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Discrete-time repetitive optimal control: Robotic manipulators

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Abstract

This paper proposes a discrete-time repetitive optimal control of electrically driven robotic manipulators using an uncertainty estimator. The proposed control method can be used for performing repetitive motion, which covers many industrial applications of robotic manipulators. This kind of control law is in the class of torque-based control in which the joint torques are generated by permanent magnet dc motors in the current mode. The motor current is regulated using a proportional-integral controller. The novelty of this paper is a modification in using the discrete-time linear quadratic control for the robot manipulator, which is a nonlinear uncertain system. For this purpose, a novel discrete linear time-variant model is introduced for the robotic system. Then, a time-delay uncertainty estimator is added to the discrete-time linear quadratic control to compensate the nonlinearity and uncertainty associated with the model. The proposed control approach is verified by stability analysis. Simulation results show the superiority of the proposed discrete-time repetitive optimal control over the discrete-time linear quadratic control.

Keywords: *Discrete-Time Linear Quadratic Control, Optimal Control, Repetitive Control, Electrically Driven Robotic Manipulators, Uncertainty Estimator*.

1. Introduction

Discrete-time control is a favorite approach since the digital processors and computers have been used as common controllers. Digital control systems have more capabilities over the traditional control systems such as flexibility to changes, immunity to noises, and less number of computations [1]. The digital control was originally developed for linear systems using the famous z transform. Considering literature confirms that the discrete-time control has been developed to cover the nonlinear systems, as well. The discrete-time control of robotic manipulators was presented in various types such as sliding mode control [2], learning control [3], adaptive control [4] and [5]. In this paper, a discrete-time repetitive optimal control (DROC) is developed.

A promising control approach to track periodic signals is repetitive control. This type of control method can be used for performing repetitive motion, which covers many industrial applications of robotic manipulators. Repetitive control has gained a great deal of research interest in various forms of control approaches applied on the robot manipulators. The control performance is related to how well the uncertainty is compensated. A discrete-time repetitive control scheme was presented using the computed-torque control link to overcome a part of uncertainty model [5]. The repetitive model reference adaptive control [6] and the adaptive repetitive learning control [7] can overcome the parametric uncertainty and the periodic external disturbance. A Lyapunov-based repetitive learning control was presented to have a good tracking performance in the presence of unknown nonlinear dynamics with a known period [8]. A robust repetitive control was developed and can compensate uncertainties including the structured uncertainty and unstructured uncertainty [9]. Time delay method [10] and uncertainty estimation [11] can be used to control the robot manipulator by estimating the unknown dynamics and disturbances. Uncertainty can be well estimated by a time-delay estimator [12] or an adaptive fuzzy system [13]. The time-delay method was effectively applied to compensate uncertainty in the robust impedance control of a suspension system [12], robust repetitive control of rigid robots [9] and robust control of flexible-joint robots [14].

The repetitive adaptive control and repetitive robust control are appreciated to overcome the structured uncertainties and unstructured uncertainties, respectively. However, they may not provide an optimal control performance. The optimal control performance is a desired control goal for repetitive control, which can be achieved by Discrete-time Linear Quadratic Control (DLQC) in linear systems with no uncertainties. However, a model of robotic system is nonlinear and uncertain. Therefore, the nonlinearity and uncertainty should be compensated.

This paper introduces a novel discrete-time linear time-variant model for the robotic system to apply the DLQC. The difference between the model and actual system is considered as a lumped uncertainty. A two-term control law is proposed in which the first term is a DLQC. The second term is a robust time-delay estimator to compensate the uncertainty and nonlinearity. The obtained control is called as the discrete-time repetitive optimal control. The permanent magnet dc motor in the current mode generates the control command as the joint torque. The motor current is regulated using a proportional-integral controller.

The rest of the paper is organized as follows: Section 2 presents the discrete-time linear time-variant model for the robot manipulator. Section 3 develops the discrete-time linear quadratic control. Section 4 presents stability analysis. Section 5 illustrates simulation results. Finally, Section 6 concludes the paper.

2. Discrete-time linear time-variant model

In order to define a model-based control, some discrete-time models were presented for the robot manipulators. However, some models such as [15] are too complex and some models such as [16] are too simple. In order to apply the DLQC, a novel discrete-time linear time-variant model is introduced as follows.

Dynamics of a robotic manipulator [17] is given by

$$\mathbf{D(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=T} \tag{1}$$

where, $\mathbf{q}\in R^n$ is the vector of generalized joint positions, $\mathbf{D(q)}$ is the complete inertia matrix, $\mathbf{C(q,\dot{q})\dot{q}}$ is the centrifugal and Coriolis torque vector, $\mathbf{g(q)}$ is the gravitational torque vector.

In the proposed approach, a permanent magnet dc motor drives each joint of the manipulator in the control system. The inserted torque on the joint to drive the manipulator is the load torque of motor, which is considered as

$$\mathbf{T_m}=\mathbf{J_m}\ddot{\boldsymbol{\theta}}_\mathbf{m}+\mathbf{B_m}\dot{\boldsymbol{\theta}}_\mathbf{m}+\mathbf{rT}+\mathbf{r}\boldsymbol{\xi} \tag{2}$$

where, $\dot{\boldsymbol{\theta}}_\mathbf{m}\in R^n$ is the vector of motor velocities, $\mathbf{T}\in R^n$ is the load torque, $\mathbf{T_m}\in R^n$ is the motors torque. The $n\times n$ positive diagonal coefficient matrices $\mathbf{J_m}$, $\mathbf{B_m}$ and $\mathbf{r}$ are the inertia, damping and reduction gear ratio, respectively. $\xi\in R^n$ presents the external disturbances.

