"Funds of hedge funds: a comparison among different portfolio optimization models implementing the zero-investment strategy"

AUTHORS	Rosella Giacometti Svetlozar T. Rachev

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Rosella Giacometti (Italy), Svetlozar T. Rachev (Germany)

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Abstract

Hedge funds are alternative investment vehicles where the fund manager's task is to generate positive returns regardless of the market conditions. A fund of hedge funds is a portfolio of hedge funds generally characterized by positive return and a high degree of diversification. In this paper, we analyze two portfolio optimization models for constructing a fund of hedge funds. Both models, which we refer to as the two-step model and the single-step model, are based on a zero-value strategy, a strategy which combines long and short selling in order to attain (ignoring margin and the cost of short selling) a zero initial investment. The principal difference between the two models is the selection techniques of the funds included in the long/short portfolio. The two-step model requires, as the first step, the classification of the funds into winner and loser groups according to their historical performance as is done in the literature on momentum strategies. After the pre-selection step, the model is solved via linear programming, the model's second step. The single-step model avoids the pre-selection of hedge funds at the cost of introducing binary variables to exclude the possibility that a hedge fund is present in both the short and long portfolios. Both models are solved with respect to a set of scenarios, based either on historical, or forecasted scenarios generated by GARCH modeling. Combining the two models and the two sets of scenarios, we end up with four strategies. Finally, we evaluate the *ex post* one-year performance of the four strategies with monthly portfolio rebalancing using hedge fund data that encompass the *sub-prime crisis*.

Keywords: hedge funds, fund of hedge funds, linear programming, zero-value strategy.
JEL Classification: C15, C61, P45, G11.

Introduction

Hedge funds are funds pursuing aggressive investment strategies, including short selling, leverage, derivatives, risk arbitrage, and securities with complex structures. As of this writing, hedge funds are exempt from many of the regulatory requirements governing types of investment companies. Hedge funds are restricted by federal securities law to limit the number of investors per fund, and the types of investors that are permitted to invest in the funds (i.e., qualified investors). Consequently, typically hedge funds impose extremely high minimum investment amounts. As with mutual funds, investors in hedge funds pay a management fee; however, hedge funds also collect a percentage of the profits (usually 20%).

The primary investment goal of most hedge funds is to reduce risk at minimal possible capital cost. The strategies formulated by hedge funds seek to generate positive returns regardless of the movement of the market for the asset class in which they invest. Empirical evidence suggests that hedge funds are scarcely correlated with traditional investment during normal market conditions and, as such, the inclusion of hedge funds into an investment portfolio may provide diversification not otherwise available in traditional investment vehicles. There are a large number of hedge fund investment styles – many uncorrelated with each other under normal market conditions – that provide investors with a wide choice of hedge fund strategies to meet their investment objectives.

The complexity of their management is mainly due to the high number of instruments that can be involved in a hedge fund, the correlation among the different instruments and the ability to implement winning selection strategies.

Knowing and understanding the characteristics of the many different hedge fund strategies are essential to capitalizing on their variety of investment opportunities. Some strategies which are not correlated with equity markets are able to deliver consistent returns with extremely low risk of loss, while others may be as or more volatile than mutual funds. A successful fund of hedge funds recognizes these differences and blends various strategies and asset classes together to create more stable long-term investment returns than any of the individual funds.

In recent years, investors' interest in funds of hedge funds has increased. In 2007, for example, funds of hedge funds received about US $60 billion of net new assets under management, increasing the amount of global capital invested in this type of fund of funds to around US $800 billion. Some market observers believe that fund of hedge funds will attract the majority of capital invested in the

fund of funds, an industry that has steadily grown since 1960s. Compared to the total amount invested in the fund of funds industry that has US $ 1,208 billion of assets under management as reported by the Barclay group, hedge funds represent about two thirds of that amount [14].

In this paper, we use various indices of hedge funds mainly based on distinct investment strategies, each offering different degrees of risk and return to determine the optimal hedge fund combinations using a zero-investment strategy. To do so, in Section 1 we introduce two portfolio optimization models based on this strategy, a strategy characterized by the value of the long portfolio equal to the value of the short portfolio so that the portfolio's net investment is zero. This self-financing strategy involves short selling which, under U.S. securities law, requires that a margin account be funded at the outset of the strategy and maintained throughout until the short sale is closed. The obligation to maintain a term deposit (in cash or securities) equal to 50% of the value of the shorted securities, has two implications. First, the strategy is no longer a self-financing strategy because of the need to post margin. Second, it effectively make it necessary to close the strategy's short positions if the losses become too high. The introduction of a marginal account will be considered in our discussion but do not affect the selection model.

The first model we refer to as the two-step model, is so named because it is based on two steps: (1) a pre-selection of the hedge funds to be included in the long portfolio (the portfolio consisting of winners) and the short portfolio (the portfolio consisting of losers), and (2) an optimization procedure. This approach is in line with the zero-investment momentum strategies discussed in the literature [2, 7] based on the empirical finding of persistence of stock returns continuation at least in the short period. The simplest decision criterion is based on the computation of compounded total monthly return in selecting the winners and losers over some defined ranking period. Given the empirical evidence that returns exhibit non-normality, it is important to incorporate this information into the pre-selection criterion in order to include not only a return but also a risk component and construct a return/risk profile of the assets. To achieve that goal, we rank all the hedge funds according to the modified Sharpe ratio [4] performance measure, and we determine the winners as the top 50% and the losers as the bottom 50%.

