A Hybrid Method Based on Fuzzy AHP and VIKOR for the Discrete Time-Cost-Quality Trade-off Problem

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Abstract

Time, cost and quality are considered as the main components in managing each project. Previous studies have mainly focused on the time-cost trade-off problems. Recently quality is considered as the most important factor in project's success, which is influenced by time acceleration. That is the less time is spent, the more success is gained. In time-cost-quality trade-off problems, each activity can be done in various execution modes and determination of these execution modes is seen as to minimize the project time and cost and maximize its quality. In this paper, three integer programming models are provided and one of the main objectives is optimized in each model by assigning the proper bound to other objectives. Following the non-dominated solutions obtained by solving models, and by means of hybrid approach of Fuzzy AHP strategy and VIKOR method regarded as multi-criteria decision making methods, the best possible alternative (from among non-dominated solutions) has been suggested.

Key words: Project management, Time-cost-quality trade-off Problems, Multi-criteria decision making, Fuzzy AHP strategy, VIKOR method.

1. Introduction

Shorter time, lower cost and higher quality are regarded as the main aims in a project. These three factors influence each other constantly. One of the main aspects of project management is understanding the appropriate information about the optimal balance between these objectives.

The critical path method (CPM) was introduced by Kelly and Walker (1959) in the late 1950 as a useful tool in planning and scheduling the projects. In the calculations of this method, it is assumed that all activities can be executed in their normal and predicted time. In some cases, however, it is necessary to complete the project even earlier than the specified date. According to various policies and aims this date is usually determined by the employer or high-level management. It is obvious that in reaching shorter accomplished time, the time of some activities should be reduced. This reduction of time (known as crash activity time) is accompanied with increasing use of resources and spending more costs. On the other hand, performing the activities in a longer duration not only decreases the activity costs but it may also lead to an increase in the project's duration that may incur certain penalties (Kelly, 1961). In relation to these advances and penalties, making a comprehensive and correct decision is a rigid challenge for managers. So, in this regard, time-cost trade-off was considered as the important issue.

In practice, one of the most fundamental measures for project success is its quality, which can be influenced by time acceleration and additional costs (Babu and Suresh, 1996). The goal of time-cost-quality trade-off is selecting a subset of activities for accelerating and selecting the proper execution modes so that the total project time and cost would minimize and its overall quality would maximize. In solving the time-cost and time-cost-quality trade-off problems, exact and heuristic methods have been mainly used. We often come to some non-dominated solutions in solving time-cost-quality trade-off problems that do not have any superiority or precedence over each other. It means that we cannot find a solution to be worse than another solution in all objective functions. In Practice, however, it is possible that the importance of the objective functions not be the same for the decision maker. For example, it is likely that the project's completion time would be more important than its quality and costs. An appropriate approach for choosing the best solution in solving the multi-objective problems can be using hybrid approaches from multi-criteria decision-

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making methods, which have not been used in the field of project management yet Therefore a hybrid approach of fuzzy hierarchical analysis and VIKOR has been applied in this paper where the significance of the objective functions is, at first, determined through Fuzzy AHP strategy and finally the best choice is picked up by VIKOR method from among the non-dominated solutions.

In the following, the paper includes these sections: in section 2, we overview the related literature. In section 3, we provide the formulation of problem model. In section 4 the used method for evaluation the solutions (non-dominated ones) are explained. In section 5, the computational experiences are given. Finally, the concluding remarks are offered in section 6.

2. Literature Review

Assuming that the direct cost of an activity is changed by altering the performance time, mathematical programming models have been developed to minimize the direct costs. In the literature, these problems are known as continuous time-cost trade-off problems. This problem was studied for the first time by Kelly (1961). They considered a linear relationship between time and cost of an activity, and proposed a mathematical model and a heuristic algorithm to solve it. Fulkerson (1961) provided a solution method to determine complete time-cost trade-off curve. In other words, for the realization of a project, efficient couples can be obtained related to the time and cost for all activities in order to minimize, under a limit on the project delivery time, the total cost of the project. In addition, other forms of activity cost-duration functions were studied by Falk and Horowitz (1972) and Kapur (1973). Moder et al. (1983) considered a general and continuous activity cost function and approximated the cost-duration curve by linear segments, then they tried to solve the simplified problem.

In many practical cases, the resources are accessible in discrete units, such as number of machine, equipment, workers and so on. In the literature this problem is known as multi-mode problem or discrete time-cost trade-off problem; the best execution modes (time, cost) are determined for activities to optimize one objective with some constraint. The basic formulation of Discrete Time-Cost Trade-off Problem (DTCTP) was provided by Meyer and Schaffer (1963). Later, other researchers began to improve the DTCTP mathematical model. Talbot (1982) introduced a zero-one programming model for standard multi-mode problems. In these problems, opposite to DTCTP that use only one non-renewable resource, several resources (including renewable and non-renewable resource) are used. Prabuddha et al. (1997) stated that DTCTP is NP-hard. Recent studies on this problem have paid much attention to solution procedures, which are classified into exact algorithms and heuristic algorithms. Exact algorithms are based on dynamic programming, branch and bound algorithms and enumeration algorithms. In addition, different metaheuristic algorithms were used to solve DTCTP, and we can point to Liu et al. (2000) research as an example.