Substituting (1) into (2) and using $\dot{\boldsymbol{\theta}}_\mathbf{m}=\mathbf{r}^{-1}\dot{\mathbf{q}}$ yields

$$\begin{aligned}\mathbf{T_m}&=\mathbf{J_m r^{-1}\ddot{q}}+\mathbf{B_m r^{-1}\dot{q}}\\&+\mathbf{r}\left(\mathbf{D(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)}\right)+\mathbf{r}\xi\end{aligned} \tag{3}$$

Equation (3) can be written as

$$\mathbf{T_m}=\mathbf{M(q)\ddot{q}+N(q,\dot{q})\dot{q}+W(q)}+\mathbf{r}\boldsymbol{\xi} \tag{4}$$

Where

$$\mathbf{M(q)}=\left(\mathbf{J_m r^{-1}}+\mathbf{rD(q)}\right) \tag{5}$$

$$\mathbf{N(q,\dot{q})}=\left(\mathbf{B_m r^{-1}}+\mathbf{rC(q,\dot{q})}\right) \tag{6}$$

$$\mathbf{W(q)}=\mathbf{rg(q)} \tag{7}$$

Then, it is easy to show that

$$\begin{aligned}\mathbf{\ddot{q}}&=-\mathbf{M}^{-1}\mathbf{(q)N(q,\dot{q})\dot{q}}-\mathbf{M}^{-1}\mathbf{(q)W(q)}-\mathbf{M}^{-1}\mathbf{(q)r}\boldsymbol{\xi}\\&+\mathbf{M}^{-1}\mathbf{(q)T_m}(t)\end{aligned} \tag{8}$$

Using nominal terms in (8) obtains that

$$\begin{aligned}\mathbf{\ddot{q}}&=-\hat{\mathbf{M}}^{-1}\mathbf{(q)}\hat{\mathbf{N}}\mathbf{(q,\dot{q})\dot{q}}-\hat{\mathbf{M}}^{-1}\mathbf{(q)}\hat{\mathbf{W}}\mathbf{(q)}+\\&\hat{\mathbf{M}}^{-1}\mathbf{(q)T_m}(t)+\boldsymbol{\varphi}\end{aligned} \tag{9}$$

where, $\hat{\mathbf{M}}\mathbf{(q)}$, $\hat{\mathbf{N}}\mathbf{(q,\dot{q})}$ and $\hat{\mathbf{W}}\mathbf{(q)}$ are the nominal terms for the real terms $\mathbf{M(q)}$, $\mathbf{N(q,\dot{q})}$ and $\mathbf{W(q)}$, respectively, and $\boldsymbol{\varphi}$ is the uncertainty.

The nominal terms have the same dynamics as the real terms with parametric errors. The uncertainty $\boldsymbol{\varphi}$ is expressed by substituting (8) into (9) as

$$\begin{aligned}\boldsymbol{\varphi}&=(\hat{\mathbf{M}}^{-1}\mathbf{(q)}\hat{\mathbf{N}}\mathbf{(q,\dot{q})}-\mathbf{M}^{-1}\mathbf{(q)N(q,\dot{q})})\mathbf{\dot{q}}\\&+\hat{\mathbf{M}}^{-1}\mathbf{(q)}\hat{\mathbf{W}}\mathbf{(q)}-\mathbf{M}^{-1}\mathbf{(q)W(q)}\\&-\mathbf{M}^{-1}\mathbf{(q)r}\boldsymbol{\xi}+(\mathbf{M}^{-1}\mathbf{(q)}-\hat{\mathbf{M}}^{-1}\mathbf{(q)})\mathbf{T_m}(t)\end{aligned} \tag{10}$$

Assume that there exists a $\mathbf{T_m}(t)=\mathbf{T_{md}}(t)$ that satisfies

$$\begin{aligned}\ddot{\mathbf{q}}_{\mathbf{d}} &= -\hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{N}}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}})\dot{\mathbf{q}}_{\mathbf{d}} \\ &-\hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{W}}(\mathbf{q}_{\mathbf{d}}) + \hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\mathbf{T}_{\mathbf{md}}(t)\end{aligned} \tag{11}$$

where, $\mathbf{q}_{\mathbf{d}}$ is the desired trajectory. Subtracting (9) from (11) yields

$$\begin{aligned}\ddot{\mathbf{q}}_{\mathbf{d}} - \ddot{\mathbf{q}} &= \hat{\mathbf{M}}^{-1}(\mathbf{q})\hat{\mathbf{N}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} - \hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{N}}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}})\dot{\mathbf{q}}_{\mathbf{d}} \\ &-\boldsymbol{\varphi} + \hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\mathbf{T}_{\mathbf{md}}(t) - \hat{\mathbf{M}}^{-1}(\mathbf{q})\mathbf{T}_{\mathbf{m}}(t) \\ &+\hat{\mathbf{M}}^{-1}(\mathbf{q})\hat{\mathbf{W}}(\mathbf{q}) - \hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{W}}(\mathbf{q}_{\mathbf{d}})\end{aligned} \tag{12}$$

Or, writing it out,

$$\begin{aligned}\ddot{\mathbf{q}}_{\mathbf{d}} - \ddot{\mathbf{q}} &= -\hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{N}}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}})(\dot{\mathbf{q}}_{\mathbf{d}} - \dot{\mathbf{q}}) + \\ &\hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})(\mathbf{T}_{\mathbf{md}}(t) - \mathbf{T}_{\mathbf{m}}(t)) + \hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\boldsymbol{\psi}\end{aligned} \tag{13}$$

where, the uncertainty $\boldsymbol{\psi}$ is expressed as

$$\begin{aligned}\boldsymbol{\psi} &= (\hat{\mathbf{M}}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{M}}^{-1}(\mathbf{q})\hat{\mathbf{N}}(\mathbf{q},\dot{\mathbf{q}}) - \hat{\mathbf{W}}(\mathbf{q}_{\mathbf{d}}) \\ &-\hat{\mathbf{N}}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}}))\dot{\mathbf{q}} - (\hat{\mathbf{M}}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{M}}^{-1}(\mathbf{q}) - \mathbf{I})\mathbf{T}_{\mathbf{m}} \\ &-\hat{\mathbf{M}}(\mathbf{q}_{\mathbf{d}})\boldsymbol{\varphi} + \hat{\mathbf{M}}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{M}}^{-1}(\mathbf{q})\hat{\mathbf{W}}(\mathbf{q})\end{aligned} \tag{14}$$

where, $\mathbf{I}$ is the identify matrix. The lumped uncertainty $\boldsymbol{\psi}$ includes the parametric uncertainty, unmodelled dynamics and external disturbances. The state space form of (13) is given by

$$\dot{\mathbf{E}} = \mathbf{A}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}})\mathbf{E} + \mathbf{B}(\mathbf{q}_{\mathbf{d}})\mathbf{U} + \mathbf{B}(\mathbf{q}_{\mathbf{d}})\boldsymbol{\psi} \tag{15}$$

