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Using NSGA II Algorithm for a Three-Objective Redundancy Allocation Problem with k-out-of-n Sub-Systems

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Abstract

In modern production systems, finding a way to improve the product and system reliability in design is very important. The reliability of the products and systems may improve using different methods. One of these methods is redundancy allocation problem. In this problem, by adding redundant components to sub-systems under some constraints, the reliability would improve. In this paper, we worked on a three-objective redundancy allocation problem. The objectives are maximizing system reliability and minimizing the system cost and weight. The structure of sub-systems are k-out-of-n and the components have constant failure rate. Because this problem belongs to "Np. Hard problems", we used NSGA II multi-objective Meta-heuristic algorithm to solve the presented problem.

Keywords: Reliability, Redundancy allocation problem, Multi-objectives problem, k-out-of-n, NSGA II algorithm.

1. Introduction

Optimizing the system reliability is one of the methods considered by many companies to use their resources more efficiently. The reliability of the system may improve using many different techniques. One of these techniques is RAP[1]. Fyffe et al. (1968) presented the mathematical model of RAP with active redundancy strategy for the first time. In their model, the objective function was maximizing system reliability under system cost and weight constraints and they solved the presented problem using dynamic programing. Nakagawa and Miyazaki (1981) worked on Fyffe's model and solved 33 different problems using Surrogate constraints algorithm and demonstrated that this algorithm has better performance versus dynamic programing for multi-constraints RAP. These 33 problems were the ones presented by Fyffe with different upper limits for system weight (from 159 to 191). Bulfin and Liu (1985) presented 3 different approaches for solving RAP. One of these approaches is a heuristic method and the others are the exact approaches based on branch and bound method. Misra and sharma (1991) considered the RAP for series-parallel structures and k-out-of-n sub-systems. In their model the sub-systems had active redundancy strategy and the components in each sub-system were identical. They solved the presented problem using zero-one programing. Pham (1992) solved the RAP for only one k-out-of-n sub-system with identical components and active redundancy strategy. The objective function of the presented model considered minimizing system total cost. Bai et al. (1991) presented a RAP with k-out-of-n sub-systems and CCF[2]. She and Pecht (1992) calculated the reliability of a k-out-of-n sub-system with warm-standby redundancy strategy in a closed form. In their model, the components were identical, the failure rate of components was constant and the switch performed correctly. The objective of the model was finding the optimal number of components to minimize the average system cost rate. Pham and Malon (1994) presented RAP for a system with k-out-of-n subsystems and identical components and active redundancy strategy with more than one failure rate. The objectives of their model were to find the optimal number of components (n) and minimum required number of components for sub-system working (k) due to minimizing system cost. They once got 'k' fixed and found the optimal number of n, then got 'n' fixed and found the optimal number of k. Coit and Smith (1995) worked on a series-parallel RAP and k-out-of-n active redundancy strategy sub-systems with the choice of allocating non-identical components to each sub-system. This model is known as RAPMC[3]. Coit and Smith (1996a) considered a series-parallel RAP and k-out-of-n active redundancy strategy sub-systems with assuming uncertainty on components reliability.

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1 Redundancy Allocation Problem

2 Common-Cause Failures

3 Redundancy allocation problem with mixing components

Chern (1992) proved that RAP belongs to "Np. Hard" problems due to time of problem solving; therefore, heuristic and meta-heuristic algorithms are more suitable for solving this problem, especially for the large scale problems. Ida et al. (1994) and Yokota et al. (1995) used a simple GA[4] for solving a RAP without choice of allocation non-identical components to each sub-systems. Coit and Smith (1996b) solved the problem, which was presented by themselves (Coit & Smith, 1995) using GA.

