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APPLICATION OF UNCERTAINTY ANALYSIS TO STABILITY PROBLEMS OF STEEL-CONCRETE STRUCTURAL MEMBERS

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Abstract. The paper deals with the fuzzy analysis of the ultimate limit state of a steel strut with an encased web in compression. The first part of the paper lists presumptions required for the determination of the theoretical load carrying capacity for the column. Stresses in the concrete and steel sections are determined according to the principles of elasticity. The ultimate limit state is given as the limit stress attained in the most stressed section of either the steel or concrete section. A general extended principle, which takes into account the epistemic uncertainty of input parameters, was utilized for the conducted analysis.

Keywords: steel, concrete, structure, design, reliability, random, fuzzy, imperfection.

1. Introduction

Concrete and steel are materials generally used in the building industry. Structures from high grade steel and concrete are more frequently used in the field of designing modern engineering structures. Steel is a high quality building material, its highest merit being its high strength. Concrete carries compressive stresses and protects tensile steel from corrosion and high temperatures. The material combination of steel-concrete brings about the occurrence of special phenomena the influence of which on structural reliability is not, at present, commonly implemented in design.

Generally, engineering procedures employed in the design of the load-bearing structures of building objects are essentially based on computational procedures that verify valid normative criteria emanating from the static, respectively the dynamic solution of a model of the real structural system, see e.g. (Ravinger and Psotný 2006), (Karkauskas and Norkus 2006). At present, according to limit state methods, i.e. the ultimate limit state and serviceability limit state. The basic problem lies in the determination of initial imperfections and in the subsequent analysis of their influence on limit states and other monitored structural properties.

The implementation and degree of the applicability of probabilistic methods for the estimation of structural reliability in design practice still remains the topic of discussion. The estimation of structural reliability through the use of probabilistic methods requires ample exclusively objective information gained during experiments. The feasibility and application limits of stochastic computational models involve both aleatory and epistemic uncertainties. The quality of reliability assessment is then given first and foremost by the manner with which the reliability method deals with the before mentioned uncertainty.

2. Theoretical Model

The subject of analysis is the ultimate limit state of a steel-concrete column of system length equal to its critical length, $L = L_{cr} = 3$ m, in compression. The column consists of steel profile HEA140 encased with high strength concrete, see Fig. 1.

The load F acting on the column consists of load F_S carried by the steel section and load F_C carried by the concrete section, i.e. $F = F_S + F_C$. Let us assume that the strut is produced in the shape affine to eventual buckling with deflection at mid length denoted as e_0.

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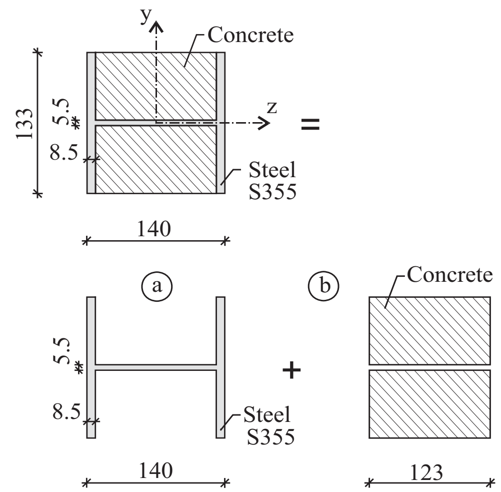

Fig. 1. Steel-concrete cross section

The maximum deflection mid-span of strut *e*, which is loaded by axial force *F* in its elastic state, may be determined according to (Timoshenko 1961) as:

$$e = \frac{e_0}{1 - \dfrac{F}{F_{cr}}} \quad (1)$$

where F is the load acting on the column and F_{cr} is Euler's critical force $F_{cr} = \pi^2 EI / L_{cr}^2$. In accordance with article 6.7.3.1 (3) of standard EN 1994-1-1: 2006, the effective elastic flexural rigidity EI of the steel-concrete column which is given according to the formula listed below may be used for short term loading.

$$EI = E_S \cdot I_S + K_E \cdot E_C \cdot I_C, \quad (2)$$

where I_S and I_C are the second moments of the area in the plane of bending structural steel and concrete (without consideration to cracking),

E_S is the modulus of steel elasticity,

E_C is the tangent modulus of the elasticity of concrete, $K_E \cdot E_C \cdot I_C$ is the effective flexural rigidity of the concrete section.

$E_S \cdot I_S$ is the effective flexural rigidity of the steel section.