Babu and Suresh (1996), as the first researchers, suggested that the project quality may be influenced by project acceleration. They assumed the cost and quality of each activity change linearly with change in project completion time. They optimized each objective by assigning desired bounds on the other objectives. Then the suggested model was applied in an actual cement factory construction project in Thailand by Khang and Myint (1999). These problems can be classified as time-cost-quality trade-off problems .El-Rayes and Kandil (2005), for the first time, studied Discrete Time-Cost-Quality Trade-off Problem (DTCQTP). They used a real world instance and suggested a new function to consider product quality in time-cost-quality optimization problems for construction industries. Then DTCQTP problem was studied by Tareghian and Tahery (2006). They developed inter-related binary linear programming models that assumed project’s activities are performed in one of several execution modes. Also, the time and quality of each activity is a non-increasing function of a non-renewable resource. Later this problem considered more and various metaheuristic algorithms based on genetic algorithm, ant colony algorithm etc. was developed to solve it. Iranmanesh et al. (2008) suggest a metaheuristic based on a genetic algorithm to solve multi-objective time-cost-quality trade-off problem and finding pareto solutions. Ravishankar et al. (2010) extended the traditional DTCTP to a new discrete resource quality constrained time cost-trade off problem (DRQTCTP), which involves renewable resources, non-renewable resources and quality constraints. Each execution mode has respective time, cost and quality and based on its nature, its devoted quality level is between zero and one. Hong and Feng (2010) suggest a fuzzy-multi-objective particle swarm optimization procedure to solve time-cost-quality trade-off problems. In this study time, cost and quality are described by means of fuzzy numbers. Shahsavari et al. (2012) develop a model for DTCQTP in which one mode is chosen for each activity among several possible execution modes and assume that the time and cost of each mode is described by means of crisp numbers but their quality is stated by linguistic variable. They developed a new hybrid genetic algorithm to solve it. A new multi-objective multi-mode model was proposed for solving preemptive time–cost–quality trade-off project scheduling problems by Tavana et al. (2014). Huan Zheng (2014) studied the time-cost-quality-environment trade-off problem of construction project and established a multi-objective decision making model under a fuzzy environment. Furthermore, a fuzzy based adaptive-hybrid genetic algorithm was developed for finding feasible solutions.

As mentioned above, in solving DTCTP and DTCQTP problem, exact solving procedures (for small instances) and heuristic and metaheuristic algorithms have been applied. Often, decision makers want to choose the best solution among various solutions (non-dominated solution) so that the desired levels of all measures are provided. Therefore, some researchers use multi-criteria decision-making methods or combination of them to compare the different solutions. Hybrid approaches of these methods have been used in the subjects such as Innovation capital indicator assessment of Taiwanese Universities (Wu, Chen and I-Shuo Chen, 2010), Marketing Strategy Selection (Mohaghar et al., 2012), benchmarking analysis in the hotel industry (Hsin-Pin Fu et al., 2011), Evaluating performance of Iranian cement firms (Rezaie et al., 2014), conservation priority assessment in coastal areas: Case of Khuzestan district (Pourebrahim et al., 2014), etc.

Usually, solving the time-cost-quality trade-off Problems eventually leads to a number of non-dominated solutions that their numbers could be many in large scale problems. The situation makes selection of the proper execution modes difficult and sometimes confusing. Unlike previous researches that have only focused on presenting non-dominated solutions, this paper tries to help decision makers to choose the best solution (among non-dominated solutions) by providing a hybrid approach of fuzzy hierarchical analysis and VIKOR method.

3. Problem Formulation

Here, the project is defined as a directed graph G(V,E), where V represents the set of nodes and E is the set of arcs. The project is displayed by activity on node network (AON) where the nodes show the project activities and their arcs represent the precedence relations. For each activity $i \in V$ in a project, M_i is a set of various execution modes of activity i where for each execution mode such as k, a threefold (t,c,q) is assigned which show the time, cost and quality of an activity in that mode, respectively, so that $t \in Z$, $c \in Z$ and $0<q \in Z<100$.

It is assumed, if r and k are two modes for execution activity i so that $k \prec r$ then $t_{ik}>t_{ir}$ and $c_{ik}<c_{ir}$ but $q_{ik} \neq q_{ir}$. The goal of this paper is to achieve the optimal compound (t_{ik}, c_{ik}, q_{ik}) of each activity to perform the project in order to minimize the time and cost and maximize the quality.

The time and cost of each activity is assumed as discrete and a function of a non-renewable resource (such as money, budget …). In defining DTCQTP, the following notations are used:

Parameters:

V: set of nodes (activities), $V = \{1, 2, \ldots, n\}$

E: set of arcs

M_i: set of execution modes for activity i ,$i \in V$

t_{ik}: duration of activity i in mode k, $i \in V$,k=1,…,$|M_i|$

c_{ik}: cost of performing activity i in mode k, $i \in V$, k=1,…,$|M_i|$

q_{ik}: quality of performing activity i in mode k, $i \in V$, k=1,…,$|M_i|$

w_i: weight of activity i such that $\sum_{i=1}^{n} w_i = 1$, $i \in V$

T_{max}: upper bound for project deadline

C_{max}: upper bound for project cost

Q_{min}: lower bound for the overall quality of project

Decision variable:

