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ASSOCIATED (SEMI)HYPERGROUPS FROM DUPLEXES

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ABSTRACT. In this paper using strongly duplexes we introduce a new class of (semi)hypergroups. The associated (semi)hypergroup from a strongly duplex is called duplex (semi)hypergroup. Two computer programs written in MATLAB show that the two groups $\mathbb{Z}_{2n}$ and $\mathbb{Z}_n \times \mathbb{Z}_2$ produce a strongly duplex and its associated hypergroup is a complementary feasible hypergroup.

1. Introduction

Hyperstructure theory was founded in 1934 at the 8th congress of Scandinavian Mathematicians. Marty [11] introduced the hypergroup notion as a generalization of groups and then he proved its utility in solving some problems of groups, algebraic functions and rational fractions. Surveys of the theory can be found in the books of Corsini [2], Davvaz and Leoreanu-Fotea [7], Corsini and Leoreanu [4] and Vougiouklis [15, 16]. Now this field of modern algebra is widely studied from the theoretical and applied viewpoints because of their applications to many subjects of pure and applied mathematics. Some applications can be used in the following areas: geometry, graphs, fuzzy sets, cryptography, automata, lattices, binary relations, codes, and artificial intelligence, see for example [1, 3, 5, 6, 8, 9, 12].
Jean-Louis Loday introduced the notion of dimonoid [10]. Dimonoids are a tool to study Leibniz algebras. A dimonoid is a set equipped with

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two binary associative operations satisfying some axioms. If the operations of a dimonoid coincide, then the dimonoid becomes a semigroup. T. Pirashvili [13] introduced the notion of a duplex which generalizes the notion of a dimonoid. In this paper we make a connection between duplexes and hyperstructures. We introduce strongly duplexes and we define and study a new class of semihypergroups. Two computer programs written in MATLAB help us to see that the two groups $\mathbb{Z}_{2n}$ (cyclic group of order $2n$) and $\mathbb{Z}_n \times \mathbb{Z}_2$ produce a strongly duplex and its associated hypergroup is a complementary feasible hypergroup.

2. Preliminaries

Let us briefly recall some basic notions and results about groups and hypergroups; for a comprehensive overview of this subject, the reader is refereed to [2, 4, 7, 14].

For a non-empty set H and a natural number n, we denote by $\mathcal{P}^*(H)$ the set of all non-empty subsets of H and $H^n = H \times H \times ... \times H$.

Definition 2.1. A non-empty set H, endowed with a mapping called *hyperoperation*, $\bullet : H^2 \longrightarrow \mathcal{P}^*(H)$ is named *hypergroupoid*. A hypergroupoid which verifies the following conditions:

(i): $(x \bullet y) \bullet z = x \bullet (y \bullet z)$, for all $x, y, z \in H$ (the *associativity*)
(ii): $x \bullet H = H = H \bullet x$, for all $x \in H$ (the *reproduction axiom*)

is called *hypergroup*. In particular, an associative hypergroupoid is called a *semihypergroup* and a hypergroupoid that verifies the reproduction axiom is called a *quasihypergroup*.
If A and B are non-empty subsets of H, then $A \bullet B = \cup_{b \in B}^{a \in A} a \bullet b$.

Definition 2.2. A *duplex* is a set D equipped with two associative operations $\cdot : D^2 \to D$ and $\circ : D^2 \to D$. We will denote by $(D, \cdot, \circ)$ the set duplex with two associative operations $(\cdot)$ and $(\circ)$. Moreover, a map $f : D \to D'$ from a duplex D to another duplex D' is a homomorphism, provided that $f(x \cdot y) = f(x) \cdot f(y)$ and $f(x \circ y) = f(x) \circ f(y)$.

Definition 2.3. A duplex $(D, \cdot, \circ)$ is called *commutative* if two operations $(\cdot)$ and $(\circ)$ are commutative.

Definition 2.4. Let $(D, \cdot, \circ)$ be a duplex. *A strong duplex* is a type of duplex that follows the condition:

$$\{(a \cdot b) \circ c,\ (a \circ b) \cdot c\} = \{a \circ (b \cdot c),\ a \cdot (b \circ c)\},$$

for all $(a, b, c) \in D^3$.

3. (Semi)hypergroups derived from strongly duplexes

Let m be a natural number and $(\mathbb{Z}_m, +)$ be the cyclic group of the set of residue classes modulo m. Moreover, suppose x is a real number and $[x]$ is the integer part of x. Then we have the following.