where, $\mathbf{E}$ is the state vector, $\mathbf{U}$ the input vector, $\mathbf{A}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}})$ the state matrix and $\mathbf{B}(\mathbf{q}_{\mathbf{d}})$ a gain matrix. The details are

$$\mathbf{A}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}}) = \begin{bmatrix}\mathbf{0} & \mathbf{I} \\ \mathbf{0} & -\hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\hat{\mathbf{N}}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}})\end{bmatrix} \quad \mathbf{E} = \begin{bmatrix}\mathbf{q}_{\mathbf{d}} - \mathbf{q} \\ \dot{\mathbf{q}}_{\mathbf{d}} - \dot{\mathbf{q}}\end{bmatrix}$$

$$\mathbf{B}(\mathbf{q}_{\mathbf{d}}) = \begin{bmatrix}\mathbf{0} \\ \hat{\mathbf{M}}^{-1}(\mathbf{q}_{\mathbf{d}})\end{bmatrix} \quad \mathbf{U} = \mathbf{T}_{\mathbf{md}}(t) - \mathbf{T}_{\mathbf{m}}(t) \tag{16}$$

The proposed model (15) has an advantage that $\mathbf{A}(\mathbf{q}_{\mathbf{d}},\dot{\mathbf{q}}_{\mathbf{d}})$ and $\mathbf{B}(\mathbf{q}_{\mathbf{d}})$ are known in advance, however, this model includes the uncertainty $\boldsymbol{\psi}$. The proposed model is an uncertain linear time-variant system with periodical coefficients. I obtain from (15) a linear discrete-time time-variant system using a sampling period σ that is a small positive constant. Substituting $k\sigma$ into t for $k = 1,2,...$ and then approximating $\dot{\mathbf{E}}$ as $\dot{\mathbf{E}} = (\mathbf{E}(t+\sigma) - \mathbf{E}(t))/\sigma$ provides a discrete-time model in the form of

$$\mathbf{E}_{k+1} = \mathbf{A}_k\mathbf{E}_k + \mathbf{B}_k\mathbf{U}_k + \mathbf{B}_k\boldsymbol{\psi}_k \tag{17}$$

where, $\mathbf{E}_k = \mathbf{E}(k\sigma)$, $\mathbf{A}_k = \mathbf{I} + \sigma\mathbf{A}(\sigma k)$, $\mathbf{B}_k = \sigma\mathbf{B}(\sigma k)$, $\mathbf{U}_k = \mathbf{U}(\sigma k)$ and $\boldsymbol{\psi}_k$ denotes the uncertainty. Since $\mathbf{A}_k$ and $\mathbf{B}_k$ are available, they can be computed in advance.

3. Discrete-time repetitive optimal control

A two-term control law is proposed to track the desired trajectory. The first term is DLQC and the second term is a robust time-delay controller. Thus, system (17) is presented as

$$\mathbf{E}_{k+1} = \mathbf{A}_k\mathbf{E}_k + \mathbf{B}_k\mathbf{U}_{1,k} + \mathbf{B}_k\mathbf{U}_{2,k} + \mathbf{B}_k\boldsymbol{\psi}_k \tag{18}$$

where, $\mathbf{U}_{1,k}$ and $\mathbf{U}_{2,k}$ are the first and second terms of control input. The control performance is improved if the lumped uncertainty $\boldsymbol{\psi}_k$ is compensated. The uncertainty is perfectly compensated if

$$\mathbf{B}\mathbf{U}_{2,k} = -\mathbf{B}\boldsymbol{\psi}_k \tag{19}$$

Since $\boldsymbol{\psi}_k$ is not known, control law (19) cannot be defined. To estimate the uncertainty, I obtain from (18)

$$\mathbf{B}\boldsymbol{\psi}_k = \mathbf{E}_{k+1} - \mathbf{A}\mathbf{E}_k - \mathbf{B}\mathbf{U}_{1,k} - \mathbf{B}\mathbf{U}_{2,k} \tag{20}$$

Since $\mathbf{E}_{k+1}$ is not available in the kth step, $\mathbf{B}\boldsymbol{\psi}_k$ cannot be calculated. Instead, the previous value of $\mathbf{B}\boldsymbol{\psi}_k$ is used as

$$\mathbf{B}\boldsymbol{\psi}_{k-1} = \mathbf{E}_k - \mathbf{A}\mathbf{E}_{k-1} - \mathbf{B}\mathbf{U}_{1,k-1} - \mathbf{B}\mathbf{U}_{2,k-1} \tag{21}$$

The term $\mathbf{B}\boldsymbol{\psi}_{k-1}$ can be calculated since all terms in the RHS of (21) are known and available. Thus, a robust control law is proposed as

$$\mathbf{B}\mathbf{U}_{2,k} = -\mathbf{B}\boldsymbol{\psi}_{k-1} \tag{22}$$

The second term in the control law is expressed by substituting (21) into (22) to yield the robust time-delay controller [9]

$$\mathbf{B}\mathbf{U}_{2,k} = -\mathbf{E}_k + \mathbf{A}\mathbf{E}_{k-1} + \mathbf{B}\mathbf{U}_{1,k-1} + \mathbf{B}\mathbf{U}_{2,k-1} \tag{23}$$

Substituting (22) into (18) yields

$$\mathbf{E}_{k+1} = \mathbf{A}\mathbf{E}_k + \mathbf{B}\mathbf{U}_{1,k} + \mathbf{B}(\boldsymbol{\psi}_k - \boldsymbol{\psi}_{k-1}) \tag{24}$$

In order to apply the DLQC, a nominal model in the form of discrete-time linear system is suggested from (24) as

$$\mathbf{E}_{k+1} = \mathbf{A}\mathbf{E}_k + \mathbf{B}\mathbf{U}_{1,k} \tag{25}$$

