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DYNAMIC EFFECTS IN THE BEAM ON AN ELASTIC FOUNDATION CAUSED BY THE SUDDEN TRANSFORMATION OF SUPPORTING CONDITIONS

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Abstract: A mathematical model of a dynamic process in a loaded beam on an elastic Winkler base with the sudden formation of a defect in the form of a change in the boundary conditions is constructed. The solution of the static problem of bending of the beam fixed at the ends serves as the initial condition for the process of forced vibrations hinged supported at the ends of the beam, which arose after a sudden break in the bonds that prevented the rotation of the end sections. Dynamic increments of stresses in the beam for various combinations of beam and foundation parameters are determined. determined.

Keywords: beam on an elastic foundation, sudden transformation of boundary conditions, deflections, bending moments, natural oscillation frequencies, forced oscillations, stress increments

ДИНАМИЧЕСКИЕ ЭФФЕКТЫ В БАЛКЕ НА УПРУГОМ ОСНОВАНИИ, ВЫЗВАННЫЕ ВНЕЗАПНЫМ ПРЕОБРАЗОВАНИЕМ УСЛОВИЙ ОПИРАНИЯ

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Аннотация: Построена математическая модель динамического процесса в нагруженной балке на упругом основании Винклера при внезапном образовании дефекта в виде изменения граничных условий. Решение статической задачи изгиба защемленной по концам балки служит начальным условием процесса вынужденных колебаний шарнирно опертой по концам балки, возникшего после внезапного обрыва связей, препятствующих повороту концевых сечений. Определены динамические приращения напряжений в балке при различных сочетаниях параметров балки и основания.

Ключевые слова: балка на упругом основании, внезапное преобразование граничных условий, прогибы, изгибающие моменты, собственные частоты колебаний, вынужденные колебания, приращения напряжений

An important problem of Structural Mechanics is the analysis of the sensitivity of load-bearing structures to structural transformations under load, such as sudden connections' removal, cracks formation, partial destruction, etc. Obtaining such information for real structures requires the development of special methods, since this problem cannot be solved by ordinary methods. In accordance with structural mechanic's provisions to solve these tasks, it becomes necessary to calculate such systems as structurally non-linear, changing the design scheme under loads, including dynamic loadings caused by sudden overlimit impacts [1-5].

And if the ordinary design situations are well analyzed and exactly are regulated in the relevant documents, the overlimit situations are not classified and the response of structural elements to such impacts is not sufficiently investigated.

There is only a few number of engineering methods to design and calculate structures at such impacts, which take into account sudden transformation and damage of structural systems, and it is not perfect. The dearth of knowledge about the deformation and stress state of structural elements during dynamic processes, initiated by sudden damage, restrains the development of the theory and design methods that take into account the possibility and potential consequences of overlimit impacts and ensure a high level of structural safety at its operating. As an example of investigations, performed in this direction, it should be noted a number of scientific publications [6–11] containing the simulation results of transient dynamic processes performing in loaded beams at sudden damage of supports, delamination, transverse or longitudinal cracks formation, partial destruction, change of connection conditions of structural elements, etc. All these investigations were performed for beams supported only at the edges that is, there is solid ground under the beams. There is theoretical and practical interest to extend similar approaches to beams on an elastic ground.

In the present paper, the problem of constructing a mathematical model of transient dynamic processes in a beam on an elastic foundation at the sudden changing of boundary conditions is described. Before the formation of a defect, the reaction of the structure is determined by a static impact. The sudden formation of a defect leads to a decrease in the overall rigidity of the structure, which does not ensure the static balance of the system. The arising inertial forces cause a dynamic reaction, redistribution and growth of deformations and stresses. As a result, there may be a violation of the normal operating of the structure or loss of bearing capacity and destruction.

Currently, in the scientific literature related with the problem of the dynamics of the "beam-ground" systems, there are many solved problems. The majority of researches on the dynamics of interaction between a beam and a foundation are devoted to the analysis of natural oscillations. At the same time, in these researches, natural and forced oscillations of beams on elastic ground are considered only for cases when the design scheme of the beam-ground system in the loaded state does not change. Appearance of constructive nonlinearity, i.e. changing in the design scheme of a loaded beam on an elastic foundation and it consequences are described only in a few papers [12–18], in which the sudden partial or complete destruction of the foundation was considered.

The paper [19] presents the results of the computational analysis of the long-term deformation of the building-foundation system of one of the nuclear power facilities — the complex of nuclear waste storage buildings, including the nuclear waste storage facility of nuclear power plants (Fig. 1 a, b). To study the dynamic effects in buildings and structures of this type, a second level design scheme - beams on an elastic foundation as an element of such a structural system [1] is considered (Fig. 1 c).

The following formulation of the problem is formulated.