Khalili-Damghani et al. (2013) proposed a new dynamic self-adaptive multi-objective particle swarm optimization (DSAMOPSO) method to solve binary-state multi-objective reliability redundancy allocation problems (MORAPs). Soltani et al. (2014) presented a model to maximize the reliability of a system by gathering various components when there are some limitations on budgeting. In their work, two models with different assumptions, including all unit discount and incremental discount strategies are considered. Garg et al. (2014) solved the bi-objective reliability redundancy allocation problem for series-parallel system where reliability of the system and the corresponding designing cost are considered as two different objectives. They used Particle swarm optimization (PSO) algorithm for solving their model. Also Zoulfaghari et al. (2014) provided a new Mixed Integer Nonlinear Programming (MINLP) model to analyze the availability optimization of a system with a given structure, using both repairable and non-repairable components, simultaneously. In order to solve this problem, they suggested an efficient Genetic Algorithm to find the solution of the introduced MINLP.

Zaretalab et al. (2015) presented an efficient multi-objective meta-heuristic algorithm based on simulated annealing (SA) in order to solve multi-objective RAP (MORAP).

One of the most important problems in solving RAP using GA is producing and selecting infeasible solutions. The penalty function was defined to avoid this problem by Coit and Smith (1996c). This function encouraged GA to select the solutions between Feasible and near-Feasible thresholds. In their method, the values of each chromosome were equal to the sum of the reliability of chromosome and the values of penalty function. Coit and Liu (2000) worked on a model with predefined active and cold-standby redundancy strategy. They considered that the components has constant failure rate and transformed the non-linear model to a linear one using alter variable. Coit (2003) presented a new model with the choice of selecting the redundancy strategy of each system (between active and cold-standby) and solved the problem with integer programing. Tavakkoli-Moghaddam et al. (2008) solved the model presented by Coit (2003) using GA and the most important characteristics of the presented algorithm were the form of chromosome and mutation operator. Safari and Tavakkoli-Moghaddam (2010) solved the model presented by Coit (2003) using Memetic algorithm. This algorithm added a local search to GA. Amari and Dill (2010) considered a series-parallel RAP with k-out-of-n sub-systems with the choice of selecting redundancy strategy (active and standby) of each sub-system. The standby redundancy strategy contains cold, warm and hot standby. They did not present a closed form for the standby section. Chambari et al. (2012) presented a bi-objective series-parallel RAP. They considered a failure rate function for the cold-standby components in the time of activation. They also considered the choice of selecting redundancy strategy and solved the presented model using NSGA II and MOPSO and compared the results of two algorithms together. Khalili Damghani and Amiri (2012) solved a binary-state multi-objective reliability redundancy allocation series-parallel problem using efficient epsilon-constraint, multi-start partial bound enumeration algorithm, and DEA.

In this paper, we work on a three-objective RAP with k-out-of-n sub-systems. The objectives of the model are maximizing system reliability and minimizing system cost and weight. The redundancy strategy of the sub-systems is active or cold standby and they are pre-defined (Coit & Liu, 2000). The presented model was solved using NSGA II[5].

The paper is divided into 5 parts. The second part provides problem definitions. In the third part, the solving algorithm is presented. Part 4 deals with the results of problem and part five presents conclusion and further studies.

2. Problem Definition

This paper deals with a RAP with S sub-systems. The structure of each sub-system is k-out-of-n and the redundancy strategy of each sub-system is active or cold-standby and is pre-defined (Coit & Liu, 2000). The objective functions of the problem are:

- Maximizing system reliability,
- Minimizing system cost,
- Minimizing system weight.

The variables of the problem are the number on allocated components in each sub-system. In k-out-of-n configuration, if the system works when at least k components are working, the configuration called k-out-of-n:G, and if the system fails when at least k components failed, the configuration called k-out-of-n:F (Sharifi et al., 2009). This configuration has more usage in electric and electronic devices in industrial design. For example consider a plane with 4 identical engines. If the plane could continue flying with at least two engines, the configuration of the plane engines is 2-out-of-4:G. The k-out-of-n:G configuration is illustrated in Figure 1.