S_i: start time of activity i

x_{ik}: if activity is done in mode k, x_{ik} =1, otherwise x_{ik}=0

Here, three inter-related integer programming models have been proposed as a framework to analyze trade-off between time, cost and quality factors, so that each model optimized one of these factors by assigning desired bounds on two other factors. The three objective functions are defined as follow:

$$f_1 = S_n + \sum_{k=1}^{|M_i|} t_{nk} x_{nk} \tag{1}$$

$$f_2 = \sum_{i=1}^{n} \sum_{k=1}^{|M_i|} c_{ik} x_{ik} \tag{2}$$

$$f_3 = \sum_{i=1}^{n} w_i \sum_{k=1}^{|M_i|} q_{ik} x_{ik} \tag{3}$$

The first model is followed as:

$$Min\, f_1 \tag{4}$$

s.t.

$$f_2 \leq C_{max} \tag{5}$$

$$f_3 \geq Q_{min} \tag{6}$$

$$\sum_{k=1}^{|M_i|} x_{ik} = 1 \qquad \forall i \in V \tag{7}$$

$$S_i + \sum_{k=1}^{|M_i|} t_{ik} x_{ik} \leq S_j \qquad \forall i,j \in V \tag{8}$$

$$S_i \geq 0 \qquad \forall i \in V, Integer \tag{9}$$

$$x_{ik} \in \{0,1\} \qquad \forall i \in V, k \in M_i \tag{10}$$

The second model, which shares constraints (6)-(10) from the first model, is:

$$Min\, f_2 \quad (11)$$

s.t.

$$f_1 \leq T_{max} \quad (12)$$

At last the third model that share constraints (5), (7)-(10) from the first model is:

$$Max\, f_3 \quad (13)$$

s.t.

$$f_1 \leq T_{max} \quad (14)$$

Objective function (1) and (2) minimize the total project time and cost respectively, while objective function (3) maximizes the overall quality of project activities. Constraint (5) shows the upper bound of the total cost, while constraint (6) represents the lower bound of overall quality of project activities. Constraint (7) ensures that one and only one execution mode is assigned to each activity. Constraint (8) represents the precedence relations between activities. Constraint (9) ensures non-negativity of decision variables, while constraint (10) is a binary mode indicator, which is 1 when mode k is assigned to activity *i*, and 0, otherwise. At last, the constraint (12) defines the project deadline.

4. Evaluation Methods

As mentioned earlier, we often come to some non-dominated solutions in solving time-cost-quality trade-off problems that do not have any superiority over each other. In this paper, our goal is to find the best solutions by means of Fuzzy AHP and VIKOR methods based on their time, cost and quality. Although the problem is multi-Objective, at first, it has been considered as the single objective and in each time, one of the objectives have been optimized with considering various levels on two other objectives as the lower and upper bounds. comparing the various solution points (threefold compounds of time, cost and quality), the non-dominated solutions – say answer A dominates answer B, if answer A in any objective be no worse than answer B and also the answer A is at least in one of the objective that is better than answer B. if there is no such situation between A and B, say answer A and B are non-dominated solutions- are detected. Then, Fuzzy AHP is used to determine the weight of each problem objectives that influence on selecting the various alternatives (from non-dominated solutions). After this, the obtained weights are used in VIKOR method calculations. Ultimately, the various alternatives are ranked by VIKOR method. The general framework of suggested method is shown in figure 1.
Following the various steps of Fuzzy AHP and VIKOR methods are illustrated, briefly.

4.1. Fuzzy AHP method

Hierarchical analysis process (AHP), as one of the most comprehensive and usable methods of decision-making with multi-criteria, reflects natural behavior and human thoughts. This technique evaluates the complex problems based on their mutual impact, and converts them into a simplified form and then solves them. The method that was introduced for the first time by Saaty (1980) is able to consider the qualitative and quantitative criteria.
However, AHP facilitates the decision making procedure by pairwise comparisons, but pairwise comparisons are done with real (crisp) numbers. On the other hand, because the human evaluations may be vague and mental judgment –which this is one of the typical features in decision-making problems- so it seems using AHP with real number to explicit evaluation of relative importance of criteria and the performance of alternative towards criteria to be insufficient. Therefore, Fuzzy AHP was introduced to evaluate the problems in ambiguity and uncertainty situations.
Numerous methods are suggested for Fuzzy AHP. In this paper Chang's extent fuzzy AHP approach is used (Chang, 1996). Such as classic AHP procedure, Fuzzy AHP has a pairwise comparison matrix in which instead of constant numbers, triangular fuzzy numbers are used. Here for pairwise comparison among criteria, we use the linguistic variable and respective fuzzy number based on the Saati's nine-point scale, which their advantage is simplicity in inexact pairwise comparisons. This linguistic variable and the corresponding fuzzy numbers are provided in Table 1.

