On the Choice of the Lightning Channel Current Decay Constant in the Modified Transmission Line Model with Exponential Decay

V. Javor and P. D. Rančić

Abstract—**For different values of the decay constant in the Modified Transmission Line Model with Exponential Decay (MTLE) and for different channel heights the results for lightning electromagnetic field (LEMF) at different distances from the channel-base are presented in this paper, so as for the spatial and temporal current distribution along the channel. The decay constant influence on LEMF above perfectly conducting ground is analyzed using new lightning channel-base current function and MTLE as an engineering model for a lightning return stroke. The proper choice of this constant can be made based on experimental results but also on the analysis of its influence on electric and magnetic field values and their shape characteristics.**

Index Terms—**Lightning electromagnetic field, return stroke, monopole antenna, time domain analysis.**

I. Introduction

Lightning modeling is important in order to explain and un-derstand lightning as a great physical phenomenon of our environment, to determine return stroke currents at the channel-base from remotely measured electric and magnetic fields, and to estimate realistic field components at different distances from the lightning channel and use them for calculating lightning induced effects. According to the type of governing equations used in lightning return stroke models they are divided into four classes [1]: the gas-dynamic or physical models, the electromagnetic models, the distributed-circuit models and the engineering models.

In engineering models spatial and temporal distribution of the channel current, or the channel charge density, is specified based on the channel-base current, the return stroke speed and the channel luminosity. The most of engineering models can be expressed by the equation relating the longitudinal channel current to the channel-base current, the return stroke speed (the upward propagating front speed) and current wave propagation speed [2], so as using the height dependent attenuation factor introduced by Rakov and Dulzon [3]. For the Modified Transmission Line Model with Exponential Decay (MTLE) this attenuation factor is determined by the decay constant [4]. This constant can be chosen according to the experimental results [5], but it is very interesting to show its influence on the results for lightning electromagnetic field (LEMF) at different distances from the channel-base, as these are usually used for the comparison of different types of models and evaluation of their results. In this paper the results for different decay constant values are offered in order to make its choice adequate when using MTLE model for different experimental results.

A brief overview of models in the first section is followed by the presentation of how to use an engineering model for the calculation of a lightning return stroke electromagnetic field, which is given in the second section. The results for the lightning channel current obtained by using the new channel-base current function and the MTLE with the decay constant of earlier proposed value [4], [5], and of the greater value proposed in this paper, are presented in the third section. The influence of the decay constant on lightning electric and magnetic field at different distances from the channel base is also analyzed. The conclusions about the choice of the decay constant in the MTLE are based on these results.

Manuscript received Jun 23, 2009; revised November 20, 2009. The material in this paper was presented in part at the 17th International Conference on Software, Telecommunications and Computer Networks (SoftCOM 2009), Split-Hvar-Korčula, Croatia, Sept. 2009.

This work was supported in part by Ministry of Sciences and Technological Development of Republic of Serbia, in 2009.

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II. Calculation of the Lightning Return Stroke Electromagnetic Field Using Engineering Models

A lightning return stroke channel is presented as a vertical finite linear antenna (Fig. 1) above perfectly conducting ground and lightning electromagnetic field is determined from the current distribution along the channel of height h and radius r_0 [6].

Electric and magnetic field components at the field point $P(r,\psi,z)$ can be expressed in cylindrical coordinates in time domain [7] as the following:

$$E_z = \frac{1}{4\pi\varepsilon_0}\int_{-h}^{h}\left[\frac{2(z-z')^2-r^2}{R^5}\int_{\tau=0}^{\tau=t} i\left(z',\tau-\frac{R}{c}\right)d\tau + \frac{2(z-z')^2-r^2}{cR^4}\, i\left(z',t-\frac{R}{c}\right) - \frac{r^2}{c^2R^3}\frac{\partial i\left(z',t-\frac{R}{c}\right)}{\partial t}\right]dz', \tag{1}$$

$$E_r = \frac{1}{4\pi\varepsilon_0}\int_{-h}^{h}\left[\frac{3r(z-z')}{R^5}\int_{\tau=0}^{\tau=t} i\left(z',\tau-\frac{R}{c}\right)d\tau + \frac{3r(z-z')}{cR^4}\, i\left(z',t-\frac{R}{c}\right) + \frac{r(z-z')}{c^2R^3}\frac{\partial i\left(z',t-\frac{R}{c}\right)}{\partial t}\right]dz', \tag{2}$$