Theorem 3.1. *Let n be a positive integer and $H_n = \{0, 1, \cdots, 2n-1\}$ be the set of natural numbers less than $2n$. The pair $(H_n, \oplus)$ is a group which is isomorphic to $(\mathbb{Z}_{2n}, +)$, where $\oplus$ is defined by:*

$$x \oplus y = x + y - 2n[(x+y)/2n], \quad \forall (x, y) \in H_n^2.$$

Proof. Let $f : H_n \to \mathbb{Z}_{2n}$ by $x \mapsto \overline{x}$ be a map. It is obvious that f is onto and one-to-one. We shall show that f is homomorphism. There are two cases:

(1) If $x + y < 2n$ then $[(x + y)/2n] = 0$ thus we have: $f(x \oplus y) = f(x + y - 2n[(x + y)/2n]) = f(x + y) = \overline{x+y} = \overline{x} + \overline{y} = f(x) + f(y)$.

(2) If $x + y \geq 2n$ then $[(x + y)/2n] = 1$ hence: $f(x \oplus y) = f(x + y - 2n[(x + y)/2n]) = f(x + y - 2n) = \overline{x + y - 2n} = \overline{x} + \overline{y} + \overline{2n} = \overline{x} + \overline{y} = f(x) + f(y)$.

Therefore $(H_n, \oplus)$ is a group isomorphic to $(\mathbb{Z}_{2n}, +)$. Symbolically, $(H_n, \oplus) \cong (\mathbb{Z}_{2n}, +)$. □

Example 3.2. Let $H_4 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set. The group $(H_4, \oplus)$ can be shown as follows:

$\oplus$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

Hence $(H_4, \oplus) \cong (\mathbb{Z}_8, +)$.

Theorem 3.3. *Let n be a natural number and $H_n = \{0, 1, \cdots, 2n-1\}$. We define an operation $\otimes$ on H_n by:*

$$x \otimes y = \begin{cases} x + y - n[(x+y)/n], & \text{if } 0 \leq x < n, \quad 0 \leq y < n \\ x + y - n[(x+y)/n] + n, & \text{if } 0 \leq x < n, \quad n \leq y < 2n \\ x + y - n[(x+y)/n], & \text{if } n \leq x < 2n, \quad n \leq y < 2n \\ x + y - n[(x+y)/n] + n, & \text{if } n \leq x < 2n, \quad 0 \leq y < n \end{cases}$$

Then $(H_n, \otimes)$ *is a group and* $(H_n, \otimes) \cong (\mathbb{Z}_2 \times \mathbb{Z}_n, +)$.

Proof. Suppose that $\mathbb{Z}_2 \times \mathbb{Z}_n = \{(\overline{x}, \overline{y}) | x \in \mathbb{Z}_2,\ y \in \mathbb{Z}_n\}$. Now consider the map

$$g : H_n \longrightarrow \mathbb{Z}_2 \times \mathbb{Z}_n$$

$$g(x) = \begin{cases} (\bar{0}, \overline{x}) & \text{if } 0 \le x < n \\ (\bar{1}, \overline{x}) & \text{if } n \le x < 2n \end{cases}$$

We prove that $g(x \otimes y) = g(x) + g(y)$. To end this first suppose that $0 \le x < n,\ 0 \le y < n$ so $0 \le x + y < 2n$.

(1) If $0 \le x + y < n$, then $[(x + y)/n] = 0$, thus $g(x \otimes y) = g(x+y-n[(x+y)/n]) = g(x+y) = (\bar{0}, \overline{x + y}) = (\bar{0}, \overline{x}) + (\bar{0}, \overline{y}) = g(x) + g(y)$.
(2) If $n \le x + y < 2n$ then $[(x + y)/n] = 1$, and so $g(x \otimes y) = g(x + y - n[(x + y)/n]) = g(x + y - n) = (\bar{0}, \overline{x + y - n}) = (\bar{0}, \overline{x}) + (\bar{0}, \overline{y}) = g(x) + g(y)$.

Now let $0 \le x < n,\ n \le y < 2n$. Then we have two cases:

(3) If $n \le x + y < 2n$, then $[(x + y)/n] = 1$ so we have $g(x \otimes y) = g(x + y - n[(x + y)/n] + n) = g(x + y - n + n) = g(x + y) = (\bar{1}, \overline{x + y}) = (\bar{0}, \overline{x}) + (\bar{1}, \overline{y}) = g(x) + g(y)$.
(4) If $2n \le x + y < 3n$, then $[(x + y)/n] = 2$, hence $g(x \otimes y) = g(x+y-n[(x+y)/n]+n) = g(x+y-2n+n) = g(x+y-n) = (1, \overline{x + y}) = (\bar{0}, \overline{x}) + (1, \overline{y}) = g(x) + g(y)$.

