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A Compromise Decision-making Model for Multi-objective Large-scale Programming Problems with a Block Angular Structure under Uncertainty

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Abstract

This paper proposes a compromise model, based on the technique for order preference through similarity ideal solution (TOPSIS) methodology, to solve the multi-objective large-scale linear programming (MOLSLP) problems with block angular structure involving fuzzy parameters. The problem involves fuzzy parameters in the objective functions and constraints. This compromise programming method is based on the assumption that the optimal alternative is closer to fuzzy positive ideal solution (FPIS) and at the same time, farther from fuzzy negative ideal solution (FNIS).An aggregating function that is developed from LP- metric is based on the particular measure of ‘‘closeness” to the ‘‘ideal” solution.An efficient distance measurement is utilized to calculate positive and negative ideal solutions. The solution process is as follows: first, the decomposition algorithm is used to divide the large-dimensional objective space into a two-dimensional space. A multi-objective identical crisp linear programming is derived from the fuzzy linear model for solving the problem. Then, a single-objective large-scale linear programming problem is solved to find the optimal solution. Finally, to illustrate the proposed method, an illustrative example is provided.

Keywords: TOPSIS; MCDM; MODM; Multi-Objective Large-Scale Linear Programming (MOLSLP); Block angular structure.

1. Introduction

Decision making is the process of selecting a course of actionfrom among several alternatives withrespect to multiple criteria. In decision making problems, the best solution is found while satisfying the constraints. Moreover, inmany decision situations, problems involve multiple objectives. In other words, multi-objective problems should be optimized simultaneously during decision making. Some objectives relate to maximization of the profit and some others deal with minimizing the cost (Abo-Sinna&Amer, 2005; Hu et al., 2009). The complexity of decision making problems is related to thenumber of variables. In other words, there are many factors in objective functions and constraints in large-scale problems. Specially, the complexity increases dramatically in large-scale linear programming problems. Furthermore, the scope of most large-scale problems is so wide that they can be solved through ordinary methods in a shorter time. But fortunately, most large-scale programming problems of practical interest usually have a special structure that can be exploited. Block angular structure is one of such structures that can be used to formulate sub-problems (Dantzig & Wolfe, 1961; Sakawa et al., 1995; Sakawa, 2000; Abo-Sinna&Amer, 2005; Heydari et al., 2010). The block angular structure problems are solved by a decomposition scheme interpreted into a lower dimension space (Dantzig& Wolfe, 1961; Ho & Sundarraj, 1981). This process is applied to solve the large-scale linear programming problems (Sakawa et al., 1995; Heydari et al., 2010). Some exact and metaheuristic approaches are proposed to solve multi-objective large-scale programming problems where the coefficients of objective functions and constraints are crisp (Augusto, 2012; Abo Sinna &Abou-El-Enien, 2014).

Recently, some compromise decision making methods are extended to solve MOLSLP problems. TOPSIS was introduced as one of the compromise solution methods by Hwang and Yoon for the first time (Hwang & Yoon,

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1981;Wang et al., 2006;Tong et al., 2007). TOPSIS is applied to sort alternatives in a decreasing order based on similarity to ideal solution which has both the shortest distance from the positive ideal solution, and at the same time, the longest distance from the negative ideal solution (Hwang & Yoon, 1981; Celik et al., 2009; Jiang et al., 2011). A variety of TOPSIS algorithms and applications has been developed in recent years. TOPSIS has been widely applied to evaluate the risk analysis problems (Chen &Tzeng, 2004; Jiang et al., 2011). TOPSIS method is applied to solve multi-objective dynamics programming problems (Abo-Sinna, 2000). An extended method is present based on TOPSIS to solve the inter-company comparison process problems (Deng et al., 2000). However, a large body of TOPSIS extensions is presented with crisp data,whereas, due to the incomplete or non-obtainable information, the real situation decision making process is based on uncertainty and vagueness data. In other words, many attributes are imprecise rather than crisp. (vahdani et al., 2010; Jolai et al., 2011).

Fuzzy concept is one of the meaningful tools to describe the imprecise content. Fuzzy set theory was proposed as a valuable tool for handling uncertainty in decision parameters (Zadeh, 1965). Then the fuzzy programming model was suggested for decisions in imprecise environment (Bellman &Zadeh, 1970). Fuzzy programming approach was developed for multipleobjective linear programming problems (Zimmermann, 1978). Some attractive methods are introduced for solving multi-objective large-scale programming problems under uncertainty (Abou-El-Enien, 2011; Abo Sinna&Abou-El-Enien, 2011; Sultan et al., 2013; Teegavarapu et al., 2013). Moreover, the fuzzy set concept and the MCDM methods were manipulated in decision-making process for solving linguistic fuzziness problems. TOPSIS is extended for solving multi-person decision making problems versus multi-criteria in fuzzy environment (Chen, 2000). Later, TOPSIS was extended to fuzzy environments for group decision making based on theconcepts of positive and negative ideal points (Chen, 2000; Mahdavi et al., 2008). TOPSIS method was extended for solving multi-objective decision making problems under fuzzy environment (Lai et al., 1994; Celik et al., 2009).

In recent years, TOPSIS method is extended as a compromise MCDM method to find the best solution for large-scale multi-objective optimization problems with block angular structure based on the Dantzig – Wolfe's decomposing algorithm (Abo-Sinna and Amer, 2005; Abo-Sinna et al., 2006). The Dantzig - Wolfe decomposing algorithm was introduced to solve large-scale linear optimization problems (Dantzig& Wolfe, 1961; El-Sawy et al., 2000). Abo-Sinna and Abou-El-Enien proposed a TOPSIS interactive algorithm to solve large scale multiple objective non-linear programming problems with crisp parameters (Abo-Sinna et al., 2008). The fuzzy LSMOLP problems are applied in many field of science but it is difficult to obtain efficient solutions for these problems in a short time and efficient manner.

In this paper, a new extended TOPSIS method is proposedfor solving LSMOLP problems with fuzzy parameters. The formulation of LSMOLP problems with block angular structure is solved using the Dantzig–Wolfe decomposition method. An aggregating function that is developed from LP- metric is based on the particular measure of ''closeness" to the ''ideal" solution. An efficient fuzzy distance measurement is utilized to calculate fuzzy positive ideal solutions and fuzzy negative ideal solutions. The solution processis as follows: first, the decomposition algorithm is utilized to divide the large dimensional objective space into a two-dimensional space. The two objective identical crisp linear programming arederived from the fuzzy programming model for solving the problem. Then, a single-objective problem is solved to find optimal solution. Finally, a numerical illustrative example is presented to clarify the main results developed in this study.