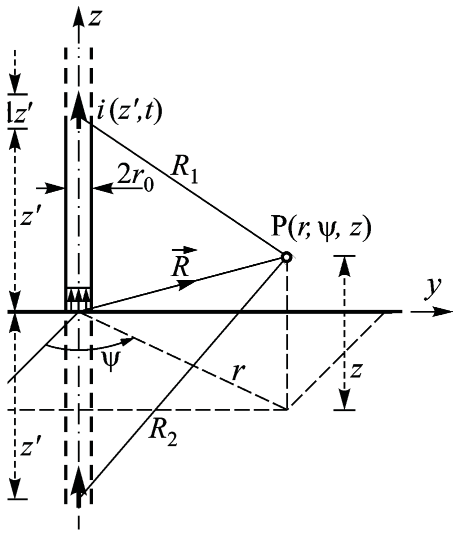

Fig. 1. Lightning return stroke scheme

$$H_\psi = \frac{1}{4\pi}\int_{-h}^{h}\left[\frac{r}{R^3} i\left(z', t - \frac{R}{c}\right) - \frac{r}{cR^2}\frac{\partial i\left(z', t - \frac{R}{c}\right)}{\partial t}\right] dz', \quad (3)$$

where E_z is the vertical electric field, E_r is the radial electric field, H_ψ is the azimuthal magnetic field, $R = \sqrt{r^2 + (z - z')^2}$ is the distance from the antenna current element or its image in plane mirror to the field point $P(r, \psi, z)$, $i(z, t)$ is the lightning channel current at the time t and the height z' from the channel-base, ε_0 is the electric permittivity of the air, μ_0 is the magnetic permeability of the air, and $c = (\varepsilon\mu_0)^{-1/2}$ is the speed of light. Transmission line model (TL) of the return stroke channel was introduced by Uman and McLain [6] who visualized the return stroke as the propagation of a current pulse from the ground end to the upper end, but assumed that the shape and amplitude of the current pulse remained the same with height, and that the propagation speed v of the current pulse was constant, so

$$i(z', t) = i(0, t - z'/v), \quad (4)$$

Most of engineering models, as the modified transmission line model with linear decay (MTLL) [8], modified transmission line model with exponential decay (MTLE) [4], the Bruce-Golde model (BG) [9], the traveling current source model (TCS) [10] and others [1], also [11], can be presented with the following equation:

$$i(z', t) = u(t - z'/v_f)P(z')i(0, t - z'/v), \quad (5)$$

where $i(z', t)$ is the longitudinal channel current expressed as the function of height z' from the channel-base and time t, $i(0, t - z'/v)$ is the channel-base current, $u(t - z'/v_f)$ is the Heaviside function equal to unity for $t \geq z'/v_f$, v_f is the return stroke speed, v is the current wave propagation speed, and $P(z')$ is the height dependent attenuation factor, having one or two constants [8]. In the MTLE introduced by Nucci et al. [4] the attenuation factor is $P(z') = \exp(-z'/\lambda)$,where λ is the decay height constant, estimated to be $\lambda = 2000$ m based on the experimental results by Nucci et al. [5].

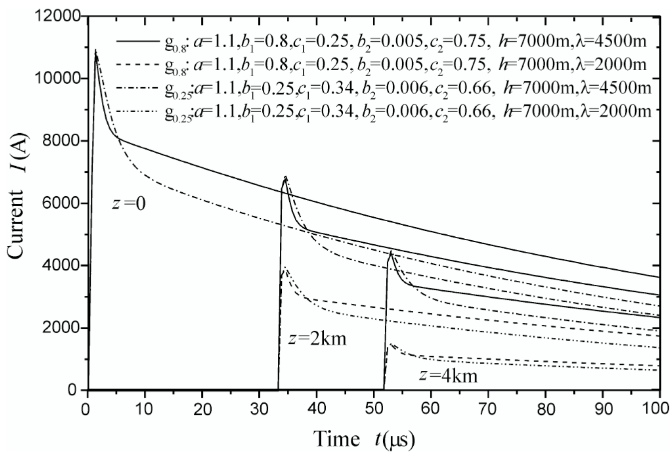

Fig. 2. Current at some heights along the channel for different channel-base current functions and decay constants