Then, the DLQC is given by

$$\mathbf{U}_{1,k} = -\mathbf{K}_k\mathbf{E}_k \tag{26}$$

The gain matrix $\mathbf{K}_k$ is calculated by minimizing a cost function of [1]

$$L = 0.5\mathbf{E}_N^*\mathbf{S}\mathbf{E}_N + \frac{1}{2}\sum_{k=0}^{N-1}\left\{\begin{aligned}&\left(\mathbf{E}_k^*\mathbf{Q}\mathbf{E}_k + \mathbf{U}_{1,k}^*\mathbf{R}\mathbf{U}_{1,k}\right) + \boldsymbol{\lambda}_{k+1}^*\left(\mathbf{A}_k\mathbf{E}_k + \mathbf{B}_k\mathbf{U}_{1,k} - \mathbf{E}_{k+1}\right)\\ &+\left(\mathbf{A}_k\mathbf{E}_k + \mathbf{B}_k\mathbf{U}_{1,k} - \mathbf{E}_{k+1}\right)^*\boldsymbol{\lambda}_{k+1}\end{aligned}\right\} \tag{27}$$

With respect to $\mathbf{E}_k$, $\mathbf{U}_{1,k}$ and $\boldsymbol{\lambda}_k$, where $\boldsymbol{\lambda}_k$ is the Lagrange multiplier, $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{S}$ are symmetric positive definite matrices. As a result,

$$\mathbf{K}_k = [\mathbf{R} + \mathbf{B}_k^*\mathbf{p}_k\mathbf{B}_k]^{-1}\mathbf{B}_k^*\mathbf{p}_k\mathbf{A}_k \tag{28}$$

Where $\mathbf{p}_k$ is calculated as

$$\mathbf{p}_k = \mathbf{Q} + \mathbf{A}_k^*\mathbf{p}_{k-1}\mathbf{A}_k - \mathbf{A}_k^*\mathbf{p}_{k-1}\mathbf{B}_k[\mathbf{R} + \mathbf{B}_k^*\mathbf{p}_{k-1}\mathbf{B}_k]^{-1}\mathbf{B}_k^*\mathbf{p}_{k-1}\mathbf{A}_k \tag{29}$$

The algorithm starts from $k = 0$ in (29), where $\mathbf{p}_{-1} = \mathbf{0}$. Then, $\mathbf{K}_k$ is calculated as (28). Next, $\mathbf{U}_{1,k}$ is computed from (26).

The discrete-time repetitive optimal control (DROC) is formed using (23) and (26) as

$$\mathbf{U}_k = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T. \left(-(\mathbf{I} + \mathbf{B}\mathbf{K}_k)\mathbf{E}_k + \mathbf{A}\mathbf{E}_{k-1} + \mathbf{B}\left(\mathbf{U}_{1,k-1} + \mathbf{U}_{2,k-1}\right)\right) \tag{30}$$

In which from (16) $\left(\mathbf{B}^T\mathbf{B}\right)^{-1} = \hat{\mathbf{M}}^2(\mathbf{q_d})$ and

$$\mathbf{U}_k = \mathbf{T}_{\mathbf{md}k} - \mathbf{T}_{\mathbf{m}k}^* \tag{31}$$

Calculating $\mathbf{T}_{\mathbf{md}k}$ from (11), $\mathbf{T}_{\mathbf{m}k}^*$ is obtained from (31) as

$$\mathbf{T}_{\mathbf{m}k}^* = \hat{\mathbf{M}}(\mathbf{q}_{\mathbf{d}k})\ddot{\mathbf{q}}_{\mathbf{d}k} + \hat{\mathbf{N}}(\mathbf{q}_{\mathbf{d}k}, \dot{\mathbf{q}}_{\mathbf{d}k})\dot{\mathbf{q}}_{\mathbf{d}k} + \hat{\mathbf{W}}(\mathbf{q}_{\mathbf{d}k}) - \mathbf{U}_k \tag{32}$$

where, $\mathbf{U}_k$ is computed by (30).

The vector of motor torques $\mathbf{T}_{\mathbf{m}k}$ is proportional to the vector of motor currents $\mathbf{I}_k$ as

$$\mathbf{T}_{\mathbf{m}k} = \mathbf{K}_\mathbf{m}\mathbf{I}_k \tag{33}$$

where, $\mathbf{K}_\mathbf{m}$ is the torque coefficient matrix. Thus

$$\mathbf{T}_{\mathbf{m}k}^* = \mathbf{K}_\mathbf{m}\mathbf{I}_{\mathbf{d},k} \tag{34}$$

Then, it is easy to show

$$\mathbf{I}_{\mathbf{d},k} = \mathbf{K}_\mathbf{m}^{-1}\mathbf{T}_{\mathbf{m}k}^* \tag{35}$$

where, $\mathbf{I}_{\mathbf{d},k}$ is the desired armature current.

A proportional integral controller is proposed to control the electric motors for generating the desired torque (34) as

$$\mathbf{V}_k = \mathbf{K}_\mathbf{p}\left(\mathbf{e}_k - \mathbf{e}_{k-1}\right) + \sigma\mathbf{K}_\mathbf{I}\mathbf{e}_k + \mathbf{V}_{k-1} \tag{36}$$

where, $\mathbf{e}_k = \mathbf{I}_{d,k} - \mathbf{I}_k$, $\mathbf{V} \in R^n$ represents a vector of motor voltages as the input of robotic system.

4. Stability analysis

To make the dynamics of tracking error well-defined in such a way that the robot can track the desired trajectory, the following assumptions are made:

Assumption 1: The desired trajectory $\mathbf{q_d}$ must be smooth in the sense that $\mathbf{q_d}$ and its derivatives up to a necessary order are available and all uniformly bounded.

Smoothness of the desired trajectory can be guaranteed by proper trajectory planning.

As a necessary condition to design a robust controller, the matching condition must be satisfied:

Matching condition: the uncertainty must be entered into the system the same channel as the control input. Then, the uncertainty is said to satisfy the matching condition [18] or equivalently it is said to be matched. I ensure the matching condition since in the system (15), the lumped uncertainty $\boldsymbol{\psi}$ enters the system the same channel as the control input $\mathbf{U}$.

As a necessary condition to design a robust control, the external disturbance $\boldsymbol{\xi}$ in (2) must be bounded.

Assumption 2: The external disturbance $\boldsymbol{\xi}$ is bounded as

$$\|\boldsymbol{\xi}\| \le \xi_{\max} \tag{37}$$

where, $\xi_{\max}$ is a positive constant.

The voltage of every motor should be limited to protect the motor against over voltages. For this purpose, every motor is equipped with a voltage limiter. Therefore, the following assumption is made:

Assumption 3: The voltages of motors are constrained as

$$\|\mathbf{V}\| \le V_{\max} \tag{38}$$

where, $V_{\max}$ is the template value of motor voltage.