4 Genetic Algorithm

5 Non-dominated Sorting Genetic Algorithm

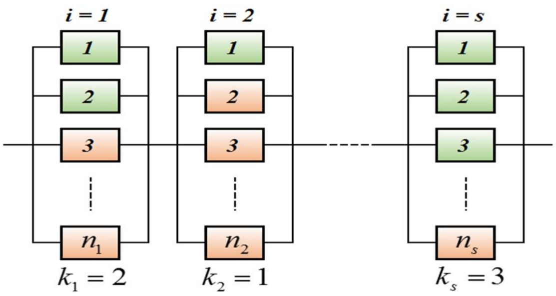

Fig. 1. System with k-out-of-n subsystems Coit and Liu (2000)

2.1. Assumptions

The assumption of the presented model in this paper are:

- The components are non-repairable,
- The failure of one component does not have any effects on system failure,
- The components have constant failure rates,
- The components have only two working and failed states,
- The components' failures are independent,
- The system parameters like the cost and the weight of components are pre-defined and constant.

2.2. Nomenclatures

i:	Sub-systems indexes, $i = 1,2,\ldots,s$
S:	Number of sub-systems,
$R(t)$:	Reliability of the system at the time t depends on design vector z and n, $n = \{n_1, n_2, \ldots, n_s\}$
n_i :	Number of components in i^{st} sub-system, $i = 1,2,\ldots,s$, $n_i \in \{1,2,\ldots,m_i\}$
$n_{Max,i}$:	Maximum limit of components in i^{st} sub-system,
c_{ij} :	The cost of component type j in i^{st} sub-system,
w_{ij} :	The weight of component type j in i^{st} sub-system,
z_i :	Index of selected components type for allocating in i^{st} sub-system, $z = \{z_1, z_2, \ldots, z_s\}$
m_i :	Number of available component types in sub-system i,
k_i :	Minimum required components in i^{st} sub-system, $k = \{k_1, k_2, \ldots, k_s\}$
T_{ij} :	Failure time of j^{st} standby component in i^{st} sub-system,
λ_{ij} :	Failure rate of j^{st} component type in i^{st} sub-system,
t :	Mission time of system.

2.3. Mathematical model

According to nomenclatures, the mathematical model is as follows:

$$Max\ R = R(t) \tag{1}$$

$$Min\ C = \sum_{i=1}^{s} n_i c_i \tag{2}$$

$$Min\ W = \sum_{i=1}^{s} n_i w_i \tag{3}$$

$$S.t: \quad k_i \le n_i \le n_{Max,i} \quad ; \quad i = 1,2,\ldots,s \tag{4}$$

$$z_i \in \{1,2,\ldots,m_i\} \quad ; \quad i = 1,2,\ldots,s \tag{5}$$

$$n_i \in W \quad ; \quad i = 1,2,\ldots,s \tag{6}$$

Equation 1 deals with maximizing the system reliability and is calculated in more detail. Equations 2 and 3 calculate the system cost and system weight that must be minimized. Equation 4 represents the lower and upper bounds for components allocated in each sub-system and equations 5 and 6 are the definitions of system variables.

The sub-systems are connected serially, so the reliability of the system can be calculated by multiplying the reliability of each sub-system (Coit & Liu, 2000). The system reliability is calculated in equation 7.

$$R(t) = \prod_{i=1}^{s} R_i(t, z_i, n_i, k_i) \tag{7}$$

In Equation 7, the reliability of i^{st} sub-system is demonstrated by $R_i(t, z_i, n_i, k_i)$. This reliability can be calculated based on the redundancy strategy of the sub-system. Coit and Liu (2000) calculated the reliability of the system as follows:

$$R(t) = \left\{ \prod_{i \in A} \sum_{l=k_i}^{n_i} \left\{ \binom{n_i}{l} \left\{ \exp\left(-\lambda_{i,z_i} t\right) \right\}^{l} \left\{ 1 - \exp\left(-\lambda_{i,z_i} t\right) \right\}^{(n_i - l)} \right\} \right\} \times \left\{ \prod_{i \in S} \left\{ \exp\left(-\lambda_{i,z_i} k_i t\right) \sum_{l=0}^{n_i - k_i} \frac{\left(\lambda_{i,z_i} k_i t\right)^{l}}{l!} \right\} \right\} \tag{8}$$

In Equation 8, the set of all sub-systems with active redundancy allocation is (A) and the set of all sub-systems with cold-standby redundancy allocation is (S).