Consider $n \le x < 2n,\ n \le y < 2n$.

(5) If $2n \le x + y < 3n$, then $[(x + y)/n] = 2$, thus $g(x \otimes y) = g(x + y - n[(x + y)/n]) = g(x + y - 2n) = (\bar{0}, \overline{x + y - 2n}) = (\bar{1}, \overline{x}) + (\bar{1}, \overline{y}) = g(x) + g(y)$.
(6) If $3n \le x + y < 4n$, then $[(x + y)/n] = 3$, hence:
$g(x \otimes y) = g(x+y-n[(x+y)/n]) = g(x+y-3n) = (0, \overline{x + y - 3n}) = (\bar{1}, \overline{x}) + (\bar{1}, \overline{y}) = g(x) + g(y)$.

Finally, if $n \le x < 2n,\ 0 \le y < n$, the proof is similar above. Therefore $(H_n, \otimes) \cong (\mathbb{Z}_2 \times \mathbb{Z}_n, +)$ and the proof is completed. $\square$

Example 3.4. Let $H_4 = \{0, 1, 2, 3, 4, 5, 6, 7\}$. We can show the Cayley table of the group $(H_4, \otimes)$ as follow:

$\otimes$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	0	5	6	7	4
2	2	3	0	1	6	7	4	5
3	3	0	1	2	7	4	5	6
4	4	5	6	7	0	1	2	3
5	5	6	7	4	1	2	3	0
6	6	7	4	5	2	3	0	1
7	7	4	5	6	3	0	1	2

In this case we have $(H_4, \otimes) \cong (\mathbb{Z}_2 \times \mathbb{Z}_n, +)$.

For a given n the following program written in MATLAB, shows that the duplex $(H_n, \oplus, \otimes)$, where two hyperoperations $\oplus$ and $\otimes$ are introduced in Theorems (3.1) and (3.3) is a strongly duplex. We could check that our claim is true for the natural numbers less than 230; of course we guess that $(H_n, \oplus, \otimes)$ is a strongly duplex for all natural numbers. The program works in this way which first for a given n as the input of $(H_n, \oplus)$, $(H_n, \otimes)$ group's tables entitled a and b which every one is isomorphic to $(\mathbb{Z}_{2n}, +)$ and $(\mathbb{Z}_2 \times \mathbb{Z}_n, +)$ groups, respectively. Then we consider table k with 7 columns that the first row shows x, the second y and the third z. In $4th$ to $7th$ rows which are related to x, y and z, equations $(x \oplus y) \otimes z$, $(x \otimes y) \oplus z$, $x \oplus (y \otimes z)$ and $x \otimes (y \oplus z)$ are obtained and finally the associativity of $(H_n, \oplus, \otimes)$ are checked.

```
n=input('please inter n=');
a=zeros(2*n);
for x=0:2*n-1
   for y=0:2*n-1
      a(x+1,y+1)=x+y-2*n*(floor((x+y)/(2*n)));
   end
end
a
b=zeros(2*n);
for x=0:n-1
   for y=0:n-1
      b(x+1,y+1)=x+y-n*(floor((x+y)/n));
   end
end
for x=n:2*n-1
```

```
    for y=0:n-1
        b(x+1,y+1)=x+y-n*(floor((x+y)/n))+n;
    end
end
for x=n:2*n-1
    for y=n:2*n-1
        b(x+1,y+1)=x+y-n*(floor((x+y)/n));
    end
end
for x=0:n-1
    for y=n:2*n-1
        b(x+1,y+1)=x+y-n*(floor((x+y)/n))+n;
    end
end
b
f=(2*n)^3;
k=zeros(f,7);
i=1;
for x=0:(2*n)-1
    for y=0:(2*n)-1
        for z=0:(2*n)-1
            d1=a(x+1,y+1);
            c1=b(1+d1,z+1);
            d2=b(x+1,y+1);
            c2=a(1+d2,z+1);
            d3=b(y+1,z+1);
            c3=a(x+1,1+d3);
            d4=a(y+1,z+1);
            c4=b(x+1,1+d4);
            k(i,1)=x;
            k(i,2)=y;
            k(i,3)=z;
            k(i,4)=c1;
            k(i,5)=c2;
            k(i,6)=c3;
            k(i,7)=c4;
            i=i+1;
        end
    end
end
k
```

```
ans1 = 1;
for i=1:f
   if (k(i,4) == k(i,5) == k(i,6) == k(i,7))
   elseif (k(i,4) == k(i,6) , k(i,5) == k(i,7))
   elseif (k(i,4) == k(i,7) , k(i,5) == k(i,6))
   else
      ans1 = 0;
      break;
   end
end
if ans1 == 1
      disp('the duplex (Hn,·,∘) is strong duplex')
else
      disp('the duplex (Hn,·,∘) is not strong duplex')
end
```