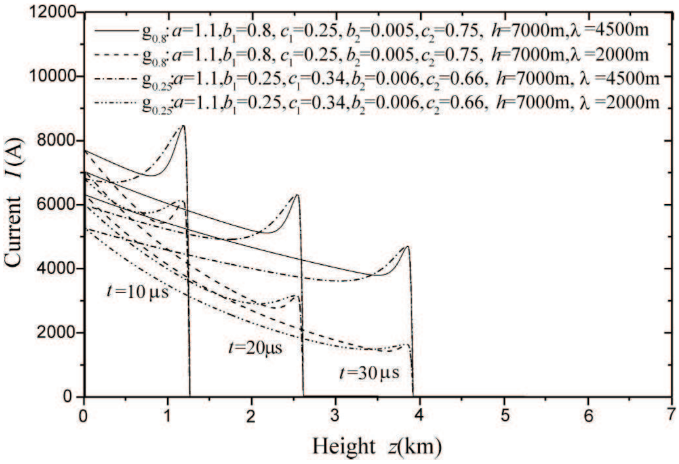

Fig. 3. Current after different time intervals for the two different channel-base current functions and decay constants

III. Results for the Lightning Channel Current and Electromagnetic Field

The new channel-base current function (NCBC) proposed in [12] and used in this paper is:

$$g_k = i(0, t) = \begin{cases} I_m\left[\tau e^{(1-\tau)}\right]^a, & 0 \leq \tau \leq 1 \\ I_m \sum_{i=1}^{2} c_i\left[e^{(1-\tau)}\right]^{b_i}, & 1 \leq \tau \leq \infty \end{cases} \quad (6)$$

for $\tau = t/t_m$, which gives the opportunity of analyzing the influence of the rising and the decaying part of the function on electromagnetic field components and has many other advantages [11]. It can be used in both engineering and electromagnetic models. NCBC function parameters are chosen according to the often used channel-base current function as given in [1] and [13]. Using the two different NCBC functions 6 of chosen pa-rameters and the MTLE 5 with the attenuation factor $P(z') = \exp(-z'/\lambda)$, for $\lambda = 2000$m and $\lambda = 4500$m, the results for the return stroke channel current along the channel are presented in Fig. 2 at different channel heights, and in Fig. 3 for different time intervals.

At three different heights from the channel-base: zero, 2km and 4km, the results are presented in Fig. 2. For $z' = 0$ the

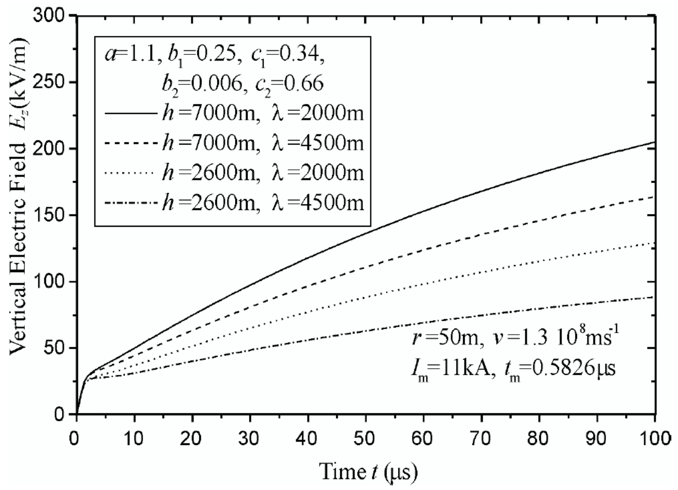

Fig. 4. Vertical electric field at 50m from the channel-base for different channel heights and different decay constants

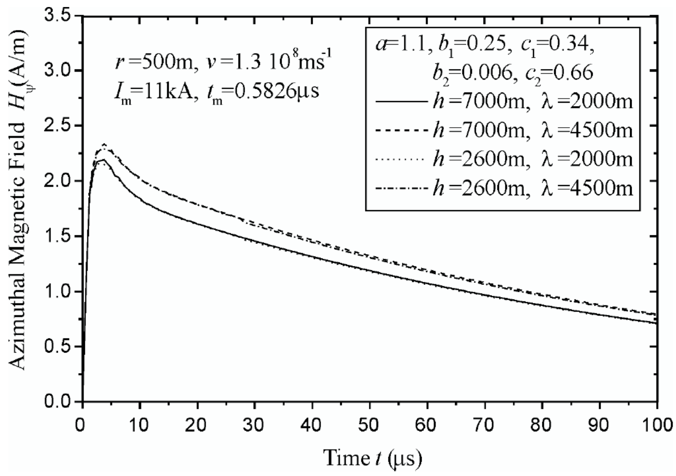

Fig. 7. Azimuthal magnetic field at 500m from the channel-base for different channel heights and different decay constants

