## "Stock options under non- expected utility theory"

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# Stock options under non-expected utility theory 


#### Abstract

This paper modifies the modeling approach of Lambert et al. (1991) and Hall and Murphy $(2000,2002)$ to assess the value of executive stock options in the presence of deviations from expected utility maximization. In particular, we specify a valuation model which incorporates elements of cumulative prospect theory. We conclude that this could potentially close the value gap identified by Lambert et al. (2002) and Hall and Murphy (2000, 2002) that results from risk aversion of executives who get part of their compensation in the form of stock options and are not able to hold sufficiently diversified portfolios. Moreover, introducing probability weighting also affects executives' willingness to take on risky investment projects. These findings are confirmed in an experiment.


Keywords: stock options, non-expected utility theory, option valuation.
JEL Classification: D81, J33, M52.

## Introduction

Stock option plans have become a standard part of executive compensation in most Western industrialized countries. In 2001, approximately $75 \%$ to $100 \%$ of the large publicly traded corporations in the US, UK, France, and Germany had stock option plans in place ${ }^{1}$. This process has been driven by a variety of factors, including the increasing globalization of firms that has led many European companies to adopt American-style compensation practices. A second factor was the boom of the 'New Economy' between 1998 and 2001 during which many Internet and technology start-ups offered their employees significant stakes in their companies. In response, a lot of 'Old Economy' companies tried to adopt similar types of compensation structures, in particular stock option plans to retain their staff. Despite the demise of the New Economy boom, the debate on the (ab-)use of stock options in recent months and the call of many politicians to abandon stock options as a means of compensation, stock option plans are still an important incentive instrument for firms to align executives' with shareholders' interests. However, an increasing need seems to exist to gain a better understanding of stock options to optimize their use, as well as their governance ${ }^{2}$.

Besides providing compensation, executive stock options can be used to create incentives for managers to improve their firm's success and to motivate them ${ }^{3}$. Two important incentive effects are discussed in the theoretical literature ${ }^{4}$ : First, stock options for executives are widely seen as an effective instrument to align managers' interests with those of the shareholders: By linking compensation

[^0]to the firm's equity executives become owners of company stocks thus motivating them to maximize shareholder value. Second, stock options may provide incentives to encourage executives' risk taking in their decisions about corporate investments: ${ }^{5}$ By rewarding managers for success without having them bear the costs of failure an asymmetry in financial payoffs is generated which may help to align managers' risk averse behavior with the preference of virtually risk neutral shareholders.

The fact that stock options link executives' compensation on the firms' stock price implies that managers are rewarded for all upward movement in the stock, regardless of the source of such movements. Because of the inability of an executive to control all the variables that influence a firm's stock price performance, some form of randomness enters his pay. One of the most fundamental insights about perform-ance-based compensation is that the executive demands a risk premium for accepting stock options as compensation if shareholders' ability to bear risk is negligible compared to his. This, of course, is fundamental for publicly traded corporations as noted by Murphy (1999, p. 2520): "the comparative advantage of the corporate form of organization is precisely that well-diversified atomistic shareholders are better able than managers to bear risk".

In designing executive stock options an optimal plan should then balance the costs of imposing risks on the executive and the benefits from providing incentives. Since the magnitude of the risk premium depends on both the riskiness of his compensation via the firm's stock price volatility and his individual degree of risk aversion, the subjective value the executive places on his stock option plan crucially determines this trade-off. The adequate governance of stock options in the provision of compensation and incentives therefore requires measuring both the subjective value and the incentives provided by executive stock option plans.

[^1]In the search for a model to estimate the subjective value of stock option plans, a recent approach by Hall and Murphy $(2000,2002)$ has aimed to estimate the value by calculating a certainty equivalent value, i.e., the amount of cash for which an executive would be willing to exchange his stock options ${ }^{1}$. This approach calculates the expected future distribution of stock prices and option pay-outs in a binominal model and uses a CRRA utility function, assuming the executive to be risk-averse. Given this specification, it is not surprising that the model arrives at the conclusion that a value gap exists, i.e., that the value of a stock option is significantly lower to the executive than what it costs the firm${ }^{2}$. To minimize the value gap, it is best to grant options with a low exercise price.
The present paper suggests that while such a methodology offers important insights the conclusions are skewed, due to the specification of their model ${ }^{3}$. While their estimation of the subjective value of stock options rests on assumptions which are in line with von Neu-mann-Morgenstern's expected utility theory, our model takes into account recent developments in the field of non-expected utility theory.

One of the main hypotheses we employ in this paper is that executives do not value stock option plans according to expected utility theory but are limitedly rational. We assume that an executive as everyone else has limited cognitive capacity so that his valuations are biased. These violations of the axioms of expected utility theory, however, do not imply that his valuation and judgment processes are limited in an unsystematic way. Recent developments in the field of utility measurement suggest that people rely on a number of simplifying strategies called heuristics. The best known proponents in this field have probably been Kahneman and Tversky and we will follow their prospect theory to estimate the subjective value of a stock option plan to an executive with limited rationality ${ }^{4}$.
We thus generalize the model by Hall and Murphy in two directions: First, the phenomenon of framing and the corresponding features of diminishing sen-

[^2]sitivity and loss aversion are taken into consideration in the value function of the certainty equivalent model. Framing is based on the observation that an executive rather than receiving stock options "for free" foregoes some other forms of compensation, in return for his stock options. In particular, he experiences a loss if the stock options expire out-of-the-money ${ }^{5}$. Second, we take care of the fact that people are limited in their ability to comprehend and evaluate probabilities of unlikely events. To consider this mental probability weighting is important as the value of stock options critically depends on an executive's expectations of large but unlikely future outcomes.
Recent research on subjective option valuation focuses on several aspects. The link between valuation and individual characteristics is studied in Malmendier and Tate $(2005,2008)$ and Malmendier et al. (2007). The observation that individuals are prone to various behavioral biases when dealing with stocks is documented in a survey by Barberis and Thaler (2003). Sautner and Weber (2009a; 2009b) empirically investigate the option valuations of top managers and study to what extent individual characteristics of these managers are correlated with their option valuation. Finally, Oyer and Schaefer (2005) and Bergman and Jenter (2007) incorporate excessive employee optimism about a firm's stock price into option compensation frameworks.
The present paper is organized as follows: In Section 1 we introduce the framing of the utility function and decision weighting in the context of option evaluation. In Section 2 we develop a certainty equivalent framework which builds on non-expected optimal valuation. Based on this model, we analyze in Section 3 and 4 the value gap and the incentives created by executive's stock options and derive a number of hypotheses that deviate from the existing perspectives on this topic. In Section 5 an experiment is reviewed that was conducted to estimate how executives value their stock options. The results are discussed and evaluated in the light of our model. The final section offers some summarized conclusions.