The implementation of the momentum strategy involves decisions on the length of the data used for the ranking of the assets and the length of the hold-

ing period. The ranking of the assets is ideally computed based on the last year data. Since we consider monthly data, we used a longer period and found that the sets of winner loser hedge funds are quite stable for different lengths of the ranking period. The holding period is one year with monthly rebalancing.

The second approach, the single-step model, avoids this *ex ante* ranking with the introduction of binary variables in the optimization model, forcing the assets to be included in only one of the sets of winners and losers.

Section 2 describes the data and the procedure for generating the scenarios. We consider both historical and forecasted scenarios. The latter are obtained by identifying a set of orthogonal factors which explain the variability of the indices' returns via principal component analysis. Isolating the most relevant factors, we estimate the residuals on which we fit univariate GARCH models. The scenarios are then obtained via simulations on a correlated set of innovations. Finally, in Section 3 we describe the empirical results and compare the different strategies.

1. The zero-investment strategy

The zero-investment strategy allows building a portfolio divided into a long portfolio and a short portfolio whose market values are the same such that the strategy involves no investment outlay.

1.1. The two-step model. With the two-step model, the optimal solution for a given (unit) period is obtained through a two-step process. In the first step we perform the pre-selection of the hedge funds in the database we use, classifying the hedge funds into winners and losers. We rank hedge funds according to a pre-specified criterion[1] and then split the ranked hedge funds in half with the top half representing the winners and the bottom half the losers. The next step, the solution of the programming problem (1) below, involves minimizing the portfolio's average value-at-risk (AVaR)[2] subject to a constraint on the expected return that must be

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[1] In order to rank the hedge funds, we use the *modified Sharpe ratio* defined as the average return over the modified Cornish-Fisher [4, 6] VaR.

[2] $VaR_\alpha(R)$, on a probability space $(\Omega, \Im, P)$, is defined as $VaR_\alpha(R) = -\inf\{r \in \Re | \Pr[R \leq r] \geq \alpha\}$ at tail probability α. For continuous return distributions, the $AVaR_\alpha(R)$ is defined as the average VaR beyond a given VaR level. Not only does AVaR have an intuitive definition, but there are also convenient ways of computing and estimating it. As a result, AVaR turns into a superior alternative to VaR suitable for managing portfolio risk and dealing with optimal portfolio problems. In general, AVaR is defined as follows:

$AVaR_\alpha(R) = \frac{1}{\varepsilon}\int_0^\varepsilon VAR_p(R)dp$, where α denotes the tail probability [15].

greater than a minimum value μ. We denote this average value-at-risk by AVaR$_\alpha$.

As Rockefellar and Uryasev [11, 12] demonstrate, it is possible to linearize the AVaR$_\alpha$ by introducing a vector of auxiliary variables, solving a linear problem where $\min AVaRa(w`r)$ is approximated by

$$\min_{w,\theta,d} \theta + \frac{1}{n\alpha} \sum_{k=1}^{n} \max(-w`r^k - \theta, 0)$$

and θ is VaR$_\alpha$,

$$\min_{w,\theta,d} \theta + \frac{1}{n\alpha} \sum_{k=1}^{n} d_k,$$

s.t.

$$-w^T r^k - \theta \le d_k, d_k \ge 0, k \in N,$$

where N is scenarios' set.

The linear programming problem that we solve is formalized in (1) below where the third and the fourth constraints require that the sum of the weights of the long and short portfolios must be equal to 1 and -1, respectively, while the last two constraints set some bounds on the weights, in order to guarantee a minimum diversification.

$$\min_{w,\theta,d} \theta + \frac{1}{n\alpha} \sum_{k=1}^{n} d_k$$

s.t.

$$-w^T r^k - \theta \le d_k, d_k \ge 0,$$
$$k \in N$$
$$w`E(r) \ge \mu \tag{1}$$
$$\sum_{i \in I^+} w_i = 1$$
$$\sum_{i \in I^-} w_i = -1$$
$$0 \le w_i \le UB, i \in I^+$$
$$LB \le w_1 \le 0, i \in I^-,$$

where w is the vector of the weights composing the zero-investment portfolio; $E(r)$ is the vector of expected returns of the hedge funds composing the portfolio; μ is the minimum expected return required for the portfolio; I^+ and I^- are the sets of the winner and the loser portfolios, respectively; UB and LB detone upper bound and lower bound on the weights of the winners and losers portfolios, respectively, in order to ensure a certain degree of diversification.

Finally, we determine the optimal asset allocation of the zero-investment portfolio for different values of μ, and then we choose the optimal portfolio which maximizes the ratio between the expected return and its $AVaR_\alpha$. This risk/return ratio is called the STARR ratio and described in [2].