The robust discrete-time linear quadratic is formed using (23) and (26) as

$$\begin{aligned}\mathbf{B}_k\mathbf{U}_k &= -\left(\mathbf{I} + \mathbf{B}_k\mathbf{K}_k\right)\mathbf{E}_k + \mathbf{A}_k\mathbf{E}_{k-1}\\ &\quad+\mathbf{B}_k\left(\mathbf{U}_{1,k-1} + \mathbf{U}_{2,k-1}\right)\end{aligned} \tag{39}$$

Applying the control law (39) on the system (17) and using (21) results in the closed-loop system

$$\mathbf{E}_{k+1} = \left(\mathbf{A}_k - \mathbf{B}_k \mathbf{K}_k\right)\mathbf{E}_k + \mathbf{B}_k\left(\boldsymbol{\psi}_k - \boldsymbol{\psi}_{k-1}\right) \quad (40)$$

The lumped uncertainty $\boldsymbol{\psi}$ is bounded as

$$\|\boldsymbol{\psi}\| \le \psi_{\max} \quad (41)$$

where, $\psi_{\max}$ is a positive scalar.

Proof: Under assumptions 1-3 and the matching condition, it is proven in [14] that for electrically driven robot manipulators, the vector of motor velocities $\dot{\boldsymbol{\theta}}_{\mathbf{m}}$ and the vector of motor currents $\mathbf{I_a}$ are bounded. Since $\dot{\mathbf{q}} = \mathbf{r}\dot{\boldsymbol{\theta}}_{\mathbf{m}}$, the vector of joint velocities $\dot{\mathbf{q}}$ is bounded. Since $\mathbf{T_m} = \mathbf{K_m I_a}$, The vector of motor torques $\mathbf{T_m}$ is bounded. For $\forall t$, $0 \le t \le T$ where T is the operating time of the desired trajectory $\mathbf{q_d}$, it can be written that $\mathbf{q} = \int_0^t \dot{\mathbf{q}}dt + \mathbf{q}(0)$. Since $\dot{\mathbf{q}}$ is bounded, the vector of joint positions $\mathbf{q}$ is bounded.

According to the properties of robot manipulator [19], $\mathbf{D(q)}$, $\mathbf{C(q,\dot{q})\dot{q}}$ and $\mathbf{g(q)}$ in (1) are bounded as $m_1\mathbf{I} \le \mathbf{D(q)} \le m_2(\mathbf{q})\mathbf{I}$, $\|\mathbf{C(q,\dot{q})\dot{q}}\| \le m_3(\mathbf{q})\|\dot{\mathbf{q}}\|$ and $\|\mathbf{g(q)}\| \le m_4(\mathbf{q})$. The matrix $\mathbf{D(q)}$ is a positive definite symmetric matrix which is invertible, m_1 is a positive constant, $m_2(\mathbf{q})$, $m_3(\mathbf{q})$, $m_4(\mathbf{q})$ are positive definite functions of $\mathbf{q}$, and $\mathbf{I}$ is an identity matrix. Since $\mathbf{r}$, $\mathbf{J_m}$ and $\mathbf{B_m}$ are constant diagonal matrices and $\mathbf{D(q)}$, $\mathbf{C(q,\dot{q})\dot{q}}$ and $\mathbf{g(q)}$ are bounded, thus $\mathbf{M(q)}$, $\mathbf{N(q,\dot{q})}$ and $\mathbf{W(q)}$ expressed in (5), (6), and (7) are bounded. The $\hat{\mathbf{M}}(\mathbf{q})$, $\hat{\mathbf{N}}(\mathbf{q},\dot{\mathbf{q}})$ and $\hat{\mathbf{W}}(\mathbf{q})$ are the nominal terms with the same structure with $\mathbf{M(q)}$, $\mathbf{N(q,\dot{q})}$ and $\mathbf{W(q)}$. Thus, they are bounded as well. It was implied that $\dot{\mathbf{q}}$, $\mathbf{q}$ and $\mathbf{T_m}$ are bounded. The external disturbance $\boldsymbol{\xi}$ is bounded in assumption 2. Therefore, the boundedness of all terms in (10) implies that $\boldsymbol{\varphi}$ is bounded. Thus, the boundedness of all terms in the right hand side of (14) proves that $\boldsymbol{\psi}$ is bounded.

Since the DLQC provides $\mathbf{K}_k$ such that $\mathbf{A}_k - \mathbf{B}_k\mathbf{K}_k$ is Hurwitz, thus system (40) is stable. In addition, the term $\mathbf{B}_k\left(\boldsymbol{\psi}_k - \boldsymbol{\psi}_{k-1}\right)$ is a bounded input to system (40) because the lumped uncertainty $\boldsymbol{\psi}$ is bounded in (41) and $\mathbf{B}_k$ is a gain matrix. Therefore, the discrete-time linear system (40) provides a bounded output $\mathbf{E}_{k+1}$ under the bounded input $\mathbf{B}_k\left(\boldsymbol{\psi}_k - \boldsymbol{\psi}_{k-1}\right)$.

The robust time-delay control law (23) plays a main role in compensating the uncertainty. If there exists a much difference between the nominal model (25) and the actual system (24), the closed-loop system (40) is subject to a large uncertainty. The residual uncertainty in the closed-loop system (40) is reduced from a large value of $\mathbf{B}_k\boldsymbol{\psi}_k$ to a small value of $\mathbf{B}_k\left(\boldsymbol{\psi}_k - \boldsymbol{\psi}_{k-1}\right)$ due to using the robust time-delay control law (23). As a result, the performance of control system is improved by reducing the residual uncertainty. The residual uncertainty $\mathbf{B}_k\left(\boldsymbol{\psi}_k - \boldsymbol{\psi}_{k-1}\right)$ will be very small when the uncertainty is smooth and sampling time is very short.

5. Simulation results

The proposed control algorithms, namely DROC in (30) are applied on an articulated robot manipulator given by [14]. The motor parameters are given in table 1, while the three motors are the same.

Table 1. Parameters of dc servomotors.

