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Published in:
IEEE Transactions on Wireless Communications

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
Link to publication record in Queen's University Belfast Research Portal

Multiuser Cognitive Relay Networks: Joint Impact of Direct and Relay Communications

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Abstract— In this paper, we propose a multiuser cognitive relay network, where multiple secondary sources communicate with a secondary destination through the assistance of a secondary relay in the presence of secondary direct links and multiple primary receivers. We consider the two relaying protocols of amplify-and-forward (AF) and decode-and-forward (DF), and take into account the availability of direct links from the secondary sources to the secondary destination. With this in mind, we propose an optimal solution for cognitive multiuser scheduling by selecting the optimal secondary source which maximizes the received signal-to-noise ratio (SNR) at the secondary destination using maximal ratio combining. This is done by taking into account both the direct and the relay link in the multiuser selection criterion. For both AF and DF relaying protocols, we first derive closed-form expressions for the outage probability, and then provide the asymptotic outage probability, which determines the diversity behavior of the multiuser cognitive relay network. Finally, our study is corroborated by representative numerical examples.

Index Terms— Cognitive radio, maximal ratio combining, multiuser scheduling, relay networks.

I. INTRODUCTION

Cognitive radio network has emerged as a promising technique to resolve the issue of sacred radio frequency spectrum for the next generation wireless communication systems [1]–[3]. In a cognitive radio network, the secondary user is allowed to occupy the licensed spectrum of the primary user in the underlay, overlay or interweave approach [2]. For the underlay approach, the secondary user is permitted to utilize the spectrum of the primary user as long as its interference is tolerated by the primary user. With this strategy, the underlay

cognitive radio network, also known as spectrum-sharing, has been extensively studied since it requires the least hardware complexity compared with the two other approaches [4], [5]. However, since the transmit power at the secondary user is limited, the underlay cognitive network suffers a relative performance loss in comparison with its non-cognitive counterpart.

Hence, to enhance the performance of cognitive networks, relaying has been incorporated to form a cognitive relay system. Relaying transmission can improve coverage area, transmission reliability, and system capacity without requiring additional powers at the transmitters [6]. As such, it has been adopted in many recent standards, such as IEEE 802.11s, IEEE 802.16j and 3GPP LTE-Advanced [7]. Some fundamental relay protocols, namely, amplify-and-forward (AF) and decode-and-forward (DF), have been studied for cognitive relay networks [8]–[10]. In addition, the performance of cognitive relay networks has been investigated for single [8]–[10] and multiple primary receivers [11], [12]. Specifically, the authors in [8] have considered cognitive relay networks with both AF and DF relaying, and derived the asymptotic outage probability in high signal-to-noise ratio (SNR) region. The authors in [9], [10] have employed the technique of relay selection to enhance the system performance of cognitive relay networks with multiple relays, and analyzed the system capacity and outage probability. The authors in [11], [12] have investigated the impact of multiple primary receivers on the system performance of cognitive relay networks.

The works in [8]–[12] considered a severe shadowing environment and ignored the direct source to destination link. However, in a moderate shadowing environment, the direct links exist and can be utilized to enhance the system performance [13]–[17]. Specifically, the authors in [13] have studied the cognitive relay networks with a single relay, and used the maximal ratio combining (MRC) technique to combine the two branch signals from the direct and relaying links. The authors in [14]–[17] have studied multi-relay cognitive relay networks, and selected one best relay to assist the data transmission. In this model, there is one source and one destination, and it is obvious that there is only a single direct link between the source and destination. Hence, the direct link does not affect the relay selection criterion, i.e., the relay selection is performed only based on the relay links.