An elastic beam with flexural stiffness EI, rigidly clamped at the ends rests along the entire length l on an elastic Winkler foundation with stiffness coefficient k (Figure 1c).

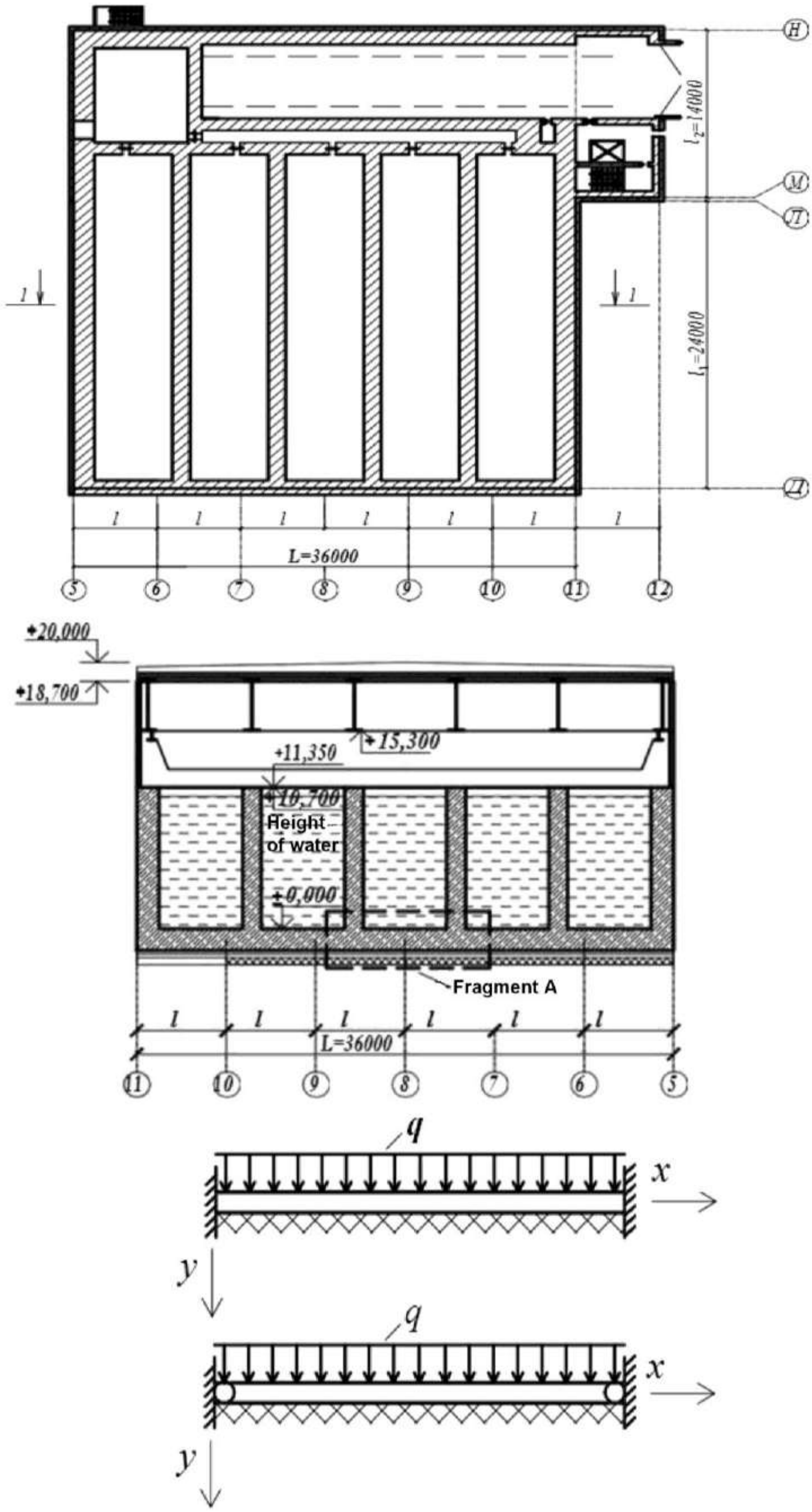


Figure 1. The design scheme of static bending of the beam on an elastic foundation.

On the outer layers of the beam acts even distributed load of q intensity and the reaction of the foundation. It is assumed that at some point in time t = 0 of the statically deforming of the beam, the connections, which impede the rotation of the end sections in the supports, suddenly collapses, forming hinges at the before clamped points. The static equilibrium of the loaded beam is interrupted and the beam begin motion $\upsilon(x,t)$, during of which the deformation and stress in the beam receive dynamic increments.

The solution to the problem is carried out in Cartesian coordinates x, y. Linear dimensions and transverse displacements are related with the length of the beam. The sought parameters are static and dynamic deflections, the frequencies and modes of the natural flexural vibrations of a hinged beam, internal bending moments. Taking into account the practical significance of the problem of ensuring the strength and survivability of structures on elastic grounds and the dearth of well-known papers by this problem, the described problems seems to be actual.