1.2. The single-step model. The single-step model avoids the *ex ante* selection by introducing binary variables (i.e., there is no need to rank the hedge fund candidates in order to obtain the sets of winners and losers). With this approach the optimization problem for a given (unit) period is solved through a single-step process, and the selection of the hedge funds to include in the long and short portfolio positions is obtained via binary variables according to programming problem (2) below:

$$\min_{w,\theta,d,\lambda} \theta + \frac{1}{n\alpha} \sum_{k=1}^{n} d_k,$$

s.t.

$$\sum_{i \in I} w_i^+ = 1$$
$$\sum_{i \in I} w_i^- = 1$$
$$w_i = w_i^+ - w_i^- ... i \in I$$
$$w`E(r) \ge \mu$$
$$-w^T r^k - \theta \le d_k, d_k \ge 0 \tag{2}$$
$$k \in N$$
$$0 \le w_i^+ \le UB...i \in I$$
$$0 \le w_i^- \le -UB...i \in I$$
$$w_i^+ - \lambda_i \le 0...i \in I$$
$$w_i^- + \lambda_i \le 1...i \in I$$
$$\lambda_i \in \{0,1\}...i \in I.$$

Many of the equations in (2) are the same as in (1). In the programming problem (2), we introduce a new set of variables w_i^+ / w_i^- which are respectively the weights of the i-th item in the long/short portfolio. Note that in this model, w_i^+ / w_i^- are always positive and λ_i is a binary variable associated with the i-th stock of the zero-investment portfolio, with the set I of all hedge funds.

Also in this case, the objective function is given by the minimization of the $AVaR_\alpha$ with confidence level α. The first two constraints simply require that the sum of the weights for both the long and the short portfolio positions must equal unity and the third constraint forces the weights of the zero-investment portfolio to be equal to the difference between those of the long and short portfolios. The fourth constraint requires that the portfolio offers a minimum level of expected return, while the fifth constraint is used to linearize the objective function. Finally, the last two constraints represent the real novelty inside this model: the introduction of the binary variables λ_i guarantees that the product

w_i^+ / w_i^- must be zero for all funds.

As in the two-step model, we solved the optimization problem for different values of expected return μ and chose the optimal asset allocation as the one maximizing the STARR ratio.

We observe that the two-step model is formulated as a linear programming problem, while the single step model is a mixed integer programming problem due to the presence of the binary variables λ_i. Since the number of binary variables equals the number of assets, in this case, the computational burden increases significantly.

2. The data and scenario generation

In the application of the two models, we focus our attention on the optimal composition of hedge funds, to be more precise of indices of hedge funds. Our dataset is the time series of monthly returns from May 1994 to October 2007 of 13 indices representative of the 13 style-based investment strategies of the hedge fund universe[1]. The database is obtained from the Credit Suisse/Tremont Hedge Fund Index Group webpage (http://www.hedgeindex.com).

We split the sample in two sets and we use a rolling window of 150 monthly returns starting from May 1994 to October 2006 for parameters estimation. The selected period includes, of course, several

financial crises – the failure of Granite Partners, a hedge fund also known as Askin Capital Management in 1994, and the major problems in the collateralized mortgage obligations market that followed the 1997 Bhat crisis, and the Russian ruble crisis in August 1998 followed by the collapse of a major hedge fund (Long-term Capital Management) in September 1998. The last 12 months of our dataset – November 2006 to October 2007 – are used to evaluate and compare the performance of the different strategies using the aggregate wealth value at the end of the holding period. Note that the holding period includes the subprime mortgage crisis that began in summer 2007.

All the hedge fund indices were ranked on the basis of their performance as measured by the modified Sharpe Ratio, as commonly done in evaluating momentum strategies. The implementation of the momentum strategy involves decisions on the length of the time period used for the ranking of the hedge funds and the length of the holding period. The ranking of the hedge funds is ideally computed using data for the prior year. However, because we consider monthly data we use a longer period and in doing so we found that the winner and loser portfolios are quite stable for different time periods

Table 1. Summary statistics for the hedge fund indices, S&P 500, J.P. Morgan U.S. Government Bond Index and CGSY commodity index

Hedge fund index	Symbol	Mean	Median	Std. deviation	Kurtosis	Asymmetry
Convertible arbitrage	CA	0.76%	1.03%	1.30%	6.87	-1.43
Dedicated short bias	DSB	-0.13%	-0.62%	4.85%	5.27	0.88
Emerging markets	EM	0.92%	1.57%	4.43%	8.87	-0.82
Equity market neutral	EMN	0.82%	0.81%	0.82%	3.54	0.31
Event driven	ED	0.98%	1.07%	1.58%	29.27	-3.60
Event driven-distressed	DIS	1.11%	1.22%	1.76%	24.64	-3.14
Event driven multi-strategy	MULTI	0.91%	0.96%	1.72%	19.89	-2.55
Event driven-risk arbitrage	RA	0.64%	0.60%	1.18%	9.80	-1.20
Fixed income arbitrage	FIA	0.54%	0.72%	1.04%	21.19	-3.19
Global macro	GM	1.19%	1.18%	2.98%	6.79	0.07
Long/short equity	LSE	1.06%	1.01%	2.84%	7.40	0.19
Managed futures	MF	0.53%	0.20%	3.50%	3.15	0.03
Multi-strategy	MULTI1	0.80%	0.84%	1.18%	6.37	-1.12
Asset class indices	Symbol					
S&P 500	S&P	0.80%	1.16%	4.10%	4.70	-0.69
J.P. Morgan US Government Bond Index	JPM	0.06%	0.12%	1.32%	4.22	-0.50
CGSY commodity index	CGSY	0.81%	0.98%	5.73%	3.29	0.10

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[1]A description of the indices and the corresponding strategies is provided at www.hedgeindex.com

In Table 1 we report the statistics for the 13 time series, Standard & Poor's 500, J.P. Morgan U.S. Government Bond Index, and the CGSY commodity index. We include the three indices in Table 1 in order to compare the performance of the hedge fund indices with the other asset classes: stocks, bonds and commodities. We observed the following: (1) the volatility of most of the hedge fund indices was less than that of the S&P 500 and CGSY commodity indices while at the same time having a higher average; (2) there is strong evidence of non-normality for most of the time series due to the high kurtosis present in the monthly data; and (3) the abnormal returns are all concentrated at the end of 1998, corresponding to the Russian crisis. We report the correlation matrix in Table 2. Note that the correlation of the hedge fund indices with the other asset classes is low, confirming the commonly held belief that hedge funds are an alternative investment vehicle.