Although cognitive relay networks have been extensively studied in the research community, most works have only considered single-user scenario. Very recently, the performance of multiuser downlink cognitive relay networks has

been addressed in [18]. It is important to note that this model is different from the model of multi-relay cognitive relay networks, since each secondary user has its own direct link and relay link to the destination. Hence, the user selection is affected by the direct links as well as the relaying links. Due to the difference in system model as well as the joint impact of direct and relay links on selection criterion, the mathematical analysis is much more involved, for example, in cognitive relay networks [19] and non-cognitive relay networks [20], [21]. The authors in [18] have considered multiuser downlink cognitive relay networks and proposed a sub-optimal user scheduling solution, where only direct links have been used for multiuser selection. In other words, the relay links have not been exploited for the cognitive multiuser selection criterion.

In this paper, we therefore consider a multiuser uplink cognitive relay network in the presence of M primary destinations in underlay spectrum-sharing. In contrast to [18], we utilize both the direct communication and the relay communication for the selection criterion of cognitive multiuser, which makes our approach an optimal solution. Particularly, for the proposed cognitive multiuser scheduling with either AF or DF relaying, we select the optimal secondary source so as to maximize the received end-to-end SNR at the secondary destination. To the best of our knowledge, no prior work has considered the joint impact of the direct and relay links on the user selection criterion for multiuser cognitive relay networks. The contributions of our paper are summarized as follows.

- We consider both AF and DF relaying for multiuser cognitive radio networks. In addition, to exploit the relay link for cognitive radio, the direct links from the secondary source to the primary and secondary destinations are taken into account. At the destination, the direct and the relay signals are maximized using MRC, which contributes to the optimal solution for cognitive multiuser scheduling.
- We investigate the multiuser cognitive performance by governing the transmit powers of both the secondary source and the relay below the peak interference power constraint inflicted by multiple primary destinations. Due to the existence of secondary direct and relay links, the transmit power constraint at both the secondary sources and the relay is essential to guarantee quality of service (QoS) for the primary networks.
- We consider the joint impact of direct and relay links on the user selection criterion in multiuser cognitive relay networks, which makes the performance analysis much involved. More specifically, due to the existence of two common random variables (RVs): 1) the channel gains of links from secondary relay to secondary destination, and 2) the channel gains of links from secondary relay to primary user, the individual SNR is no longer independent although the fading channels are uncorrelated[1]. To get around this troublesome, we propose a novel analytical

framework by deriving a lower bound, which is extremely tight to the actual outage probability with AF relaying, while deriving an exact closed-form expression for the outage probability with DF relaying.
- We present the asymptotic outage probabilities for AF and DF relaying. From the asymptotic expressions, we show that the system diversity order is $N + 1$ for both AF and DF relaying.

Notation: The notation $\mathcal{CN}(0, \sigma^2)$ denotes a circularly symmetric complex Gaussian RV with zero mean and variance σ^2. We use $f_X(\cdot)$ and $F_X(\cdot)$ to represent the probability density function (PDF) and cumulative distribution function (CDF) of RV X, respectively. The notation A→B denotes the link from A to B, and $\Pr[\cdot]$ returns the probability.

II. SYSTEM MODEL

We consider a multiuser uplink[2] communication for a dual-hop cooperative spectrum sharing system where multiple secondary users share the same frequency spectrum with the multiple primary users[3]. Specifically, the underlay cognitive relay network is composed of N secondary users $\{SU_n | n = 1, \cdots, N\}$, an AF or DF relay R, a secondary destination D, and M primary destinations $\{PD_m | m = 1, \cdots, M\}$, as shown

Fig. 1. System model of multiuser cognitive relay network in the presence of direct links.

