DETERMINATION OF OPTIMAL WORK START TIME

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Abstract. The question is: whether the system total travel cost and travel time are reduced by adjusting the work start time or not? This paper proposes the two-level model for answering the question; the upper-level minimizes the system travel cost and travel time by using the work start time as a decision variable, the lower-level models the stochastic dynamic simultaneous route/departure time equilibrium problem. Finally, numerical results of a small network are provided to illustrate the behavior of the model.

Keywords: optimal work start time, stochastic dynamic simultaneous route/departure time equilibrium.

1. Introduction

In almost all Asian metropolises, as road infrastructure is far below the satisfactory level, we are suffering from heavy road congestion and environmental problems due to exhaust emission. Considering both monetary and space constraints, we cannot expect a large amount of road facilities expansion any more. Therefore, the "Flexible Work Hours (FWH) system" has been introduced recently in order to reduce the peak of commuting demand. FWH is one of the transportation demand management measures to change the distribution of home departure time, as well as that of the work start time.

T. Friesz [1], H.-J. Huang and W. H. K. Lam [2] model the route and departure time choice behavior of travellers by the fixed work start time. C. Hendrickson and G. Kocur [3] propose a model to determine the work start time and the flexibility of the arrival time of employees in order to maximize the employer's benefits. M. Yoshimura and M. Okumura [4] formulate an optimal control model of flexible work hours system under the conditions of system optimization and user equilibrium.

The question arises whether the system total travel cost and travel time are reduced by adjusting the work start time in the view of system manager and travellers or not? The two-level model is used to answer the question. The upper-level minimizes the system travel cost and travel time by using the work start time as a decision variable; the lower-level models the stochastic dynamic simultaneous route/departure time equilibrium problem. The other section is organized as follows. In Section 2, we propose a two-level program model. In Section 3, the heuristic procedure is proposed. Finally, numerical results of a small network are provided to illustrate the behavior of the model.

2. Model formulation

Consider a network $G = (N; A)$, where N is the set of nodes and A is the set of links in the network. Let RS be the set of all OD pairs in the network and R_{rs} be the set of routes between OD pair $rs \in RS$. The studies horizon is discretized into m intervals of length δ such that $T = m \cdot \delta$. Here, we assume the study horizon is long enough to ensure all travellers can exit from the network after the time T. On the other hand, it is also assumed that the value of δ is small enough so that the discrete time model can approximate its continuous time counterpart.

$f_p^{rs}(k)$ – the inflow rate on route p between OD pair rs. $c_p^{rs*}(k)$ – the actual route travel time experienced on route p between OD pair rs. $c_p^{rs}(k)$ – the actual route travel cost experienced on route p between OD pair rs. $\hat{c}_p^{rs}(k)$ – the perceived route travel cost on route p between OD pair rs. q^{rs} – the demand between OD pair rs. y_1 – the parameter reflected information quality.

The stochastic dynamic route and departure time choice conditions state for each OD pair that the perceived route travel costs for travellers regardless of departure time are equal and minimal. In other words, the perceived route travel costs of any unused route and departure time for each OD pair is greater than or equal to the minimal perceived route travel costs. The stochastic dynamic simultaneous route/departure time user equilibrium condition can be expressed as follows:

$$f_p^{rs}(k) = \frac{\exp(-y1 \cdot c_p^{rs}(k))}{\sum_p \sum_k \exp(-y1 \cdot c_p^{rs}(k))} \cdot q^{rs} \quad \forall rs, p, k. \ (1)$$

This condition can also be expressed as shown below:

$$\hat{c}_p^{rs}(k) \begin{cases} = \pi_{\min}^{rs}, if \quad f_p^{rs}(k) > 0 \\ \geq \pi_{\min}^{rs}, if \quad f_p^{rs}(k) = 0 \end{cases} \forall rs, p, k,$$

$$\pi_{\min}^{rs} = \min\left\{\hat{c}_p^{rs}(k) : \forall p, k\right\} \qquad (2)$$

The above expression can further be described by a variational inequality formulation as follows:

$$\sum_{rs}\sum_{p}\sum_{k}\hat{c}_p^{rs'}(k) \cdot (f_p^{rs}(k) - f_p^{rs'}(k)) \geq 0 \quad \forall rs, p, k , \quad (3)$$

where

$$\sum_{p}\sum_{k} f_p^{rs}(k) \cdot \delta = q^{rs} . \qquad (4)$$

$$\hat{c}_p^{rs}(k) = c_p^{rs}(k) + \frac{1}{y_1} \cdot \ln f_p^{rs}(k) \quad \forall rs, p, k. \qquad (5)$$

$$c_p^{rs^*}(k) = \sum_{a}\sum_{t} c_a(t) \cdot \delta_{apk}^{rs}(t) \quad \forall rs, p, k. \qquad (6)$$

$$c_p^{rs}(k) = \alpha \cdot c_p^{rs^*}(k) + sch_s(k) \quad \forall rs, p, k. \qquad (7)$$