3. Solving Algorithm

As mentioned earlier, Chern (1992) proved that RAP belongs to “Np. Hard” problems so the exact methods are not suitable for solving this problem and the heuristic and meta-heuristic algorithms have more performance. In this paper, we used NSGA II for solving the presented problem.

3.1. NSGA II

This algorithm is one of the most effective multi-objective algorithms presented by Deb et al. (2000). The algorithm mechanism was presented by Deb et al. (2000) and is shown in Figure 2.

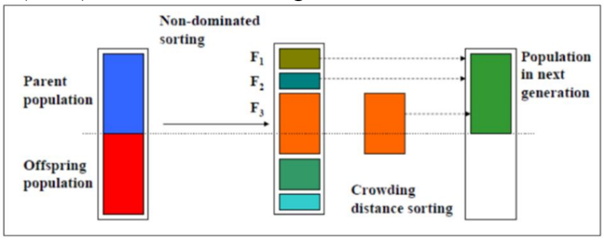

Fig. 2. Mechanism of NSGA II (Deb et Al., 2000)

The pseudo-code of proposed NSGA II algorithm presented in Figure 3.

```
procedure NSGA-II(N', g, f_k(x_k))   ▷ N' members evolved g generations to solve f_k(x)
    Initialize Population P'
    Generate random population - size N'
    Evaluate Objective Values
    Assign Rank (level) Based on Pareto dominance - sort
    Generate Child Population
      Binary Tournament Selection
      Recombination and Mutation
    for i = 1 to g do
        for each Parent and Child in Population do
            Assign Rank (level) based on Pareto - sort
            Generate sets of nondominated vectors along PF_known
            Loop (inside) by adding solutions to next generation starting from the first front until N' individuals found determine crowding distance between points on each front
        end for
        Select points (elitist) on the lower front (with lower rank) and are outside a crowding distance
        Create next generation
            Binary Tournament Selection
            Recombination and Mutation
    end for
end procedure
```

Fig. 3. The pseudo-code of proposed NSGA II algorithm (Deb et Al., 2000)

3.1.1. Initialization

The parameters of NSGA II are:

- Initial population size $(npop)$,
- Probability of crossover operator (p_c),
- Probability of mutation operator (p_m),
- Number of algorithm iterations $(MaxIt)$.

3.1.2. Problem chromosome

Because the NSGA II is a population-based algorithm, each solution of the problem (the algorithm chromosome) is considered as a matrix with rank $2\times s$ in which S is the number of sub-systems and the first and second rows represent the number and type of allocated components to each sub-system, respectively. The structure of the problem chromosome is presented in Figure 4.

n_1	n_2	$\dots$	n_{s-1}	n_s
Z_1	z_2	$\dots$	z_{s-1}	z_s

Fig. 4. Structure of presented chromosome

3.1.3. Crossover operator

The crossover operator in this paper is uniform crossover Gen and Cheng (1997) and Tavakkoli-Moghaddam et al. (2008). In this type of crossover, operator two parents are selected using roulette wheel and for each genome of the parents' chromosome a binary random number is generated. If this number is equal to 1, the genome of parents will be replaced by each other's and if this number is equal to 0, the genome of the parents will not change. This type of crossover operator is illustrated in Figure 5.

3.1.4. Mutation operator

For mutation operator, after selection of a chromosome for each genome of chromosome a random number is generated. If this number is less than mutation rate (in this paper the mutation rate is considered 0.1), the genome will be mutated randomly; otherwise, the genome will not change (Gen and Cheng, 1997). This type of mutation is illustrated in Figure 6.