The solution to the problem is constructed in the following sequence:

1) determination of a static deflection of the beam clamped at the ends (“intact”) on an elastic foundation. These parameters will be used later as the initial condition of a dynamic process initiated in the system by a sudden transformation of boundary conditions;

2) determination of the frequencies and forms of natural flexural oscillations of hinged at the ends of the (“damaged”) beam on an elastic base;

3) investigation of forced bending vibrations of a loaded beam. In this case, the load, the static deflection of the "intact" beam and the unknown dynamic deflection are arranged in rows according to the natural vibration forms of the "damaged" beam.

PROBLEM SOLUTION

1. Static bending of clamped at the edges beam on an elastic Winkler foundation is described by equation with dimensionless parameters [20, 21]

$$\frac{d^4 w_{cm}}{d\xi^4} + 4\alpha^4 w_{cm} = \bar{q}, \qquad (1)$$

where

$$\xi = \frac{x}{l}, \quad w_{cm} = \frac{\upsilon}{l}, \quad \bar{q} = \frac{ql^3}{EI}, \quad \alpha = \sqrt[4]{\frac{kl^4}{4EI}}.$$

General solution to the equation (1) for the case, when edges are clamped [20, 21]

$$w_{cm} = \frac{\bar{q}}{4\alpha^4}\left(1 - k_n(\alpha\xi)\right) + w_0'' k_2(\alpha\xi) + w_0''' k_1(\alpha\xi), \qquad (2)$$

where $k_i(\alpha\xi)$ $(i = 1 \div 4)$ – Krylov function, that takes the following form

$$k_1(\alpha\xi) = \frac{\sin\alpha\xi ch\alpha\xi - \cos\alpha\xi sh\alpha\xi}{4\alpha^3}$$

$$k_2(\alpha\xi) = \frac{\sin\alpha\xi sh\alpha\xi}{2\alpha^2}$$

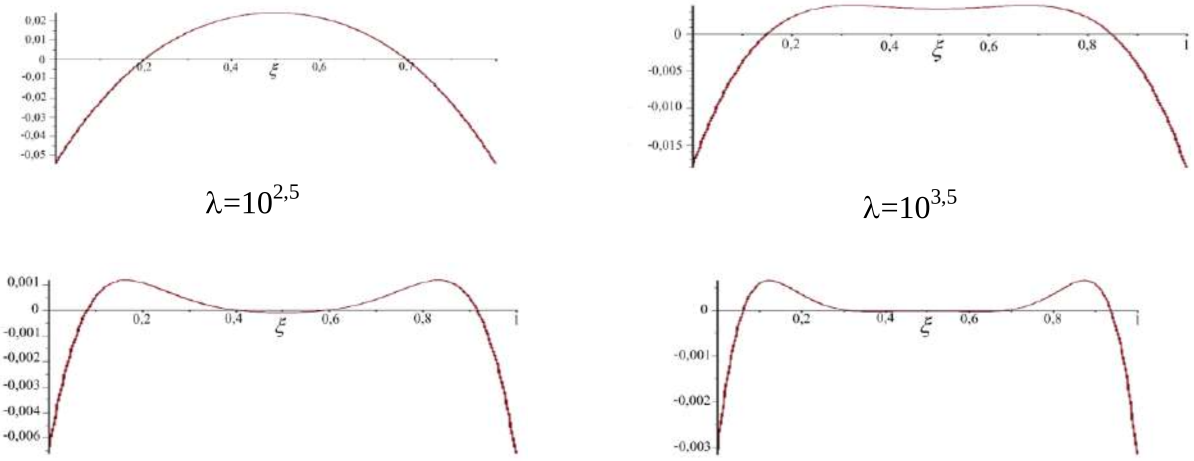
$$k_3(\alpha\xi) = \frac{\sin\alpha\xi ch\alpha\xi + \cos\alpha\xi sh\alpha\xi}{2\alpha}$$

$$k_4(\alpha\xi) = \cos\alpha\xi ch\alpha\xi$$

w_0'', w_0''' – initial parameters, dimensionless bending moment and shear force in a cross section at the point $\xi = 0$

$$w_0'' = \frac{\bar{q}}{k_2^2(\alpha) - k_1(\alpha)k_3(\alpha)} \times \left(\frac{k_4(\alpha) - 1}{4\alpha^4} k_2(\alpha) + k_1^2(\alpha)\right),$$

$$w_0''' = \frac{\bar{q}}{k_1(\alpha)k_3(\alpha) - k_2^2(\alpha)} \times \left(\frac{k_4(\alpha) - 1}{4\alpha^4} k_3(\alpha) + k_1(\alpha)k_2(\alpha)\right).$$



λ=10^{2,5} λ=10^{3,5}

λ=10^{4,5} λ=10^{5}

Figure 2. Diagrams of bending moments at the initial static state, depending on the stiffness indicator of the beam-foundation system λ.