Therefore, the travel cost of a trip from origin r to destination s on path p for a commuter leaving origin at time interval k is:

$$c_p^{rs}(k) = \alpha t_p^{rs}(k) + \begin{cases} \beta[t_s - \Delta_s - k\delta] & if \ t_s - \Delta_s > k\delta, \\ \gamma[k\delta - t_s - \Delta_s] & if \ t_s + \Delta_s < k\delta, (8) \\ 0 & otherwise \end{cases}$$

where: α is a convention factor to transform the path travel time into travel cost. Denote $[t_s - \Delta_s, t_s + \Delta_s]$ as the desired time interval for arrival at the destination s in the network, where: $t_s - \Delta_s$ is the commuter's desired earliest arrival time, $t_s + \Delta_s$ is the desired latest arrival time at the destination s. β, γ is the unit cost of schedule delay (early, late) at the destination s. In accordance with the empirical results, we assume that $\gamma > \alpha > \beta$ holds.

At the upper level, the objective is to minimize total system travel time and total travel time by using the work start time as the decision variable.

Total travel cost:

$$TSC = \sum_{rs}\sum_{p}\sum_{k} f_p^{rs}(k) c_p^{rs'}(k) .$$

Total travel time:

$$TTC = \sum_{rs}\sum_{p}\sum_{k} f_p^{rs}(k) t_p^{rs}(k) .$$

Total travel cost (total travel time) is obtained by summing the actual travel costs (travel time) of all used paths. This can be expressed as:

$$\min J_{t_s} = TTC \ or \ TSC ,$$

$$t_{s\min} \leq t_s \leq t_{s\max},$$

where: $[t_{s\min}, t_{s\max}]$ is the range of work start time.

Here, we use the discrete-time network model proposed by I. Chabini [5].

3. Algorithm

The algorithm processes can be depicted as follows:
Step 0: Set initial iteration number $t = 1$, choose initial path inflow rate $f_p^{rs}(k)_t$.

Step 1: Perform dynamic network loading, compute $c_p^{rs}(k)_t$.

Step 2: Calculate an auxiliary path flow $\hat{f}_p^{rs}(k)_t$ by Equation (1).

Step 3: Move. Find the new path flow pattern
$$f_p^{rs}(k)_{t+1} = f_p^{rs}(k)_t + \gamma_t \cdot (\hat{f}_p^{rs}(k)_t - f_p^{rs}(k)_t).$$

Step 4: Convergence criterion.

If $\sum_{rs}\sum_{i} \dfrac{\sum_{p}\sum_{k}\left|f_{p,i}^{rs}(k)_{t+1} - f_{p,i}^{rs}(k)_t\right|}{q_i^{rs}} \leq \varepsilon$,

then stop, otherwise, go to step 1 and set $t = t + 1$.

In the algorithm, the step size γ_t could be determined a priori. For example, the following simplest move size sequences could be applied $\gamma_t = 1/t$.

The two-level program problem is transformed to the following two-level unconstrainted minimization:

$$\min J_{t_s}(f) = TTC \ or \ TSC,$$

$$t_{s\min} \leq t_{rs} \leq t_{s\max},$$

$$f \in S(t_{rs}),$$

where

$$S(t_{rs}) = \left\{f \in \Omega(t_{rs}) : c(f^* - f) \geq 0, \forall f^* \in \Omega(t_{rs})\right\}.$$

Since the upper level involves only a single decision variable, this two-level problem can be solved by setting t_{rs} as a parameter of the lower problem. We repeatedly have to solve the lower unconstrained minimization by varying value of t_{rs} until we cannot lower the upper level objective.
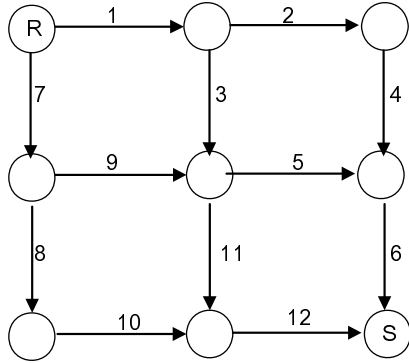
4. Numerical example