## 1. Option valuation under non-expected utility theory

If an executive was rational his valuation of a stock option plan would be based on two criteria: First, on his expected personal gain in wealth according to the realized stock price at the time of exercise. Second, on his subjective probabilities he attaches to different realizations of the stock price. Given these two rationality requirements, an executive would value a stock option plan in line with von Neumann-

[^3]Morgenstern's expected utility theory, that is, his expected utility is derived from the utilities of payouts weighted by their probabilities.
However, in real life, executives as most other individuals rarely obey the axioms of expected utility theory. For the valuation of stock option plans, the work by Kahneman and Tverksy (1979; 1992) and others in non-expected utility theory suggests two major modifications from von Neumann-Morgenstern: The value function and the weighting function.

## The value function

One of the key deviations in non-expected utility theory is the assumption that framing of events matters, i.e. people seem to evaluate not absolute outcomes but look at outcomes in terms of gains and losses relative to a neutral reference point ${ }^{1}$. As a result, the value $v(\cdot)$ of an outcome is treated as a function of two arguments: The reference point and the magnitude of change from that reference point.
To include framing in the evaluation of his stock options, we assume that an executive does not assign value to final pay-outs of his stock options this would imply a non-negative value independent of the realized stock price. Instead, the executive sees his stock options as a substitute for some other form of compensation and is conscious of the fact that his stock options have a real cost to him ${ }^{2}$.

As under prospect theory, executives evaluate gains and losses relative to this reference point. Following Kahneman and Tversky (1979), executives are assumed to be risk-averse for gains, while it is believed that they are risk-seeking below the reference point ${ }^{3}$. It is further assumed that they are loss-averse, i.e., that in their minds, losses loom larger than gains.

## The weighting function

The second essential deviation of non-expected utility theory from von Neumann-Morgenstern's utility theory is the assumption that the probability of individual outcomes is transformed into specific

[^4]"decision weights"; see Tversky and Kahneman (1992) ${ }^{4}$. In general, these decision weights $\pi(\cdot)$ do not coincide with the subjective or objective probabilities of the outcomes. Instead, people overvalue small probabilities and undervalue large probabilities.
In the context of our discussion, an executive will not weight the value of a particular pay-out of the stock option by the probability associated with the realized stock price but by a decision weight. With this modification we can incorporate two phenomena in the evaluation of his stock option plans: First, an executive may overemphasize the small chance of both hitting a very large payoff and receiving nothing at all. Second, an executive may generally be overly positive about future outcomes ${ }^{5}$. This would imply that preferred outcomes are overweighted, while unattractive outcomes are largely ignored.

## 2. Non-expected option valuation in a certainty equivalence model

To estimate the value a stock option plan has for an executive in a non-expected utility framework, we follow Lambert et al. (1991) and Hall and Murphy (2000, 2002) and use the certainty equivalence approach. Suppose an executive is granted $n$ options to buy $n$ shares of stock at exercise price X in T years ${ }^{6}$. The certainty equivalent then is defined as the amount of compensation, payable for certain, that the executive regards as equivalent in value to the stock option plan ( $\mathrm{n}, \mathrm{T}, \mathrm{X}$ ).

Let $\mathrm{S}_{\mathrm{T}}$ be the realized stock price at time T . The pay-out to the executive at time T, i.e. his gain in wealth, then is equal to max $\left(0, \mathrm{n} \cdot\left(\mathrm{S}_{\mathrm{T}}-\mathrm{X}\right)\right)$ and his value over wealth is v ( $\mathrm{n} \cdot \max \left[0, \mathrm{~S}_{\mathrm{T}}-\mathrm{X}\right]$ ). Let $\mathrm{f}\left(\mathrm{S}_{\mathrm{T}}\right)$ be the distribution of future stock prices at date T and the decision weight the executive attaches to a particular stock price $\mathrm{S}_{\mathrm{T}}$ be then

[^5]$\pi\left(f\left(S_{T}\right)\right)$. His anticipated utility $E U(\cdot)$ at time T of the stock option plan ( $\mathrm{n}, \mathrm{T}, \mathrm{X}$ ) is then equal to
$$
E U(n, T, X)=\int_{S_{T}} v\left(n \cdot \operatorname { m a x } \left[0, S_{T}-X \| \cdot \pi\left(f\left(S_{T}\right)\right) d S_{T} .\right.\right.
$$

If, instead of stock options, the executive had received a deferred cash compensation V that was securely invested at the risk free rate r , his pay-out at time T would have been $V \cdot(1+r)^{T}$ and his value $v\left(V \cdot(1+r)^{T}\right)$. Altogether, the certainty equivalent V then is implicitly defined by the following equality:

$$
\int_{S_{T}} v\left(n \cdot \max \left[0, S_{T}-X\right]\right) \cdot \pi\left(f\left(S_{T}\right)\right) d S_{T}=v\left(V(1+r)^{T}\right) .
$$

To solve this equation numerically requires assumptions about the form of the value function $v(\cdot)$, the weighting function $\pi(\cdot)$ and the distribution of future stock prices $f(\cdot)$.

