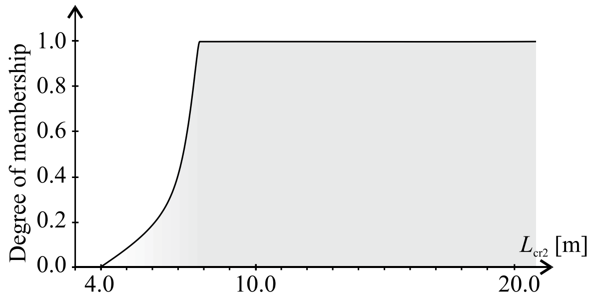

Fig. 11. Fuzzy buckling length of the right column

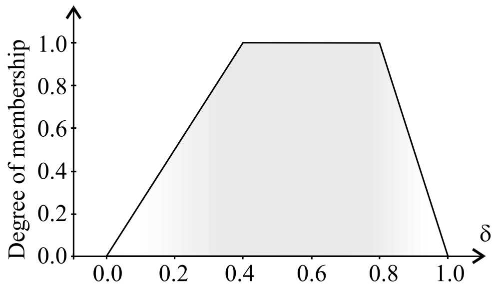

Fig. 12. Fuzzy parameter δ of the right load action

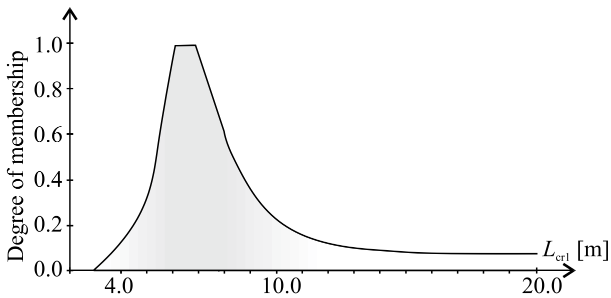

Fig. 13. Fuzzy buckling length of the left column

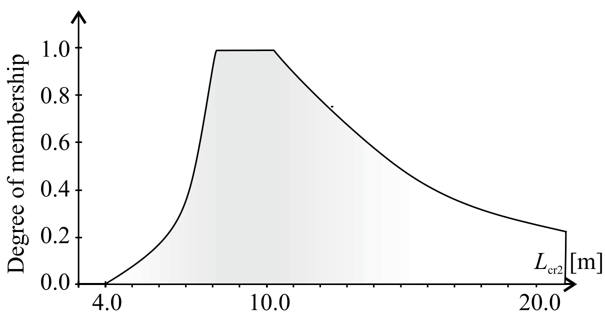

Fig. 14. Fuzzy buckling length of the right column

The paradox about buckling lengths is that if axial critical Euler force approaches zero, buckling length transcends above all limits. The defuzzification of buckling lengths would require the elimination of zero joint stiffness and the zero loading of the right column. Infinite buckling lengths have a zero degree of membership and are on the boundaries of the fuzzy sets. The problem requires an elaborate mathematical analysis, the basis of which could be an analytical derivation of the dependence between the stiffness and buckling lengths of the frame.

Even though technical regulations are apparently strict, their realizations are always performed by people, i.e. more or less vaguely. This example demonstrates a clear and specific application of fuzzy sets in modelling uncertainties emanating from vagueness. This phenomenon occurs virtually during all human activities, even though we might not like to admit it.

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FORMALIOSIOS LOGIKOS ANALIZĖS PRITAIKYMAS PLOKŠČIŲ RĖMŲ SKAIČIUOJAMŲJŲ ILGIŲ NUSTATYMUI

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Santrauka. Straipsnis pateikia siūlomą skaičiuojamojo ilgio nustatymo metodą, grįstą formaliąja logika, sukeltą esant nepastoviam jungčių standumui ir elemento atramų ribinėms sąlygoms. Pateiktas pavyzdys demonstruoja aiškų ir specifinį formaliosios aibės pritaikomumą modeliuojant nepastovumą projektavimo metu. Skaičiuojamasis ilgis buvo analizuojamas panaudojant stabilumo sprendinius taikant antros eilės teoriją. Sijos baigtinių elementų metodas su sinuso ir hiperbolinio sinuso formos funkcijomis buvo pritaikytas analizėje. Įvedamų duomenų nepastovumas taip pat buvo įvertintas analizėje.

Reikšminiai žodžiai: plienas, betonas, konstrukcija, projektavimas, patikimumas, atsitiktinis, formalioji logika, trūkumas.

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