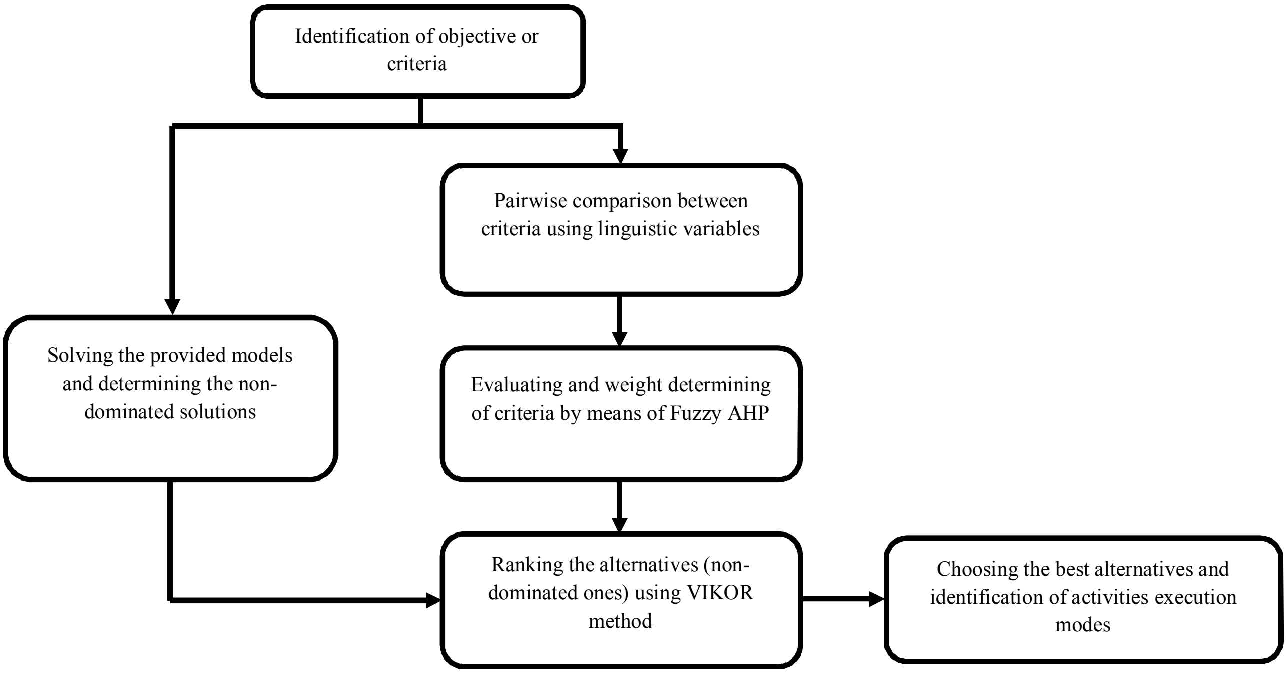

Fig. 1: General framework of suggested method

Table 1
Fuzzy comparison measures

Linguistic terms	Triangular fuzzy numbers
Equal importance	(1,1,1)
Middle value between 1 and 3	(1,2,3)
Weak importance	(2,3,4)
Middle value between 3 and 5	(3,4,5)
Strong importance	(4,5,6)
Middle value between 5 and 7	(5,6,7)
Very strong importance	(6,7,8)
Middle value of 7 and 9	(7,8,9)
Absolute importance	(8,9,10)

Let Ã is considered as a n×n pairwise comparison matrix including triangular fuzzy numbers $\tilde{a}_{ij}$, $i,j \in \{1,2,\ldots,n\}$ as following:

$$\tilde{A} = \begin{bmatrix} (1,1,1)\tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21}(1,1,1) & \cdots & \tilde{a}_{2n} \\ \vdots \quad \vdots & \ddots & \vdots \\ \tilde{a}_{n1}\tilde{a}_{n2} & \cdots & (1,1,1) \end{bmatrix}$$

Where $\tilde{a}_{ij}= (l_{ij}, m_{ij}, u_{ij})$ are triangular fuzzy numbers. Assume that $\widetilde{M}_1= (l_1, m_1, u_1)$ and $\widetilde{M}_2=(l_2, m_2, u_2)$ are two triangular fuzzy number, the basic operations are defined as:

$$\widetilde{M}_1 \oplus \widetilde{M}_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \tag{15}$$

$$\widetilde{M}_1 \otimes \widetilde{M}_2 = (l_1 l_2, m_1 m_2, u_1 u_2) \tag{16}$$

$$\widetilde{M}_1^{-1} = (\frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1}) \tag{17}$$

The general steps of Chang's extent fuzzy AHP approach are given as following:
Step 1- Calculating the summation of each row of Fuzzy decision matrix to get the fuzzy number vector RS.

$$RS=\begin{bmatrix}rs_1\\rs_2\\\vdots\\rs_n\end{bmatrix}=\begin{bmatrix}\sum_{j=1}^{n}\tilde{a}_{1j}\\\sum_{j=1}^{n}\tilde{a}_{2j}\\\vdots\\\sum_{j=1}^{n}\tilde{a}_{nj}\end{bmatrix}=\begin{bmatrix}(\sum_{j=1}^{n}l_{1j},\sum_{j=1}^{n}m_{1j},\sum_{j=1}^{n}u_{1j})\\(\sum_{j=1}^{n}l_{2j},\sum_{j=1}^{n}m_{2j},\sum_{j=1}^{n}u_{2j})\\\vdots\\(\sum_{j=1}^{n}l_{nj},\sum_{j=1}^{n}m_{nj},\sum_{j=1}^{n}u_{nj})\end{bmatrix}\quad(18)$$

Step 2- normalizing the rows of RS fuzzy vector to get fuzzy synthetic extent value vector S.

$$\tilde{S}=\begin{bmatrix}\tilde{s}_1\\\tilde{s}_2\\\vdots\\\tilde{s}_n\end{bmatrix}=\begin{bmatrix}rs_1\otimes\left(\sum_{i=1}^{n}rs_i\right)^{-1}\\rs_2\otimes\left(\sum_{i=1}^{n}rs_i\right)^{-1}\\\vdots\\rs_n\otimes\left(\sum_{i=1}^{n}rs_i\right)^{-1}\end{bmatrix}\quad(19)$$