The remainder of this paper is organized as follows. The problem statement is presented in the next section. In this section, the decomposed problem is introduced and then the parameters and variables are described. In section 3, the TOPSIS solution method for fuzzy MOLSLP is introduced. In section 4, an example is proposed to illustrate the process of proposed method step by step. The last section is devoted to conclusion.

2. Problem Statement

In this paper, the fuzzy MOLSLP problem with the block angular structure is constructed as follows.

$$
\begin{aligned}
P&: \text{Max (Min)} f_1(X, \tilde{U_1}) \\
&\text{Max (Min)} f_2(X, \tilde{U_2}) \qquad (1) \\
&\;\;. \\
&\;\;. \\
&\text{Max (Min)} f_L(X, \tilde{U_L}) \\
&S.t. \qquad FS
\end{aligned}
$$

$$
= \begin{cases}
\tilde{g_m}(x_1) \leq \tilde{B_1} m = 1, 2, \dots, \quad s_1 \\
\tilde{g_m}(x_2) \leq \tilde{B_2} m = s_1 + 1, \dots, \quad s_2 \\
\quad \vdots \\
\tilde{g_m}(x_N) \leq \tilde{B_N} m = s_r + 1, \dots, \quad s_N \\
\tilde{H_i}(X) = \sum_{j=1}^{N} \tilde{h_{ij}}(X_j) \leq \tilde{B} i = 1, 2, \dots, \mathrm{w}
\end{cases}
$$

$$
f_i(X, \tilde{U_i}) = \tilde{U_i} C_i X = \textstyle\sum_{j=1}^{N} \tilde{U_{ij}} C_{ij} X_j \, i = 1, 2, \dots, \mathrm{L} \qquad (2)
$$

$\tilde{g_m}(xi) = V\sim_{mi} d_{mi} X_i$; $i = 1, 2, \ldots, s_1$ are the inequality constraint functions and $\tilde{H_i}(X)$ are the common constraints functions on R^n which can be constrained as: $\tilde{H_i}(X) = \sum_{j=1}^{N} O\sim_{ij} e_{ij} X_j i = 1, 2, \ldots,$
Where $V\sim_{mi} = (v_{m1}, v_{m2}, v_{m3})$, $O\sim_{ij} = (o_{ij1}, o_{ij2}, o_{ij3})$, $\tilde{B_m} = (b_{m1}, b_{m2}, b_{m3})$, $\tilde{B} = (r_i, s_i, t_i)$
Model parameters:
L the number of objective functions
q the number of sub problems
N the number of variables
N_i the set of variables of the ith sub problem, i= 1,2,. . . ,q
p_i ith sub problem
R the set of all real numbers
C_i an N-dimensional row vector of fuzzy parameters for the ith objective function
C_{ij} the crisp coefficient for the jth variableof $i$thobjective function
d_{ij} the crisp coefficient for the $j$thconstraintof $i$thvariable
e_{ij} the crisp coefficient for the ith common constraint for the jth variable
$\tilde{U_i}$ an N-dimensional row vector of fuzzy parameters for the ith objective function
$\tilde{U_{ij}}$ the fuzzy parameters for the jth variable of the$i$thobjective function
$V\sim_{ij}$ the fuzzy parameters for the $i$thconstraint of the jth variable
$O_{ij}\sim$ the fuzzy parameters for the jth variable of the$i$th common constraint
W the number of common constraints on R^N
S_i maximum amount of index for the constraints for the ith variable
$\tilde{B}$ an w-dimensional column vector of right-hand sides of the common constraints whose elements are constants
$\tilde{B_j}$ an S_i -dimensional column vector of independent constraints right-hand sides whose elements are fuzzy parameters for the ith sub problem, i = 1,2,. . . ,q.
Where $X = (x_1, x_2, .., x_N)$ is the N-dimensional decision vector. $f_i(X, \tilde{U_i})$, i=1,2,…, $L$are the objective functions. It is assumed that the objective functions have an additively separable form. Using Dantzig-Wolfe decomposition algorithm, the fuzzy MOLSLP problem can be decomposed into q sub-problems. The ith sub-problem for i = 1, . . , q is defined as:

$$P_i \begin{cases} \text{Max (Min)} f_1(X, \tilde{U_1}) = \sum_{j \in N_i} f_1(X_j, \tilde{U_1}) = \sum_{j \in N_i} \tilde{U_1} \tilde{C_1} X_j \\ \text{Max (Min)} f_2(X, \tilde{U_2}) = \sum_{j \in N_i} f_2(X_j, \tilde{U_2}) = \sum_{j \in N_i} \tilde{U_2} \tilde{C_2} X_j \\ \vdots \\ \text{Max (Min)} f_L(X, \tilde{U_L}) = \sum_{j \in N_i} f_L(X_j, \tilde{U_L}) = \sum_{j \in N_i} \tilde{U_L} \tilde{C_L} X_j \\ S.t. \quad FS_i = \begin{cases} \sum_{j \in N_i} \tilde{g_m}(X_j) \le \tilde{B_m} m = s_{j-1} + 1, \ldots, s_j \\ \tilde{H_i}(X) = \sum_{j=1}^{N} \tilde{h_{ij}}(X_j) \le \tilde{B} i = 1, 2, \ldots, w \end{cases} \end{cases} \tag{3}$$

As shown in problem (3), the ith sub problem consists of L objective functions. Moreover, $\tilde{h_{ij}}(X_j) = O\sim_{ij} e_{ij} X_{ij}$ where h_{ij} is the function of jth variable in ith common constraint and $\tilde{U}$ is the coefficient of the objective functionand $\tilde{B}$ is the coefficient of the right-hand side of constraintsin problem (3). It is pointed out that all of the coefficients are presented as triangular fuzzy numbers.