Dimensionless bending moment at static state is determined by function

$$w_{cm} = \bar{q}k_2\left(\alpha\xi\right) + w_0''k_4\left(\alpha\xi\right) + w_0'''k_3\left(\alpha\xi\right). \quad (3)$$

In figure 2, diagrams of bending moments in the beam with clamped edges at different values of generalized stiffness $\lambda = 4\alpha^4$. of "beam-foundation" system are presented. It should be noted the "extraordinary" [22] view of diagrams of bending moments at growing of stiffness of the system. Bending moments in the middle of the beam is significantly lower, than in the quarters of the span. This is result of complex action of external load and reaction of an elasti foundation to the beam.

2. Appeared motion $\upsilon_{\partial ин} = \upsilon\left(x, t\right)$ after sudden transformation of clamped supports of the beam to hinges is described by equation [10]

$$\frac{\partial^4 w_{\partial ин}}{\partial \xi^4} + 4\alpha^4\left(w_{\partial ин} + \frac{\partial^2 w_{\partial ин}}{\partial \tau^2}\right) = \bar{q}, \quad (4)$$

where

$$w_{\partial ин} = \frac{\upsilon\left(x, t\right)}{l}, \quad \tau = w_0 t.$$

$$w_0 = \sqrt{\frac{k}{\rho A}}$$

is parameter, that has frequency dimension, and called as "conventional" frequency.

Equation (4) describes forced vibrations of the loaded beam. Winkler model does not suppose dynamic effects in an elastic foundation. Necessary for the further calculation eigen functions can be obtained from the equation (4) with zero padded right part. After parameters division by performance

$$w_{\partial ин} = W\left(\xi\right)\sin\bar{\omega}\tau, \quad (5)$$

takes the form

$$\frac{d^4 W}{d\xi^4} + 4\alpha^4\left(1 - \bar{\omega}^2\right)W = 0, \quad (6)$$

where

$$\bar{\omega} = \frac{\omega}{\omega_0}$$

is dimensionless natural frequency of vibration of the beam on an elastic foundation.

Using "conventional" frequency ω_0, characterizing stiffness and insertional properties of the system "beam – foundation", and known the basic frequency of flexural oscillations of the beam with the same support at the edges, but without an elastic foundation

$$\omega_{1cв} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}},$$

We transform equation (6) to the form

$$\frac{d^4W}{d\xi^4} = \pi^4 \left(-\bar{\omega}_0^2 + \tilde{\omega}^2\right) W, \qquad (7)$$

where

$$\bar{\omega}_0 = \frac{\omega_0}{\omega_{1cв}}$$

is relative "conventional" frequency;

$$\tilde{\omega} = \frac{\omega}{\omega_{1cв}}$$

is relative finding frequency.

Using Euler substitution

$$W = Ae^{r\xi}, \qquad (8)$$

we obtain characteristic equation for differential equation (7),

$$r^4 + \pi^4 \left(\bar{\omega}_0^2 - \tilde{\omega}^2\right) = 0, \qquad (9)$$

The roots of which can be represented in both forms in accordance with frequencies ratio $\bar{\omega}_0$ and $\tilde{\omega}$:

– if $\tilde{\omega} > \bar{\omega}_0$, then roots of the equation (9) are real and imaginary

$$r_{1,2} = \pm\beta_1, \quad r_{3,4} = \pm i\beta_1, \quad \beta_1 = \pi\sqrt[4]{\tilde{\omega}^2 - \bar{\omega}_0^2}, \qquad (10)$$

In this case a deflection function (8) takes the form

$$W = A_1 ch\beta_1\xi + A_2 sh\beta_1\xi + A_3 \cos\beta_1\xi + A_4 \sin\beta_1\xi; \qquad (11)$$

– if $\tilde{\omega} < \bar{\omega}_0$, then roots of the equation (9) are complex

$$r_{1,2,3,4} = (\pm i \pm 1)\beta_2, \quad \beta_2 = \frac{\pi}{\sqrt{2}}\sqrt[4]{\bar{\omega}^2 - \tilde{\omega}^2} \qquad (12)$$

and a deflection function takes the form

$$W = A_1 sh\beta_2\xi \sin\beta_2\xi + A_2 sh\beta_2\xi \cos\beta_2\xi + A_3 ch\beta_2\xi \sin\beta_2\xi + A_4 ch\beta_2\xi \cos\beta_2\xi. \qquad (13)$$

In the papers [23, 24] it is shown that for a beam fully supported on the elastic Winkler foundation, in the case of canonical symmetric boundary conditions: clamping-clamping, hinge-hinge, free ends, only option (10) is realized.