The functional form of the value function for gains and losses is borrowed from Tversky and Kahneman (1992, p. 309). The value for a gain or loss x under this model is given by a power function so that:
$v(x)=\left\{\begin{array}{cl}x^{\alpha} & \text { for } \mathrm{x} \text { above the reference point } \\ -\lambda\left(-\mathrm{x}^{\beta}\right) & \text { for } \mathrm{x} \text { below the reference point }\end{array}\right.$
where $\alpha, \beta$ are the parameters of the value function and $\lambda$ is the loss aversion parameter. To preserve some level of parsimony in the model we assume $\alpha=\beta^{1}$. The value function parameter settings were taken from the paper by Tversky and Kahneman (1992, p. 311), see also Gonzales and Wu (1999, p. 142): The risk tolerance parameter $\alpha$ is set at 0.88 , suggesting slight risk aversion. The loss aversion factor $\lambda$ is set at 2.25 , which implies significant discounts for possible outcomes below the reference point.
When estimating the cost of the stock options in terms of the amount of other forms of compensation foregone, the executive can use the risk-free value of these options ${ }^{2}$. Based on this proposition, we assume that the reference point for the executive is the Black-Scholes value of the options at the time of grant plus the riskfree interest earned on this amount during the option

[^6]period. It is assumed that the value function for the reference point is equal to the value function for gains.
The functional form of the weighting function draws on recent work by Gonzales and Wu (1999) by employing a two-parameter rank-dependent decision weighting function ${ }^{3}$. The decision weight $\pi\left(p_{i}\right)$ of a probability $p_{i}$, belonging to an outcome $\mathrm{x}_{\mathrm{i}}$, is equal to
$$
\pi\left(\mathrm{p}_{\mathrm{i}}\right)=\sum_{\mathrm{j}=1}^{\mathrm{i}} \mathrm{w}\left(\mathrm{p}_{\mathrm{j}}\right)-\sum_{\mathrm{j}=1}^{\mathrm{i}-1} \mathrm{w}\left(\mathrm{p}_{\mathrm{j}}\right)
$$
where $\mathrm{x}_{1}>\mathrm{x}_{2}>\ldots>\mathrm{x}_{\mathrm{i}}>\ldots>\mathrm{x}_{\mathrm{n}} . w($.$) is the correspond-$ ing probability weighting function, see Gonzales and Wu (1999),
$$
\mathrm{w}\left(p_{i}\right)=\frac{\delta p_{i}^{\gamma}}{\delta p^{\gamma}+\left(1-p_{i}\right)^{\gamma}} .
$$

The parameter $\delta$ controls the elevation of the probability weighting function and, as such, the attractiveness, resp. optimism, of the decision weighting process. $\gamma$, on the other hand, controls the curvature of the probability weighting function, i.e., the degree of sensitivity of the decision weighting process. If $\delta$ is set larger than 1 , the individual has a positively skewed set of expectations. For the parameter $\gamma$, values below 1 lead to an overweighting of small probabilities in general and an underweighting of larger probabilities. To illustrate the link between real probabilities, probability weighting and decision weights, two examples are outlined in the following figure:


Fig. 1. The impact of decision weighting

- Normal sensitivity but optimistic: $(\delta=1,5 ; \gamma=1)$ Here, the probability weighting function assigns higher weights to outcomes with a high ranking number. The probability function is, consequently, shifted toward the left.
- More sensitive but balanced expectations: ( $\delta=1$; $\gamma=0,6$ ). Here, the probability weighting function shows the inverse-S shaped form ${ }^{4}$. The decision weighting, in turn, leads to an overweighting of small probabilities, resulting in a 'flattening' of the curve.

[^7]The selection of the appropriate parameter settings is difficult, due to the wide spread of existing estimates. Drawing on the literature, the following parameter settings will be assumed ${ }^{1}$. The elevation parameter is set between 0.8 to 1.6 and the range of the curvature parameter is set between 0.6 and 1 . Whereas $\gamma$ slightly below 1 reflects the hypothesis that there will be some overweighting of the small probabilities but only of a limited nature ${ }^{2}, \delta$ slightly above 1 reflects the fact that it is believed that executives are generally optimistic about their own future, as well as that of their employer.

To model the distribution of the underlying stock price, a binominal stock price model is used, specified in line with the relevant literature. ${ }^{3}$ Under such a model, it is assumed that the stock price S at the beginning of a period can either go up or down by a fixed percentage. In the model, this is expressed as S being either multiplied by a factor $u$ (up-tick) or $d$ (down-tick). The probability of the up-tick is p and for d is 1-p. One additional condition is that $u=1 / d^{4}$. The parameters $u, d, p$ are chosen to recreate the expected rate of return as well as volatility of the stock. Assuming that the future, expected average rate of return per period $(\Delta t)$ is $r$, and the volatility is $\sigma$, the parameters are calculated to $\mathrm{be}^{5}$ :
$u=e^{\sigma \sqrt{\Delta t}}$
$d=e^{-\sigma \sqrt{\Delta t}}$ where $a=e^{r \Delta t}{ }_{6}$.
$p=\frac{a-d}{u-d}$
The model was implemented using a PC-based spreadsheet. The model was set to include 1000 steps to secure sufficient accuracy in the calculation.

## 3. The value gap

Of course, the assumption of risk averse executives by itself implies that the certainty equivalent to an executive's stock option plan differs from the expected pay-out of the stock option plan. The difference, that is the amount the executive would pay to have a certain compensation for sure rather than the risky stock option pay-out, is the risk premium the firm has to pay to keep the executive indifferent. One of the central results of decision theory is that the risk premium is increasing in a person's degree of risk aversion. In particular, the

[^8]risk premium for a risk neutral option holder is zero, that is, he is unwilling to pay any premium to avoid the risk associated with his stock options.
If we rely on option pricing formulae, such as Black-Scholes, to measure the certainty equivalent the risk premium corresponds to a value gap between the executive's evaluation of his stock option plan and the firm's cost of this plan ${ }^{7}$. Thus, granting stock options to risk averse executives necessarily creates a value gap between the costs to the firm and the value to the executive. This is the central result of Hall and Murphy $(2000,2002)$.

The framing and decision weighting processes introduced in the previous section in the executive's evaluation of stock options extend this result in different directions. The introduction of the reference point together with 'loss aversion' reduces the value of the stock option to the executive in a similar way as does the assumption of risk aversion. The decision weighting process, however, can yield opposite results. A tendency to be overly optimistic would increase the subjective value of the stock options. This effect might well be compounded by an emotional overemphasis of the unlikely extreme outcomes, given the asymmetric pay-outs from a stock option.

To study the interplay of these different factors, the model of Section 4 is specified and applied to a representative hypothetical employment situation at a company ${ }^{8}$. In particular, we assumed that the option would become exercisable after three years ${ }^{9}$. The number of stock options was set such that the cost to the company from stock option compensation measured by the Black-Scholes formula is equal to $€ 100.000^{10}$. The stock price at the time of grant was fixed at $€ 100$, the exercise price was taken as a variable.

