“The fair price of Guaranteed Lifelong Withdrawal Benefit option in Variable Annuity”

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Abstract

In this paper we use the No Arbitrage pricing theory in order to derive the fair insurance fee for the Guaranteed Lifelong Withdrawal Benefit (GLWB) option embedded in Variable Annuity contracts (VA); moreover, we verify if the current GLWB fees on the USA market are fair. The typical VA is a unit-linked annuity contract, which is normally purchased by a single premium payment up-front; the premium is invested in one of several funds. The VA also typically contains some embedded guarantees. One of these is the GLWB option: it gives the policyholder the possibility to withdraw annually a certain percentage of the single premium; if the fund value drops to zero the insurer has to pay the guaranteed amount to the policyholder. The guarantee is lifelong. Any remaining account value at the time of death is paid to the beneficiary as death benefit. In line with the actuarial literature, we assume that the fund follows a Geometric Brownian Motion and the insurance fee is paid ongoing as fraction of assets. We take a static approach that hypothesizes the withdrawal amount is always equal to the guaranteed amount. In this case we calculate the fair insurance fee with Monte Carlo simulations under different scenarios and verify that the product is underpriced on the USA market.

Keywords: Guaranteed Lifelong Withdrawal Benefit option, insurance fee, Monte Carlo simulation, static approach, Variable Annuity.
JEL Classification: G22.

Introduction

Variable Annuities (VA) were introduced in the 1970s in the United States (see Sloane, 1970). The typical VA is a unit-linked annuity contract or managed fund product offering either accumulation or decumulation benefits and at least one optional guarantee (see Ledlie et al., 2007). Two kinds of embedded guarantees are offered in such policies (see Hanif, 2007): Guaranteed Minimum Death Benefit (GMDB) as well as Guaranteed Minimum Living Benefit (GMLB). One of the GMLB options is the Guaranteed Minimum Withdrawal Benefit (GMWB), which gives the insured the possibility of withdrawing a pre-specified amount annually, even if the account value has fallen below this amount. The latest financial innovation introduced on the VA market is the Guaranteed Minimum Withdrawal Benefit for Life or Guaranteed Lifelong Withdrawal Benefit option (GLWB). As the name suggests, it offers a lifelong withdrawal guarantee; therefore, there is no limit for the total amount that is withdrawn over the term of the policy, because if the account value becomes zero while the insured is still alive, the insurer has to pay the guaranteed amount to policyholder annually until death. The first VA with a withdrawal benefit guaranteed for the life was introduced in the USA market in 2003. In light of the growing importance of this market, the aim of this paper is to use a pricing model in order to verify if the current GLWB price on the USA market is fair. Our work uses the standard No-arbitrage models of mathematical finance, in line with the tradition of Boyle and Schwartz (1997) that extend the Black-Scholes framework to insurance contract. The main difference is that for the option embedded in VA products the fee is deducted ongoing as fraction of the asset, whereas in the Black and Scholes approach the premium is paid up-front. The approach follows the recent actuarial literature on the valuation of VA products: Bauer et al. (2006), Chen et al. (2008), Coleman et al. (2006), Holz (2006), Milevsky and Posner (2001), Milevsky M.A and Promislow S.D (2001), Milevsky and Salisbury (2002). In order to price options embedded in Variable Annuity contracts many authors use numerical PDE methods (see Dai, 2008; Chen et al., 2008; Milevsky and Posner, 1998; Milevsky and Salisbury, 2006; Nielsen and Sandmann, 2003), others exploit Monte Carlo simulations (Milevsky and Panyagometh, 2001; Milevsky and Posner, 2000). We choose to follow the latter approach. We adopt a static approach that assumes policyholders follow a static strategy, i.e. the withdrawal amount is always equal to the guaranteed amount. The insurer can induce the policyholder to assume this behavior by introducing a penalty charge on the amount of withdrawal that exceeds the guaranteed amount. We develop an application of the model to the USA market and derive the fair insurance fee for an illustrative policyholder. Our conclusion is that the GLWB issued on the USA market are underpriced. Similar conclusions have been reached for the GMWB option by Milevsky and Salisbury (2006). Also, Chen et al. (2008) verify that the market fees are inadequate if the underlying risky asset follows a jump diffusion process. The framework developed can be used by insurance companies to evaluate the appropriate fees on GLWB they want to issue on new markets; it can also be considered the starting point for a more complex management system for this kind of product.

The remainder of the paper is organized as follows. In section 1 we describe the pricing model. In section 2 we show some numerical results with an application to the USA market. Concluding remarks are offered in the final section.

1. The model

The VA contract with a GLWB option gives the policyholder the possibility of annual withdrawal of a certain percentage of the single premium, which is invested in one or several mutual funds. The guarantee consists of the entitlement to withdrawal until an amount equal to the premium paid, even if the account value falls to zero while the policyholder is still alive. Moreover, any remaining account value at the time of death is paid to the beneficiary as death benefit. The insurer charges a fee for this guarantee, which is usually a pre-specified annual percentage of the account value. In the following, we describe the stochastic model used to derive the fair insurance fee.