The principally possible case of fourfold root

$$r_{1,2,3,4} = 0, \qquad (14)$$

when $\bar{\omega}_0 = \tilde{\omega}$ and deflection function takes the form

$$W = A_1 + A_2\xi + A_3\frac{\xi^2}{2} + A_4 \sin\frac{\xi^3}{6} \qquad (15)$$

in the case of limiting the deflections of the ends of the beam, as it is in our case with the hinged supports, also is not implemented. Let us note that with partial support of the beam on an elastic foundation and at various boundary

conditions, all three modes of oscillations (11), (13) and (15) are possible.
Using initial parameters

$$W_0 = W(0),\quad W_0' = W'(0),\quad W_0'' = W''(0),\quad W_0''' = W'''(0)$$

instead constants of integration $A_i\left(i=1\div 4\right)$, let us write relationships, characterizing state of arbitrary section ξ of the beam, using variant (10), (11). The deflection function in this case takes the form

$$W(\xi) = W_0 R_4(\beta_1\xi) + W_0' R_3(\beta_1\xi) + W_0'' R_2(\beta_1\xi) + W_0''' R_1(\beta_1\xi), \tag{16}$$

where $R_i\left(i=1\div 4\right)$ is Krylov function, that takes the form

$$R_1(\beta_1\xi) = \frac{sh\beta_1\xi - \sin\beta_1\xi}{2\beta_1\xi}$$
$$R_2(\beta_1\xi) = \frac{ch\beta_1\xi - \cos\beta_1\xi}{2\beta_1^2}$$
$$R_3(\beta_1\xi) = \frac{sh\beta_1\xi + \sin\beta_1\xi}{2\beta_1}$$
$$R_4(\beta_1\xi) = \frac{ch\beta_1\xi + \cos\beta_1\xi}{2}$$

The state of an arbitrary section of the beam is described by equation in the matrix form

$$\bar{W}(\xi) = V_1(\xi)\bar{W}_0, \tag{17}$$

where $\bar{W}(\xi)$ is vector of the state of an arbitrary section ξ

$$\bar{W}(\xi) = \left\{W(\xi)\ W'(\xi)\ W''(\xi)\ W'''(\xi)\right\};$$

$\bar{W}_0(\xi)$ is vector of initial parametrs

$$\bar{W}_0 = \left\{W_0\ W_0'\ W_0''\ W_0'''\right\};$$

$V_1(\xi) = \left\{\upsilon_{ij}\right\}$ – functional matrix, characterizes affecting of initial parameters to state of a section ξ

$$V_1(\xi) = \begin{pmatrix} R_4(\beta_1\xi) & R_3(\beta_1\xi) & R_2(\beta_1\xi) & R_1(\beta_1\xi) \\ \beta_1^4 R_1(\beta_1\xi) & R_4(\beta_1\xi) & R_3(\beta_1\xi) & R_2(\beta_1\xi) \\ \beta_1^4 R_2(\beta_1\xi) & \beta_1^4 R_1(\beta_1\xi) & R_4(\beta_1\xi) & R_3(\beta_1\xi) \\ \beta_1^4 R_3(\beta_1\xi) & \beta_1^4 R_2(\beta_1\xi) & \beta_1^4 R_1(\beta_1\xi) & R_4(\beta_1\xi) \end{pmatrix}.$$

3. Let us perform an analysis of the natural frequencies and modes of bending oscillations of the beam on an elastic foundation with hinged ends. In this case, the boundary conditions and the deflection function are

$$W_0 = W_0'' = 0$$
$$W(1) = W''(1) = 0. \tag{18}$$

$$W(\xi) = W_0' R_3(\beta_1\xi) + W_0''' R_1(\beta_1\xi). \tag{19}$$

Satisfying the second pair of boundary conditions (18), from function (19) and its second derivative, we obtain a system of algebraic equations for the unknown initial parameters W_0' and W_0'''

$$\begin{cases} W_0' R_3(\beta_1) + W_0''' R_1(\beta_1) = 0, \\ W_0' \beta_1^4 R_1(\beta_1) + W_0''' R_3(\beta_1) = 0. \end{cases} \tag{20}$$

The condition for the existence of nonzero solutions of a given homogeneous system is the equality to zero of the determinant of the coefficient matrix of this system

$$\begin{vmatrix} R_3(\beta_1) & R_1(\beta_1) \\ \beta_1^4 R_1(\beta_1) & R_3(\beta_1) \end{vmatrix} = 0. \tag{21}$$

Expanding the determinant, we obtain the frequency equation

$$4\sin\beta_1 sh\beta_1 = 0,$$

From where follows

$$\sin\beta_1 = 0$$

$$\beta_{1n} = n\pi \quad (n = 1, 2, 3, ...).$$

Taking in account the formula (10), we obtain frequencies spectrum

$$\tilde{\omega}_n = \sqrt{\bar{\omega}_0^2 + n^4}. \qquad (22)$$

From any equation of the system (20) when $\beta_n = n\pi$ follows

$$\frac{W_0'''}{W_0'} = -(n\pi)^2,$$

then, in accordance with (19), *n*-th mode with frequency ω_n, takes the form

$$W_n(\xi) = A_n \sin n\pi\xi, \qquad (23)$$

where n is a number of half-waves of sinusoid along the beam length *l*; A_n is unknown amplitude of oscillations for *n*-th mode.