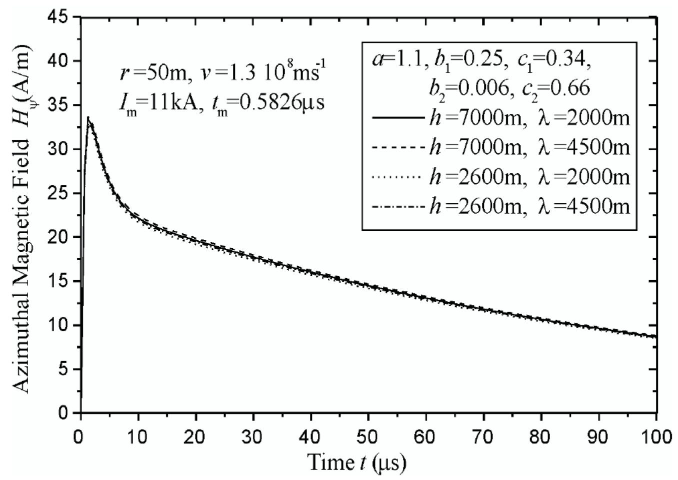

Fig. 5. Azimuthal magnetic field at 50m from the channel-base for different channel heights and different decay constants

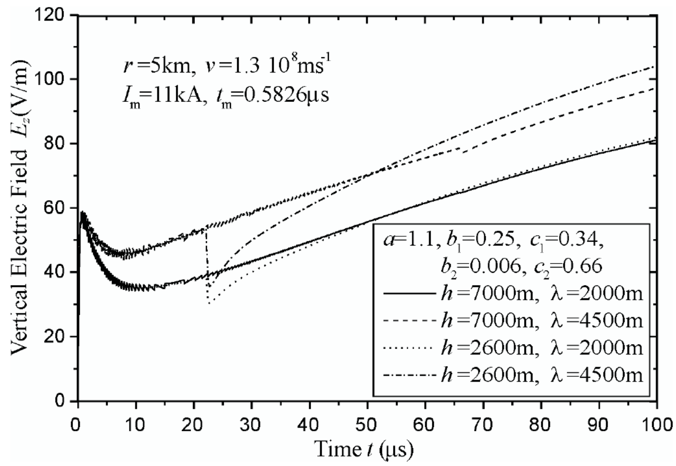

Fig. 8. Vertical electric field at 5km from the channel-base for different channel heights and different decay constants

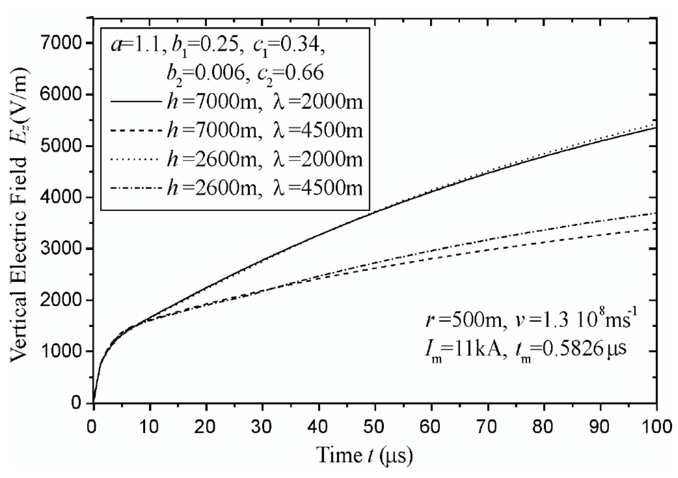

Fig. 6. Vertical electric field at 500m from the channel- base for different channel heights and different decay constants