Where $(\sum_{i=1}^{n}rs_i)^{-1}$ is calculated as follow:

$$\left(\sum_{i=1}^{n}rs_i\right)^{-1}=\left(\frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n}u_{ij}},\frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n}m_{ij}},\frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n}l_{ij}}\right)\quad(20)$$

Step 3- Computing the degree of possibility to get the non-fuzzy weight vector V.$\tilde{s}_1$

$$V=\begin{bmatrix}v_1\\v_2\\\vdots\\v_n\end{bmatrix}=\begin{bmatrix}\min V(\tilde{s}_1\geq\tilde{s}_i)\\\min V(\tilde{s}_2\geq\tilde{s}_i)\\\vdots\\\min V(\tilde{s}_n\geq\tilde{s}_i)\end{bmatrix},\quad i\in\{1,2,\dots,n\}\quad(21)$$

Where the degree of possibility of $\tilde{s}_2=(l_2, m_2, u_2)\geq\tilde{s}_1=(l_1, m_1, u_1)$ is obtained by (22).

$$V(\tilde{s}_2\geq\tilde{s}_1)=\begin{cases}1, & if\quad m_2\geq m_1\\0, & if\quad l_1\geq u_2\\\frac{l_1-u_2}{(m_2-u_2)-(m_1-l_1)}, & otherwise\end{cases}\quad(22)$$

Step 4- Define the final non-fuzzy normalization weight vector W.

$$w=\begin{bmatrix}w_1\\w_2\\\vdots\\w_n\end{bmatrix}=\begin{bmatrix}v_1/\sum_{i=1}^{n}v_i\\v_2/\sum_{i=1}^{n}v_i\\\vdots\\v_n/\sum_{i=1}^{n}v_i\end{bmatrix}\quad(23)$$

4.2. VIKOR method

VIKOR is a multi-criteria decision-making in solving discrete decision-making problems with conflicting and non-commensurable criteria (various measurement units). This method innovated by Opricovic and Tzeng (2007) focused on the ranking and selecting from a set of alternatives and determine the compromise solutions for a problem with conflicting criteria. The compromise solution is a feasible solution that is the nearest solution to the ideal solution. The compromise solution means it is based on mutual agreement between criteria. VIKOR method has been developed based on Lp-metric:

$$L_{p,i}=\left\{\sum_{j=1}^{n}\left[\mu_j\times\frac{(f_j^*-f_{ij})}{(f_j^*-f_j^-)}\right]^p\right\}^{1/p}\quad 1\leq p\leq+\infty\ ,\quad i=1,2,\dots,m\quad(24)$$

Where μ_j is the weight of the jth criterion, f_{ij} is the rating (score) of the jth criterion for ith alternative and f_j^* and f_j^- denote the best (positive ideal) and the worst (negative ideal) value of the scores, respectively. This method can provide a maximum group utility for the majority and a minimum of an individual regret for the opponent .If p is small, the group utility is concerned and if p increases, the individual regrets receive more weight. The advantage of this method over other methods, mainly TOPSIS, is that VIKOR method uses the linear normalization. So the normalized values in VIKOR are independent of criteria measurement unit. On the other hand, in VIKOR method, always compromise solution is the closest alternative to ideal solution, while TOPSIS does not consider the distance relative importance from positive and negative ideal solutions, that is why the best solution in TOPSIS is not necessarily the closest alternative to positive ideal solution.

The steps of this method are as follows:

Step 1: Construct comparison matrix: this matrix is constructed with regard to the evaluation of all alternatives based on various criteria. Assume that having a multi-criteria decision-making problem with m alternatives and n criteria, x_{ij} denotes the performance of ith alternative based on jth criteria.

$$X=\begin{bmatrix}x_{11}&\cdots&x_{1n}\\\vdots&\ddots&\vdots\\x_{m1}&\cdots&x_{mn}\end{bmatrix}$$

Step 2: determination of criteria weight vector: According to the importance and rate of various criteria in decision-making the criteria weight vector is calculated using different methods. Here Fuzzy AHP method has been used for determining the weight of criteria.
Step 3: determining the best (positive ideal) and the worst (negative ideal) values: For each criterion, the best and worst alternatives were detected and were called them f_j^* and f_j^- respectively. Assuming that the criteria are profit type, then:

$$f_j^* = Max\, f_{ij} \quad , i=1,2,\ldots,m \ , \ j=1,2,\ldots,n \tag{25}$$

$$f_j^- = Min\, f_{ij} \ , i=1,2,\ldots,m \ , \ j=1,2,\ldots,n \tag{26}$$

With associates all f_j^*, we would obtain an optimal combination with the highest score (positive ideal solution). Similarly, for f_j^- a negative ideal solution is obtained.
Step 4: Computing the distance of alternatives to ideal solution.

$$S_i = \sum_{j=1}^{n} w_j \times \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \tag{27}$$

$$R_i = Max\left\{w_j \times \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)}\right\} \tag{28}$$

Where S_i represents the relative distance of the ith alternative to the positive ideal solution and R_i represents the relative distance of the ith alternative to the negative ideal solution.
Step 5: VIKOR index calculation:

$$Q_i = v\left[\frac{S_i - S^*}{S^- - S^*}\right] + (1-v)\left[\frac{R_i - R^*}{R^- - R^*}\right] \quad , \quad v \in [0,1] \tag{29}$$