Thus, the forms of natural oscillations of a beam on an elastic foundation remain the same as that of a free beam, but with frequencies ω_n, greater than the corresponding frequencies of a free beam $\omega_{св_n}$ in $\sqrt{\bar{\omega}_0^2 + n^4}$ times, i.e. in accordance with (22)

$$\omega_n = \sqrt{\bar{\omega}_0^2 + n^4}\,\omega_{св_n}.$$

4. The solution of the differential equation of forced oscillations (4) can be obtained by expanding the function $w_{дин}(\xi,\tau)$ in a series in eigenfunctions $W_n(\xi)$ (23) with coefficients in the form of unknown functions of time $Q_n(\tau)$

$$w_{дин} = \sum_{n=1}^{\infty} Q_n(\tau) W_n(\xi). \qquad (24)$$

The functions $Q_n(\tau)$ can be found, applying the following procedures: substituting series (24) and expression (7) into equation (4), multiplying both sides of this equation by $W_n(\xi)$, integrating both sides by ξ from 0 to 1 and, using the orthogonality property of the natural vibration forms $W_n(\xi)$, we obtain the differential equation for function $Q_n(\tau)$

$$\frac{d^2 Q_n}{d\tau^2} + \bar{\omega}_n^2 Q_n = S_n, \qquad (25)$$

where

$$S_n = \frac{2\bar{q}}{\pi^4 \bar{\omega}_0^2} \frac{\sin^2 \frac{n\pi}{2}}{\frac{n\pi}{2}}.$$

General solution of inhomogeneous equation (25)

$$Q_n = D_{1n} \cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + D_{2n} \sin\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + \frac{S_n}{\bar{\omega}_n^2} \qquad (26)$$

is a sum of relative homogeneous solution (first and second additives) and partial solution, corresponding to the right part of (25) (third additive).

Now, according to (24), the dynamic deflection function takes the form

$$w_{дин} = \sum_{n=1}^{\infty} \left(D_{1n} \cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + D_{2n} \sin\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + \frac{S_n}{\bar{\omega}_n^2} \right) \sin n\pi\xi. \qquad (27)$$

Integration constants D_{1n} and D_{2n} is determined from initial condition

$$w_{дин}(\xi,0) = w_{cm}(\xi)$$
$$\left.\frac{\partial w_{дин}}{\partial \tau}\right|_{\xi,0} = 0. \tag{28}$$

From the second condition (28) we define one constant

$$D_{2n} = 0. \tag{29}$$

Multiplying both sides of the first condition (28) in accordance with (27) and (29) to $\sin n\pi\xi$ and integrating by ξ from 0 to 1, we obtain another constant

$$D_{1n} = B_n - \frac{S_n}{\bar{\omega}_n^2}, \tag{30}$$

where

$$B_n = 2\int_0^1 w_{cm}(\xi)\sin n\pi\xi d\xi.$$

Substituting (29) and (30) into (27) and taking into account the trigonometric identity

$$1-\cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau = 2\sin^2\frac{\tilde{\omega}_n}{2\bar{\omega}_0}\tau,$$

we obtain

$$w_{дин} = \\ = \sum_{n=1}^{\infty}\left(B_n\cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + C_n\sin^2\frac{\tilde{\omega}_n}{2\bar{\omega}_0}\tau\right)\sin n\pi\xi, \tag{31}$$

where

$$C_n = \frac{4\bar{q}}{\pi^4\tilde{\omega}_n^2}\,\frac{\sin^2\frac{n\pi}{2}}{\frac{n\pi}{2}}.$$

The dimensionless bending moment is obtained by twice differentiating the series (31)

$$w''_{дин} = \\ = -\pi^2\sum_{n=1}^{\infty}n^2\left(B_n\cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + C_n\sin^2\frac{\tilde{\omega}_n}{2\bar{\omega}_0}\tau\right)\sin n\pi\xi \tag{32}$$

5. Numerical example.

Using the Maple software package, we calculated the dimensionless deflections $w(\xi)$ and bending moments $w''(\xi)$ in the beam loaded with a even distributed load of $\bar{q}=1$ intensity on an elastic Winkler foundation

– in the initial static state with clamping of its ends: $w_{cm}(\xi)$, $w''_{cm}(\xi)$;

– in the static state, formed after quasi-static transformation of clamping to hinges $w_{кв}(\xi)$, $w''_{кв}(\xi)$;

– in the dynamic process that occurs at a sudden transformation of clamping to hinges: $w_{дин}(\xi,\tau)$, $w''_{дин}(\xi)$.