[1]This is one of the important aspects between our selection criterion compared to [18]. Obviously, this leads to the fact that the mathematical derivation in [18] involves only a common RV, i.e., the channel gain from secondary source to primary user, whereas our approach consists of two common terms, which is much more involved.
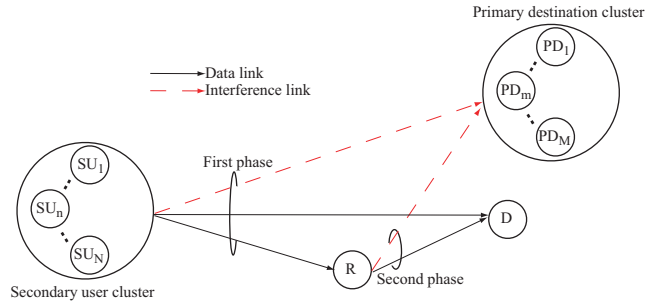
[2]In [18], downlink system is considered and multiple users share the same interference link with the primary destination, which leads to the fact that the cognitive multiuser scheduling is based only on the direct data channels. In contrast, we consider uplink system and each user has independent interference link with the primary destinations. For this reason, both the secondary data links and primary interference links affect the cognitive multiuser scheduling, which makes the cognitive multiuser selection in our work optimal.

[3]Hereafter, we will use the terms secondary user and secondary source, interchangeably. Likewise, primary user and primary destination can be interchangeably used.

in Fig. 1[4]. The N secondary users are close together and form a small cluster, which experiences the same scale fading to other nodes. This assumption also holds for the M primary destinations. We consider a moderate shadow environment so that the secondary users have direct links with the primary and secondary destinations. For either AF or DF relaying, we select one best SU_n amongst N to maximize the received SNR at D. The communication reliability is significantly improved by employing the MRC technique for the selected SU_n's signals through both direct and relay links to maximize the end-to-end SNR at D. The selection criterion is based on both direct and relay links, which is the optimal solution for cognitive multiuser scheduling. To guarantee the communication quality at the primary network, the interference at each of the M primary destinations imposed by the secondary user and the relay should not exceed a given threshold I_P [5]. All terminals in the network have a single antenna due to size limitation, and operate in a half-duplex mode. All links in the system undergo Rayleigh flat fading. In the following, we first present two-phase data transmission for both AF and DF relaying. We then discuss the criterion of user selection.

A. AF Relaying

Suppose that the secondary source SU_n is selected to transmit its information to D. In the first phase, SU_n sends normalized signal x_n to R and D. The corresponding received signals at R and D are respectively written as

$$y_R = \sqrt{P_n} h_{SU_n,R} x_n + n_R, \tag{1}$$

$$y_D^{(1)} = \sqrt{P_n} h_{SU_n,D} x_n + n_D^{(1)}, \tag{2}$$

where $h_{SU_n,R} \sim \mathcal{CN}(0,\alpha)$ and $h_{SU_n,D} \sim \mathcal{CN}(0,\varepsilon)$ are the channel coefficients of the $SU_n \to R$ and $SU_n \to D$ links, respectively. Moreover, $n_R \sim \mathcal{CN}(0,\sigma^2)$ and $n_D^{(1)} \sim \mathcal{CN}(0,\sigma^2)$ are the additive white Gaussian noise (AWGN) at R and D, respectively, with noise variance σ^2. Here, P_n denotes the transmit power at SU_n. It is important to note that the interference on each primary destination is maintained below a given threshold I_P as

$$P_n = \frac{I_P}{\max\limits_{m=1,\cdots,M} |h_{SU_n,PD_m}|^2}, \tag{3}$$

where $h_{SU_n,PD_m} \sim \mathcal{CN}(0,\eta)$ is the channel coefficient of the $SU_n \to PD_m$ link. Relay R amplifies the received signal y_R by the factor κ

$$\kappa = \sqrt{\frac{P_R}{P_n |h_{SU_n,R}|^2 + \sigma^2}} \tag{4}$$

where P_R is the transmit power at R. Similarly as in SU_n, relay R should not impose the interference on each primary destination above I_P, which then yields

$$P_R = \frac{I_P}{\max\limits_{m=1,\cdots,M} |h_{R,PD_m}|^2}, \tag{5}$$

where $h_{R,PD_m} \sim \mathcal{CN}(0,\zeta)$ is the channel coefficient of the $R \to PD_m$ link. Then, in the second phase, R forwards the amplifying signal to D