3. The TOPSIS Solution Method for Fuzzy MOLSLP

In this section, the Dantzig-wolf decomposition method is successfully applied to decompose the original problem into$q$ independent linear sub-problems. In other words, the L-dimensional problem space is reduced into a one-dimensional space by applying Dantzig-Wolfe decomposition algorithm. Then, the TOPSIS method is applied as a compromised method to aggregate the objectives of each sub-problem. To obtain compromise solution of original problem, the individual positive ideal solution (PIS) and negative ideal solution (NIS) are calculated for each objective. Applying PIS and NIS, the bi-objective problems are constructed for $j$thsub-problem. Afterwards, the final single-objective problem is constructed for each sub-problem. The mentioned single programming problem is solved to obtain the final optimal solution. The proposed method has the following steps:

Step 1.Decompose the original problem in to q sub problems by applying the Dantzig-wolf decomposition method for objective functions and constraints to reduce the dimension of primal problem. The ith sub problem can be stated as:

$$p_i \begin{cases} \text{Max (Min)} f_1(X, \tilde{U_1}) = (a_{i1}, b_{i1}, c_{i1}) C_{i1} X_i \\ \text{Max (Min)} f_2(X, \tilde{U_2}) = (a_{i1}, b_{i1}, c_{i1}) C_{i1} X_i \\ \vdots \\ \text{Max (Min)} f_L(X, \tilde{U_L}) = (a_{i1}, b_{i1}, c_{i1}) C_{i1} X_i \\ S.t. \quad FS_i = \begin{cases} (v_{im1}, v_{im2}, v_{im3}) d_{im} X_i \leq (b_{im1}, b_{im2}, b_{im3}) m = s_{i-1}+1, \dots, s_i \\ \tilde{H_i}(X) = \sum_{j=1}^{N} (o_{ij1}, o_{ij2}, o_{ij3}) e_{ij} X_i \leq (r_i, s_i, t_i) \ i = 1, 2, \dots, w \end{cases} \end{cases} \tag{4}$$

s.t .$(X_1, X_2, \dots, X_N) \in FS$.

Step 2.Use a simple method to transfereach fuzzy programming problem in to three crisp problems. This method is proposed and extended to defuzing some fuzzy problems (Lia& Hwang, 1992; Wang & Liang, 2005; Torabi&Hassini, 2008). Because the coefficients of objective functions and constraints are assumed as triangular fuzzy numbers, there are three crisp objective functions for each fuzzy objective function.Moreover, each fuzzy constraint can be changed in to three crisp constrains. The ith sub problem is transferred as:

$$P_i:$$

$$P_{i1}: \begin{cases} \text{Min(Max)} (b_{i1} - a_{i1}) C_{i1} X_i \\ \text{Max (Min)} (b_{i1}) C_{i1} X_i \\ \text{Max (Min)} (c_{i1} - b_{i1}) C_{i1} X_i \end{cases} \tag{5}$$

$$Pi_2: \begin{cases} \text{Min(Max)} (b_{i2} - a_{i2}) C_{i2} X_i \\ \text{Max (Min)} (b_{i2}) C_{i2} X_i \\ \text{Max (Min)} (c_{i2} - b_{i2}) C_{i2} X_i \end{cases} \tag{6}$$

$$\vdots$$

$$P_{iL}: \begin{cases} \text{Min(Max)} (b_{iL} - a_{iL}) C_{iL} X_i \\ \text{Max (Min)} (b_{iL}) C_{iL} X_i \\ \text{Max (Min)} (c_{iL} - b_{iL}) C_{iL} X_i \end{cases} \tag{7}$$

$$S.t. \begin{cases} \begin{cases} v_{im1} d_{im}(xi) \leq b_{im1} \\ v_{im2} d_{im}(xi) \leq b_{im2} \ m = s_{i-1}+1, \dots, s_i \\ v_{im3} d_{im}(xi) \leq b_{im3} \end{cases} \\ \begin{cases} \sum_{j=1}^{N} o_{ij1} e_{ij} X_j \leq r_i \\ \sum_{j=1}^{N} o_{ij2} e_{ij} X_j \leq s_i \ i = 1, 2, \dots, w \\ \sum_{j=1}^{N} o_{ij3} e_{ij} X_j \leq t_i \end{cases} \end{cases} \tag{8}$$

Step 3. Calculate the positive ideal solution (PIS) and the negative ideal solution (NIS) of each objective function with fuzzy coefficient under the given constraints. Note that the values of PIS and NIS are calculated through solving the multi-objective problem as a single objective using, each time, only one objective.

$$\text{PIS}: f_{bj}^{*} = \{\text{Max (Min)} f_{bj}(X_j) \ (f_{cj}(X_j), \ \forall b \ (\forall c)\} \tag{9}$$

$$\text{NIS}: f_{bj}^{-} = \{\text{Min (Max)} f_{bj}(X_j) \ (f_{cj}(X_j), \ \forall b \ (\forall c)\} \tag{10}$$

$f_{bj}(X_j)$Benefit objective for maximization

$f_{cj}(X_j)$Cost objective for maximization

Step 4.Applying PISand NISfrom the results of step 3, Constructthefunctions of d^{PIS}as shorter distance from the PISand d^{NIS} as farther distance from NIS for each sub problem.

$$d_i^{PIS} = \sum_{j \in B_i} w_j \left(\frac{f_{ij}^{*} - f_{ij}}{f_{ij}^{*} - f_{ij}^{-}} \right) + \sum_{j \in C_i} w_j \left(\frac{f_{ij} - f_{ij}^{*}}{f_{ij}^{-} - f_{ij}^{*}} \right) \tag{11}$$

$$d_i^{NIS} = \sum_{j \in B_i} w_j \left(\frac{f_{ij} - f_{ij}^{-}}{f_{ij}^{*} - f_{ij}^{-}} \right) + \sum_{j \in C_i} w_j \left(\frac{f_{ij}^{-} - f_{ij}}{f_{ij}^{-} - f_{ij}^{*}} \right) \tag{12}$$

In order to obtain a compromise solution, the following bi-objective problem is introduced:

$$\text{Min } d_i^{PIS} \tag{13}$$
$$\text{Max } d_i^{NIS}$$
$$X \in FS_i$$

We can utilize a single objective instead of problem (13) based on a max-min decision making model. This method is proposed by Bellman and Zadehand extended by Zimmermann (Bellman &Zadeh, 1970; Zimmermann, 1987; Abo-Sinna et al., 2008). The steps of this model are shown in following steps:

Step 4-1. Construct the two membership functions for$d^{PIS}$and d^{NIS},respectively. As shown in Fig. 1.