Assume a frictionless market with continuous trading, no taxes, no transaction costs, no restrictions on borrowing and perfectly divisible securities. The insurance company invests the single premium ω_0 paid by the policyholder in an equity fund W, whose dynamic under the risk-neutral equivalent martingale measure Q on the risk neutral probability space $(\Omega, \Im, \{F_t\}_{t\geq 0}, Q)$ is described by the following equation:

$$\begin{aligned} dW_t &= (r-\delta)W_t dt - \gamma_t dt + \sigma W_t d\tilde{Z}_t \\ W_0 &= \omega_0, \end{aligned} \tag{1}$$

where $\tilde{Z}_t$ is a standard Brownian motion under the measure Q, r is the risk free rate, σ is the fund volatility, δ is the insurance fee paid ongoing as fraction of assets, γ_t is the withdrawal from the fund at time *t*. Equation (2) holds while $W_t>0$.

This dynamic model for the underlying investment is consistent with the actuarial literature on pricing insurance guarantees (Chen and Forsyth, 2008; Gerber and Shiu, 2003; Milevsky and Salisbury, 2006; Windcliff et al., 2001). Following the literature, we are assuming that there exists a risk neutral measure, under which payment streams can be valued as expected discounted value using the risk-neutral valuation formula (cf. Bingham and Kiesel, 2004); existence of this measure implies the existence of an arbitrage free market.

Assume the policyholder takes a static strategy and the withdrawal amount is always equal to the guaranteed amount. Let g be the guaranteed rate, so that the evolution of the fund becomes:

$$dW_t = (r-\delta)W_t dt - G dt + \sigma W_t d\tilde{Z}_t^Q, \tag{2}$$

where $G = g\omega_0$.

Assume further that the mortality events are independent of the financial events. Let T_x be a random variable which represents the remaining lifetime of the policyholder aged x at the inception of the contract. The survival function of the random variable T_x is given by:

$${}_t p_x = P(T_x > t).$$

This is the probability of an individual of age x being alive at age $x+t$, $t = 0,1,\cdots n-x$. We denote with the symbol q_{x+t} the probability of an individual of age $x+t$ dying within one year, formally

$$q_{x+t} = P(T_{x+t} < 1).$$

Hence, ${}_t p_x q_{x+t}$ is the probability of an individual currently aged x being alive until age $x+t$ and dying between ages $x+t$ and $x+t+1$; in symbols:

$${}_{t/1}q_x = {}_t p_x q_{x+t} = P(t < T_x < t+1).$$

The GLWB offers both living and death benefits. Let V_0 be the discounted value at *t*=0 of the GLWB; it is the sum of the discounted values of the living and death benefits:

$$V_0 = LB_0 + DB_0. \tag{3}$$

LB_0 is the discounted value of a life annuity, i.e. the present value of the sequence of amounts G weighted with the probability to receive this amounts:

$$V_0 = \sum_{t=1}^{n-x} g\omega_0 e^{-rt} {}_t p_x. \tag{4}$$

DB_0 can be calculated considering the payoff that the beneficiary will receive at the random time of death τ:

$$DB_{\tau} = Max(W_{\tau};0). \quad (5)$$

Since the maturity is stochastic and τ and W_{τ} are independent, the discounted value at t=0 of the death benefit is given by the expectation under τ and W_{τ} :

$$DB_0 = E_t\left\{E\left\{e^{-r\tau}DB_{\tau}^{p}\middle|\tau = t\right\}\right\}. \quad (6)$$

If we fix the date $\tau = T$, the death benefit can be calculated by Ito's lemma; the solution to equation is:

$$DB_T = e^{(r-\delta-\frac{\sigma^2}{2})T+\sigma\tilde{Z}_T} \max\left[\left(\omega_0 - G\int_0^T e^{-(r-\delta-\frac{\sigma^2}{2})t+\sigma\tilde{Z}_t} dt\right);0\right] \quad . (7)$$

Using a standard technique in literature, the no-arbitrage time-zero value of death benefit at time t is:

$$DB_0(\tau = T) = E_0^Q\left[e^{(r-\delta-\frac{\sigma^2}{2})T+\sigma\tilde{Z}_T} \max\left[\left(\omega_0 - G\int_0^T e^{-(r-\delta-\frac{\sigma^2}{2})t-\sigma\tilde{Z}_t} dt\right);0\right]\right] = E_0^Q(DB_T), \quad ...(8)$$

where the expectation is taken under the risk neutral measure Q.