Where
$S^- = \underset{i}{Max}\, S_i \quad , \quad S^* = \underset{i}{Min}\, S_i \quad ,$
$R^- = \underset{i}{Max}\, R_i \quad , \quad R^* = \underset{i}{Min}\, R_i$
when the v is bigger than 0.5, the Q_i index will lead to majority agreement, and when v is less than 0.5 the Q_i index will indicate majority negative attitude, generally when v is equal to 0.5, this shows the compromise attitude of evaluation experts.
Step 6: ranking the alternatives by sorting out each S, R and Q values in a decreasing order. Alternative a′ is proposed as a compromise solution if it has first rank based on Q value and the following two conditions are satisfied:
Condition 1: Acceptable advantage

$$Q(a'') - Q(a') \geq \frac{1}{i-1} \tag{30}$$

Where a'' is the alternative with second position in the ranking list by Q and i is the number of alternatives.
Condition 2: Acceptable stability in decision-making: Alternative a' must also be the best ranked based on S or R value or both of them.
If one of the above conditions is not satisfied, then a set of compromise solutions are suggested:
i. if only condition 2 is not satisfied then alternatives a' and a'' are suggested.
ii. If condition 1 is not satisfied then alternatives a', a'',…,a^m are suggested. And a^m is determined by the following relation for the maximum of m:

$$Q(a^m) - Q(a') < \frac{1}{i-1} \tag{31}$$

5. Computational Experiments

In order to solve the time-cost-quality trade-off problem by means of hybrid approach of Fuzzy AHP and VIKOR method, an example including a project with 15 activities is provided in an AON network (see Fig. 2). Each activity has a weight based on its importance. In this example the weight of each activity has been randomly chosen from {1/30, 2/30, 3/30, 4/30}, with considering this constraint that the summation of all weight must be equal to one. In this network, the precedence relations between activities are finish to start relation with zero lag time. For each activity there are several execution modes that is randomly chosen from the discrete uniform distribution $|M_i| \in [2,8]$. Each execution mode has three parameters (time, cost and quality). The duration of each mode is randomly sampled from DU(20, 100). Then, for each activity, the generated durations of execution modes, are sorted in an ascending order. For each activity, the corresponding cost with the activity's mode, with the shortest time which includes the maximum cost mode are selected randomly from DU(100,200). The costs for other modes are determined as follows; if c_k is the cost of an activity in mode k and t_k is the execution time of that activity in mode k, then the execution cost in mode k is sampled randomly from DU(c_{k-1}+(t_k-t_{k-1}),c_{k-1}+4(t_k-t_{k-1})). It is assumed that the acceleration of activity duration do not necessarily result in reducing its quality, so the execution quality of each mode is randomly sampled from DU(60, 99). Table 2 shows the activity weight and also time, cost and quality values of various execution modes.

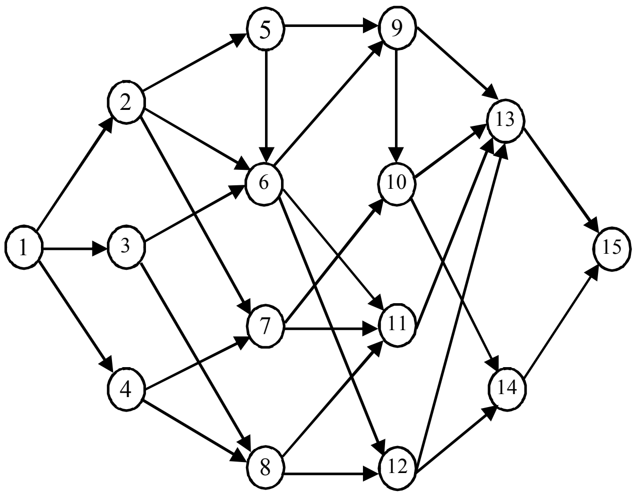

Fig.2. Project network

The suggested models in this paper are coded in GAMS 24.0.2 optimizer software on a machine with Intel Pentium processor Core i7 2.93 GHz, 6 GB of RAM. The models are solved by allocating the various bounds into two objectives and finding the optimal solution of the other objective. Some of the obtained results from solving the models are shown in Tables 3, 4 and 5. Then, among the obtained solutions from solving the models, 186 non-dominated solution were detected which some of them are shown in Table 6.

Table 2
Detailed data of the example

Activity	weight	*Mode 1*			*Mode 2*			*Mode 3*			*Mode 4*			*Mode 5*			*Mode 6*			*Mode 7*		
		T	C	Q	T	C	Q	T	C	Q	T	C	Q	T	C	Q	T	C	Q	T	C	Q
1	1/30	89	136	95	81	145	79	75	160	87												
2	2/30	81	188	97	74	198	80	71	215	86	66	229	75	59	240	67	56	253	70	54	269	65
3	1/30	59	113	82	53	128	96	47	140	76	43	155	68									
4	3/30	75	133	87	64	149	65															
5	2/30	54	134	92	48	151	81	44	168	74												
6	1/30	50	104	97	42	111	81	38	125	87	34	137	74	31	151	76	28	165	65			
7	1/30	69	148	94	61	157	77	54	161	73												
8	2/30	50	152	97	46	159	89	41	172	78	37	189	66	34	201	71						
9	2/30	89	158	84	79	170	93	72	183	74												
10	1/30	88	145	85	80	156	96	73	169	86	67	180	72									
11	3/30	85	110	86	72	123	96															
12	2/30	64	195	73	56	209	95	50	221	81	46	243	66									
13	4/30	60	104	99	51	113	79	45	125	86	40	139	76	36	158	78	31	169	66	29	187	62
14	3/30	53	148	96	50	162	81	46	175	86	41	190	72	37	211	63						
15	2/30	76	165	99	68	179	83	61	187	72	55	203	79	51	219	68	47	235	61			