In this two-level program model, the control variable is the work start time t_s; the parameters are the uncertainty of travel cost $y1$, total travel demand q^{rs}, and the flexibility of arrival time TW (these values are exogenous to the program); and the output variables determined by the program are total travel cost TSC and travel time TTC. We use a simple grid network to the idea of using the work start time as a policy instrument to reduce total travel cost TSC or travel time TTC. A simple grid network shown in Fig 1 is used for testing. The network consists of 12 links and 9 nodes, one origin-destination as shown. The four-hour horizon from 6 to 10 a.m is

divided into 400-time interval of 0.6-minute length. Here we think 6 a.m to be the most early departure time, $\alpha = 6.4$, $\beta = 3.9$, $\gamma = 15.21$. We express the travel time function in the following simple form: $c_a(k) = \phi_a + \lambda_a x_a(k)$, where ϕ_a is free flow time in the running segment of link, λ_a is the parameter. The values of these parameters are given in Table.

path 1 = (1, 2, 4, 6);   path 2 = (1, 3, 5, 6)
path 3 = (1, 3, 11, 12); path 4 = (7, 9, 11, 12)
path 5 = (7, 9, 5, 6);   path 6 = (7, 8, 10, 12)

Fig 1. Example network

Parameters of link travel time function

Link	Φ_a	λ_a
1	0.2	0.0002
2	0.3	0.0004
3	0.2	0.00025
4	0.2	0.0002
5	0.3	0.00045
6	0.2	0.00025
7	0.2	0.0002
8	0.3	0.0004
9	0.2	0.00025
10	0.2	0.0002
11	0.3	0.00045
12	0.2	0.00025

We examine the total travel cost and travel time by the varying the values of work start time t_s with different parameter values of y_1, q^{rs}, *TW*. For brevity we show only two demand cases in Figs 2, 3. Demands are 20 000 and 25 000 vehicles. We give the value of flexibility of the arrival time *TW* in the legend of each graph for each demand level. The total travel cost and travel time cost value with respect to one value of *y1* and two values of *TW* are given in Fig 2, $y_1 = 0.01$. We see the
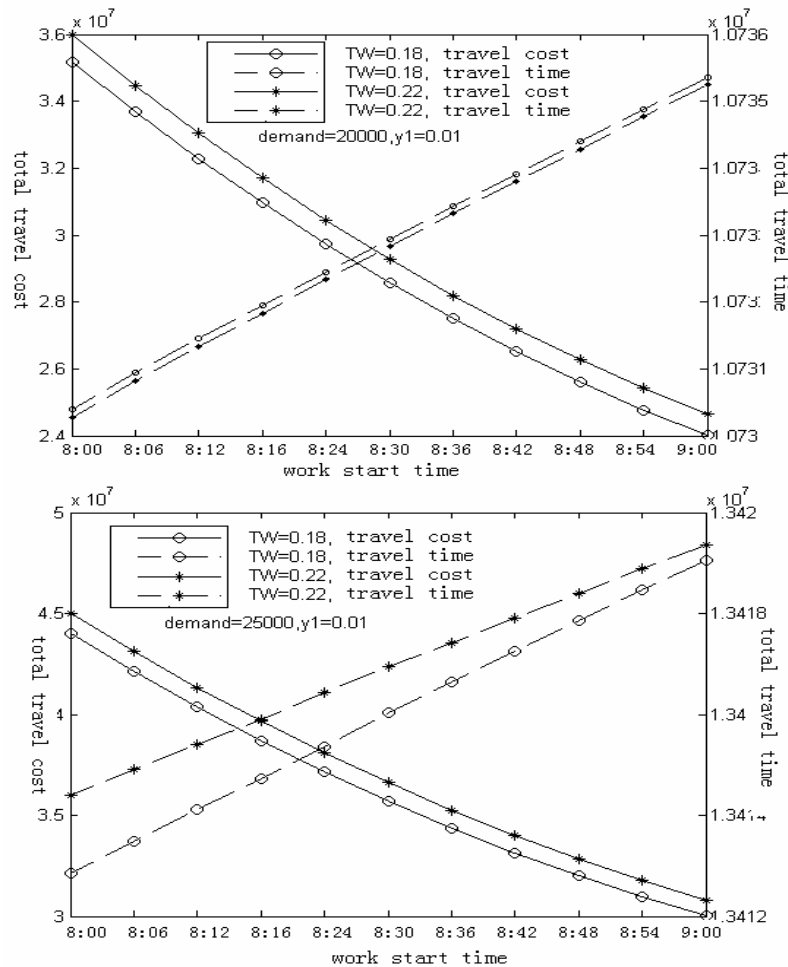