Table 2. Correlation between the hedge fund indices and asset class indices

	CA	DSB	EM	EMN	ED	DIS	MULTI	RA	FIA	GM	LSE	MF	MULTI1	S&P	JPM	CGSY
CA	1.00															
DSB	-0.24	1.00														
EM	0.28	-0.54	1.00													
EMN	0.33	-0.31	0.24	1.00												
ED	0.57	-0.61	0.66	0.37	1.00											
DIS	0.49	-0.61	0.57	0.35	0.93	1.00										
MULTI	0.57	-0.53	0.66	0.34	0.94	0.74	1.00									
RA	0.41	-0.51	0.43	0.32	0.68	0.57	0.65	1.00								
FIA	0.55	-0.07	0.25	0.13	0.37	0.31	0.41	0.16	1.00							
GM	0.28	-0.12	0.42	0.21	0.38	0.31	0.42	0.14	0.41	1.00						
LSE	0.27	-0.71	0.60	0.34	0.67	0.59	0.65	0.53	0.19	0.41	1.00					
MF	-0.09	0.08	-0.06	0.14	-0.09	-0.06	-0.11	-0.13	-0.02	0.27	0.05	1.00				
MULTI1	0.43	-0.12	0.05	0.24	0.27	0.18	0.32	0.14	0.34	0.19	0.24	0.09	1.00			
S&P	0.18	-0.67	0.44	0.37	0.55	0.54	0.48	0.50	0.03	0.20	0.49	-0.06	0.11	1.00		
JPM	-0.04	0.17	-0.16	0.03	-0.18	-0.14	-0.19	-0.12	-0.02	0.15	-0.04	0.28	-0.05	-0.17	1.00	
CGSY	-0.02	-0.05	0.06	0.06	0.09	0.07	0.09	0.03	0.05	0.04	0.20	0.21	0.09	-0.01	0.07	1.00

The optimization problems described in the previous section are solved for the 12 consecutive months for two sets of scenarios: historical scenarios and simulated ones. In the historical approach we considered as scenarios the monthly data using a window of 150 months; the k-th historical scenario is the vector of monthly returns of the 13 indices in month k: this means that a scenario is given by the vector $r^K = \left[r_1^K, r_2^K, ..., r_{13}^K \right]$ with $k \in N$, where the first month considered as the first scenario is May 1994 and r_i^K denotes the k-th historical monthly return for the i-th index.

On the basis of these scenarios we solve the optimization problems presented above, repeating this procedure for 12 months using a moving windows of length 150. Each time we select as the optimal solution the one maximizing the STARR ratio. In other words, we repeat the optimization procedure 12 times, and every time we use the solution to the problem to compute the final wealth accumulated by the portfolio.

For the simulated scenarios, an econometric model is used to generate the forecasted scenarios for the next unit period. We use the data in the rolling window to estimate the parameters of the econometric model in order to generate the 150 forecasted scenarios which are the input into the optimization model. We generated for each month from November 2006 to October 2007, 150 different one-step ahead scenarios and, on the basis of these scenarios, we solved the two models. This procedure has the advantage of reducing the volatilities of the scenarios since we generate 150 future realizations of the returns in the next month and not the dynamics of an asset in the next 150 months as when the historical scenarios are used.

The procedure we employed in the forecasting approach for scenario generation basically involves two distinct steps. Commencing with the historical returns, we utilize principal component analysis (PCA) to find three orthogonal factors that have an explanatory power of more than 80% of the variability for the 13 hedge fund indices.

Thus, we compute the residuals not explained by the three factors obtained from the PCA and we then model the variance of the residuals with an ARMA-GARCH model in order to capture the dependence of

returns. These models incorporate (1) a *dependence effect* given by the relevance of the observations of the immediate past (conditional term), and (2) a *feedback mechanism* with which past observations are taken into consideration to explain the present volatility value (autoregressive part). Roughly speaking, if a time series exhibits GARCH effects, it means it is *heteroscedastic*; that is, its variance may be well described by a time-varying process.

The importance of GARCH modelling is due to the fact that it considers two important characteristics of most time series: the *excess kurtosis effect* and *volatility clustering*. Hence, in general a GARCH model is able to forecast (*ex ante* analysis) or to explain (*ex post* analysis) quite accurately the variances derived from different hedge fund index returns.