, Fig. 2.

The linear membership function for the negative (ord^{PIS}) objective can be defined as:

$$\mu_1(x) = \frac{(d_i{}^{PIS}) - (d_i{}^{PIS})^*}{(d_i{}^{PIS})^- - (d_i{}^{PIS})^*} \quad (14)$$

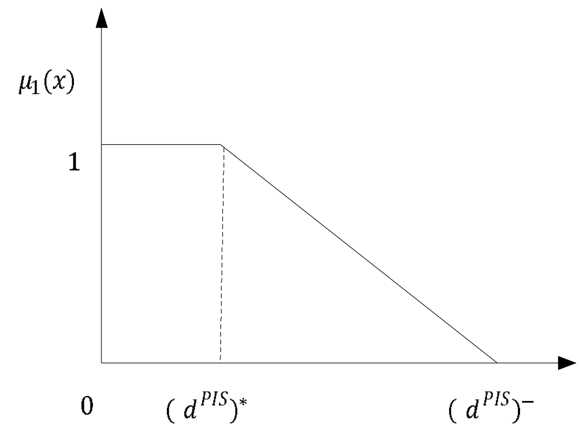

Fig. 1. The membership function of $\mu_1(\chi)$

The linear membership function for the positive (ord^{NIS}) objective can be defined as:

$$\mu_2(x) = \frac{(d_i{}^{NIS})^* - (d_i{}^{NIS})}{(d_i{}^{NIS})^* - (d_i{}^{NIS})^-} \quad (15)$$

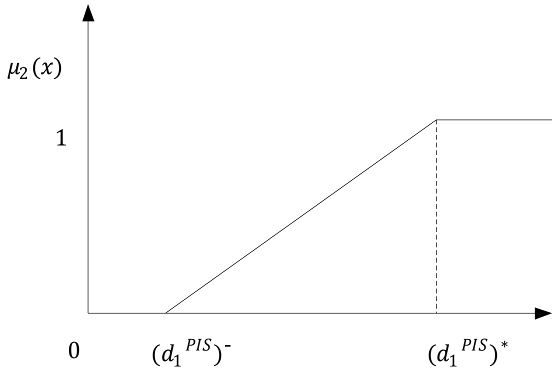

Fig. 2. The membership function of $\mu_2(\chi)$

Step 4-2. Construct the final single objective problem for each sub problem based on the membership functions.Then solve it to obtain the final optimal solution.The problem(13) is equivalent to the form of following problem as:

$$\max \lambda$$

$$S.t. \begin{cases} \dfrac{(d_1{}^{PIS}) - (d_1{}^{PIS})^*}{(d_1{}^{PIS})^- - (d_1{}^{PIS})^*} \geq \lambda \\ \dfrac{(d_1{}^{PIS})^* - (d_1{}^{NIS})}{(d_1{}^{NIS})^* - (d_1{}^{NIS})^-} \geq \lambda \end{cases} \quad (16)$$

$$0 \leq \lambda \leq 1, \qquad X \in FS_1$$

The final compromised solution and satisfactory level are obtained by solving problem (16). The flowchart of proposed TOPSIS method based on Dantzig-wolf decomposition method is depicted in Fig. 3.

4. Illustrative Numerical Example

The proposed compromised method is demonstrated by an illustrative example in this paperthat has three objective functions. The objective functions and constrains are proposed as linear on R^3 where the coefficient of the objective functions and constraints are assumed as triangular fuzzy numbers. Moreover the weights of objective functions are samefor all sub problems.The linear programming example is proposed as:

P:

$$\max f_1(x) = (1,2,3)x_1 + (2,4,6)x_2 + (1,3,5)x_3$$

$$\max f_2(x) = (1,3,5)x_1 - (2,5,7)x_2 - (1,2,3)\,x_3 \quad (17)$$

$$\max f_3(x) = (2,4,6)x_1 + (1,3,5)x_2 - (3,6,9)\,x_3$$

Subjectto:

$$FS = \begin{Bmatrix} (1,3,5)x_1 + (2,4,6)x_2 - (1,2,3)x_3 \leq (4,8,12) \\ (0,0,0) \leq (2,4,6)x_1 \leq (5,10,15) \\ (0,0,0) \leq (1,2,3)x_2 \leq (2,5,8) \\ (0,0,0) \leq (1,3,5)x_3 \leq (1,5,9) \end{Bmatrix}$$

Then Step by Step solutionof the problem is given below.

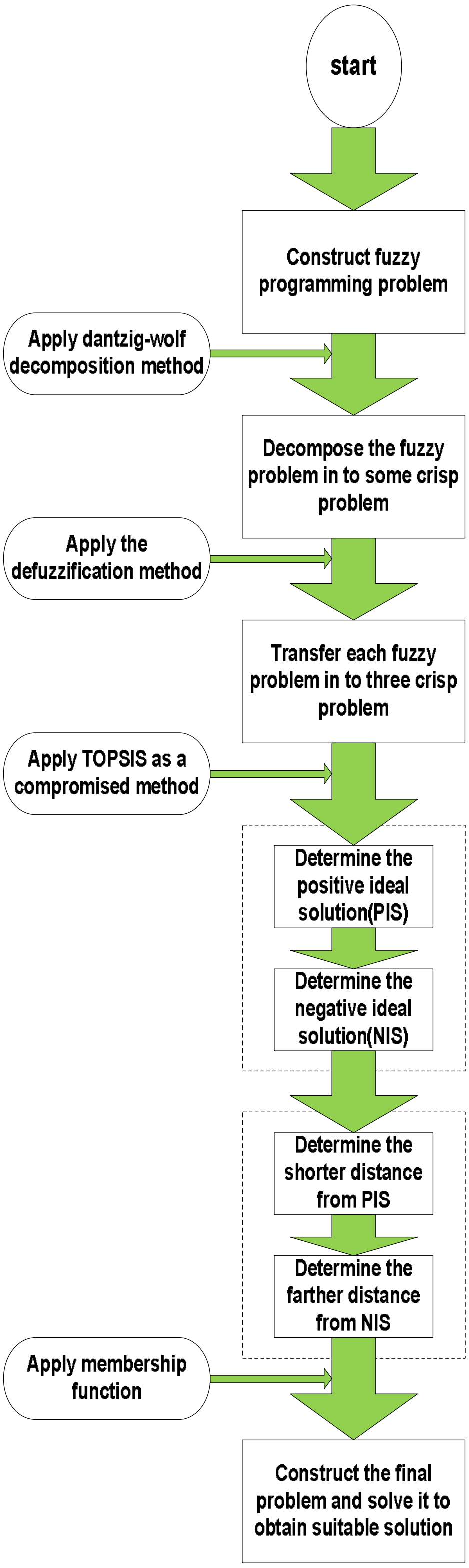

Fig. 3. The flowchart of proposed TOPSIS solution method

Step 1.Decompose the original programming problem in to three sub problems, because the programming problem is introduced on R^3 . The decomposed sub problemsP_1, P_2andP_3 are proposed as:

P:

P_1:

$$\max f_1(x) = (1,2,3)x_1$$
$$\max f_2(x) = (1,3,5)x_1 \quad (18)$$
$$\max f_3(x) = (2,4,6)x_1$$

$$FS_1 = \begin{Bmatrix} (1,3,5)x_1 + (2,4,6)x_2 - (1,2,3)x_3 \leq (4,8,12) \\ (0,0,0) \leq (2,4,6)x_1 \leq (5,10,15) \end{Bmatrix}$$

P_2:

$$\max f_1(x) = (2,4,6)x_2 \quad (19)$$
$$\max f_2(x) = -(2,5,7)x_2$$
$$\max f_3(x) = (1,3,5)x_2$$

$$FS_2 = \begin{Bmatrix} (1,3,5)x_1 + (2,4,6)x_2 - (1,2,3)x_3 \leq (4,8,12) \\ (0,0,0) \leq (1,2,3)x_2 \leq (2,5,8) \end{Bmatrix}$$

P_3:

$$\max f_1(x) = (1,3,5)x_3$$
$$\max f_2(x) = -(1,2,3)\,x_3 \quad (20)$$
$$\max f_3(x) = -(3,6,9)\,x_3$$

$$FS_3 = \begin{Bmatrix} (1,3,5)x_1 + (2,4,6)x_2 - (1,2,3)x_3 \leq (4,8,12) \\ (0,0,0) \leq (1,3,5)x_3 \leq (1,5,9) \end{Bmatrix}$$

Step 2.UsingEqs. (5)–(8), Transfereach fuzzy programming problem in to three crisp problems.Because the coefficients of objective functions and constraints are assumed as triangular fuzzy numbers, Sub problems P_1, P_2andP_3 are transferred in to three crisp objective functions programming problems. The sub problem P_1 can be transfer as follow:

P_1:

$P_{11}: \min f_1(x) = x_1$	$P_{12}: \min f_1(x) = 2x_1$	$P_{13}: \min f_1(x) = 2x_1$
$\max f_2(x) = 2x_1$	$\max f_2(x) = 3x_1$	$\max f_2(x) = 4x_1$
$\max f_3(x) = x_1$	$\max f_3(x) = 2x_1$	$\max f_3(x) = 2x_1$
Subject to:	Subject to:	Subject to:
$X \in FS_1$(21)	$X \in FS_1$(22)	$X \in FS_1$(23)

Similar to first problem, the second problem can be transfer in to three crisp sub problems as:

P_2:

$P_{21}: \min f_1(x) = 2\,x_2$	$P_{22}: \min f_1(x) = -3\,x_2$	$P_{23}: \min f_1(x) = 2\,x_2$
$\max f_2(x) = 4x_2$	$\max f_2(x) = -5x_2$	$\max f_2(x) = 3x_2$
$\max f_3(x) = 2\,x_2$	$\max f_3(x) = -2\,x_2$	$\max f_3(x) = 5\,x_2$
Subject to:	Subject to:	Subject to:
$X \in FS_2$(24)	$X \in FS_2$(25)	$X \in FS_2$(26)

The following crisp sub problems are transferred from third fuzzy sub problem as:

P_3:

P_{31}: $\min f_1(x) = 2x_3$	P_{32}: $\min f_1(x) = -x_3$	P_{33}: $\min f_1(x) = -3x_3$
$\max f_2(x) = 3x_3$	$\max f_2(x) = -2x_3$	$\max f_2(x) = -6x_3$
$\max f_3(x) = 5x_3$	$\max f_3(x) = -3x_3$	$\max f_3(x) = -9x_3$
Subject to:	Subject to:	Subject to:
$X \in FS_3$(27)	$X \in FS_3$(28)	$X \in FS_3$(29)

Step 3. Appling TOPSIS method, calculate the individual PIS and NIS of each objective function for sub problems P_1, P_2 and P_3. The obtained PIS, NIS of sub problem P_1 are shown in Tables1,2.

PIS: $f_{11}^* = (f_1^*, f_2^*, f_3^*) = (0.0000, 5.0000, 2.5000)$.

$f_{12}^* = (f_1^*, f_2^*, f_3^*) = (0.0000, 7.5000, 5.0000)$

$f_{13}^* = (f_1^*, f_2^*, f_3^*) = (0.0000, 10.0000, 5.0000)$.

Table 1
PIS payoff table of (P_1)

		f_1	f_2	f_3	x_1	x_2	x_3
	$\min f_1$	0.0000*	0.0000	0.0000	0.0000	0.0000	0.0000
P_{11}	$\max f_2$	2.5000	5.0000*	2.5000	2.5000	0.0000	0.1667
	$\max f_3$	2.5000	5.0000	2.5000*	2.5000	0.0000	0.1667
	$\min f_1$	0.0000*	0.0000	0.0000	0.0000	0.0000	0.0000
P_{12}	$\max f_2$	5.0000	7.5000*	5.0000	2.5000	0.0000	0.1667
	$\max f_3$	5.0000	7.5000	5.0000*	2.5000	0.0000	0.1667
	$\min f_1$	0.0000*	0.0000	0.0000	0.0000	0.0000	0.0000
P_{13}	$\max f_2$	5.0000	10.0000*	5.0000	2.5000	0.0000	0.1667
	$\max f_3$	5.0000	10.0000	5.0000*	2.5000	0.0000	0.1667

Table 2
NIS payoff table of (P_1)

		f_1	f_2	f_3	x_1	x_2	x_3
	$\max f_1$	2.5000⁻	5.0000	2.5000	2.5555	0.0000	0.1667
P_{11}	$\min f_2$	0.0000	0.0000⁻	0.0000	0.0000	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000⁻	0.0000	0.0000	0.0000
	$\max f_1$	5.000⁻	7.5000	5.0000	2.5000	0.0000	0.1667
P_{12}	$\min f_2$	0.0000	0.0000⁻	0.0000	0.0000	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000⁻	0.0000	0.0000	0.0000
	$\max f_1$	5.0000⁻	10.0000	5.0000	2.5000	0.0000	0.1667
P_{13}	$\min f_2$	0.0000	0.0000⁻	0.0000	0.0000	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000⁻	0.0000	0.0000	0.0000

NIS: $f_{11}^- = (f_1^-, f_2^-, f_3^-) = (2.5000, 0.0000, 0.0000)$.