The values of forces F_C and F_S can be obtained from the condition that deflection at strut mid span e equals the deflection of steel section e_S which is equal to the deflection e_C of the concrete section, i.e. $e=e_S=e_C$.

$$e = \frac{e_0}{1 - \dfrac{F}{F_{cr}}} = \frac{e_0}{1 - \dfrac{F_S}{F_{cr,S}}} = \frac{e_0}{1 - \dfrac{F_C}{F_{cr,C}}}, \quad (3)$$

where $F_{cr,S}$ is Euler's critical force of the steel section $F_{cr,S} = \pi^2 E_S I_S / L_{cr}^2$, $F_{cr,C}$ is Euler's critical force of the concrete section $F_{cr,C} = \pi^2 \cdot K_E \cdot E_C I_C / L_{cr}^2$. The load carried by the steel section F_S and the load carried by the concrete section F_C with parameter K_E can be determined from the above listed mathematical dependencies:

$$F_S = F \frac{E_S \cdot I_S}{EI} \quad (4)$$

$$F_C = F \cdot K_E \frac{E_C \cdot I_C}{EI} \quad (5)$$

Stresses in steel and concrete sections are determined according to the principles of elasticity. The load-carrying capacity of the steel member is given by yield strength attained in the most stressed section. The load-carrying capacity of the concrete section is given as cubic strength in the most compressed section or as 10 % of cubic strength in the most tensioned part of the section. The load-carrying capacity of the steel-concrete column is given as the minimum of the above listed values.

3. Input Quantities

The value of initial imperfection e_0 was considered according to the tolerance limits of standard EN 10034 (1995). The steel grade S355 and concrete grade C50/60 were used. The modulus of the elasticity of concrete was considered in compliance with standard Eurocode 2 (2006) according to the formula:

$$E_C = 22 \cdot \left(\frac{5}{6} \cdot \frac{f_{cc}}{10} \right)^{0.3} \cdot \Theta_{E_C}, \quad (6)$$

where:

E_C – the tangent modulus of elasticity [GPa],

f_{cc} – cubic strength [MPa],

Θ_{E_C} – the non-dimensional coefficient of the (i) influence of concrete constituents (namely aggregates) and (ii) the variance of cylindrical and cubic strength.

We have incomplete information at our disposal without corresponding to the mathematical description of parameters K_E and Θ_{E_C}.

Table 1. Input random quantities

Quantity			Characteristic value
Steel	Cross-s. height	h	133 mm
	Flange width	b	140 mm
	Web thickness	t_1	5.5 mm
	Flange thick.	t_2	8.5 mm
	Yield strength	f_y	355 MPa
	Young's m.	E_S	210 GPa
Conc.	Cubic strength	f_{cc}	60 MPa
	Elasticity coef.	Θ_{Ec}	Fuzzy number
Imperfection		e_0	4.5 mm
Coefficient K_E		K_E	Fuzzy number

The characteristic value 355 MPa (5 % quantil) of yield strength was used. The experimentally obtained values of the yield strength of profiles HEA 140 of the compressed members occurred within the interval from 436 MPa to 473 MPa. The results were obtained from sixteen samples taken from the flanges of two members of the same test sets. It is desirable to repeat experiments in the future for a higher number of members.

4. Fuzzy Analysis of Load-Carrying Capacity

4.1. Fuzzy Set Theory

The term 'fuzzy' was first proposed by prof. Lotfi A. Zadeh in 1962 in (Zadeh 1962). In 1965, L. Zadeh published his famous paper (Zadeh 1965) where he formulated the foundation of this theory. The fuzzy set theory proved very useful for the problems the input parameters of which are imprecise or ill-defined and often represented by linguistic expressions. The theory of fuzzy sets, or fuzzy logic, has been developed to replace simple models, which could not handle more complex problems, with a practical instrument capable solving recent more robust and complex problems involving human aspects (Štemberk 2000). Furthermore, it helps decision-making and control systems not only to find optimal solution under given constraints but also can develop new alternatives.

Traditionally, the probability theory has been used to handle this incomplete information and uncertainty. The probability theory is certainly a good approach to process knowledge and information the boundaries of which are clearly defined. For example, throwing coins or dice represents the case in which the possible results are known in advance and the boundaries are distinctly set. The result of throwing coin cannot be half-head and half-tail; a dice cannot show 1.5 or 5.9. But there are cases in which the boundaries are not precisely marked. The fuzzy set theory is developed to define and solve problems without sharp/crisp boundaries, i.e. it considers partial membership. Fuzzy sets have already been used in a large field of applications (Kala and Omishore 2006), (Kala 2007), (Štemberk and Kalafutová 2008), (Tanyildizi 2007), (Unal et al 2007), (Ross 1995), (Akkurt et al 2004), (Ferracuti et al 2005).