$$y_D^{(2)} = h_{R,D} \kappa y_R + n_D^{(2)}, \tag{6}$$

where $h_{R,D} \sim \mathcal{CN}(0,\beta)$ is the channel of $R \to D$ link, and $n_D^{(2)} \sim \mathcal{CN}(0,\sigma^2)$ is the additive white noise at D in the second phase. By employing the MRC on (2) and (6), the received SNR at D for AF relaying can be shown as

$$\mathsf{SNR}_n^{\mathsf{AF}} = \frac{\tilde{I}_P^2 \frac{u_n}{z_n} \cdot \frac{v_1}{v_2}}{1 + \tilde{I}_P \frac{u_n}{z_n} + \tilde{I}_P \frac{v_1}{v_2}} + \tilde{I}_P \frac{w_n}{z_n}, \tag{7}$$

where $\tilde{I}_P = I_P/\sigma^2$ is the peak interference to noise power ratio, $u_n = |h_{SU_n,R}|^2$, $z_n = \max\limits_{m=1,\cdots,M} |h_{SU_n,PD_m}|^2$, $w_n = |h_{SU_n,D}|^2$, $v_1 = |h_{R,D}|^2$ and $v_2 = \max_{m=1,\cdots,M} |h_{R,PD_m}|^2$ denote the associated instantaneous channel gains. From (7), the best user SU_{n^*} is selected to maximize the received SNR at D[6]

$$n^* = \arg \max_{n=1,\cdots,N} \left(\frac{\tilde{I}_P^2 \frac{u_n}{z_n} \cdot \frac{v_1}{v_2}}{1 + \tilde{I}_P \frac{u_n}{z_n} + \tilde{I}_P \frac{v_1}{v_2}} + \tilde{I}_P \frac{w_n}{z_n} \right), \tag{8}$$

The above selection involves all links of the system, and it is much more complicated than the selection in the system without direct links[7].

B. DF Relaying

Suppose that SU_n is selected to transmit its information to D. In the first phase, SU_n transmits its normalized signal x_n to R and D, as shown in (1) and (2). The DF relay R will then keep silent if the received SNR at R falls below a given SNR threshold γ_{th},

$$\tilde{I}_P \frac{u_n}{z_n} < \gamma_{th}. \tag{9}$$

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[4]In this work, we consider a multiuser scenario where multiple sources communicate with a single destination, which is a general uplink communication. The use of a relay is encouraged to improve the reliability of the main channel. Multiuser uplink has been considered for non-cognitive spectrum-sharing systems in the literature [20] [21]. Different from [20] [21] where cognitive radio was not incorporated, we address the multiuser uplink communication in a limited spectrum environment by considering the cognitive spectrum sharing system where secondary users can utilize the radio spectrum dedicated to primary users. More importantly, we consider the joint impact of direct and relay links on the secondary user selection criterion, which makes our approach optimal. In contrast, [18] considered a sub-optimal user scheduling solution, where only direct links have been used for multiuser selection. In summary, our proposed model for cognitive multiuser uplink is general and practically applicable. This scenario can be directly applied to cellular networks where multiple cognitive mobile stations communicate with the base station.

[5]In this work, we neglect the interference on the secondary receiver from the primary transmitter. This assumption is reasonable when the primary transmitter is located far from the secondary receiver [9].

[6]The secondary destination D can be used to implement user selection. The primary and secondary destinations can firstly estimate the channel parameters of their links with the secondary users and relay, with the help of some pilot signals [8], [9]. Here we assume the error-free channel estimation. Then D gathers the channels of the interfering links from the primary destinations by many mechanisms, e.g., direct feedback from PU, indirect feedback from band manager [8], [9], [22]. After that, D performs user selection and broadcasts the index of the selected user to other nodes in the network.