$f_{12}^- = (f_1^-, f_2^-, f_3^-) = (5.0000, 0.0000, 0.0000)$

$f_{13}^- = (f_1^-, f_2^-, f_3^-) = (5.0000, 0.0000, 0.0000)$

Step 4. Applying PISand NISfrom the results of step 3, Construct the functions of d^{PIS} as shorter distance from the PISand d^{NIS} as farther distance from NIS for each sub problem.The values d^{PIS} and d^{NIS} for problem d_1 are calculated as follows:

$$d_1^{PIS} = \frac{1}{3}\left(\frac{x_1-0.0000}{2.5000-0.0000}\right) + \frac{1}{3}\left(\frac{5.0000-2x_1}{5.0000-0.0000}\right) + \frac{1}{3}\left(\frac{2.5000-x_1}{2.5000-0.0000}\right) + \frac{1}{3}\left(\frac{2x_1-0.0000}{5.0000-0.0000}\right) + \frac{1}{3}\left(\frac{5.0000-3x_1}{7.5000-0.0000}\right) + \frac{1}{3}\left(\frac{2.5000-2x_1}{5.0000-0.0000}\right) + \frac{1}{3}\left(\frac{2x_1-0.0000}{5.0000-0.0000}\right) + \frac{1}{3}\left(\frac{5.0000-3x_1}{7.5000-0.0000}\right) + \frac{1}{3}\left(\frac{2.5000-2x_1}{5.0000-0.0000}\right) \quad (30)$$

$$d_1^{NIS} = \frac{1}{3}\left(\frac{2.5000-x_1}{2.5000-0.0000}\right) + \frac{1}{3}\left(\frac{2x_1-0.0000}{5.0000-0.0000}\right) + \frac{1}{3}\left(\frac{x_1-0.0000}{2.5000-0.0000}\right) +$$

$$\frac{1}{3}\left(\frac{5.0000-2x_1}{5.0000-0.0000}\right)+\frac{1}{3}\left(\frac{3x_1-0.0000}{7.5000-0.0000}\right)+\frac{1}{3}\left(\frac{2x_1-0.0000}{5.0000-0.0000}\right)+\frac{1}{3}\left(\frac{5.0000-2x_1}{5.0000-0.0000}\right)+\frac{1}{3}\left(\frac{4x_1-0.0000}{10.0000-0.0000}\right)+\frac{1}{3}\left(\frac{2x_1-0.0000}{5.0000-0.0000}\right) \quad (31)$$

The calculated values of$d^{PIS}$and d^{NIS} are proposed in Table 3.

Step 4-1. Applying $d^{PIS}$and $d^{NIS}$from the results of Table 7,the two membership function for the positive (ord^{NIS}) objective and negative (ord^{PIS}) objective can be defined as:

$\mu_1(x) = -0.2667x_1 + 0.6667$
$\mu_2(x) = -0.1333x_1 + 0.4000$

Step 4-2. Final solution is obtained by solving the single problem (34) as:

$$\max \lambda$$

$$-0.8000x_1 + 2.8889 \geq \lambda \quad (34)$$
$$-0.1333x_1 + 0.8333 \geq \lambda$$
$$0 \leq \lambda \leq 1, \qquad X \in FS_1$$

$$\lambda^* = 0.8333 x_1{}^* = 0$$

$\lambda^*$is the maximum satisfactory level and $x_1{}^*$is the final compromised solution for first sub problem.

Now, we solve the second sub problem by using the proposed method.The individual PIS and NIS of each objective function for sub problems P_2 as shown in Tables4, 5.

PIS: $f_{21}{}^* = (f_1{}^*, f_2{}^*, f_3{}^*) = (0.0000, 8.0000, 4.0000)$. (32)

$f_{22}{}^* = (f_1{}^*, f_2{}^*, f_3{}^*) = (-6.0000, 0.0000, 0.0000)$ (33)

$f_{23}{}^* = (f_1{}^*, f_2{}^*, f_3{}^*) = (0.0000, 6.0000, 4.0000)$

Table 3
PIS payoff table of (d_1)

	${d_1}^{PIS}$	${d_1}^{NIS}$	x_1	x_2	x_3
$\min {d_1}^{PIS}$	0.4444*	1.4444⁻	2.5000	0.0000	0.1667
$\max {d_1}^{NIS}$	0.8333⁻	1.8333*	2.5000	0.0000	0.1667

Table 4
PIS payoff table of (P_2)

		f_1	f_2	f_3	x_1	x_2	x_3
P_{21}	$\min f_1$	0.0000*	0.0000	0.0000	0.0000	0.0000	0.0000
	$\max f_2$	4.0000	8.0000*	4.0000	0.0000	2.0000	0.0000
	$\max f_3$	4.0000	8.0000	4.0000*	0.0000	2.0000	0.0000
P_{22}	$\min f_1$	−6.0000*	-10.0000	-4.0000	0.0000	2.0000	0.0000
	$\max f_2$	0.0000	0.0000*	0.0000	0.0000	0.0000	0.0000
	$\max f_3$	0.0000	0.0000	0.0000*	0.0000	0.0000	0.0000
P_{23}	$\min f_1$	0.0000 *	0.0000	0.0000	0.0000	0.0000	0.0000
	$\max f_2$	4.0000	6.0000*	4.0000	0.0000	2.0000	0.0000
	$\max f_3$	4.0000	6.0000	4.0000*	0.0000	2.0000	0.0000