The subjective values of the stock option grant are shown in Figures 2 and $3^{11}$. Focusing on the role of the decision weighting process, both exhibits assume fixed risk tolerance and loss aversion. Figure 2 shows the impact of optimism resp. pessimism on

[^9]the executive's value of the grant. Unsurprisingly, regardless of the exercise price, the value of the stock option raises with increasing optimism, i.e. the higher his $\delta$, the higher his subjective value for each exercise price is. While the results suggest that optimism may close the value gap, at least for options for which the exercise price is not too high, they also show that, without some subjective optimism ceteris paribus, the value gap for an individual with reasonable risk aversion would be dramatic ${ }^{1}$.


Stock price at time of grant

Fig. 2. Subjective value of total stock option grant with fixed sensitivity, $\gamma=1$
The effects of changes on the subjective sensitivity towards changes in small probabilities are shown in Figure 3. For restricted stock, the gain of the downside and upside risks partly balance out. For a fair-market-value option, or even a premium stock option, this effect can be dramatic. At a $\gamma$ of 0.7 , the subjective value of a stock option grant with an exercise price per option, set at twice the current market price of the stock, is close to $200 \%$ of the Black Scholes cost of the grant.


## Stock price at time of grant

Fig. 3. Subjective value of total stock option grant with fixed optimism, $\delta=1$

[^10]Figure 4 shows the combined effects of optimism and sensitivity.


Stock price at time of grant
Fig. 4. Subjective value of total stock option grant
As can be seen in this case, the value gap of the stock option is actually negative! If, the executives were, while still favorably biased, less optimistic and less prone to overemphasizing the outliers as in case (B) $(\gamma=0.9, \delta=1.1)$, the resulting pattern of the executive value line would show marked differences. The value gap now emerges and grows with a higher exercise price.
These results have important implications. First, the value of a stock option grant to the executive depends importantly on the subjective perspective of the probability distribution of future stock prices. Second, assuming the likely existence of risk and loss aversion, executives who take an objective view of the odds would be expected to perceive a significant gap between their subjective value of the stock options and their costs to the firm $^{2}$. Third, decision weighting could lead the executives to have a higher perceived value of the stock option grants. This effect may remove the value gap or even lead to a value surplus. Finally, unless a very high tendency exists to overweight the small chance of high outcomes, the value gap is always minimized by setting a medium to low exercise price.

## 4. Incentive effects

As outlined in the Introduction, stock option plans can be designed to induce at least two incentives effects: First, stock options can affect the executive's motivation to increase the firm's stock price. Second, stock options can increase the executive's incentive for risk-taking.
To assess the first incentive effect of a stock option grant in our framework, its pay-performance intensity (PPI) was calculated: The pay-performance intensity is

[^11]defined as the total change in the $€$ value of the stock option grant for $\mathrm{a} € 1$ increase in the initial stock price ${ }^{1}$.
In the case of a hedged investor, the PPI rises monotonously with a higher exercise price (see Figure $5^{2}$ ). In the case of risk and loss aversion, but no decision weighting, the PPI consequently deviates from the 'risk-free' curve: The PPI for this case is always below the PPI for the risk-neutral case. Furthermore, increasing the exercise price initially raises the PPI, as the increase in the leverage of the grant more than outbalances the risk aversion. For higher exercise prices, the PPI declines as the risk and loss aversion cause the executive to increasingly discount the upside potential.


Fig. 5. Subjective pay-performance intensity
Decision weighting may raise the subjective PPI in two ways. First, optimism shifts the mean of the probability distribution. The result is a shift of the high point to the right and the effect of heightened sensitivity towards small probabilities is more dramatic. If $\gamma$ is set at 0.6 , the subjective PPI curve becomes similar to the risk-neutral curve. As this conclusion does not appear to make a lot of sense, it can inversely be hypothesized that the degree of sensitivity may be less pronounced or relevant for the case of subjective stock option valuation ${ }^{3}$. In a mixed case in which the executives are assumed to be a little optimistic as well as sensitive, the PPI curve slopes upwards to an exercise price of about $160 \%$ of the current market price of the stock. Relative to the risk-free PPI, the subjective PPI is actu-

[^12]ally larger for exercise prices above the current market value. These results imply that the subjective PPI might be maximized for stock options with premium exercise prices.
To analyze the second incentive effect of a stock option grant in our framework, its pay-volatility intensity (PVI) was calculated. Identifying risk with the volatility of the share price, the pay-volatility intensity is defined as the increase in the value of the stock option grant for a one percentage point increase in the future volatility of the stock price ${ }^{4}$.

From the perspective of a risk-neutral investor, see Figure 6 , the PVI is a monotonously increasing function of the exercise price starting at zero. For a restricted stock grant, the impact of an increase in the volatility, ceteris paribus, is zero. The higher the exercise price, the higher the incentive to become more risk-seeking. For a fair-market-value stock option grant in our example, the risk-neutral PVI is about $€ 2.300$, for a stock option grant with a $€ 200$ exercise price, it is $€ 10.000$.


Fig. 6. Subjective pay-volatility intensity
Assuming that the executive possesses both risk aversion and loss aversion but applies the 'objective' probabilities in the evaluation of his stock option grant, the current model suggests that the executive would have no incentive to increase the volatility, due to the declining marginal utility of the executive towards wealth paired with a strong aversion to potential losses. Executives, therefore, would, ceteris paribus, try to reduce rather than increase volatility.
With decision weighting, the executive's incentives towards raising resp. lowering the volatility of the companies' stock change significantly. Under optimism, the executive's subjective PVI creates some positive incentive to increase volatility in the case of higher exercise prices. Despite this, the subjective PVI of an executive remains far below that of a diversified investor. At an exercise price of $€ 200$, it is less than half of that of the risk-free

[^13]PVI. Changes in the executive's sensitivity towards small probabilities have a more dramatic effect on the subjective PVI. The heightened appetite for risk, implied by increased sensitivity, naturally favors increases in volatility. In the mixed case, which assumes some optimism as well as some increase in sensitivity, the result is, unsurprisingly, mixed as well. The PVI remains negative for very low exercise prices, creating some, if limited, incentive to raise volatility for fair-market-value as well as premium stock option grants.

Based on these observations, the following conclusions can be drawn. First, contrary to the common notion that stock options necessarily cause executives to become more risk-seeking, they may actually, depending on how much they are in-themoney, create an incentive to lower volatility. This is due to the fact that the executive will try to 'secure' the existing value rather than risking losing it. Second, stock options can, based on the model presented here, only create an incentive to raise volatility if the executive is either optimistic or overly sensitive to small probabilities. Finally, if a company wants to create volatility by increasing incentives, it should set a high exercise price. Here, it is, however, to trade off the increase in the subjective PVI against the potential of causing a value gap and a reduction in the pay-performance intensity.