[7]In the absence of direct links, the received SNR at D becomes $\frac{\tilde{I}_P^2 \frac{u_n}{z_n} \cdot \frac{v_1}{v_2}}{1+\tilde{I}_P \frac{u_n}{z_n}+\tilde{I}_P \frac{v_1}{v_2}}$. Hence, the best user can be selected by maximizing u_n/z_n, which is much simpler than our selection in (8).

In this case, the data transmission is only through the direct link, and the received SNR at D is

$$\mathsf{SNR}_n^{\mathsf{DF}} = \tilde{I}_P \frac{w_n}{z_n}. \qquad (10)$$

Otherwise, when $\tilde{I}_P \frac{u_n}{z_n} \geq \gamma_{th}$, the DF relay R will forward the decoded message to D in the second phase. After MRC, the received SNR at D can be obtained as

$$\mathsf{SNR}_n^{\mathsf{DF}} = \tilde{I}_P \frac{w_n}{z_n} + \tilde{I}_P \frac{v_1}{v_2}. \qquad (11)$$

By combining the two cases, the received SNR at D for DF relaying can be written as

$$\mathsf{SNR}_n^{\mathsf{DF}} = \begin{cases} \tilde{I}_P \dfrac{w_n}{z_n}, & \text{If } \tilde{I}_P \dfrac{u_n}{z_n} < \gamma_{th} \\ \tilde{I}_P \dfrac{w_n}{z_n} + \tilde{I}_P \dfrac{v_1}{v_2}, & \text{If } \tilde{I}_P \dfrac{u_n}{z_n} \geq \gamma_{th} \end{cases}. \qquad (12)$$

Let Ω denote the cardinality of the decoding set at the secondary sources. Then the best user SU_{n^*} is selected according to

$$n^* = \begin{cases} \arg \max\limits_{1 \leq n \leq N} \tilde{I}_P \dfrac{w_n}{z_n}, & \text{If } |\Omega| = 0 \\ \arg \max\limits_{n \in \Omega} \left(\tilde{I}_P \dfrac{w_n}{z_n} \right) + \tilde{I}_P \dfrac{v_1}{v_2}, & \text{If } |\Omega| > 0 \end{cases}, \qquad (13)$$

which maximizes the received SNR at D for DF relaying. Similar to the user selection with AF relaying, the above selection involves all links of the system[8].

III. OUTAGE PROBABILITY OF AF RELAYING

In this section, we derive the outage probability for multiuser cognitive relay networks with AF and DF relaying. As can clearly be observed from (7) and (12), each individual SNR term for AF and DF relaying contains the two common RVs: 1) the channel gains of links from secondary relay to secondary destination, and 2) the channel gains of links from secondary relay to primary destination. As such, $\mathsf{SNR}_n^{\mathsf{A}}$, for $\mathsf{A} \in \{\mathsf{AF}, \mathsf{DF}\}$ and $n = 1, 2, \ldots, N$, are correlated RVs although the channels are independently distributed[9].

A. Closed-form Lower Bound of Outage Probability

The outage probability, an important metric of quality of service, is defined as the probability that the received SNR at

[8]Again, it is important to note that since the selection combining (SC) is applied in [18], the selection criterion may only be related to the data channel of the secondary direct link, which makes the cognitive multiuser scheduling a sub-optimal solution. In contrast, as can be clearly seen from (8) and (13), both data and interference channels, i.e., the secondary and the primary links, are exploited to select the best secondary user. As such, our proposed cognitive multiuser scheduling is an optimal solution, which enhances the system performance compared to [18].

[9]The statistical correlation in cognitive relay networks has been observed in [18], [19], [23]. However, in these works, there is only one common RV, which is the channel gain for the link from secondary source to primary destination. More significantly, we have considered the impact of direct link in the cognitive relay networks. As such, in this paper, there exist two common RVs, namely, v_1 and v_2 in (7) and (12).