Table 5
NIS payoff table of (P_2)

		f_1	f_2	f_3	x_1	x_2	x_3
P_{21}	$\max f_1$	4.0000⁻	8.0000	4.0000	0.0000	2.0000	0.0000
	$\min f_2$	0.0000	0.0000⁻	0.0000	0.0000	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000⁻	0.0000	0.0000	0.0000
P_{22}	$\max f_1$	0.0000 ⁻	0.0000	0.0000	0.0000	0.0000	0.0000
	$\min f_2$	-6.0000	−10.0000⁻	-4.0000	0.0000	2.0000	0.0000
	$\min f_3$	-6.0000	-10.0000	−4.0000⁻	0.0000	2.0000	0.0000
P_{23}	$\max f_1$	4.0000⁻	6.0000	4.0000	0.0000	2.0000	0.0000
	$\min f_2$	0.0000	0.0000⁻	0.0000	0.0000	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000⁻	0.0000	0.0000	0.0000

NIS: $f_{21}{}^{-} = (f_1{}^{-}, f_2{}^{-}, f_3{}^{-}) = (4.0000, 0.0000, 0.0000)$.

$f_{22}{}^{-} = (f_1{}^{-}, f_2{}^{-}, f_3{}^{-}) = (0.0000, -10.0000, -4.0000)$

$f_{23}{}^{-} = (f_1{}^{-}, f_2{}^{-}, f_3{}^{-}) = (4.0000, 0.0000, 0.0000)$

Now calculate the amount of d^{PIS}as shorter distance from the PISand d^{NIS} as farther distance from NIS for second sub problem as:

$$d_2{}^{PIS} = \frac{1}{3}\left(\frac{2x_2-0.0000}{4.0000-0.0000}\right) + \frac{1}{3}\left(\frac{8.0000-4x_2}{8.0000-0.0000}\right) + \frac{1}{3}\left(\frac{4.0000-2x_2}{4.0000-0.0000}\right) + \frac{1}{3}\left(\frac{-3x_2+6.0000}{0.0000+6.0000}\right) + \frac{1}{3}\left(\frac{8.0000+5x_2}{8.0000+10.0000}\right) + \frac{1}{3}\left(\frac{4.0000+2x_2}{4.0000+4.0000}\right) + \frac{1}{3}\left(\frac{2x_2-0.0000}{4.0000-0.0000}\right) + \frac{1}{3}\left(\frac{6.0000-3x_2}{6.0000-0.0000}\right) + \frac{1}{3}\left(\frac{4.0000-2x_2}{4.0000-0.0000}\right) \quad (35)$$

$$d_2{}^{NIS} = \frac{1}{3}\left(\frac{4.0000-2x_2}{4.0000-0.0000}\right) + \frac{1}{3}\left(\frac{4x_2-0.0000}{8.0000-0.0000}\right) + \frac{1}{3}\left(\frac{2x_2-0.0000}{4.0000-0.0000}\right)\frac{1}{3}\left(\frac{0.0000+3x_2}{0.0000+6.0000}\right) + \frac{1}{3}\left(\frac{-5x_2-0.0000}{8.0000+10.0000}\right) + \frac{1}{3}\left(\frac{-2x_2-0.0000}{4.0000+4.0000}\right) + \frac{1}{3}\left(\frac{4.0000-2x_2}{4.0000-0.0000}\right) + \frac{1}{3}\left(\frac{3x_2-0.0000}{6.0000-0.0000}\right) + \frac{1}{3}\left(\frac{2x_2-0.0000}{4.0000-0.0000}\right) \quad (36)$$

The values of$d^{PIS}$and d^{NIS} of second sub problem are proposed in Table 6.

The membership functions for $d_2{}^{PIS}$ and $d_2{}^{NIS}$ are proposed respectively in Eqs (37), (38).

$$\mu_1(x) = -0.7334x_2 + 0.6266 \quad (37)$$
$$\mu_2(x) = -0.5001x_2 + 1 \quad (38)$$

Solving final single objective programming problem, the compromised solution for second sub problem is obtained.

$$\max \lambda$$
$$-0.7334x_2 + 0.6266 \geq \lambda (39)$$
$$-0.5001x_2 + 1 \geq \lambda$$
$$0 \leq \lambda \leq 1, \qquad X \in FS_2$$
$$\lambda^* = 0.5556 x_2{}^* = 0$$

Similar to sub problemsP_1,P_2, the values of PIS and NIS for sub problem P_3,are proposed in Tables7, 8.

PIS: $f_{31}{}^{*} = (f_1{}^{*}, f_2{}^{*}, f_3{}^{*}) = (0.0000, 3.0000, 2.0000)$.

$f_{32}{}^{*} = (f_1{}^{*}, f_2{}^{*}, f_3{}^{*}) = (-1.0000, 0.0000, 0.0000)$

$f_{33}{}^{*} = (f_1{}^{*}, f_2{}^{*}, f_3{}^{*}) = (-3.0000, 0.0000, 0.0000)$

NIS: $f_{31}{}^{-} = (f_1{}^{-}, f_2{}^{-}, f_3{}^{-}) = (2.0000, 0.0000, 0.0000)$

$f_{32}{}^{-} = (f_1{}^{-}, f_2{}^{-}, f_3{}^{-}) = (0.0000, -2.0000, -3.0000)$

$f_{33}{}^{-} = (f_1{}^{-}, f_2{}^{-}, f_3{}^{-}) = (0.0000, -6.0000, -3.0000)$

Appling Eqs (11), (12), we compute the values$d_3{}^{PIS}$and $d_3{}^{NIS}$ as:

$$d_3{}^{PIS} = \frac{1}{3}\left(\frac{2x_3-0.0000}{2.0000-0.0000}\right) + \frac{1}{3}\left(\frac{3.0000-3x_3}{3.0000-0.0000}\right) + \frac{1}{3}\left(\frac{2.0000-2x_3}{2.0000-0.0000}\right) + \frac{1}{3}\left(\frac{-x_3+10.0000}{0.0000+10.0000}\right) + \frac{1}{3}\left(\frac{8.0000+2x_3}{0.0000+2.0000}\right) + \frac{1}{3}\left(\frac{4.0000+3x_3}{0.0000+3.0000}\right) + \frac{1}{3}\left(\frac{-3x_3+3.0000}{0.0000+3.0000}\right) + \frac{1}{3}\left(\frac{0.0000+6x_3}{0.0000+2.0000}\right) + \frac{1}{3}\left(\frac{0.0000+3x_3}{0.0000+3.0000}\right) \quad (40)$$