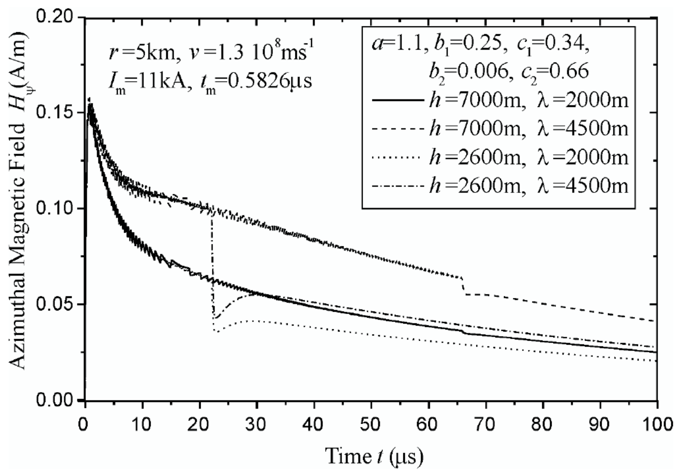

Fig. 9. Azimuthal magnetic field at 5km from the channel-base for different channel heights and different decay constants

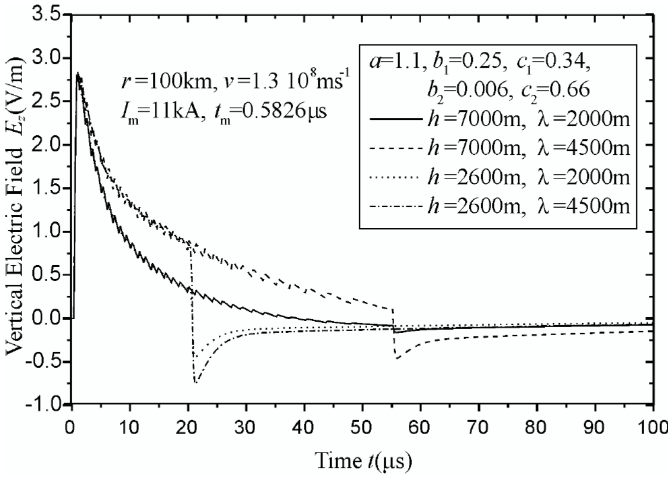

Fig. 10. Vertical electric field at 100km from the channel-base for different channel heights and different decay constants

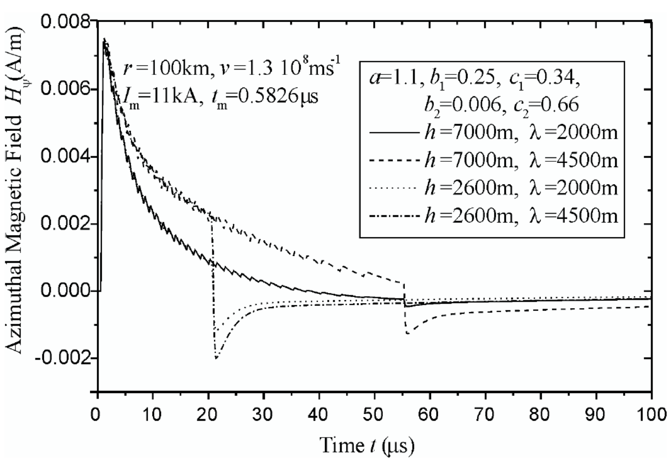

Fig. 11. Azimuthal magnetic field at 100km from the channel-base for different channel heights and different decay constants

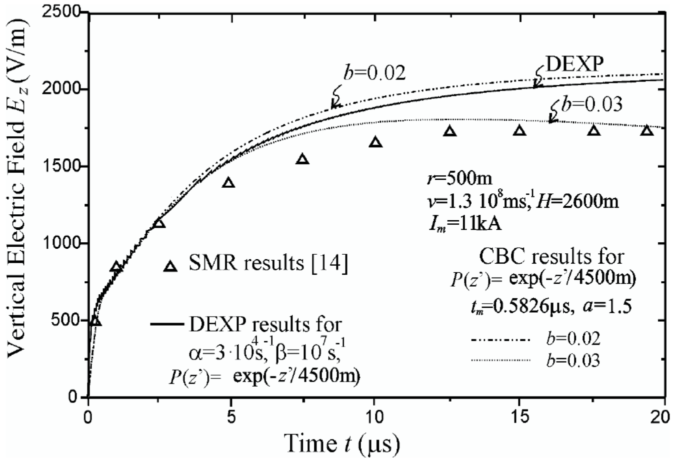

Fig. 12. Vertical electric field at r=500m for the MTLE model and different channel-base currents functions