Table 3
Project deadline when its quality and cost is varied

upper bound for project cost	lower bound for the overall quality of project																	
	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99
2133	587	587	587	587	587	587	587	587	587	587	587	587	587	587	INF	INF	INF	INF
2160	563	563	563	563	563	563	563	563	563	563	563	563	563	569	577	INF	INF	INF
2190	542	542	542	542	542	542	542	542	542	542	545	545	545	547	561	INF	INF	INF
2220	524	524	524	524	524	524	524	524	524	524	524	524	529	536	543	569	INF	INF
2250	504	504	504	504	504	504	504	504	504	504	511	511	513	521	534	569	INF	INF
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
2760	424	424	424	424	424	424	424	424	430	433	445	454	467	489	519	569	INF	INF
2790	424	424	424	424	424	424	424	424	430	433	445	454	467	489	519	569	INF	INF

INF: Infeasible

Table 4
Project cost when its quality and deadline is varied

upper bound for project deadline	lower bound for the overall quality of project																	
	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99
424	2580	2580	2580	2580	2580	2580	2593	2607	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF
430	2524	2524	2524	2524	2524	2524	2524	2538	2585	INF	INF	INF	INF	INF	INF	INF	INF	INF
440	2470	2470	2470	2470	2470	2470	2470	2470	2478	2505	INF	INF	INF	INF	INF	INF	INF	INF
450	2423	2423	2423	2423	2423	2423	2423	2423	2423	2437	2459	INF	INF	INF	INF	INF	INF	INF
460	2379	2379	2379	2379	2379	2379	2379	2379	2379	2390	2405	2427	INF	INF	INF	INF	INF	INF
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
580	2140	2140	2140	2140	2140	2140	2140	2140	2140	2140	2140	2140	2140	2144	2158	2198	INF	INF
587	2133	2133	2133	2133	2133	2133	2133	2133	2133	2133	2133	2133	2133	2133	2158	2198	INF	INF

INF: Infeasible

Table 5
Project quality when its deadline and cost is varied

upper bound for project deadline	upper bound for project cost																	
	424	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	587
2133	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	91.3
2160	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	92	93.4	93.8
2190	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	91.6	92.9	94.8	94.9	94.9
2220	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	INF	89.5	92.6	94.2	94.9	95.2	95.2	95.2
2250	INF	INF	INF	INF	INF	INF	INF	INF	INF	84	90.6	92.4	94.1	94.6	94.9	95.2	95.2	95.2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
2760	79.9	81.1	84.5	86.3	88	89.3	90.4	91.2	91.9	92.6	93.2	93.9	94.3	94.6	94.9	95.2	95.2	95.2
2790	79.9	81.1	84.5	86.3	88	89.3	90.4	91.2	91.9	92.6	93.2	93.9	94.3	94.6	94.9	95.2	95.2	95.2

INF: Infeasible

Table 6
Some of the non-dominated solutions of the example

Alternatives	Time	Cost	Quality
1	497	2280	85
2	519	2310	93
3	520	2250	90.6
4	448	2430	81
5	587	2160	93.8
⋮	⋮	⋮	⋮
181	460	2460	88
182	450	2490	86.3
183	490	2340	90
184	464	2370	83
185	430	2610	81.1
186	424	2640	79.9

As mentioned in section 3.4, we want to compare the importance rate of criteria or objectives related to each other. Here we encountered with time, cost and quality criteria. Pairwise comparisons between the objectives are done by linguistic variables and corresponding Fuzzy numbers with decision-maker opinion. In this example, based on the decision-maker's opinion, pairwise comparison matrix between criteria is according to Table 7.

Table 7
Pairwise comparison between criteria

	Time	Cost	Quality
Time	(1,1,1)	(1,2,3)	(2,3,4)
Cost	(1/3,1/2,1)	(1,1,1)	(1,2,3)
Quality	(1/4,1/3,1/2)	(1/3,1/2,1)	(1,1,1)

Then, we calculate the value of fuzzy synthetic extent:

rs_1= (4, 6, 8)
rs_2= (2.33, 3.5, 5)
rs_3= (1.58, 1.83, 2.5) and $(\sum_{i=1}^{n} rs_i)^{-1}$= (0.0645, 0.0882, 0.1263)

As a result, we have:
$\tilde{s}_1$= (0.258, 0.529, 1.01)
$\tilde{s}_2$= (0.151, 0.309, 0.632)
$\tilde{s}_3$= (0.102, 0.162, 0.316)

Now for each possible double state, we calculate the degree of possibility:

$V(\tilde{s}_1 \geq \tilde{s}_2)$ =1	$V(\tilde{s}_2 \geq \tilde{s}_1)$ =0.629	$V(\tilde{s}_3 \geq \tilde{s}_1)$ =0.136
$V(\tilde{s}_1 \geq \tilde{s}_3)$ =1	$V(\tilde{s}_2 \geq \tilde{s}_3)$ =1	$V(\tilde{s}_3 \geq \tilde{s}_2)$ =0.529

At least, the minimum degree of possibility for each criteria towards other criteria and also the final non-fuzzy normalization weights of criteria are shown in Table 8.