Proposition 3.5. *Let* $(D, \cdot, \circ)$ *be a strong duplex. Then* $(D, \odot)$ *is a semihypergroup, where the hyperoperation* $\odot$ *is defined on* D *as follows:*

$$x \odot y = \{x \cdot y,\ x \circ y\}, \quad \textit{for all } (x,\ y) \in D^2.$$

Proof.

$$\begin{aligned}(x \odot y) \odot z &= \cup_{t \in x \odot y} t \odot z \\ &= (x \cdot y) \odot z \cup (x \circ y) \odot z \\ &= \{(x \cdot y) \cdot z, (x \cdot y) \circ z, (x \circ y) \cdot z, (x \circ y) \circ z\}.\end{aligned}$$

On the other hand

$$\begin{aligned}x \odot (y \odot z) &= \cup_{t \in y \odot z} x \odot t \\ &= x \odot (y \cdot z) \cup x \odot (y \circ z) \\ &= \{x \cdot (y \cdot z), x \circ (y \cdot z), x \cdot (y \circ z), x \circ (y \circ z)\}.\end{aligned}$$

Since $(D, \cdot, \circ)$ is a strong duplex, we have:

$$\{(x \circ y) \cdot z, (x \cdot y) \circ z\} = \{x \circ (y \cdot z), x \cdot (y \circ z)\}.$$

Therefore $(x \odot y) \odot z = x \odot (y \odot z)$. □

The associated semihypergroup $(D, \odot)$ is called *duplex semihypergroup.*

Example 3.6. $(H_4, \odot)$ is a hypergroup, where $\odot$ is defined as follows:

$\odot$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	0, 4	5	6	7	0, 4
2	2	3	0, 4	1, 5	6	7	0, 4	1, 5
3	3	0, 4	1, 5	2, 6	7	0, 4	1, 5	2, 6
4	4	5	6	7	0	1	2	3
5	5	6	7	0, 4	1	2	3	0, 4
6	6	7	0, 4	1, 5	2	3	0, 4	1, 5
7	7	0, 4	1, 5	2, 6	3	0, 4	1, 5	2, 6

Proposition 3.7. *The duplex semihypergroup* $(D, \odot)$ *is commutative if and only if* $(D, \cdot, \circ)$ *is commutative duplex.*

Example 3.8. For every natural number n, $(H_n, \oplus, \otimes)$ is a commutative duplex.

Definition 3.9. Let $*$ be a hyperoperation on H such that $x * y \neq H$, for all $(x, y) \in H^2$. We define the hyperoperation $*^c$ on H as follows:

$$x *^c y = H \setminus (x * y).$$

From now on we call $*^c$, *the complementary hyperoperation* of $*$ on H.

Definition 3.10. A (semi)hypergroup $(H, *)$ is called a *complementary feasible* (semi)hypergroup if $(H, *^c)$ is a (semi)hypergroup.

Example 3.11. Let $H = \{0, 1, 2, 3\}$. Then the semihypergroup $(H, *)$, where the hyperoperation $(*)$ is as follows:

$*$	0	1	2	3
0	0	1	2, 3	3
1	1	0, 1	2, 3	2, 3
2	2	2, 3	0, 1	0, 1
3	2, 3	2, 3	0, 1	0, 1

is not a complementary feasible semihypergroup.

Example 3.12. Every group is a complementary feasible hypergroup.