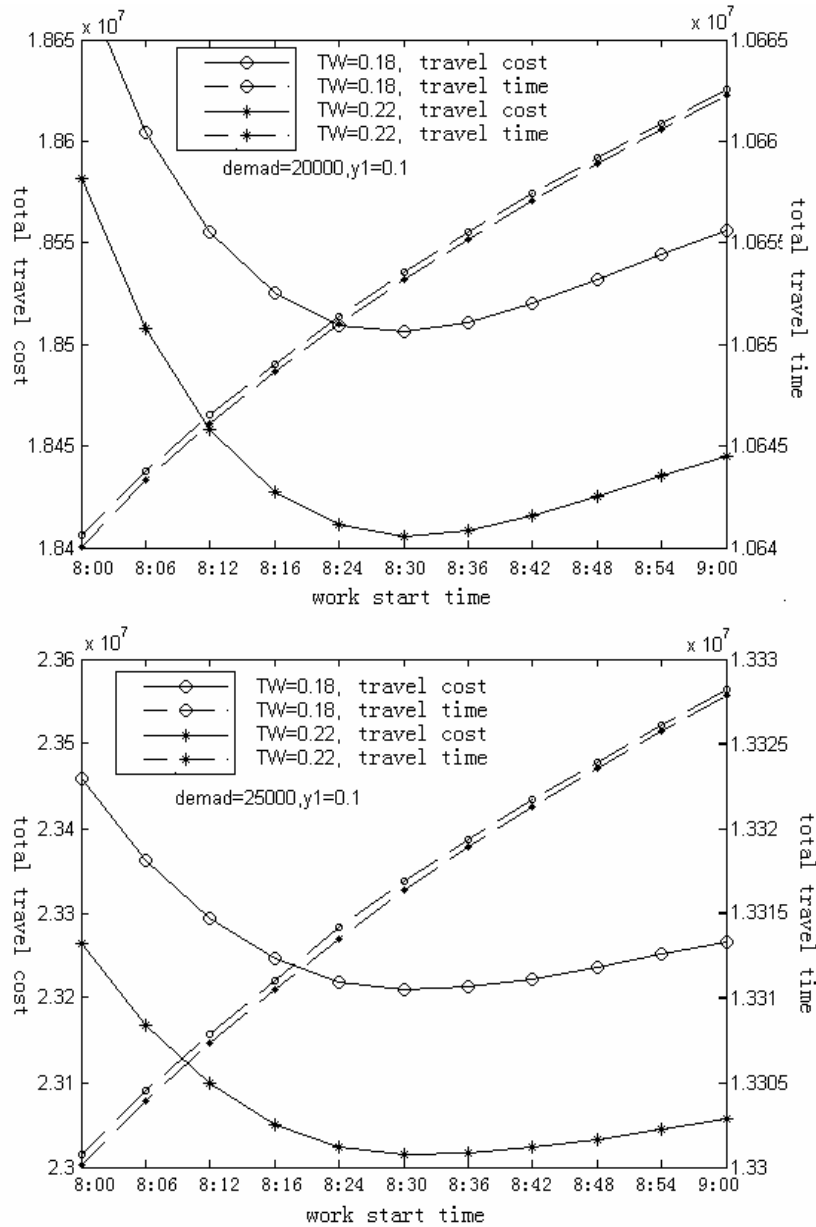**Fig 2**. Total system time and cost with respect to work start time

Fig 3. Total system time and cost with respect to work start time

decrease of the total travel cost with the increase of the work start time, as well as the increase of the total travel time with the increase of the work start time. Since the travellers' possibility to be late for work becomes small with the postponement of the work start time, thus the total travel cost is decreased.

In Fig 3, total travel costs do not decrease with the increase of the work start time as $y_1 = 0.01$. When work start time is more than a value, total travel cost is increasing. The change curve of total travel time is similar in Fig 2.

From the change curves of total travel cost and time, we find that the work start time can influence system traffic conditions. In other words, we can improve

the traffic conditions by setting the value of the work start time.

5. Conclusion

This paper proposes a two-level program model to determine the optimal work start time. The upper level minimizes the total travel cost and travel time by using work start time as a decision variable, the lower level models the dynamic simultaneous route/departure time equilibrium problem. The numerical results, based on a small network, are given. The example shows we can improve the traffic conditions by setting the optimal value of the work start time. In further studies, a detailed simulation is given in actual networks.

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