$$d_3{}^{NIS} = \frac{1}{3}\left(\frac{2.0000-2x_3}{2.0000-0.0000}\right) + \frac{1}{3}\left(\frac{3x_3-0.0000}{3.0000-0.0000}\right) + \frac{1}{3}\left(\frac{2x_3-0.0000}{2.0000-0.0000}\right) + \frac{1}{3}\left(\frac{0.0000+x_3}{0.0000+10.0000}\right) + \frac{1}{3}\left(\frac{-2x_3+2.0000}{0.0000+2.0000}\right) + \frac{1}{3}\left(\frac{-3x_3+3.0000}{0.0000+3.0000}\right) + \frac{1}{3}\left(\frac{0.0000+3x_3}{0.0000+3.0000}\right) + \frac{1}{3}\left(\frac{-6x_3+6.0000}{0.0000+6.0000}\right) + \frac{1}{3}\left(\frac{-3x_3+3.0000}{0.0000+3.0000}\right) \quad (41)$$

The values of $d_3{}^{PIS}$and $d_3{}^{NIS}$are proposed in Table 9.

Using Eqs(14), (15),$\mu_1(x)$and$\mu_2(x)$ can be obtained as follows:

$$\mu_1(x) = x_3 + 3.3333 \quad (42)$$
$$\mu_2(x) = 0.6551x_3 + 0.3448 \quad (43)$$

After using the proposed method, the resulting solution and the maximum satisfactory level is obtained for sub problem 3 as:

$$\max \lambda$$
$$x_3 + 3.3333 \geq \lambda (44)$$
$$0.6551x_3 + 0.3448 \geq \lambda$$
$$0 \leq \lambda \leq 1, \qquad X \in FS_3$$
$$\lambda^* = 0.99996 x_3{}^* = 1$$

The maximum satisfactory level ($\lambda^* = 0.99996$) is achieved for the compromised solution $x_3{}^* = 1$.

Table 6
PIS payoff table of (d_2)

	$d_2{}^{PIS}$	$d_2{}^{NIS}$	x_1	x_2	x_3
min $d_2{}^{PIS}$	1.7500*	2.1667⁻	0.0000	2.0000	0.0000
max $d_2{}^{NIS}$	0.6667⁻	1.3148*	0.0000	2.0000	0.0000

Table 7
PIS payoff table of (P_3)

		f_1	f_2	f_3	x_1	x_2	x_3
	$\min f_1$	0.0000^*	0.0000	0.0000	0.0000	0.0000	0.0000
P_{31}	$\max f_2$	2.0000	3.0000^*	2.0000	0.0000	0.0000	1.0000
	$\max f_3$	2.0000	3.0000	2.0000^*	0.0000	0.0000	1.0000
	$\min f_1$	-1.0000^*	-2.0000	-3.0000	0.0000	0.0000	1.0000
P_{32}	$\max f_2$	0.0000	0.0000^*	0.0000	0.0000	0.0000	0.0000
	$\max f_3$	0.0000	0.5000	0.0000^*	0.0000	0.0000	0.0000
	$\min f_1$	-3.0000^*	-6.0000	-3.0000	0.0000	0.0000	1.0000
P_{33}	$\max f_2$	0.0000	0.0000^*	0.0000	0.0000	0.0000	0.0000
	$\max f_3$	0.0000	0.5000	0.0000^*	0.0000	0.0000	0.0000

Table 8
NIS payoff table of (P_3)

		f_1	f_2	f_3	x_1	x_2	x_3
	$\max f_1$	2.0000^-	3.0000	2.0000	0.0000	0.0000	1.0000
P_{31}	$\min f_2$	0.0000	0.0000^-	0.0000	0.0000	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000^-	0.0000	0.0000	0.0000
	$\max f_1$	0.000^-	0.0000	0.0000	0.0000	0.0000	0.0000
P_{32}	$\min f_2$	-1.0000	-2.0000^-	-3.0000	0.0000	0.0000	1.0000
	$\min f_3$	-1.0000	-2.0000	-3.0000^-	0.0000	0.0000	1.0000
	$\max f_1$	0.0000^-	0.0000	0.0000	0.0000	0.0000	0.0000
P_{33}	$\min f_2$	-3.0000	-6.0000^-	-3.0000	0.0000	0.0000	1.0000
	$\min f_3$	-3.0000	-6.0000	-3.0000^-	0.0000	0.0000	1.0000

Table 9
PIS payoff table of (d_3)

	${d_3}^{PIS}$	${d_3}^{NIS}$	x_1	x_2	x_3
$\min {d_3}^{PIS}$	3.1111^*	4.4111^-	0.0000	0.0000	0.0000
$\max {d_3}^{NIS}$	1.0333^-	2.0000^*	0.0000	0.0000	0.0000

5. Conclusion

In this paper, the focus was on applying a TOPSIS approach as a compromise decision making method to deal with MOLSLP problems with block angular structure. Sincethe decision making parameters are not usually crisp to deal with the real word situation problems, the value of decision matrix can be presented with uncertainty. The dantzig-wolf decomposition method is utilized to decompose a*N*-dimension problem into some single-space sub-problems. Then a useful method was applied to transfer each fuzzy sub-problem to three crisp sub-problems. Moreover, the fuzzy constraints were changed into crisp constraints. Then the proposed TOPSIS method was applied to obtain a suitable compromise solution. To obtain compromise solution of original problem, the individual positive ideal solution (PIS) and negative ideal solution (NIS) were calculated for each objectiveas presented in Tables 1,2,4,5,7,8. The concept of membership function was introduced and applied to aggregate the objective functions in each sub-problemas shown in Tables 3, 6, 9. Therefore,this method can help the decision maker when the coefficient of objective functions and constraint is not crisp and the problem is large-scale. Hence, it can be argued that this method can be applied to a large number of issues dealing with the real world problems.Finally, to justify the proposed method, an illustrative example was provided. The objective functions and constraints may be proposed as a fuzzy non-linear programming problem. In addition, the programming problem can be proposed as a non-convex problem. These subjects provide a new opportunity for further research.

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