For a given n, the following program written in MATLAB shows that the hypergroup $(H_n, \odot)$, where the hyperoperation $\odot$ introduced in Proposition (3.5) is complementary feasible for the natural numbers less than 31; of course we also guess $(H_n, \odot)$ is a complementary feasible hypergroup for all natural numbers such that $n \geq 2$. This program works in this way for a given n Cayley tables $(H_n, \oplus)$, $(H_n, \otimes)$ entitled a and b then the Cayley table $(H_n, \odot^c)$ entitled c. The functions *compute* and *compute*1 obtain $(x \odot^c y) \odot^c z$ and $x \odot^c (y \odot^c z)$, for all (x, y, z) and the function *equal* checks the associativity.

```
n=input('please inter n=');
a=zeros(2*n);
for x=0:2*n-1
   for y=0:2*n-1
      a(x+1,y+1)=x+y-2*n*(floor((x+y)/(2*n)));
   end
end
a
b=zeros(2*n);
for x=0:n-1
   for y=0:n-1
      b(x+1,y+1)=x+y-n*(floor((x+y)/n));
   end
end
for x=n:2*n-1
   for y=0:n-1
      b(x+1,y+1)=x+y-n*(floor((x+y)/n))+n;
   end
end
for x=n:2*n-1
   for y=n:2*n-1
      b(x+1,y+1)=x+y-n*(floor((x+y)/n));
   end
end
for x=0:n-1
   for y=n:2*n-1
      b(x+1,y+1)=x+y-n*(floor((x+y)/n))+n;
   end
end
b
d=zeros(1,2*n);
for i=1:2*n
   d(i)=i-1;
end
c=zeros(2*n,(2*n-1)*(2*n));
for i=1:2*n
   s=1;
   for j=1:2*n
      k=a(i,j);
```

```
        k1=b(i,j);
        for t=1:2*n
            if (d(t)~=k && d(t)~=k1)
                c(i,s)=d(t);
                s=s+1;
            end
        end
            if a(i,j) =b(i,j)
                c(i,s)=-1;
                s=s+1;
            end
    end
end
c
T=zeros(1,(2*n)^3);
y=1;
f=zeros(1,2*n-1);
h=zeros(1,(2*n-1)*(2*n-1));
h1=zeros(1,(2*n-1)*(2*n-1));
for i=1:2*n
    for j=1:2*n
        for k=1:2*n
            h=compute(c,i,j,k,n);
            h1=compute1(c,i,j,k,n);
            if (equal(h,h1,n))
                T(y)=1;
                y=y+1;
            else
                T(y)=0;
                y=y+1;
            end
        end
    end
end
if(isequal(T,ones(1,(2*n)^3)))
    disp('the hypergroup (Hn,⊙) is a complementary feasible');
else
    disp('the hypergroup (Hn,⊙) is not a complementary feasible');
end
```

In this step the function compute calculate $a\odot(b\odot c)$, for all $(a,b,c)\in(H_n)^3$.

```
function h=compute(c,i,j,k,n)
t=k*(2*n-1);
t1=t-(2*n-1)+1;
s=1;
for m=t1:t;
   f(s)=c(j,m);
   s=s+1;
end
s1=1;
for m=1:(s-1)
   g=f(m)+1;
   if(g =0)
      t=g*(2*n-1);
      t1=t-(2*n-1)+1;
      for x=t1:t
         h(s1)=c(i,x);
         s1=s1+1;
      end
   end
end
```

In this step the function compute1, calculate $(a \odot b) \odot c$ for all $(a, b, c) \in (H_n)^3$.

```
function h1=compute1(c,i,j,k,n) t=j*(2*n-1);
t1=t-(2*n-1)+1;
s=1;
for m=t1:t;
   f(s)=c(i,m);
   s=s+1;
end
s1=1;
for m=1:(s-1)
   g=f(m)+1;
   if(g =0)
      t=k*(2*n-1);
      t1=t-(2*n-1)+1;
      for x=t1:t
         h1(s1)=c(g,x);
         s1=s1+1;
      end
```

```
   end
end
```

In this step the function equals shows the equality between two sets $(a \odot b) \odot c$ and $a \odot (b \odot c)$, for all $(a, b, c) \in (H_n)^3$.

```
function e=equal(h,h1,n)
r(1)=h(1);
k=2;
find=0;
for i=2:length(h)
   find=0;
   if(h(i) =-1)
   for j=1:k-1
      if (h(i)==r(j))
         find=1;
         break;
      end
   end
   if(find==0)
         r(k)=h(i);
         k=k+1;
      end
      end
   end
r1(1)=h1(1);
k=2;
find=0;
for i=2:length(h1)
      find=0;
      if(h1(i) =-1)
      for j=1:k-1
         if (h1(i)==r1(j))
            find=1;
            break;
         end
      end
      if(find==0)
         r1(k)=h1(i);
         k=k+1;
      end
      end
```

```
  end
r2=sort(r);
r3=sort(r1);
if (isequal(r2,r3))
    e=1;
  else
    e=0;
  end
```

4. CONCLUSION

In this paper we introduce and analyze a new class of (semi)hypergroups that derived from strongly duplexes. Several properties of the derived (semi)hypergroups are investigated. This research can be continued in the study of some particular classes of (semi)hypergroups, for instance. Two open problems are raised that they can be incipience of a new work. The problems are:
1) Let n be a natural number. Then $(H_n, \oplus, \otimes)$ is a strongly duplex.
2) Let $n \geq 2$ be a natural number. Then $(H_n, \odot)$ is a complementary feasible hypergroup.

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