## 5. Experimental estimation of the subjective values

The focus chosen for the experiment was fourfold. First, to find out what value executives assign to stock options when asked directly. Second, to understand the impact of the exercise price on the subjective valuation. Third, to calibrate our model. Additionally it was intended to see if a strong national or company/industry-specific component exists that drives the parameter settings.
5.1. Experimental design. The experimental design was based on the single-response sequential method $(\mathrm{BDM})^{1}$. The method has been modified for the current context using an approach similar to that recently chosen by Wu and Gonzales (1999). Participants were asked to indicate the necessary minimum amount of cash required to induce them to give up a (hypothetical) stock option grant ${ }^{2}$. Op-

[^14]erationally, the experiment was implemented using an email-based automated questionnaire. The participants in the experiment were selected from the client service staff of an international management consulting firm ${ }^{3}$. In the experiments, the sample was split into a "German" (GE) and an "American" (US) group. The two national groups were each split into two subgroups. The experiments were identical for each subgroup, except for the description of the situation of the company. The first subgroup, for both Germans and Americans, was presented with the case of a solid, steadily performing branded goods company ( BG ) with low risk (beta: $0,53$; annual volatility $30 \%)^{4}$. The second company profile, given to the other two subgroups, portrayed a 'beaten down' e-commerce company (EC), which offers potentially high upside but is also very risky (beta: 1,82 ; annual volatility $82 \%$ ).
As regards the employment situation, the experiment describes a recruitment scenario for all the sample groups. The participating consultants were not offered any direct compensation for participating in the experiment but were all entered in a raffle.
5.2. Experimental results. The survey was sent out to a total of 1,640 consultants being evenly split between the subgroups. As shown in Appendix B, a total of 206 consultants responded to the survey ${ }^{1}$. This corresponds to an overall $12 \%$ net response rate, the rate being slightly higher in Germany than for the US. In the survey, the participants were asked to give their subjective value of (1) a restricted stock grant, (2) a stock option grant with exercise prices at the current market level, and (3) a stock option grant with exercise prices at twice current market level.

The results (see Figure 7) are quite intriguing. Both, for the restricted stock as well as for the fairmarket value option, the mean subjective value is actually above the Black-Scholes cost of the grants. Only for the premium option with a strike price set high, at twice the current market value, does the subjective value fall below the risk-free valuation.

[^15]

Fig. 7. Empirical average size of the certainty equivalents

When analyzing the feedback on the experiments by subgroups, a similar pattern of the distribution of the subjective valuations prevails in all of them. In all cases, the restricted stock and the fair-market value options are quoted above their risk-free cost (up to a $20 \%$ premium), while the stock options, with an exercise price set at twice the current market price, are valued with a discount between 15 and $25 \%$.

The experimental results of the subgroups were analyzed in some detail. It was found that no statistically significant difference between the evaluations of the stock options exists (for any of the three stock option varieties) between Germany and the US. On the other hand, a small but statistically significant difference between the two industry cases exists. The valuation for the fair-market value stock option grant in the Branded Goods case is significantly higher than in the E-commerce case. This would suggest that, in some cases, stock options may actually carry a higher subjective value when they are issued by a company with performance stability, rather than a risky one. This may suggest that risk aversion is quite high and people attach less importance to the 'outliers', i.e., the very large but relatively unlikely chance of the underlying share price becoming very high.
5.3. Implications for the valuation model. According to the experimental data, executives value at par or even overvalue stock option grants with low to medium exercise prices ${ }^{2}$. Only for the very high exercise prices does the value gap open up. Given the structure of the model developed in section 1, the empirical findings would suggest that, assuming risk as well as loss aversion, 'optimism' exists, i.e., some shift of the probability distribu-

[^16]tion towards the left. Sensitivity, i.e., an overweighting of the small probabilities is less apparent. To confirm this intuitive evaluation of the implications of the experimental results for the model, the data gathered was used to calibrate the model parameters ${ }^{3}$ (see Figure 8). It was found that it is possible to achieve a good fit with the model for all subsamples by means of the calibration. This is remarkable as the settings for the movement in the underlying stock price are very different in both cases.

The individual calibrated parameters offer some room for an interesting interpretation of their economic and psychological meaning:

- The risk tolerance parameter $(\alpha)$ is calculated as 0.82 , a result that is close to the 0.88 number found by Tversky and Kahneman (1992). As such, the model confirms the existence of some moderate risk aversion.
- The loss aversion parameter $(\lambda)$ comes out as 1.96, again fairly close to the 2.25 estimate by Kahneman and Tversky. Loss aversion, therefore, is also found to play a role for stock options. Here the findings again support the findings reported by the current literature.

For the probability decision weighting process, the findings shed some new light on the individual evaluation of probabilities. The elevation/optimism parameter ( $\delta$ ) that offers the best fit is calculated as being 1.56. This would suggest significant optimism as regards the future of the stock price. The overweighting of small probabilities seems less important and the corresponding parameter $(\gamma)$ is, consequently, close to one at 0.95 .

[^17]

Fig. 8. Survey averages versus calibrated valuation model outputs

The parameter settings identified in the experiment suggest that the decision weights are the product of a transformation that shifts the probability distribution function of future stock prices to the left while slightly 'squashing it downwards and outwards'. The former effect being the dominant one, while the latter is only mildly noticeable.