The variables to be modelled are the residuals series obtained by the PCA. We applied the following univariate ARMA(1,1)-GARCH(1,1) to each of them:

$$RES_t = C + AR * RES_{t-1} + \varepsilon_t + MA * \varepsilon_{t-1}$$

$$\sigma_t^2 = K + GARCH * \sigma_{t-1}^2 + ARCH * \varepsilon_{t-1}^2,$$

where RES_t are the residuals from the PCA; C – constant in the ARMA(1,1) model; AR – autoregressive term in the ARMA(1,1) model; MA – moving average in the ARMA(1,1) model; $GARCH$ – GARCH coefficient in the GARCH(1,1) model; $ARCH$ – ARCH coefficient in the GARCH(1,1) model; σ_t^2 – conditional variance process of the residuals; ε_t – innovation of the time series process with $\varepsilon_t = z_t \sigma_t$ and z_t is a Gaussian iid process with zero mean and unit variance.

In Table 3 we report the estimate of the ARMA(1,1)-GARCH(1,1) models and asymptotic *t*-statistics.

Table 3. Parameters estimate of the ARMA(1,1)-GARCH(1,1) model for the last month of the *ex-post* analysis (October 2007)

Hedge fund index	C	AR(1)	MA(1)	K	GARCH (1)	ARCH (2)
Equity market neutral	-0.0002 (-0.4246)	0.4464 (1.5536)	-0.2329 (-0.7212)	0.0000 (1.1224)	0.5230 (1.5531)	0.1620 (1.6224)
Event driven distressed	0.0004 (0.6649)	0.6795 (3.0933)	-0.4945 (-1.7891)	0.0001 (4.4146)	0.0000 (0.0000)	0.4417 (4.7217)
Event driven	0.0003 (0.6349)	0.7513 (4.7352)	-0.5608 (-2.9877)	0.0000 (1.7087)	0.6833 (9.1228)	0.2386 (3.9111)
Global macro	0.0025 (1.3384)	-0.8831 (-21.516)	1.0000 (50.8795)	0.0000 (0.9212)	0.8324 (13.3215)	0.1676 (2.6308)
Long/short equity	0.0004 (0.3245)	0.2829 (0.5444)	-0.0789 (-0.1507)	0.0000 (2.176)	0.7430 (9.3411)	0.1679 (2.9999)
Multi strategy	0.0004 (0.641)	0.7294 (2.863)	-0.6085 (-2.0153)	0.0000 (1.4995)	0.7534 (10.2735)	0.2096 (2.6651)
Event driven multi strategy	0.0007 (0.7941)	0.2434 (0.3225)	-0.0944 (-0.1228)	0.0000 (0.7827)	0.9170 (20.725)	0.0830 (2.0926)
Convertible arbitrage	0.0004 (0.8224)	0.5256 (4.1543)	-0.0760 (-0.4819)	0.0000 (4.7083)	0.0000 (0.000)	0.7769 (3.0449)
Event driven risk arbitrage	0.0002 (0.1382)	-0.5986 (-2.1629)	0.7632 (3.538)	0.0000 (1.048)	0.7982 (7.1823)	0.1321 (2.1157)
Fixed income arbitrage	0.0002 (0.1545)	-0.0230 (-0.0512)	0.3343 (0.7934)	0.0001 (0.5917)	0.0000 (0.0000)	0.0629 (0.4445)
Emerging markets	0.0047 (1.2573)	-0.7332 (-4.8475)	0.8857 (9.1835)	0.0000 (0.8729)	0.8574 (17.2709)	0.1426 (2.9918)
Managed futures	0.0006 (0.4348)	-0.4181 (-1.6732)	0.6908 (3.4714)	0.0000 (0.8147)	0.5670 (1.1518)	0.0799 (0.7724)
Dedicated short bias	-0.0015 (-1.1259)	-0.6154 (-1.8134)	0.7415 (2.5503)	0.0000 (0.8567)	0.8427 (11.5752)	0.1389 (2.4452)

Note: t-statistics are in parentheses.

We generate the scenarios using the one step forecast of the variance of the residuals for the next month obtained with the GARCH(1,1) model, updating the diagonal of the next month covariance matrix of the residuals and adjusting the off diagonal elements with the historical correlations.

We draw a vector of residuals simulating the innovations from the multivariate Gaussian with a zero-mean and the updated covariance matrix. Once we obtain the simulated residuals, we reconstruct the forecasted scenarios using the estimated three principal factors.

3. Empirical result

Overall, we had four strategies to evaluate:

Strategy 1: Two-step model with historical scenarios.

Strategy 2: Two-step model with simulated scenarios.

Strategy 3: Single-step model with historical scenarios.

Strategy 4: Single-step model with simulated scenarios.

In order to implement the first two strategies, we determine the winner and the loser sets for each month. In Table 4 we report the two sets of winners and losers for the first month of the analysis. We compute each month the modified Sharpe ratio and observe the relative ranking changes, but the sets of winner and losers remain unchanged in all the periods of the analysis. Most of the assets with the highest kurtosis are included in the winners set.