D falls below the given SNR threshold γ_{th}, i.e.,

$$\mathcal{P}_{\mathsf{out}}^{\mathsf{AF}} = \Pr \left(\max_{n=1,\cdots,N} \mathsf{SNR}_n^{\mathsf{AF}} < \gamma_{th} \right) \qquad (14)$$

$$= \Pr \left[\max_{n=1,\cdots,N} \left(\frac{\tilde{I}_P^2 \frac{u_n}{z_n} \cdot \frac{v_1}{v_2}}{1 + \tilde{I}_P \frac{u_n}{z_n} + \tilde{I}_P \frac{v_1}{v_2}} + \tilde{I}_P \frac{w_n}{z_n} \right) < \gamma_{th} \right]. \qquad (15)$$

From these two equations, we can find that the received SNRs with N secondary users are correlated with each other, because of two common RVs v_1 and v_2. To deal with this troublesome, we need to first solve the CDF of $\mathsf{SNR}_n^{\mathsf{AF}}$ conditioned on a given v_1/v_2. Then by statistically averaging the N-th power of the conditional CDF of $\mathsf{SNR}_n^{\mathsf{AF}}$ with respect to v_1/v_2, we can obtain the analytical expression of outage probability for the considered system with AF relaying.

The derivation process is detailed as follows. Firstly, since further manipulations of exact outage probability given in (15) are cumbersome, we apply the upper bound of the harmonic mean of two positive numbers by the minimum of those two numbers [24]

$$\frac{\tilde{I}_P^2 \frac{u_n}{z_n} \cdot \frac{v_1}{v_2}}{1 + \tilde{I}_P \frac{u_n}{z_n} + \tilde{I}_P \frac{v_1}{v_2}} < \tilde{I}_P \min(\frac{u_n}{z_n}, v), \qquad (16)$$

where $v = \frac{v_1}{v_2}$ represents the channel gain ratio of the R→D link to the link of R with PD. Then a lower bound of outage probability is obtained as

$$\mathcal{P}_{\mathsf{low}}^{\mathsf{AF}} = \Pr \left[\max_{n=1,\cdots,N} \left(\min(\frac{u_n}{z_n}, v) + \frac{w_n}{z_n} \right) < \frac{\gamma_{th}}{\tilde{I}_P} \right]. \qquad (17)$$

Let $\theta_n = \min(\frac{u_n}{z_n}, v) + \frac{w_n}{z_n}$, where $\tilde{I}_P \theta_n$ is the bound of received SNR with the n-th secondary user. The CDF of θ_n conditioned on a given v is provided by the following theorem.

Theorem 1: The CDF of θ_n conditioned on v can be shown as

$$F_{\theta_n}(\theta|v) = \begin{cases} G_1(\theta) = 1 + \frac{1}{\varepsilon-\alpha} \sum\limits_{m=1}^{M} (-1)^{m-1} \binom{M}{m} \\ \times \left(\frac{m\alpha^2}{m\alpha+\eta\theta} - \frac{m\varepsilon^2}{m\varepsilon+\eta\theta} \right), & \text{If } \theta < v \\ G_2(\theta) = 1 + \frac{1}{\varepsilon-\alpha} \sum\limits_{m=1}^{M} (-1)^{m-1} \binom{M}{m} \\ \times \left(\frac{m\alpha^2\varepsilon}{m\alpha\varepsilon+\eta\alpha\theta+\eta(\varepsilon-\alpha)v} - \frac{m\varepsilon^2}{m\varepsilon+\eta\theta} \right), & \text{If } \theta \geq v \end{cases} \qquad (18)$$

Proof: See Appendix I. ∎

We now turn to compute the PDF of $v = \frac{v_1}{v_2}$, where v_1 follows the exponential distribution with mean β and the PDF of v_2 is given by [25]

$$f_{v_2}(v_2) = \sum_{m=1}^{M} (-1)^{m-1} \binom{M}{m} \frac{m}{\eta} e^{-\frac{mv_2}{\eta}}. \qquad (19)$$