## Conclusion

The question of how executive stock options should be valued is important for two reasons. For one thing, it is necessary to determine the economic cost of stock options to a company and its shareholders to create transparency and to enable their controlled usage. Second, it is important to understand the value of stock options to their recipients, i.e. the executives. This is relevant because it is possible that the executives' "subjective value" of a stock option grant deviates significantly from the cost of such a grant to the company. The reason why the "subjective value" can deviate from the cost is because executives are generally not well diversified nor are they able to or allowed to hedge their exposure. This makes it impossible to apply risk free-valuation techniques such as Black-Scholes.
The aim of this article was to offer a new model to derive an estimate for the subjective value of a stock option grant. The new model draws on recent research done in the field of non-expected utility and is, thus, able to account for some characteristics
of human decision-making that are of critical importance in the evaluation of a stock option but that are neglected by standard expected utility theory. The resulting model output shows that a value gap exists, i.e., that the value of a stock option is significantly lower to the executive than it costs the firm. The reason for this is that people, largely due to optimism, overestimate the likelihood of a positive outcome.
Our findings suggest that stock options might actually be less inefficient, as some authors have argued in the past ${ }^{1}$. Given sufficient optimism, they may even be a 'cheap' method of compensation. Inversely, in situations of pessimism, they may be a very expensive method. The insight here is that it is always necessary to take the subjective preferences of the executives into account, because, depending on their risk aversion and more importantly their level of optimism, the executive's evaluation of stock options is likely to vary significantly. As such, there can be no single answer to fit all situations. The second insight, from the data gathered here, is that it appears as if to minimize the value gap and to maximize the incentive to raise the share price, it is best to choose either restricted stock or stock options with an exercise price set at the market value of the stock at the time of grant. In these cases, the 'excessive' optimism of the executives more than compensates for their risk aversion.

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## Appendix A. The two cases

Case


Description
The company is an international branded goods company with a focus in cosmetics. It is well known and has a successful product offering. Sales have grown continuously in recent years and profitability has been solid. The company's stock has performed quite well in the past (currently trading at $\$ 100$ per share). One important characteristic of the stock has been its low level of volatility. Most analysts believe the company's stock will grow in line with the overall market and have given it a "moderate buy" rating.

Stock Price Chart

The company is a leading global e-commerce player. It is well known and has a successful product offering. Sales have grown rapidly in recent years but the company has been struggling to reach profitability. The company's stock price has declined sharply over the last two year together with the whole internet sector (currently trading at $\$ 10$ per share). One important characteristic of the stock has been its high level of volatility. Most analysts believe that this is still a risky investment but some see significant upside for the stock in the medium-term future


Appendix B. Response analysis

## Questionnaires sent (number, \% total)

Responses
(number, $\%$ total)

| GE | US | Total |
| :---: | :---: | :---: |
|  |  |  |
| $65^{*}$ | 35 | 100 |
| $15 \%$ | $8 \%$ | $11 \%$ |
|  |  |  |
| 64 | 40 | 104 |
| $14 \%$ | $13 \%$ | $14 \%$ |
|  |  |  |
|  |  | 204 |
| 129 | $10 \%$ | $\mathbf{1 2 \%}$ |
|  |  |  |

## Appendix C. Experiment

## Sample text for experiment (e-commerce case) How would you value your stock options?

We would like you to participate in an experiment conducted as part of a Ph.D. by a member of the CF\&S practice. The purpose is to find out how you personally would value stock options if they were offered to you as part of a compensation package. (It might - at some point - come in handy for you to know something about this topic!)
In the experiment, we will briefly outline a hypothetical but realistic situation followed by three short questions. The description may seem a little long and complex at first but please do not be put off by this (stock options are a tricky subject). The whole experiment will take you about five minutes to complete.
All participants also take part in a lottery. The six lucky winners will receive a bottle of champagne each.
Your answers are important to us. Thank you very much in advance for your participation!

## Description of situation:

Imagine you have been offered an attractive management position in a company and are discussing your compensation package.


Here are some facts:

- The company is a leading global e-commerce player. It is well known and has a successful product offering. Sales have grown rapidly in recent years but the company has been struggling to reach profitability. The company's stock price has declined sharply over the last two years together with the whole Internet sector (currently trading at $\mathbf{\$ 1 0}$ per share). One important characteristic of the stock has been its high level of volatility. Most analysts believe that this is still a risky investment but some see significant upside for the stock in the medium-term future.
- The compensation package offered to you includes a comfortable annual cash salary $(\$ 150,000)$. On top of this, you have been offered a stock option plan. The stock options you would receive under the plan can be exercised 3 years after they are granted to you (at which point they also have to be exercised). Each year you will receive stock options with a market value of $\$ 100,000$, i.e., that is the price at which they could be sold in the market at the time of grant. (However, you are explicitly not allowed to sell or hedge your options).
- During the discussion you are told that rather than stock options you may elect to receive an annual amount of deferred cash compensation instead. In this case, the agreed amount of deferred cash will be placed in a secure bank account for 3 years after which it will be paid out to you.


## Question:

We want you to indicate the minimum (annual) amount of deferred cash compensation you personally would accept instead of receiving stock options.

We will ask you this question for three alternative types of stock option plans. The difference between the alternatives is the level of the exercise price of the stock options. (The number of stock options has been calculated to maintain the same current market value of $\$ 100,000$ in each alternative).

1. Alternative: Exercise price $\mathbf{\$ 0}$. In this case you would receive 10.000 shares for free which you can sell in three years' time.
2. Alternative: Exercise price equal to the current market price of the stock. In this case you would receive the right to buy 18.000 shares at a price of $\$ 10$ each in three years' time.
3. Alternative: Exercise price equal to twice the current market price of the stock. In this case you would receive the right to buy 25.600 shares at a price of $\$ 20$ each in three years' time.
The chart underneath illustrates the possible payouts from a single annual stock option grant for the three stated alternatives in year 3 depending on the level of the stock price at that time.

Total pay-out in Year $\mathbf{3}$ depending on stock price


For each of the three alternatives indicate the minimum level of deferred cash compensation you would demand instead of receiving the stock options.

1. Alternative: Exercise price $\mathbf{\$ 0}$. In this case you would receive 10,000 shares for free which you are allowed to sell in three years time (current share price \$10).

| Level of deferred cash <br> compensation | Indicate the minimum amount of deferred cash compensation at which you prefer it to the stock options |
| :---: | :---: |
| $\$ 200,000$ |  |
| $\$ 180,000$ |  |
| $\$ 160,000$ |  |
| $\$ 140,000$ |  |
| $\$ 120,000$ |  |
| $\$ 100,000$ |  |
| $\$ 80,000$ |  |
| $\$ 60,000$ |  |
| $\$ 40,000$ |  |
| $\$ 20,000$ |  |

2. Alternative: Exercise price equal to the current market. In this case you would receive the right to buy 18,000 shares for $\$ 10$ each in three years time (current share price $\$ 10$ ).