Table 4. Pre-selection according to the modified Sharpe ratio

	Symbol	Description	E(R)	MVaR*	Modified Sharpe ratio
Winner	EMN	Equity market neutral	0.82%	2.11%	0.1914
	DIS	Distressed	1.09%	4.55%	0.1475
	ED	Event driven	0.94%	3.96%	0.1313
	GM	Global macro	1.19%	5.98%	0.1299
	LSE	Long/short equity	1.03%	5.38%	0.1146
	MULTI	Multi-strategy	0.77%	3.11%	0.1138
	MULTI1	Multi-strategy 1	0.86%	4.14%	0.1063
Loser	CA	Convertible arbitrage	0.74%	3.35%	0.0974
	RA	Risk arbitrage	0.62%	2.79%	0.0713
	FIA	Fixed income arbitrage	0.55%	2.61%	0.0509
	EM	Emerging markets	0.80%	8.83%	0.0440
	MF	Managed futures	0.53%	6.13%	0.0193
	DSB	Dedicated short bias	-0.11%	6.50%	-0.0804

In Table 5 we report the average composition of the long and short portfolios and the standard deviation of the composition. The results for the two-step model and single-step model are in Panels (A) and (B), respectively. Even if the compositions among the different strategies for each month differ significantly, it is interesting to notice that the selection via binary variables on average is concentrated on the same indices of the pre-selection procedure when we consider simulated scenarios. In contract, the composition of the optimal portfolios via binary variables with historical scenarios leads to a completely different selection of long and short portfolios. Three of the hedge fund indices (GM, MULTI, MULTI1) classified as winners according to the momentum strategy are always selected in the short portfolios and two hedge fund indices (CA, FIA), classified as losers, are always in the long portfolio.

Table 5. Average compositions and standard deviation of the optimal portfolios

Panel A. Average compositions and standard deviation of the optimal portfolios for the two-step model. The indices are classified as winners and losers

	Historical scenarios		Simulated scenarios			Historical scenarios		Simulated scenarios	
Winner	Mean	Std. dev.	Mean	Std. dev.	Loser	Mean	Std. dev.	Mean	Std. dev.
EMN	35.00%	-	11.19%	14.19%	CA	-0.53%	1.84%	-11.28%	15.05%
DIS	19.65%	12.77%	15.47%	15.73%	RA	-34.99%	0.03%	-22.35%	16.31%
ED	-	-	11.98%	15.12%	FIA	-35.00%	-	-31.06%	8.39%
GM	20.41%	4.03%	31.33%	6.48%	EM	-7.00%	0.99%	-15.22%	13.19%
LSE	11.99%	6.91%	3.05%	10.07%	MF	-6.27%	1.10%	-9.65%	10.68%
MULTI	12.94%	9.95%	6.09%	12.13%	DSB	-16.20%	1.29%	-10.43%	5.36%
MULTI1	-	-	20.90%	15.04%					

Panel B. Average compositions and standard deviation of the optimal portfolios for the single-step model

Winner	Historical scenarios		Simulated scenarios		Loser	Historical scenarios		Simulated scenarios	
	Mean	Std. dev.	Mean	Std. dev.		Mean	Std. dev.	Mean	Std. dev.
EMN	35.00%	-	11.19%	14.19%	CA	-0.53%	1.84%	-11.28%	15.05%
DIS	19.65%	12.77%	15.47%	15.73%	RA	-34.99%	0.03%	-22.35%	16.31%
ED	-	-	11.98%	15.12%	FIA	-35.00%	-	-31.06%	8.39%
GM	20.41%	4.03%	31.33%	6.48%	EM	-7.00%	0.99%	-15.22%	13.19%
LSE	11.99%	6.91%	3.05%	10.07%	MF	-6.27%	1.10%	-9.65%	10.68%
MULTI	12.94%	9.95%	6.09%	12.13%	DSB	-16.20%	1.29%	-10.43%	5.36%
MULTI1	-	-	20.90%	15.04%					

Table 6 shows the performance of the four strategies in terms of expected return and risk. The *ex ante* STARR ratios are significantly smaller in the case of the two-step model with historical scenarios due to a considerably higher value for the optimal AVaR.

In order to evaluate each strategies' performance on the basis of an *ex post analysis*, we solved the programming problems (1) and (2) using the historical and simulated scenarios for 12 consecutive months, from November 2006 to October 2007, rebalancing each period. We solve the problems using Gams [3], a high-level modeling system for mathematical programming and optimization, and in particular the CPLEX solver for the mixed programming problems.

Table 6. *Ex ante* expected return and risk of the optimal portfolios

	Two-step model				Single-step model			
	Historical scenarios		*Simulated scenarios*		*Historical scenarios*		*Simulated scenarios*	
	Return	AVaR	Return	AVaR	Return	AVaR	Return	AVaR
Nov 2006	0.50%	2.439%	1.70%	0.573%	0.70%	0.250%	0.60%	1.863%
Dec 2006	0.50%	2.424%	1.60%	0.525%	0.40%	0.067%	0.40%	1.823%
Jan 2007	0.50%	2.383%	1.70%	0.555%	0.70%	1.112%	0.50%	0.803%
Feb 2007	0.50%	2.353%	1.70%	0.549%	0.90%	0.748%	0.80%	0.666%
Mar 2007	0.50%	2.337%	1.70%	0.427%	0.70%	0.079%	0.70%	2.094%
Apr 2007	0.50%	2.275%	1.70%	0.410%	0.80%	0.863%	0.50%	0.661%
May 2007	0.50%	2.250%	1.70%	0.378%	0.50%	0.385%	0.40%	0.997%
Jun 2007	0.50%	2.146%	1.70%	0.368%	1.20%	0.979%	1.00%	1.720%
Jul 2007	0.60%	2.602%	1.80%	0.389%	0.60%	0.214%	0.70%	0.748%
Aug 2007	0.60%	2.604%	1.70%	0.368%	1.20%	0.871%	1.00%	0.844%
Sep 2007	0.60%	2.652%	1.70%	0.368%	0.80%	0.317%	0.70%	0.842%
Oct 2007	0.50%	2.264%	1.70%	0.424%	0.50%	0.466%	0.40%	1.892%

We computed cumulative returns for the different strategies, the Sharpe ratio, and the aggregated wealth per unit capital invested at the end of the holding period. From a practical point of view a margin account of 50% on the short part must be paid to the broker for short selling activities.