In practical calculations, 20 members of the series (31) and (32) were taken into account. In this case, we obtain a practical coincidence of the diagrams of dynamic deflections $w_{дин}(\xi, 0)$ and static deflection $w_{cm}(\xi)$, that is

$$\sum_{n=1}^{20} B_n \sin n\pi\xi \approx w_{cm}(\xi).$$

The calculation results are shown in figures 3 and 4, as well as in table 1. In figures 3 and 4 are shown respectively: diagrams of bending moments $w''_{кв}(\xi)$ in the beam after quasi-static transformation of clamped points into hinges and during oscillations $w''_{дин}(\xi,\tau_0)$ after sudden transformation of clamped points into hinges at the moment τ_0 of reaching the highest values.

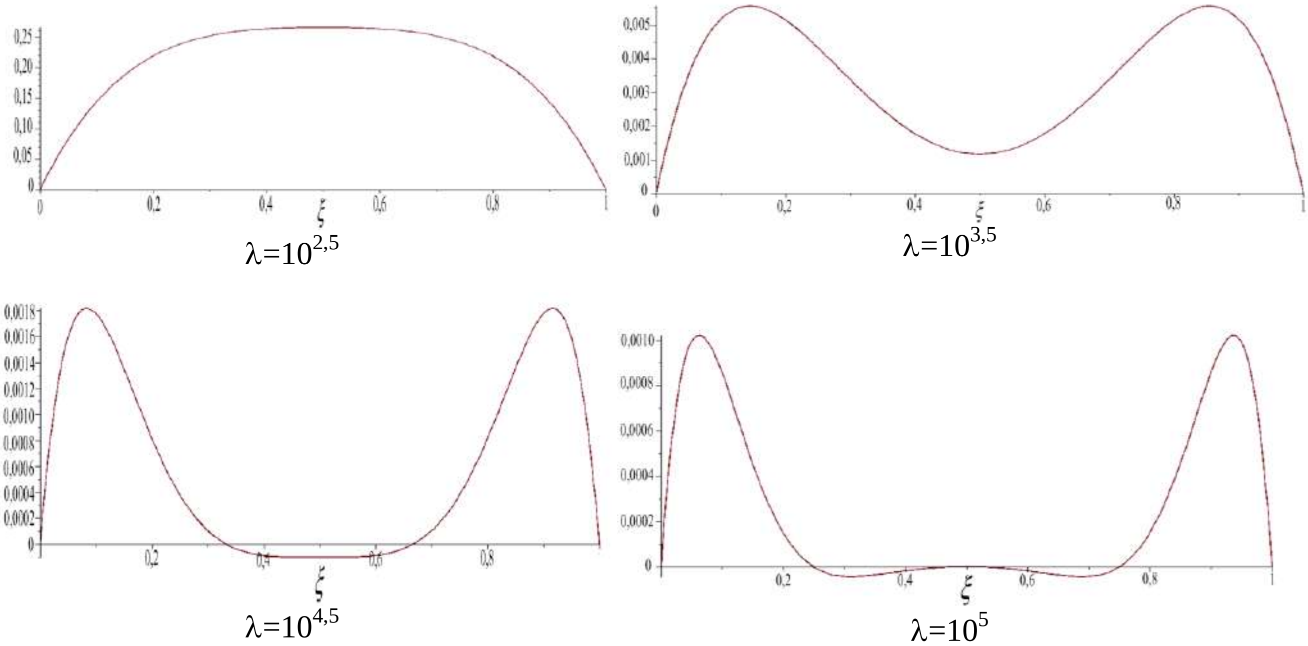


Figure 3. Diagrams of bending moments after quasistatic transformation of the boundary conditions depending on the stiffness indicator of the "beam-foundation" system λ.

Table 1. Affecting of stiffness of the "beam – foundation" system to the increment of bending moments.