Then the PDF of v can readily be written as

$$f_v(v) = \int_0^\infty v_2 f_{v_1}(vv_2) f_{v_2}(v_2) dv_2$$

$$= \frac{1}{\beta\zeta} \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \int_0^\infty v_2 e^{-(\frac{m}{\zeta}+\frac{v}{\beta})v_2} dv_2$$

$$= \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \frac{m\lambda}{(v+m\lambda)^2}, \qquad (20)$$

where $\lambda = \frac{\beta}{\zeta}$ is the average channel gain ratio of the R→D link to the link of R with PD. By applying the obtained results of $f_v(v)$ in (20) and $F_{\theta_n}(\theta|v)$ in (18) into (17), the lower bound of $\mathcal{P}_{\text{out}}^{\text{AF}}$ is as follows

$$\mathcal{P}_{\text{low}}^{\text{AF}} = \int_0^\infty F_{\theta_n}^N\left(\frac{\gamma_{th}}{\tilde{I}_P}|v\right) f_v(v) dv$$

$$= \int_0^{\frac{\gamma_{th}}{\tilde{I}_P}} G_2^N\left(\frac{\gamma_{th}}{\tilde{I}_P}\right) f_v(v) dv + \int_{\frac{\gamma_{th}}{\tilde{I}_P}}^\infty G_1^N\left(\frac{\gamma_{th}}{\tilde{I}_P}\right) f_v(v) dv$$

$$= G_1^N\left(\frac{\gamma_{th}}{\tilde{I}_P}\right) \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \frac{m\lambda}{m\lambda + \frac{\gamma_{th}}{\tilde{I}_P}}$$

$$+ \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} m\lambda \int_0^{\frac{\gamma_{th}}{\tilde{I}_P}} G_2^N\left(\frac{\gamma_{th}}{\tilde{I}_P}\right) \frac{1}{(v+m\lambda)^2} dv. \qquad (21)$$

To solve the integral in (21), we rewrite $G_2(\frac{\gamma_{th}}{\tilde{I}_P})$ in (18) in a more compact form as

$$G_2\left(\frac{\gamma_{th}}{\tilde{I}_P}\right) = c_1 + \sum_{m=1}^M \frac{c_{2m}}{v+c_{3m}}, \qquad (22)$$

where

$$c_1 = 1 - \frac{1}{\varepsilon-\alpha} \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \frac{m\varepsilon^2}{m\varepsilon + \eta\frac{\gamma_{th}}{\tilde{I}_P}},$$

$$c_{2m} = (-1)^{m-1} \binom{M}{m} \frac{m\alpha^2\varepsilon}{\eta(\varepsilon-\alpha)^2},$$

$$c_{3m} = \frac{m\alpha\varepsilon + \frac{\alpha\eta\gamma_{th}}{\tilde{I}_P}}{\eta(\varepsilon-\alpha)}.$$

Substituting (22) into (21) yields

$$\int_0^{\frac{\gamma_{th}}{\tilde{I}_P}} G_2^N\left(\frac{\gamma_{th}}{\tilde{I}_P}\right) \frac{1}{(v+m\lambda)^2} dv$$

$$= \Xi(N, m, c_1, c_{21}, \cdots, c_{2M}, c_{31}, \cdots, c_{3M}), \qquad (23)$$

where

$$\Xi(n, l, c_1, c_{21}, \cdots, c_{2M}, c_{31}, \cdots, c_{3M})$$

$$= \int_0^{\frac{\gamma_{th}}{\tilde{I}_P}} \left(c_1 + \sum_{m=1}^M \frac{c_{2m}}{v+c_{3m}}\right)^n \frac{1}{(v+l\lambda)^2} dv \qquad (24)$$

and its closed-form solution is given by (B.7) of Appendix II. By applying the result of (23) into (21), we get the closed-form

expression for the lower bound of outage probability as[10]

$$\mathcal{P}_{\text{low}}^{\text{AF}} = \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \left[\frac{m\lambda G_1^N\left(\frac{\gamma_{th}}{\tilde{I}_P}\right)}{m\lambda + \frac{\gamma_{th}}{\tilde{I}_P}}\right.$$

$$\left. + m\lambda \Xi(N, m, c_1, c_{21}, \cdots, c_{2M}, c_{31}, \cdots, c_{3M})\right]. \qquad (25)$$