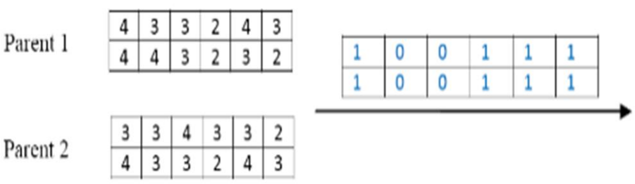

Fig. 5. Crossover operation

Fig. 6. Mutation operator

3.2. Numerical example

For evaluation of the presented NSGA II, a numerical example is solved. This example is proposed by Coit and Liu (2000). In this example, a system with 14 sub-systems exists. For each sub-system, 3 or 4 different types of components are available to allocate. The cost, weight, failure rate of components and minimum required components in each sub-system are presented in Table 1. In this example, the first 7 sub-systems have active redundancy strategy and the second 7 sub-systems have cold-standby redundancy strategy. The mission time considered 100 hours and the maximum number of allocated components in each sub-system considered 6 ($n_{Max,i} = 6$).

Table 1
Component data for example (Coit & Liu, 2000)

subsystem			Component Choice 1			Component Choice 2			Component Choice 3			Component Choice 4		
i	k_i	type	λ_{ij}	c_{ij}	w_{ij}	λ_{ij}	c_{ij}	w_{ij}	λ_{ij}	c_{ij}	w_{ij}	λ_{ij}	c_{ij}	w_{ij}
1	1	A	0.001054	1	3	0.000726	1	4	0.000943	2	2	0.000513	2	5
2	2	A	0.000513	2	8	0.000619	1	10	0.000726	1	9	-	-	-
3	1	A	0.001625	2	7	0.001054	3	5	0.001393	1	6	0.000834	4	4
4	2	A	0. 001863	3	5	0.001393	4	6	0.001625	5	4	-	-	-
5	1	A	0.000619	2	4	0.000726	2	3	0.000513	3	5	-	-	-
6	2	A	0.000101	3	5	0.000202	3	4	0.000305	2	5	0.000408	2	4
7	1	A	0.000943	4	7	0.000834	4	8	0.000619	5	9	-	-	-
8	2	S	0.002107	3	4	0.001054	5	7	0.000943	6	6	-	-	-
9	3	S	0.000305	2	8	0.000101	3	9	0.000408	4	7	0.000943	3	8
10	3	S	0.001863	4	6	0.001625	4	5	0.001054	5	6	-	-	-
11	3	S	0.000619	3	5	0.000513	4	6	0.000408	5	6	-	-	-
12	1	S	0.002357	2	4	0.001985	3	5	0.001625	4	6	0.001054	5	7
13	2	S	0.000202	2	5	0.000101	3	5	0.000305	2	6	-	-	-
14	3	S	0.001054	4	6	0.000834	4	7	0.000513	5	6	0.000101	6	9

Notes: A = active redundancy, S = cold-standby redundancy, units for λ_{ij} are failures/hour

3.3. The evaluation metrics of multi-objectives meta-heuristic algorithms

Generally, in single objective problems, the target is to find optimal solution of objective function. Whereas in multi-objective problems, the objectives may be in conflict and most of the time finding the optimal solution is impossible. Some metrics are used to compare the solving methods. Five metrics to evaluate the performance of multi-objective algorithms are as follow.

3.3.1. Diversity

This metric was presented by Zitzler (1999) and calculated the terminal points spatial cube diagonal of objectives in non-dominated solutions. Equation 9 calculates this scale:

$$D = \sqrt{\sum_{j=1}^{m} \left(\underset{i}{Max}\ f_i^j - \underset{i}{Min}\ f_i^j \right)^2} \tag{9}$$

This scale is equal to Euclidian distance between two boundaries solutions in target space. The algorithm with greater values of this scale is a better algorithm.

3.3.2. Spacing

This scale that was originally presented by Schott (1995) calculates the comparative distance of consecutive solutions using Equation 10.

$$S = \sqrt{\frac{1}{n-1}\sum_{j=1}^{m}\left(d_i - \bar{d}\right)^2}$$
$$d_i = \underset{(k \in n \cap k \neq i)}{Min} \left| f_j^i - f_j^k \right| \tag{10}$$
$$\bar{d} = \frac{1}{n}\sum_{j=1}^{m} d_i$$

This metric is equal to the sum of the absolute value of objective function between i[th] solution and the set of final non-dominated solutions and is different to Euclidian distance between two boundaries solutions in the target space. The algorithm with lower values of spacing is a better algorithm.