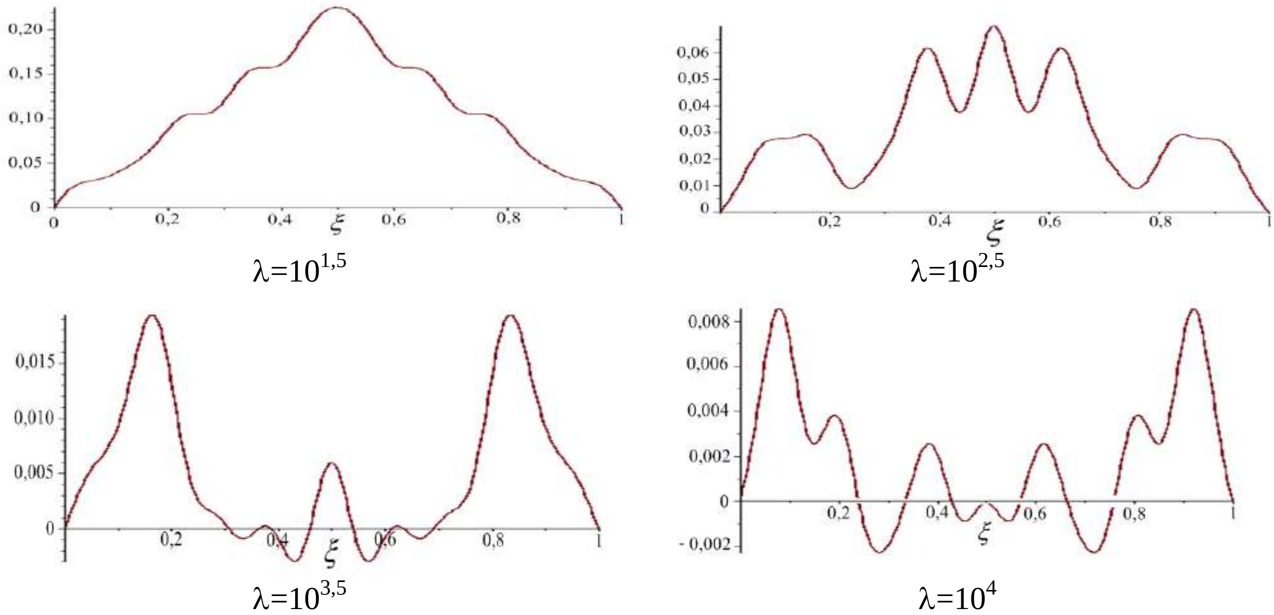
λ	$\left\lvert w''_{\substack{ст.\\ \max}} \right\rvert$	$w''_{\substack{кв.\\ \max}}$	$K_{ст}$	$w''_{\substack{дин.\\ \max}}$	$K_{дин}$
0	0.083	0.125	1.506	0.3	3.614
10	0.082	0.112	1.366	0.269	3.28
$10^{1.5}$	0.079	0.093	1.177	0.225	2.848
10^{2}	0.071	0.06	0.845	1.156	2.197
$10^{2.5}$	0.0544	0.0265	0.487	0.07	1.287
10^{3}	0.0334	0.0104	0.311	0.0346	1.036
$10^{3.5}$	0.018	0.0055	0.305	0.0186	1.033
10^{4}	0.01	0.003	0.3	0.01	1

Diagrams are constructed for different values of the stiffness parameter of the "beam-foundation" system λ.

Table 1 contains the values of the largest bending moments (dimensionless stresses) in three states $w''_{\substack{ст.\\ \max}}$, $w''_{\substack{кв.\\ \max}}$, $w''_{\substack{дин.\\ \max}}$ with different parameters of the stiffness of the "beam-foundation" system λ, as well as the coefficients

$$K_{ст} = \frac{w''_{\substack{кв.\\ \max}}}{w''_{\substack{ст.\\ \max}}}; \quad K_{дин} = \frac{w''_{\substack{дин.\\ \max}}}{w''_{\substack{ст.\\ \max}}},$$

characterizing increasing (decreasing) order of the maximum bending moment at quasi-static ($K_{ст}$) and dynamic ($K_{дин}$) changing of boundary conditions.



<u>Figure 4.</u> Bending moment diagrams after a sudden transformation of the boundary conditions at the time of reaching the highest values depending on the rigidity indicator of "beam – foundation" systems λ.

CONCLUSION

If we consider the transformation of the boundary conditions in this "beam-foundation" system under load caused by a damage, then the provided study shows that quasi-static defect formation, that is, a decrease in the stiffness of the end supports, leads to an insignificant increase in the stresses in the beam ($K_{cm} > 1$) when there is not foundation ($\lambda = 0$) and low indicator values of the "beam-foundation" system ($0 < \lambda \leq 10^{1,79}$). For beams based on more rigid bases ($\lambda > 10^{1,79}$), the formation of the same defect, on the contrary, leads to a decrease in the greatest stresses ($K_{cm} < 1$).

The sudden formation of a defect gives a more than threefold ($K_{дин} = 3{,}614$) increase in the greatest stress in the free beam ($\lambda = 0$). For systems with higher stiffness, the effect of transforming the boundary conditions is reduced. There is a redistribution of stresses along the span, but the greatest stress at $\lambda > 10^4$ does not exceed the value of the initial static one($K_{дин} = 1$). In addition, regardless of the speed of formation of a defect, with the increase in the rigidity of the system, the greatest stresses move from the center of the beam to the periphery of the span.

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