In Table 7 we report the net return and the cumulative net return for the different strategies and for the Tremont AllHedge index, the geometric mean, the volatility, the Sharpe ratio, and β considering the Tremont index as the market benchmark. The Tremont Index is an asset-weighted hedge fund index derived from the market leading Credit Suisse/Tremont Hedge Fund Index. In order to compare the strategies, we consider the final net cumulative return and the volatility of the return. Unfortunately, we do not have a sufficient number of data points to compute the ex-post STARR ratio. We observe that the best strategy is the two-step model with historical scenarios and the worst is the one-step model, with historical scenarios. The simulated scenarios for both models lead to very good results.

Table 7. Net returns and cumulative returns for the four strategies and for the Tremont AllHedge Index

Month	Two-step model				Single-step model				Tremont index	
	Historical scenarios		Simulated scenarios		Historical scenarios		Simulated scenarios			
	Monthly	Cum	Monthly	Cum	Monthly	Cum	Monthly	Cum	Monthly	Cum
	Return	Return	Return	Return	Return	Return	Return	Return	Return	Return
Nov-06	1.20%	1.20%	1.53%	1.53%	-1.24%	-1.24%	-0.08%	-0.08%	2.07%	2.07%
Dec-06	1.22%	2.43%	-0.60%	0.92%	0.83%	-0.42%	-0.88%	-0.96%	1.83%	3.94%
Jan-07	2.19%	4.68%	3.67%	4.62%	-0.13%	-0.55%	2.56%	1.58%	1.33%	3.19%
Feb-07	0.01%	4.69%	1.16%	5.84%	-0.71%	-1.26%	0.38%	1.96%	0.74%	2.08%
Mar-07	2.57%	7.38%	3.95%	10.02%	-0.14%	-1.39%	3.19%	5.21%	1.24%	1.98%
Apr-07	1.76%	9.27%	1.25%	11.39%	-0.93%	-2.31%	-0.30%	4.90%	2.02%	3.28%
May-07	4.70%	14.40%	1.63%	13.21%	2.86%	0.48%	3.33%	8.39%	2.31%	4.38%
Jun-07	-0.05%	14.35%	0.62%	13.91%	0.79%	1.28%	-0.73%	7.60%	0.78%	3.11%
Jul-07	-1.57%	12.55%	-4.50%	8.79%	1.80%	3.10%	2.75%	10.56%	0.00%	0.77%
Aug-07	0.68%	13.32%	-0.17%	8.60%	-0.90%	2.17%	-1.29%	9.13%	-1.53%	-1.53%
Sep-07	1.75%	15.30%	0.51%	9.15%	-2.97%	-0.86%	0.40%	9.57%	2.71%	1.13%
Oct-07	3.82%	19.70%	2.72%	12.12%	0.82%	-0.05%	3.56%	13.47%	3.16%	5.95%
Geometric mean		1.51%		0.96%		0.00%		1.06%		0.48%
Volatility		1.71%		2.22%		1.54%		1.85%		1.28%
Sharpe ratio		0.83		0.39		-0.05		0.53		0.31
β		0.79		0.68		-0.04		0.44		1

Note: We assume an annual risk free rate of 1%.

Figure 1 shows the computed cumulative returns. We find that, apart from the single-step model with historical simulation, the other strategies have a similar behavior and all outperform the Tremont index. Up until the subprime crisis, the two-step model with simulated scenarios outperforms the other strategies, but the forecasting procedure fails as the market volatility dramatically increases. This call for a more accurate modelling of the innovation in order to capture extreme movements in the mar-