3.3.3. Number of Pareto solutions (NOS)

This scale is equal to the number of different Pareto optimal solutions of an algorithm.

3.3.4. Mean ideal distance (MID)

This scale calculates the distance of the fronts and the better populations using Equation 11, (Zitzler, 1998).

$$Mid = \frac{1}{NOS}\sum_{i=1}^{NOS} c_i \tag{11}$$

Where c_i is the distance of a population member from the best value.

3.3.5. CPU time of the algorithm

The CPU time of algorithms is one of the most important scales for comparison.

3.4. Parameter tuning

To reach better solutions, RSM is used in the next section to calibrate the algorithm parameters. As the results of meta-heuristic algorithm highly depends on its parameters, these parameters must be tuned to achieve the better solutions. If the results of a response of process y is affected by many variables (X), the objective function defined as $y=f(x_1,x_2,\ldots,x_n)+\varepsilon$ that ε is the observation error in response y. If the expected value of response represented by $E(y)=f(x_1,x_2,\ldots,x_n)=\eta$ then the $\eta=f(x_1,x_2,\ldots,x_n)$ is a surface that is called response surface. The target of RSM is to find an appropriate approximation between the response y and the independent variables (X). The NSGA-II has three parameters that must be tuned. These parameters are population size $(npop)$, crossover probability (p_c), and mutation probability (p_m). Using MINITAB 16 and considering $\left(MID/_{Diversity}\right)$ as objective function $(2^3+2\times3+5=19)$ problem solved and the optimal parameters of algorithm calculated. We added 5 central points to test any curve in response surface and the results are presented in Table 2.

Table 2
The optimal values of NSGA-II parameters

parameter	Optimal values
$npop$	100
p_c	0.7
p_m	0.3

4. Problem results

Using the optimal parameters of algorithm, an example that presented by Coit and Liu (2000) solved to analyzing the results of NSGA-II algorithm. In optimal solution of this example, the system cost and weight are 118 and 177 and the reliability of the system is 0.4466. The Pareto solutions of NSGA-II algorithm is presented in Table 3. In this table, the solution number 20 (that has red color) is the same result obtained by Coit and Liu and the other solutions were not dominated by this solution and all results in this table are non-dominated solutions.

The solutions presented in this table are appropriate for decision makers. If increasing of the system reliability is important for decision makers, the results that are in yellow color in Table 1 have the reliability of more than 0.4466, if decreasing of the system cost is important for decision makers, the results that are green color in Table 1 have the cost less than 118 and if decreasing of the system weight is important for decision makers, the results that are in blue color in Table 1 have the cost less than 177. The Pareto front of the solutions are presented in Figure 7. At the end, by changing the weight of component type one in the first sub-system from 1 to 10, the example solved ten times and the results of algorithm metrics are presented in Table 4.

4. Conclusion and Suggestions for Further Studies

In this paper, we presented an NSGA-II algorithm for solving MORAP for a series-parallel problem and k-out-of-n sub-systems with 3 objectives. The objectives were reliability, cost, and weight. The components were CFR and the sub-system redundancy strategy was active and cold-standby. The results of the algorithm prepared a wide range of solution for decision makers. The further studies divided in-to two categories. The first category is the solving methodology and the other multi-objective meta-heuristic algorithm like NRGA, MOPSO, and MOSA that may be used for solving the problem. The other category deals with problem specification. For example, this algorithm can be used for solving the problem with multi-state components, time dependent failure rate components, and repairable components.