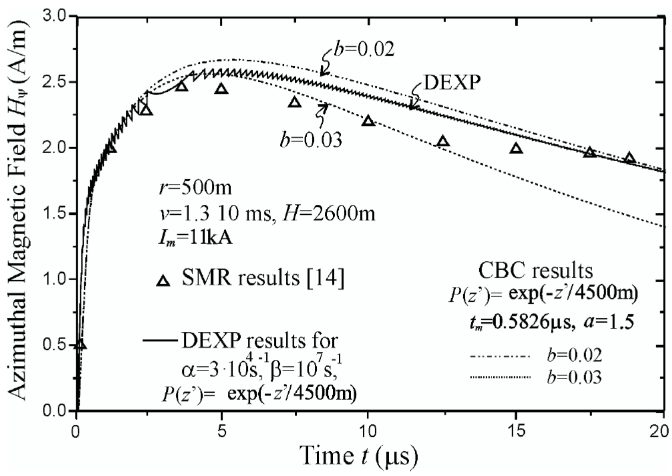

Fig. 13. Azimuthal magnetic field at r=500m for the MTLE model and different channel-base currents functions

values of the return stroke current do not depend on λ, and these are the two different channel-base current functions in Fig. 2 of which $g_{0.8}$ is decaying faster in the first 5 μs, and then slower in the next 5 μs than $g_{0.25}$, and later having greater current values.

The chosen channel height is $h = 7$km, the maximum current value at the channel-base $I_m = 11$kA at $t_m = 0.5826$μs, the current wave propagation speed equal to the return stroke speed $v_f = v = 1.3 \cdot 10^8$m/s. For $\lambda = 2000$m at the height of 2km the maximum current value decays to $e^{-1} I_m$, and for $\lambda = 4500$m to $e^{-4/9} I_m$. For $\lambda = 2000$m the maximum current value is equal to $e^{-2} I_m$ at the height of 4km from the channel base and for $\lambda = 4500$m its maximum is $e^{-8/9} I_m$ at the same height. If the decay constant λ is smaller then the maxima of the current values at different heights along the channel are smaller. The results for the current along the channel after three different time intervals 10μs, 20μs, and 30μs, are presented in Fig. 3, for the two different channel-base currents.

The influence of the decay constant on electric and magnetic field values at different distances is presented in Figs. 4-11, for the first 100μs of the pulse appearing at the chosen distance. At tens of meters greater λ gives smaller values of vertical electric field (Fig. 4), so as obtained from other models presented in [13], but neither h nor λ influence magnetic field (Fig. 5).

The influence of the channel height on electric and magnetic field at hundreds of meters is negligible (Fig. 6), but λ has the similar effect on electric field as at tens of meters (the greater λ - the smaller electric field), but the greater λ gives greater H_ψ (Fig. 7). At 5km from the channel-base both electric (Fig. 8) and magnetic field are of greater values (Fig. 9) for greater λ. It can be also noticed in Figs. 10 and 11, for the distance r=100km from the base.

MTLE with $\lambda > 2000$m gives the results more similar to those of other engineering models [13], at both smaller and greater distances from the lightning channel-base.

The peaks occurring in Fig. 8 and Fig. 9 for the distance of 5km at approximately 22μs are due to the channel interruption at the height of 2600m, and for the channel height of the 7000m at approximately 66μs. The same can be noticed in Fig. 10 and Fig. 11, but for the distance of 100km they occur at approximately 20 s for the channel height of 2600m, and at approximately 55μs for the height of 7000m.

There is a comparison of the results from [14] and the verti-cal electric field at r=500m from the channel-base (Fig. 12), and the azimuthal magnetic field (Fig. 13), obtained for the NCBC function and i=1 in (6). For the same model and the double-exponential function (DEXP), as the channel-base function [9], the results are presented also in Figs. 12 and 13. The results at other distances from the channel-base are also in good agreement with the results from literature [1], [13], and [14].

IV. Concluding Remarks

The influence of the decay constant in the Modified trans-mission line model with exponential decay on the results for lightning electromagnetic field at different distances from the channel-base and for different channel heights is presented in this paper. The comparison to the LEMF results from litera-ture is also presented in the paper.

The influence of this constant on the shape and values of the current along the channel is also presented.

The decay constant can be chosen on the basis of experi-mental results, but also using the presented analysis in order to make its choice adequate to different experimental results and to achieve better results with MTLE. Greater value of the decay constant could give some LEMF characteristics better in comparison to other models.

This analysis can be used also for improving other types of engineering models characteristics and for the better insight into lightning as the powerful phenomenon with great envi-ronmental influence.

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