K_m	J_m	B_m	$1/r$	R_a	L_a
0.26	0.002	0.001	100	1.26	0.001

The desired repetitive trajectory is given by

$$\mathbf{q_d} = \left[\cos(0.1\pi t) \quad \cos(0.1\pi t) \quad \cos(0.1\pi t)\right]^T \quad (42)$$

where, $\mathbf{q_d}$ is a vector of desired joint angles with a period of $20\sec$.

Simulations are presented to show the performance of proposed control laws DROC in (39) and DLQC in (26).

The desired trajectory is sufficiently smooth and the motors are sufficiently strong such that the robot can track the desired trajectory. I run the simulations for two periods to illustrate the repetitive motion.

The uncertainty may include the external disturbances, unmodelled dynamics, and parametric uncertainty. To consider the parametric uncertainty, all parameters of the nominal model used in the control law are given as 95% of the real one. The external disturbance is given to load torque of the third joint by $100\,N.m$. The uncertainty is unknown; however, I have to use an example of a bounded uncertainty to check the performance of the control system. The matrices $\mathbf{Q}$ and $\mathbf{R}$ in (28) and (29) are given by trial and error method to have a good performance through using $\mathbf{Q} = 10^8\mathbf{I}_{6\times6}$ and $\mathbf{R} = 10\mathbf{I}_{3\times3}$ where $\mathbf{I}_{n\times n}$ is the

$n \times n$ identity matrix. The matrices $\mathbf{K_p}$ and $\mathbf{K_I}$ in (36) are given by

$$\mathbf{K_p} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{K_I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}.$$

Simulation 1: The DROC in (30) for tracking control with the zero initial error is simulated. Using the sampling time of 0.001s, the tracking performance is very well such that the tracking error is under $9\times10^{-5} rad$ shown in figure 1. The sampling time of 0.001s may be too short in real-time control. Thus, the sampling time is set to 0.01s. As a result, the tracking error is under $7\times10^{-4} rad$ shown in figure 2. Compared with figure 1, the tracking error is increased if used longer sampling time. The real-time control needs a sufficient time for computation and implementation. The control efforts behave well under the permitted values shown in figure 3.

To see the effect of initial error, it is set to $\mathbf{e}(0) = \mathbf{q_d}(0) - \mathbf{q}(0) = [0.5 \quad 1.5 \quad 2]^T\ rad$.

The tracking error is reduced well from initial value to be under $2.7\times10^{-5} rad$ at the end shown in figure 4.

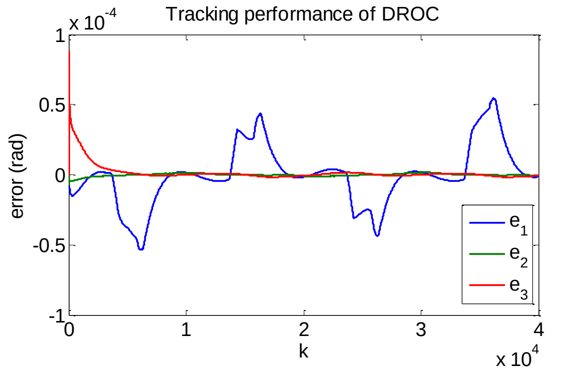

Figure 1. Tracking performance of DROC in the sampling time of 0.001s.

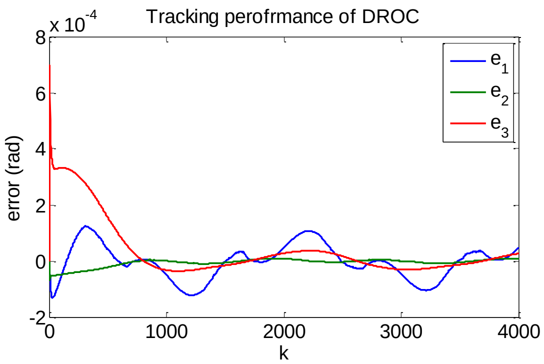

Figure 2. Tracking performance of DROC in the sampling time of 0.01s.

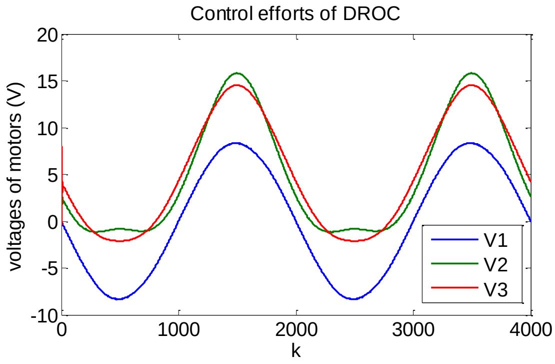

Figure 3. Control efforts of DROC.

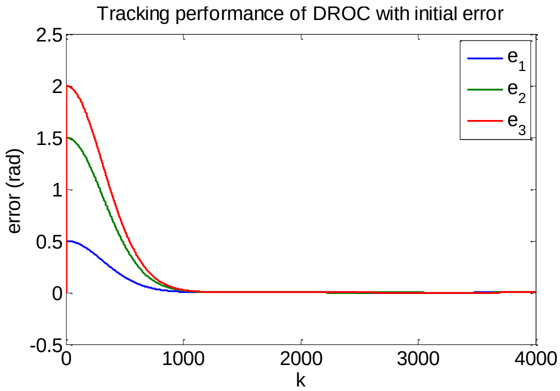

Figure 4. Tracking performance of DROC with initial error.

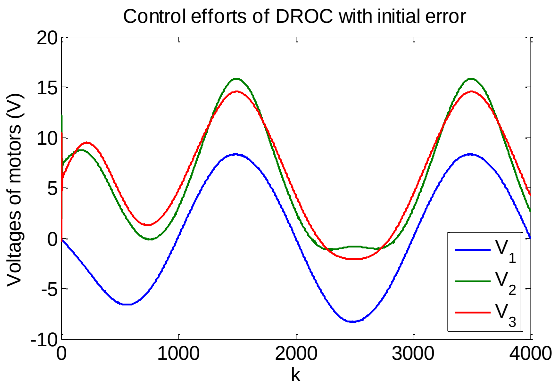

Figure 5. Control efforts of DROC with Initial error.

The control efforts behave well under the permitted values shown in figure 5.