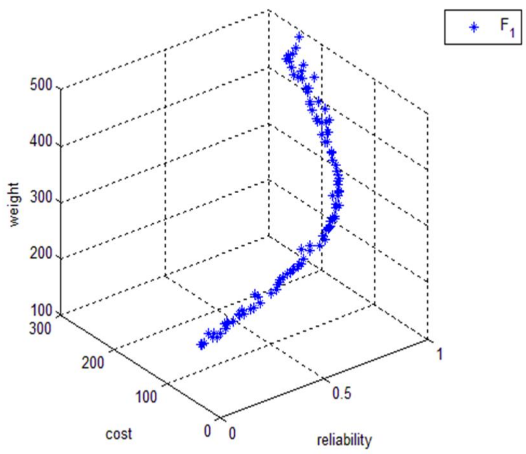

Fig. 7: Pareto front solutions of NSGA-II using optimal parameters

Table 3
The Pareto solutions obtained by NSGA-II

Solution	*R*	*C*	*W*	Solution	*R*	*C*	*W*
1	0.1086	78	169	51	0.9993	239	431
2	0.2030	86	162	52	0.9991	225	409
3	0.1074	78	168	53	0.9933	192	334
4	0.9994	267	435	54	0.8540	130	238
5	0.9994	263	443	55	0.9784	161	296
6	0.9993	242	488	56	0.9990	227	401
7	0.9994	249	465	57	0.9767	159	292
8	0.7642	113	231	58	0.9658	157	289
9	0.9863	175	312	59	0.5114	95	199
10	0.9856	169	307	60	0.9994	261	435
11	0.9948	199	342	61	0.7264	116	224
12	0.4975	99	192	62	0.2861	88	178
13	0.9993	242	488	63	0.9991	214	434
14	0.9929	180	325	64	0.9993	236	419
15	0.9994	255	450	65	0.8914	134	252
16	0.5403	104	197	66	0.9641	155	285
17	0.9792	164	301	67	0.9992	253	414
18	0.9268	140	262	68	0.9992	245	416
19	0.9174	142	259	69	0.4226	95	198
20	0.4466	118	170	70	0.1845	85	172
21	0.3799	92	185	71	0.9954	189	352
22	0.8083	119	232	72	0.8713	130	255
23	0.9984	206	397	73	0.9473	148	281
24	0.4450	100	182	74	0.2316	83	185
25	0.9540	149	276	75	0.8883	132	249
26	0.7429	123	227	76	0.9984	221	383
27	0.9338	154	270	77	0.2793	97	171
28	0.9059	136	270	78	0.8360	121	248
29	0.6751	116	209	79	0.3358	90	190
30	0.8333	127	241	80	0.2619	94	174
31	0.3492	92	181	81	0.9993	234	444
32	0.6605	107	217	82	0.6177	110	204
33	0.1258	80	166	83	0.9969	208	361
34	0.9994	257	425	84	0.9986	221	387
35	0.3125	87	189	85	0.2612	88	177
36	0.7094	126	223	86	0.9962	198	365
37	0.6990	118	213	87	0.9972	210	363
38	0.1716	82	170	88	0.2413	85	173
39	0.5974	108	206	89	0.6212	105	215
40	0.9423	147	273	90	0.9950	186	355
41	0.1450	79	183	91	0.2209	86	166
42	0.9982	194	391	92	0.6449	117	202
43	0.9963	185	376	93	0.9938	188	337
44	0.8093	123	235	94	0.8713	133	246
45	0.9991	231	405	95	0.9965	190	369
46	0.4085	96	181	96	0.8495	126	244
47	0.9899	177	325	97	0.5557	106	203
48	0.5666	110	195	98	0.3237	90	183
49	0.9978	218	373	99	0.6428	106	215
50	0.5866	112	199	100	0.2030	86	162

Mean

R	*C*	*W*
0.7361	150.59	279.56

Table 4
The results of metrics in the ten time problem solving.

example	Diversity	Spacing	NOS	MID	Time(s)
1	370.0954	5.3895	100	155.2505	198.554296
2	363.4085	4.6697	100	159.5707	199.438622
3	369.5250	4.2956	100	160.1108	214.988428
4	342.4774	5.3706	100	144.1408	198.674896
5	333.3989	4.3993	100	140.6809	197.099027
6	331.5084	4.7061	100	143.7109	200.728362
7	386.9094	5.5234	100	153.4909	204.411265
8	406.9861	4.5990	100	162.2407	202.433352
9	350.3624	4.8913	100	147.7609	200.919858
10	368.0907	4.6249	100	151.2407	201.550561

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