| Level of deferred cash <br> compensation | Indicate the minimum amount of deferred cash compensation at which you prefer it to the stock options |
| :---: | :---: |
| $\$ 200,000$ |  |
| $\$ 180,000$ |  |
| $\$ 160,000$ |  |
| $\$ 140,000$ |  |
| $\$ 120,000$ |  |
| $\$ 100,000$ |  |
| $\$ 80,000$ |  |
| $\$ 60,000$ |  |
| $\$ 40,000$ |  |
| $\$ 20,000$ |  |

3. Alternative: Exercise price equal to twice the current market price. In this case you would have the right to buy 25,600 shares for $\$ 20$ each in three years time (current share price $\$ 10$ )

| Level of deferred cash <br> compensation | Indicate the minimum amount of deferred cash compensation at which you prefer it to the stock options |
| :---: | :---: |
| $\$ 200,000$ |  |
| $\$ 180,000$ |  |
| $\$ 160,000$ |  |
| $\$ 140,000$ |  |
| $\$ 120,000$ |  |
| $\$ 100,000$ |  |
| $\$ 80,000$ |  |
| $\$ 60,000$ |  |
| $\$ 40,000$ |  |
| $\$ 20,000$ |  |


[^0]:    © Peter-J. Jost, Florian C. Wolff, 2010.
    ${ }^{1}$ See Towers-Perrin (2001).
    ${ }^{2}$ Compare Gillian (2001).
    ${ }^{3}$ For the conflict of interests between management and shareholders and the use of stock options as resolution device see Jensen and Meckling (1976), Haugen and Senbet (1981) or Feltham and Wu (2001).
    ${ }^{4}$ Stock options can play an additional incentive role in curbing overly aggressive behavior by managers, see Reitman (1993) as well in reducing dividend yields, see Murphy (1999).

[^1]:    ${ }^{5}$ For an example taken from the oil industry, see Rajgopal and Shevlin (2000).

[^2]:    ${ }^{1}$ This approach follows Lambert, Larcker, and Verrechia (1991). See also Kutatilaka and Marcus (1994), Rubenstein (1995), Carpenter (1998), Meulbroek (2000) or Hull and White (2004).
    ${ }^{2}$ Hall and Murphy (2002) in their base case, estimate that the risk premium for a fair-market-value option grant is close to $70 \%$.
    ${ }^{3}$ It can also be shown that the choice of parameter settings has important implications. For example, their model assumes relatively little volatility for the underlying stock price. Using average levels of volatility from the high-tech sector would, in their model, reduce the value of the stock option to close to zero.
    ${ }^{4}$ Despite the current progress in the understanding of how stock options work, there is still relatively little empirical evidence about how managers and employees subjectively value the stock options they are holding. In their survey on equity-based compensation, Core, Guay and Larcker (2003, p. 43) therefore conclude that "an interesting question for future research is to examine how executives actually value their stock options".

[^3]:    ${ }^{5}$ As such, he does experience a sense of loss if the stock options expire out-of-the-money. For framing refer, for example, to Tversky and Kahneman (1981), Tversky and Kahneman (1986) and Weber, Keppe and Meyer-Delius (2000).

[^4]:    ${ }^{1}$ "The location of the reference point, and the manner in which problems are coded and edited emerge as critical factors in the analysis of decisions" see Kahneman and Tversky (1979, p. 288). Unfortunately, today neither a comprehensive nor a proven set of rules of how people frame outcomes exists. However, among researchers, some basic common principles of framing are assumed, see Tversky and Kahneman (1986, p. 257), March and Shapira (1987) or Tversky and Kahneman (1992, p. 299).
    ${ }^{2}$ This follows the proposition by Berkelaar, Kouwenberg and Post (2004) that the aspiration level of wealth forms the reference point.
    ${ }^{3}$ A convex utility function below the reference point helps to explain why people hold on to declining stocks too long, while selling their winners too early, see Odean (1998, p. 1783), Barber and Odean (1999, p. 20) for an empirical study or Barberis and Huang (2001, p. 1247) for a more conceptual treatment. A recent study by Barberis and Huang (2009) shows that this is only true in a dynamic framework under an additional assumption dubbed "realization utility". The specific risk aversion for gains for executives has actually been the subject of a variety of studies, which confirm their risk aversion, see, for example, March and Shapira (1987, p. 1409) or Wehrung (1989).

[^5]:    ${ }^{4}$ Conceptually, such a transformation process consists of two separate stages, see Kilka and Weber (2001, p. 6): In the first stage, the individual agent forms a view of what he believes the real probabilities are. In the second stage, the agent then assigns an 'emotional weight' to the assumed probabilities. In the current model, however, only the second effect is included whereas the first step is omitted as no available set of rules exists on how to estimate such distorted probabilities. Consequently, the model assumes that the perceived probabilities are equal to the objective probabilities. This is not because it is believed that executives necessarily share the "objective" view of future stock price distributions that can be derived from the prices of publicly traded stock options. Quite the contrary, it seems likely that their evaluation may in many instances deviate significantly from this market-average perspective. Like most people, executives are likely to be "taking a view" on their firm's stock price and the market in general, see Bloomfield and Hales (2002).
    ${ }^{5}$ This was documented in a study by Weinstein (1980, p. 807, p. 818). The same is true, in a more specific sense, for investors and managers. For investors, see Moore, Kurtzerg, Fox and Bazerman (1999, p. 95), for managers the case is made by Hirshleifer (2001, p. 1562).
    ${ }^{6}$ To make the model more manageable, it is assumed that the option period equals the vesting period, i.e., the options must be exercised at time T. Moreover, we assume that the executive cannot trade his options or hedge the risks associated with his options by short-selling firm stock, see Hall and Murphy (2000, p. 5).

[^6]:    ${ }^{1}$ This simplification has no significant impact on the results of the model, see Tversky and Kahneman (1992, p. 312). However, it allows for an easier calculation of the certainty equivalent.
    ${ }^{2}$ He knows that a firm will normally first set the total value of his compensation and, afterwards, split this amount into separate components (base pay, performance pay, fringe benefits, stock options, etc.). The value of the stock option grant normally is a certain percentage of the total compensation. The cost of the options from the perspective of the firm can be estimated using risk-free option pricing formulae. This work is often supported by compensation consultants who publish reports that list the level and composition of the compensation for different positions. For popular examples, refer to TowersPerrin (2000) or HayGroup (2001).