Simulation 2: I apply the DROC in (30) for the set point control in the sampling time of 0.01s. The initial positions of the joint angles are set to $\mathbf{q}(0) = [0 \quad 0.5 \quad 2]^T\ rad$ while the position of the desired trajectory is given by $\mathbf{q_d} = [1 \quad 1 \quad 1]^T\ rad$. The initial error is calculated as

$\mathbf{e}(0) = \mathbf{q_d}(0) - \mathbf{q}(0) = [1 \quad 0.5 \quad -1]^T\ rad$.

The motor voltages are practically limited to the maximum value of $40\,V$ to protect the motors from over voltages. The set point performance is very well such that the norm of errors is vanished

well after 10s and comes under the $2.2\times10^{-7}\,rad$ in the end shown in figure 6. The motor voltages are under the permitted value of 40V and behave well without any problems shown in figure 7.
The control efforts behave well under the permitted values. As a result, the uncertainties are compensated well.
Simulation 3: I apply the DLQC in (23) for tracking control with zero initial error and the sampling time of 0.01s.
The tracking errors are under the $0.085\,rad$ shown in figure 8 and the control efforts behave well under the permitted value of $40\,V$ shown in figure 9. The maximum value of errors for the DLQC is about 121 times larger than one for the DROC.
Simulation 4: The set point performance of the DLQC is simulated with the sampling time of 0.01s. The initial errors and desired trajectories are given the same as the DROC for comparing the results. The tracking errors are vanished after $10s$ and come under $0.033\,rad$ in the end shown in figure 10.
The motor voltages are under the permitted value of 40V and behave well without any problems shown in figure 11. The maximum value of errors for the DLQC is about 1.5×10^{5} times larger than one for the DROC at the end.

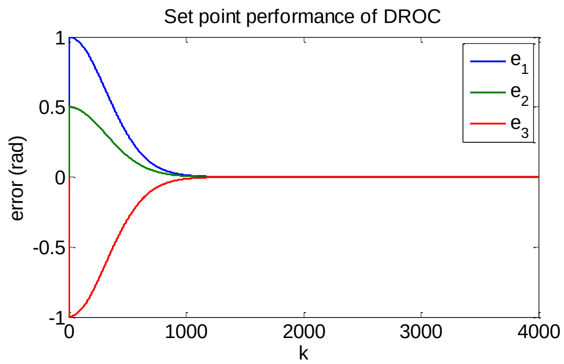

Figure 6. Set point performance of DROC.

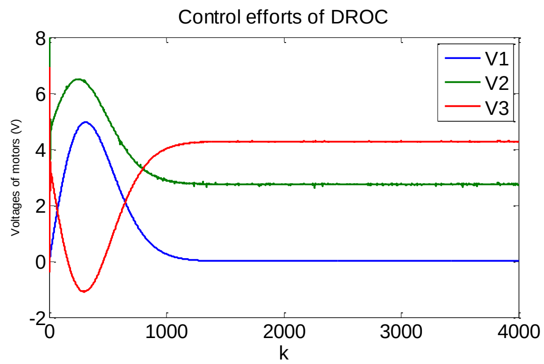

Figure 7. Control efforts of Set point DROC.

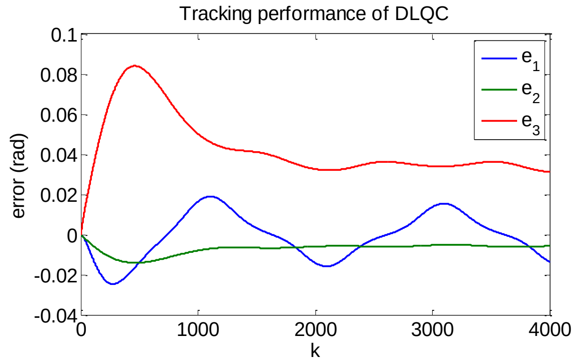

Figure 8. Tracking performance of DLQC.

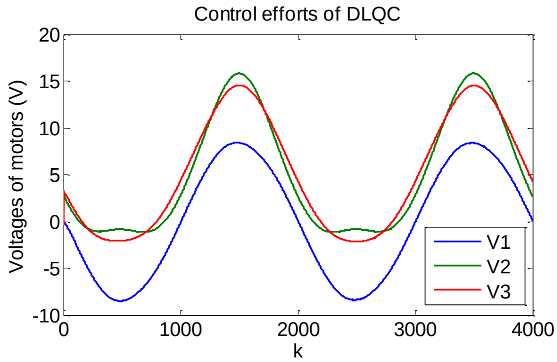

Figure 9. Control efforts of DLQC.

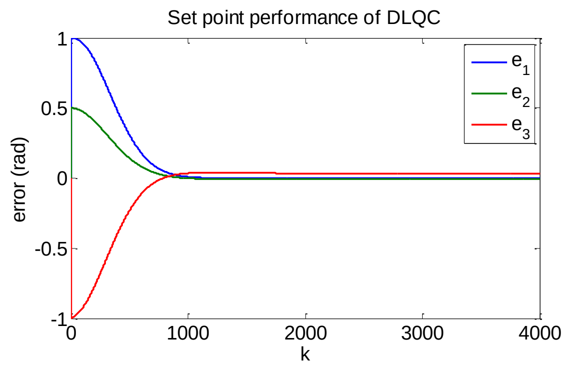

Figure 10. Set point performance of DLQC.

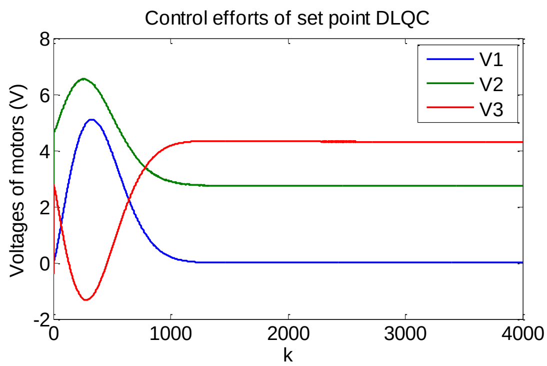

Figure 11. Control efforts of DLQC.

6. Conclusion

A novel discrete-time repetitive optimal control of electrically driven robot manipulators has been developed by a modification on the discrete linear quadratic control. The proposed control law includes two terms: The discrete linear quadratic controller and the robust time-delay controller. In order to apply the discrete linear quadratic control, a control-oriented discrete-time linear time-variant model has been proposed for the robotic system. The control-oriented model highly differs from the actual system. To compensate the model imprecision, I have used the time-delay controller. The proposed control approach has been verified by stability analysis. Simulation results have shown the superiority of the proposed control method over the discrete linear quadratic control. The time-delay controller efficiently compensates the uncertainty and nonlinearity.