4.2. The Principal of General Extension

One of the most important principles enabling the transformation of any arbitrary operation in the classic set into operation in the fuzzy set is that of the principle of general extension; see e.g. (Dubois 1980). For fuzzy numbers K_E and $\Theta_{E_C} \subseteq$ R, the principle may be written as: Let K_E, Θ_{E_C} be convex fuzzy numbers and f: $y=f(K_E, \Theta_{E_C})$ be a given binary function. The degree of the membership μ_y of fuzzy number y may then be obtained according to relation (7).

$$\mu_y(K_E, \Theta_{E_C}) = \bigvee_{K_E, \Theta_{E_C} / f(K_E, \Theta_{E_C}) = y} \left(\mu_1(K_E) \wedge \mu_2(\Theta_{E_C}) \right) \quad (7)$$

The obtained result is fuzzy number y containing elements with a degree of membership μ_f given as the supremum (maximum) from the minima (K_E, Θ_{E_C}) for all K_E, Θ_{E_C}, for which $f(K_E, \Theta_{E_C})=y$.

Three basic operations are introduced for fuzzy sets – intersection, union and supplement (a number of other operations are sometimes listed, e.g. limited sum, limited difference, power, probabilistic sum, Lukasiewic's operations of intersection and union).

Relation (7) is however quite inadequate for direct evaluation. Due to this fact, basic fuzzy arithmetic was worked out for simple problems in which the response of the system (structure) may be expressed by a polynomial function. Fuzzy analysis in the form of the principle of general extension (similarly as stochastic methods) have some limitations in more complex problems due to the need to perform a high number of combinations of input data. A solution for this problem is provided through the approximation of the structural response, the so-called surface response function in the simplest possible form. This makes fuzzy analysis a very strong tool utilizable whenever provision for uncertainty, which is not of stochastic character, is needed.

4.3. Principle of Incompatibility

The contradiction between precision and relevance was first pointed out by L. A. Zadeh (Zadeh 1973) in 1973 when he formulated the principle of incompatibility. As the complexity of the system increases, our ability to formulate precise and yet significant judgments about its behaviour decreases until a threshold is reached, beyond which precision and relevance become practically mutually exclusive characteristics.

In this context, the precision of the concurrent standard EUROCODE is in sufficient concord with the accuracy that is to be attained. The ability of self-assignment, robustness and easy implementation is supported at the expense of precision. Ample exclusively objective information from experiments on the input random variables is generally absent for the utilization of more accurate stochastic methods. At present, it is therefore topical to strive for the refinement of procedures for the verification of reliability and improvement of the quality of manufactured steel structures.

4.4. Fuzzification

The membership functions defined on the input variables are applied to their actual values to determine the degree of truth for each rule premise at this stage. No relevant information is available for parameter the K_E because it cannot be determined from measurement on a higher number of steel-concrete columns. The parameter K_E is given in EN 1994-1-1: 2006 by $K_E = 0.6$. No relevant information is also available for the parameter Θ_{E_C}. For fuzzy analysis, we shall consider that parameters K_E, Θ_{E_C} have fuzzy numbers with symmetrical triangular (linear) membership functions, see Fig. 2 and 3.

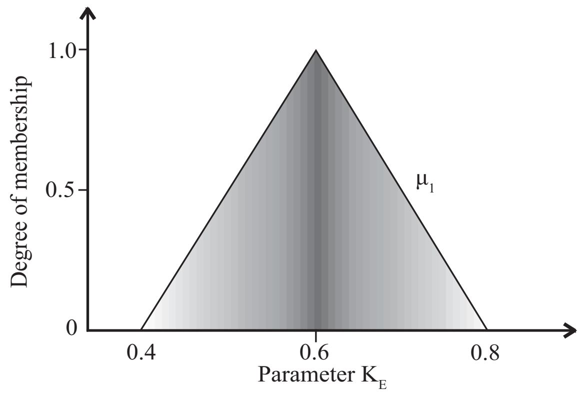

Fig. 2. The fuzzy number of parameter K_E

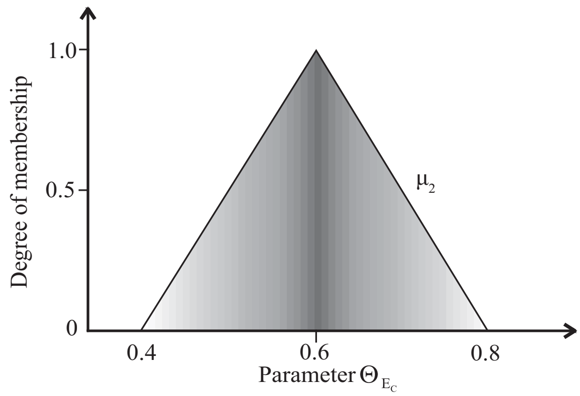

Fig. 3. The fuzzy number of parameter Θ_{E_C}

It is apparent from Fig. 4 that the output membership function is slightly nonlinear despite linear input membership functions (see Fig. 2 and Fig. 3). The membership function in the obtained load-carrying capacity point out to the need for further studies in calibration..