If we consider both the expectations in the equation (6), we obtain:

$$DB_0 = \int_0^{n-x} f_x(t)E_0^Q(DB_t)dt. \quad (9)$$

In the discrete case we have:

$$DB_0 = \sum_{t=0}^{n-x} {}_t p_x q_{x+t} E_0^Q(DB_t) \quad (10)$$

From equations (4) and (10) we obtain:

$$V_0 = \sum_{t=0}^{n-x}\left[{}_t p_x g\omega_0 e^{-rt} + {}_{t/1}q_x E_0^Q(DB_t)\right] \quad (11)$$

Giving the value of the other parameters, the fair insurance fee can be obtained making the single premium ω_0 equal to zero value of the all future cash flows V_0.

2. Numerical results

We apply our pricing model to GLWB options issued in the USA market. The main product futures in this market are summarized as follows. The guaranteed rate offered increases with the age of policyholder at the inception of the contract; for a policyholder aged 60 at the inception of the contract the average guaranteed rate is equal to 5%. The insurance fee in the market ranges from 50 to 70 b.p. According to Morningstar Principia Pro, the average of the sub-account volatility for the universe of variable annuity products is 18%, the 25th percentile is 16% and the 90th percentile is 25%.

We consider a policyholder aged 60 at the inception of the contract, the final age is n=110; in order to price the GLWB option we use the latest USA mortality table downloaded from the Human Mortality Database. We set $\omega_0 = 100$ and $r = 5\%$. We carry out many Monte Carlo simulations under different scenarios generating for each of them 10000 paths of evolution of the fund. We calculate the fair insurance fee according to the pricing model developed in the previous section. Giving the value of the other parameters, the fair insurance fee can be obtained making equal the single premium ω_0 to the zero value of the all future cash flows V_0. The following table displays the fair insurance fee under different guaranteed rate and sub-account volatility.

Table 1. The impact of the guaranteed rate and sub-account volatility on the fair insurance fee for a policyholder aged 60

g	σ = 18%	σ = 20%	σ = 25%
4%	43 b.p.	54,5 b.p.	83 b.p
5%	79 b.p.	96,5 b.p	138 b.p
6%	143 b.p.	167 b.p	226 b.p
7%	270 b.p.	308 b.p	389 b.p

The results are obtained with Monte Carlo simulation. Once the interest rate, the volatility and the guaranteed rate have been fixed, we have sought the fair value of the fee with an iterative procedure: if the time-zero cost of the whole product turned out to be higher than ω_0 we increased the fee up to decrease the cost to ω_0; vice versa, if the time-zero cost of the whole product turned out to be smaller than ω_0 we decreased the fee. As expected, the fair guarantee is increasing in the volatility, because guarantees are more expensive when volatility increases. In the same way, we can verify that if, ceteris paribus, we increase the interest rate, the fair fee decreases because the risk neutral value of the guarantee decreases. We pay attention to the fair insurance fee under the hypothesis $g = 0.05$ and $\sigma = 0.18$, which are consistent with the market. In this case the fair insurance is equal to 79 b.p., whereas the current market fee ranges between 50 b.p. and 70 b.p. Although there is a common belief that the guarantees embedded in variable annuity policies are overpriced (Clements, 2004), our analysis shows that the USA market of GLWB is underpriced, in line with the results obtained by Milevsky and Salisbury (2006) for the GMWB market.

Conclusion

In this paper we have used the No arbitrage pricing technique in order to derive the fair insurance fee for a GLWB option embedded in VA issued on USA market. We have considered a static approach that assumes policyholders take a static strategy, i.e. the withdrawal amount is always equal to the guaranteed amount. In this case we have bifurcated the product into a life annuity plus a derivatives component given by a death benefit linked to the fund value.

Up to this time the literature has not offered a specific model for GLWB pricing, but only a general pricing-framework for the universe of VA or papers on the pricing of other particular embedded options, like GMWB and GMDB. Our paper fits into the actuarial literature on VA and investigates the fairness of the current GLWB price on the USA market. On a practical side, our numerical results show that the No Arbitrage cost of a GLWB issued to a policyholder aged 60 would be equal to 79 b.p. assuming a sub-account volatility in line with the average of the sub-account volatility for the universe of variable annuity products, while most products in the USA market only charge 50-70 basis points. This result indicates that the market fees are not sufficient to cover the market hedging cost of the guarantee. Of course, our pricing model does not allow for more sophisticated financial hypothesis, such as stochastic volatility or jumps in the fund process and term-structure effects, but, as Milevsky and Salisbury (2006), we are confident that these considerations will only increase the price of the embedded option. The same effect would be obtained with the introduction of a dynamic strategy, according to which, the policyholder chooses to withdraw more or less than the amount guaranteed trying to maximize the present value of the product. So we conclude by arguing that the current price of GLWB is not sustainable for insurers and the fees have to increase in order to avoid arbitrage opportunities. Future researches will examine optimal withdrawal schemes for rational policyholders and realistic hedging strategy for GLWB options.

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