References

[1] Ogata, K. (1987). Discrete Time Control Systems, Prentice-Hall.

[2] Corradini, M. L., Fossi, V., Giantomassi, A., Ippoliti, G., Longhi, S. & Orlando, G. (2012). Discrete Time Sliding Mode Control of Robotic Manipulators: Development and Experimental Validation. Control Engineering Practice, vol. 20, no. 8, pp. 816-822.

[3] Poo, A. N., Lim, K. B. & Ma, Y. X. (1996). Application of Discrete Learning Control to a Robotic Manipulator. Robotics and Computer-Integrated Manufacturing, vol. 12, no. 1, pp. 55-64.

[4] Mahmoud, M. S. & Bahnasawi, A. A. (1993). Indirect Discrete-Time Adaptive Algorithm for Manipulator Control. Applied Mathematical Modelling, vol. 17, no. 8, pp. 423-429.

[5] Tsai, M. C., Anwar, G. & Tomizuka, M. (1988) Discrete Time Repetitive Control for Robot Manipulators. IEEE International Conference on Robotics and Automation, Philadelphia, PA, 1988.

[6] Tsai, M. C. & Tomizuka, M. (1989). Model Reference Adaptive Control and Repetitive Control for Robot Manipulators. IEEE International Conference on Robotics and Automation, Scottsdale, AZ, 1989.

[7] Sun, M., Ge, S. S. & Moreels, I. M. Y. (2006). Adaptive Repetitive Learning Control of Robotic Manipulators without the Requirement for Initial Repositioning. IEEE Transactions on Robotics, vol. 22, no. 3, pp. 563-568.

[8] Dixon, W. E., Zergeroglu, E., Dawson, D. M. & Costic, B. T. (2002). Repetitive Learning Control: A Lyapunov-based Approach. IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, vol. 32, no. 4, pp. 538-545.

[9] Fateh, M. M., Ahsani Tehrani, H. & Karbassi, S.M. (2013). Repetitive Control of Electrically Driven Robot Manipulators. International Journal of Systems Science, vol. 44, no. 4, pp. 775-785.

[10] Youcef-Toumi, K. & Shortlidge, C. (1991). Control of Robot Manipulators Using Time Delay, Proc. IEEE International Conference on Robotics and Automation, Sacramento, CA, 1991.

[11] Talole, S. E. & Phadke, S. B. (2008). Model Following Sliding Mode Control Based on Uncertainty and Disturbance Estimator. ASME Journal of Dynamic, Systems, Measurement and Control, vol. 130, no. 3, pp. 034501 (5 pages).

[12] Fateh, M. M. (2009). Robust Impedance Control of a Hydraulic Suspension System. International Journal of Robust and Nonlinear Control, vol. 2, no. 8, pp. 858-872.

[13] Fateh, M. M. & Khorashadizadeh S. (2012). Robust Control of Electrically Driven Robots by Adaptive Fuzzy Estimation of Uncertainty. Nonlinear Dynamics, vol. 69, no. 3, pp. 1465-1477.

[14] Fateh, M. M. (2012). Robust Control of Flexible-Joint Robots Using Voltage Control Strategy. Nonlinear Dynamics, vol. 6, no. 2, pp. 1525–1537.

[15] Neuman, C. P. & Tourassis, V. D. (1985). Discrete dynamic robot models. IEEE Transactions on Systems Man and Cybernetics, vol. 15, no. 2, pp.193–204.

[16] Mareels, I. M. Y., Penfold, H. B. & Evans, R. J. (1992). Controlling Nonlinear Time-Varying Systems via Euler Approximations. Autotomatica, vol. 28, no. 4, pp. 681–696.

[17] Spong, M. W., Hutchinson, S. & Vidyasagar, M. (2006). Robot Modelling and Control. John Wiley & Sons, Inc..

[18] Corless, M. & Leitmann, G. (1981). Continuous State Feedback Guaranteeing Uniform Ultimate Boundedness for Uncertain Dynamics Systems. IEEE Transactions on Automatic Control, vol. 26, no. 5, pp. 1139-1144.

[19] Qu, Z. & Dawson, D. M. (1996). Robust tracking control of robot manipulators. IEEE Press, Inc., New York.

نشریه هوش مصنوعی و داده کاوی

دانشگاه شاهرود

کنترل بهینه تکراری گسسته-زمان: بازوهای رباتیک

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چکیده:

این مقاله، کنترل بهینه تکراری گسسته-زمان بازوهای رباتیک الکتریکی با بکارگیری تخمین‌گر عدم قطعیت را پیشنهاد می‌دهد. روش کنترل پیشنهادی برای اجرای حرکت تکراری به‌کار می‌رود که بسیاری از کاربردهای صنعتی بازوهای رباتیک را دربر می‌گیرد. این نوع کنترل، در گروه کنترل بر مبنای گشتاور قرار دارد که در آن گشتاور مفاصل توسط موتورهای جریان-مستقیم در مود جریان تولید می‌شود. جریان موتور توسط کنترل‌کننده تناسبی-انتگرالی تنظیم می‌شود. نوآوری مقاله، بهبود بکارگیری کنترل درجه دوم خطی گسسته-زمان برای بازوی رباتیک است که سیستمی نامعین و غیرخطی می‌باشد. برای این منظور، مدل تغییر پذیر با زمان خطی گسسته-زمان جدید برای سیستم رباتیک معرفی می‌شود. سپس، تخمین‌گر عدم قطعیت تاخیر زمانی به کنترل درجه دوم خطی گسسته-زمان اضافه می‌شود تا عدم قطعیت و بخش غیرخطی مدل را جبران نماید. روش کنترل پیشنهادی توسط تحلیل پایداری تایید می‌گردد. نتایج شبیه‌سازی، برتری کنترل بهینه تکراری گسسته-زمان پیشنهادی را بر کنترل درجه دوم خطی گسسته-زمان نشان می‌دهد.

کلمات کلیدی: کنترل درجه دوم خطی گسسته-زمان، کنترل بهینه، کنترل تکراری، بازوهای رباتیک الکتریکی، تخمین‌گر عدم قطعیت.