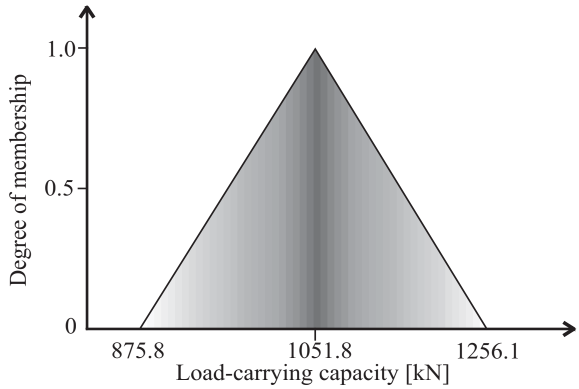

Fig. 4. The fuzzy number of load carrying capacity

4.5. Fuzzy analysis

The fuzzy number of load-carrying capacity was determined using the principle of general extension (7), see Fig. 4. The membership function has an asymmetrical form and the support of the fuzzy number is in the interval 875.8 kN to 1256.1 kN.

5. Conclusions

The study clearly determines the fuzzy uncertainty of standard code requirements. The results of fuzzy analysis quantify the dependence of load-carrying capacity on the change of coefficients K_E, Θ_{E_C} . The output low asymmetric and low non –linear membership function vs. triangular symmetric membership functions of coefficients K_E, Θ_{E_C} is obtained. This information is very valuable because it quantifies the value of non –linear dependence between the coefficients of model uncertainties K_E, Θ_{E_C} and theoretical load-carrying capacity.

The influence of residual stresses was not taken into consideration in the analysis presented in the article. For a more detailed description of the influence of this structural deviation, it would be necessary to model the elements using shell finite elements and employ the geometrical nonlinear solution for the evaluation of load carrying capacity.

At present, the basic method for the evaluation and verification of limit states according to EUROCODE is the partial method of safety factors. In this regard, the elaboration of uncertainty studies from the perspectives of transparency and the verification of procedures implemented in practice are topical (Kala 2007), (Vičan and Koteš 2003), (Koteš and Vičan 2005). The fuzzy, probabilistic, and fuzzy-probabilistic assessments of reliability enable the comparison, generalization and further improvement of normative procedures.

In the future, it is necessary to continue the theoretical studies of the reliability of bearing elements and structures. Limitation to purely probabilistic methods may be misleading. In order to make provision for all types of uncertainty encountered in design, it is necessary to pay more attention to constantly more frequently employed and mathematically well worked out alternate approaches to representing uncertainty based on the theory of fuzzy sets, the theory of possibilities and the Dempster-Shafer theory in addition to traditional probability methods (Möller *et al.* 2005), (Möller and Reuter 2007).

It is also necessary to focus our attention to other occurring cases of the strain of individual members, namely thin walled structures (Kotełko and Kołakowski 2000), (Kotełko *et al.* 2007), (Kotełko *et al.* 2008) and also to the cases of more complex systems comprised of more struts. Further research will deal with the sensitivity analysis of columns and structures with two, three columns etc. Other factors that influence the behaviour of steel-concrete members could be discussed (Gribniak 2008). Interaction effects in the mathematical models of structure optimisation may be important from the point of view of reliability and attention is to be drawn to those in safety and serviceability assessments of all types of building structures (Karkauskas and Norkus 2001), (Karkauskas 2007).

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NEAPIBRĖŽTUMO ANALIZĖS TAIKYMAS PLIENBETONIO KONSTRUKCINIŲ ELEMENTŲ STABILUMO UŽDAVINIUOSE

L. Puklický, Z. Kala

Santrauka. Straipsnyje nagrinėjamas plienbetonio konstrukcinių elementų stipris remiantis *fuzzy* analize. Pirmoje straipsnio dalyje aprašomos prielaidos, reikalingos teoriniam kolonos laikomosios galios nustatymui. Įtempiai plieniniuose ir betoniniuose skerspjūviuose nustatomi pagal tamprumo teorijos principus. Ribinis pavojingiausio pjūvio būvis pasiekiamas tada, kai įtempiai pasiekia ribinių įtempių reikšmes plieno apvalkale arba betono užpilde. Analizei buvo pasitelktas išplėstinis principas, kuris įvertina pradinių parametrų „epistematinį" neapibrėžtumą.

Reikšminiai žodžiai: plienas, betonas, konstrukcija, projektavimas, tvirtumas, *fuzzy*, yda.

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