ket [8, 9, 10, 16]. These extreme movements are present in the historical data which include many financial crises (the failure of Granite Partners, a hedge fund also known as Askin Capital Management in 1994, and the major problems in the collateralized mortgage obligations market that followed the 1997 Bhat crisis, and the Russian ruble crisis in August 1998 followed by the collapse of a major hedge fund (Long-term capital management) in September 1998.
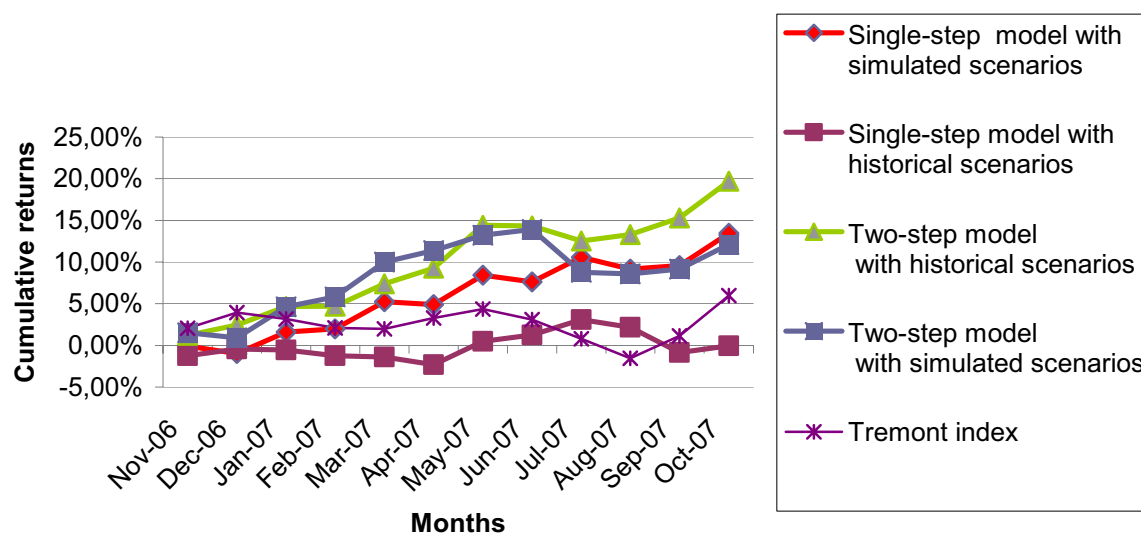

Fig. 1. Cumulative returns

Finally, we perform the Fama decomposition [5] in Table 8. That is, we decompose the excess realized return in the risk premium that the portfolio should have earned given its beta, market return, and risk-free rate and the risk premium due to the manager's skill. The manager's skill is the sum of two components: diversification and pure selectivity. The diversification premium is the difference between the return explained by the total risk of the portfolio and the systematic risk of the portfolio. This will be zero for per-

fectly diversified portfolios. The pure selectivity premium is the difference between the risk premium due to selectivity and diversification. We observe that the single-step model using historical scenarios completely fails in the selection of the losers and the winners. We can confirm that the two-step model is the model with the best behaviors performance using both historical and simulated data, while the single-step model is not able to properly select the winner/loser hedging at least on historical scenarios.

Table 8. Fama decomposition

Inputs				Return decomposition				
	Average return	Standard deviation	Portfolio beta	Risk pre-mium	Risk premium due to		Risk premium due to selectivity is the sum of	
Strategy					beta	selectivity	diversification	net selectivity
1. Two-step model with historical scenarios	1.51%	1.71%	0.79	1.43%	0.31%	1.11%	0.22%	0.90%
2. Two-step model with simulated scenarios	0.96%	2.22%	0.68	0.88%	0.27%	0.61%	0.42%	0.19%
3. One-step model with historical scenarios	0.00%	1.54%	-0.04	-0.08%	-0.02%	-0.07%	0.49%	-0.56%
4. One-step model with simulated scenarios	1.06%	1.85%	0.44	0.98%	0.18%	0.80%	0.40%	0.40%

Note: We assume an annual risk free rate of 1%.

We compute the aggregated wealth at the end of the holding period per unit of invested capital, considering a margin account of 50% to be deposited with the broker for short selling activity. We do not consider the influence of the transaction costs for monthly rebalancing since they affect pretty much all strategies in the same way, resulting in no change in the final ranking of the best approaches.

If we look at the net realized returns, the two-step model with historical scenarios offers the best performance with a final wealth per unit of invested capital[1] of 39.39%. The single-step model with simulated scenarios provides a return of 26.94%, followed by the 24.22% return offered by the two-step model with simulated scenarios. Finally, the single-step model with historical scenarios basically offers a return of close to zero (-0.10%).

Conclusions

In this paper we quantified and analyzed the results of two different optimization models: one is based on an *ex ante* ranking of the best and worst hedge funds, while the other introduces binary variables avoiding the pre-selection prior to any optimization. Both models seek to find the optimal hedge fund allocation of a zero-investment portfolio strategies

for a funds of hedge funds indices. The solution to both models requires the generation of scenarios in order to predict the future. We solved the two optimization problems using two scenario generation techniques: (1) assuming that the past could be considered as a good forecast of the future (historical scenarios), and (2) using principal component analysis via an ARMA-GARCH model, we simulated the future scenarios (simulated scenarios).

With the exception of the single-step model using historical scenarios, the other strategies considerably overperformed the Tremont index. The single-step model using simulated scenarios selects the same set of winners and losers for the short and long portfolios that were explicitly selected by the *ex ante* ranking procedure. These results suggest that the introduction of binary variables and the related complex mixed integer programming does not lead to an improvement in the performance over the two-step model. The two-step model is the best performing model and furthermore (I) for the first period the one solved on simulated scenarios (see (2) has the best performance, but (II) after the subprime crisis, as soon as a dramatic increase in the market volatility occurs, the historical method (see (1) outperformed (2). The GARCH model with normal innovations seems to be incapable of adequately capturing the changes in volatility that are present in the historical dataset which includes the Russian crisis, as shown by the high value of the STARR *ratio*. These results suggest the line for future researches with the introduction of non Gaussian innovation.

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[1] The return of the strategy over the period is computed assuming that the size of the short portfolio is S, and that 0.30% S is the initial investment. After 12 months, the margin is refunded and the final wealth is:

$$R = \frac{(\text{CumNetReturn})S + 0,5S - 0,5S}{0,5S} = \frac{\text{CumNetReturn}}{0,5}.$$

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