Table 8
Minimum degree of possibility and final weights of the criteria

Criteria	minimum degree of possibility	final weights
Time	v_1=1	w_1=0.567
Cost	v_1=0.629	w_1=0.356
Quality	v_1=0.136	w_1=0.077

Until now, the weight of each criterion has been calculated using Fuzzy AHP. In the following, these values are used in VIKOR method to determine the best alternative.

At first, we construct the decision matrix, which has 186 rows or alternative (equal to number of non-dominated solutions) and 3 columns (equal to number of criteria). The first column represents the time criteria and the second and third columns represent the cost and mean quality of performing project, respectively.

$$X = \begin{bmatrix} 497 & 2280 & 85 \\ 519 & 2310 & 93 \\ \vdots & \vdots & \vdots \\ 464 & 2370 & 83 \\ 430 & 2610 & 81.1 \\ 424 & 2640 & 79.9 \end{bmatrix}$$

For each criterion, we choose the best and worst alternatives:

$f_1^* = 424$ $\quad f_2^* = 2133$ $\quad f_3^* = 95.2$
$f_1^{\,*} = 587$ $\quad f_2^{*} = 2640$ $\quad f_3^{\,*} = 76.9$

Then we calculate the S_i and R_i values for all alternatives. After determining the maximum and minimum values of S_i and R_i, we calculate the VIKOR index. In Table 9 the S_i, R_i and Q_i indexes are shown for some alternatives.

$S^* = 0.3518$ $\quad R^* = 0.1600$

$S^- = 0.5918$ $\quad R^- = 0.5670$

Ultimately, we rank the alternatives and then check the two mentioned condition in section 2-3. Table 10 shows the ranking of number of eight alternatives based on S_i, R_i and Q_i indexes.

Table 9
S_i, R_i and Q_i values of some alternatives

Alternative	Time	Cost	Quality	S_i	R_i	Q_i
1	497	2280	85	0.4001	0.2539	0.2160
2	519	2310	93	0.4640	0.3305	0.4431
⋮	⋮	⋮	⋮	⋮	⋮	⋮
184	464	2370	83	0.3596	0.1664	0.0185
185	430	2610	81/1	0.4151	0.3349	0.3469
186	424	2640	79.9	0.4204	0.3560	0.3837

Table 10
Ranking the alternatives according to Q_i, S_i and R_i values

Rank	Based on Q_i values	Based on S_i values	Based on R_i values
1	84	44	83
2	34	76	84
3	35	172	28
4	83	38	33
5	79	77	34
6	33	169	35
7	165	34	165
8	80	80	29
⋮	⋮	⋮	⋮

Examining the two conditions, we conclude that none of them are satisfied. So, the numbers of alternatives are chosen based on the minimum value of VIKOR index while the following relation is satisfied considering the maximum value of m.
Here m is equal to 3.

$$Q(a^m) - Q(a') < \frac{1}{i-1}$$

Finally, the proposed alternatives are according to Table 11. These three alternatives are among the best possible alternatives based on the determined weight for each objective. The alternative 84 has a higher time than two other alternatives, while its cost is lower than both of them and its quality is higher than alternative 34 and it is almost equal to alternative 35. In deed though alternative 34 has a lower quality rather than two other alternatives, its execution time is lower than them too.

Table 11
Proposed alternatives

Alternative	Time	Cost	Quality
84	470	2358	83.2
34	464	2368	82.9
35	467	2371	83.3

As seen in the sample example, 186 solutions were identified as non-dominated solutions with different amounts of time, cost and quality. But the number of solutions makes decision on choosing the appropriate execution mode difficult, since the level of the objective functions' importance could not be equal for decision makers. Therefore, the importance rate of each objective function was determined by Fuzzy AHP method. Finally, using VIKOR method, 3 solutions were selected, from among 186 possible alternatives, as the best solutions. Table 12 shows the activity execution modes based on the 3 suggested alternatives.

Table 12
The execution modes of activities for proposed alternatives

Proposed alternatives	Execution mode for activity i														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
84	3	5	1	1	2	4	1	1	3	4	1	1	1	1	4
34	3	6	1	1	2	3	1	1	3	4	1	1	2	1	4
35	3	6	1	1	2	4	1	1	3	4	1	1	1	1	4

6- Conclusion

Project manager should deliver the project in time, with the lowest cost and highest quality. Making decisions about these conflicting objectives is an essential issue. In this paper the discrete time-cost-quality trade-off problem is studied in which for each activity several execution modes have been defined and each execution mode has its own time, cost and quality. So, three integer programming models were presented. Each model optimized one objective (minimizing the total project time or cost and or maximizing the mean quality of project) with considering proper bounds for two other objectives. Then non-dominated solutions were detected and the best possible solutions were determined using the hybrid approach of Fuzzy AHP and VIKOR methods. Fuzzy AHP method has been used to determine the importance rate of each objective. In this method linguistic variables were used which take us closer to reality. At the end, through applying these weights in VIKOR method, the best possible alternatives (among non-dominated solutions) were found. Using this hybrid approach can help managers, to a great extent, in selecting the appropriate solution so that maximum desirability is obtained due to the importance rate of the objective functions from the viewpoint of decision maker.
Using other hybrid approaches of multi-criteria decision-making methods, considering time, cost and quality values as fuzzy numbers, developing the expressed model by generalized precedence relations and using the real world examples can be the proposed topics for future researches.

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