[^7]:    ${ }^{3}$ Their findings represent an evolution, as well as synthesis of the earlier rank-dependent utility work by Quiggin (1982) and cumulative prospect theory (Tversky and Kahneman (1992). See also Abdellaoui (2000).
    ${ }^{4}$ Asknown firmCumulative Prospect Theory, see Tversky and Kahneman(1992,p.310).

[^8]:    ${ }^{1}$ These studies will be introduced later in the text. Compare Gonzales and Wu (1999, p. 157), Kilka and Weber (2001, p. 30) or Bleichrodt and Pinto (2000, p. 1494).
    ${ }^{2}$ This is because executives, according to March and Shapira (1987, p. 1411), are prone to ignoring the very small probabilities altogether.
    ${ }^{3}$ The general approach used here was first developed by Cox, Ross and Rubinstein (1979). In formulating the details, the model also draws on Hull (2000), Wilmott (1998).
    ${ }^{4}$ This condition was suggested by Cox, Ross and Rubinstein (1979), p. 236.
    ${ }^{5}$ This is actually an approximation, see Hull (2000, p. 390). For derivation, also refer to Wilmott (1998, p. 164).
    ${ }^{6}$ The expected drift of the model was based on the CAPM, see Sharpe (1964) or Brealey and Myers (2000). The expected volatility was based on the volatility implicit in traded option contracts.

[^9]:    ${ }^{7}$ Since shareholders as outside investors in general hold freely traded options and are able to fully hedge the risk of options by short-selling stock, Black and Scholes (1973) demonstrated, that they value options as if they were risk neutral and all assets appreciate at the risk-free rate. Option price formulae such as Black and Scholes therefore can be used to measure the cost of stock options for the firm. See also Hall and Murphy (2000, p. 1 1ff) for a discussion.
    ${ }^{8}$ The parameter settings are chosen to reflect the settings for an executive in an "average" large German company. To make the case realistic, the company data was actually based on a large diversified manufacturer. The example was disguised to avoid potential sensitivity. The compensation level was based on that of a senior executive who receives a relatively large share of his salary in the form of stock option.
    ${ }^{9}$ This is in line with the practice of most German DAX and NEMAX companies.
    ${ }^{10}$ This figure was based on data provided in the report of a compensation consultancy, see HayGroup (2001, p. 74) or TowersPerrin (2000). The $€ 100.000$ level was taken as an average estimate. It would represent between 10 and $20 \%$ of the average total compensation of a senior executive.
    ${ }^{11}$ Note that the Black-Scholes value of the grant is independent of the chosen exercise price equal to $€ 100.000$.

[^10]:    ${ }^{1}$ Pessimism (e.g., $\delta=0.8$ ) could further reduce the value dramatically. Now, the subjective value of a stock grant would be only about $60 \%$ of the cost to the company and a fair-market-value option would receive roughly an $80 \%$ markdown.

[^11]:    ${ }^{2}$ This is in line with the findings of Hall and Murphy $(2000,2002)$.

[^12]:    ${ }^{1}$ This definition is based on the concept defined by Jensen and Murphy (1990). In the model, the initial stock price was set at $€ 100$. The increase is, therefore, equal to a $1 \%$ rise.
    ${ }^{2}$ This is necessarily so, as the increase in the exercise price leads to the granting of more highly leveraged stock options. In the case of a riskneutral valuation it would, therefore, be optimal in terms of maximizing the PPI to grant an infinite amount of stock options with an infinitely high exercise price. The fact that this conclusion is obviously nonsensical serves to support the assumption that executives do not value stock options in the same way as diversified investor.
    ${ }^{3}$ Numerous discussions with executives from various companies suggest that the incentive effect of the stock option grant will eventually decline if the exercise price is set too high.

[^13]:    ${ }^{4}$ As long as this does not increase the systemic risk of the stock, i.e., the beta, this can raise the value of the firm, in particular for firms with few current earnings.

[^14]:    ${ }^{1}$ See Becker, DeGroot and Marschak (1964). The method is commonly referred to as BDM. In it, participants are presented with the option of receiving a risky wager or selling that wager for cash. Each participant is asked individually to state the smallest amount of cash he would be willing to accept in lieu of receiving the wager. According to Kagel and Roth (1995 p. 79), BDM "has the property that it gives the utility maximizers the incentive to reveal their true reservation price for an object that is the price at which they would be indifferent to selling it and not selling it (or buying it and not buying it)".
    ${ }^{2}$ To make this question more plausible, the participants are asked to put themselves in the position of an applicant for a job offer who, having been offered the position, is asked to select between different compensation models. To support the case, each participant is given a short

[^15]:    briefing of the company he applied for, as well as some key facts about the compensation package. In the experiment, each participant is asked to select his reservation price for three different scenarios, the difference between the scenarios being the level of the exercise price of the stock options. The total Black-Scholes cost of the stock option plan, however, is kept at a fixed level for all three scenarios. See Appendix C.
    ${ }^{3}$ For a discussion of the pros and cons of using different sample groups (students, professionals, etc.) see Friedman and Sunder (1994, p. 38).
    ${ }^{4}$ Both cases are based on real companies using publicly available information. Their names are disguised to avoid possible sensitivity. See Appendix A.
    ${ }^{5}$ The two company cases were randomly assigned to different offices, e.g., in Germany, the Berlin-, Cologne-, Dusseldorf-, Frankfurt-, and Stuttgart-based consultants received the E-commerce case, while the Hamburg-, Munich- and Vienna-based consultants received the Branded Goods case.

[^16]:    ${ }^{1}$ The survey feedback was automatically gathered in a Lotus Notes database. From there it was exported, first into Excel, and then into SPSS version 10 which was used for all the data analysis.
    ${ }^{2}$ Meaning restricted stock or exercise prices set at the market price at the time of grant.

[^17]:    ${ }^{3}$ This was done using the solver function included in Excel. The solver was set up to minimize the sum of the squares of the differences between the average results from the experiment for the three stock option grant types in the Branded Goods and E-commerce cases. To minimize the sum of the squares, the solver function numerically tries different combinations of the input variables $\alpha, \lambda, \gamma$ and $\delta$. The solver was set to conduct 10,000 iterations. The constraints included in the model were $\alpha \leq 1, \lambda \geq 1, \gamma \leq 1$ and $\delta \geq 1$.

[^18]:    ${ }^{1}$ A recent paper by Bettis et al. (2005) e.g. shows that the failure to adjust for observed exercise patterns can also significantly overstate the cost of stock options.