B. Asymptotic Outage Probability

To gain additional insights on the system, we now aim at deriving the asymptotic outage probability. By using the Taylor's series expansion of $(1+x)^{-1} \simeq 1-x+x^2$ for small $|x|$ [26], we can obtain the asymptotic $F_{\theta_n}(\theta|v)$ as

$$F_{\theta_n}(\theta|v) \approx \begin{cases} \dfrac{A_{2M}\eta^2\theta^2}{\alpha\varepsilon}, & \text{If } \theta < v \\ \dfrac{A_{1M}\eta}{\varepsilon}(\theta-v), & \text{If } \theta \geq v \end{cases}, \qquad (26)$$

where

$$A_{1M} = \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \frac{1}{m}, \qquad (27)$$

and

$$A_{2M} = \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \frac{1}{m^2}. \qquad (28)$$

The asymptotic outage probability of the considered system can be shown as

$$\mathcal{P}_{\text{out}}^{\text{AF}} = \int_0^\infty \left[F_{\theta_n}\left(\frac{\gamma_{th}}{\tilde{I}_P}|v\right)\right]^N f_v(v) dv \qquad (29)$$

$$\approx \frac{A_{2M}^{N+1}\eta^N\zeta}{\beta\varepsilon^N(N+1)} \left(\frac{\gamma_{th}}{\tilde{I}_P}\right)^{N+1} + \frac{A_{1M}^N\eta^{2N}}{\alpha^N\varepsilon^N} \left(\frac{\gamma_{th}}{\tilde{I}_P}\right)^{2N}. \qquad (30)$$

For different values of N, we can further specify the asymptotic outage probability with AF relaying as

$$\mathcal{P}_{\text{out}}^{\text{AF}} \approx \begin{cases} \left(\dfrac{A_{2M}^2\eta\zeta}{2\beta\varepsilon} + \dfrac{A_{1M}\eta^2}{\alpha\varepsilon}\right)\left(\dfrac{\gamma_{th}}{\tilde{I}_P}\right)^2, & \text{If } N = 1 \\ \dfrac{A_{2M}^{N+1}\eta^N\zeta}{\beta\varepsilon^N(N+1)}\left(\dfrac{\gamma_{th}}{\tilde{I}_P}\right)^{N+1}, & \text{If } N \geq 2 \end{cases}. \qquad (31)$$

From the asymptotic expression[11], we can find that the system diversity order is $N+1$ for AF relaying. Note that the

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[10]It can be readily obtained that $c_{3m} = \frac{m\alpha\varepsilon + \frac{\alpha\eta\gamma_{th}}{\tilde{I}_P}}{\eta(\varepsilon-\alpha)} < -\frac{\gamma_{th}}{\tilde{I}_P}$ when $\varepsilon < \alpha$, as c_{3m} can be rewritten as $c_{3m} = -\frac{\gamma_{th}}{\tilde{I}_P} + \frac{m\alpha\varepsilon + \eta\varepsilon\frac{\gamma_{th}}{\tilde{I}_P}}{\eta(\varepsilon-\alpha)} < -\frac{\gamma_{th}}{\tilde{I}_P}$ for $\varepsilon < \alpha$.

[11]Note that this asymptotic expression is derived from the lower bound of outage probability. By using the SNR bound of $\frac{\tilde{I}_P^2 \frac{u_n}{z_n} \cdot \frac{v_1}{v_2}}{1 + \tilde{I}_P \frac{u_n}{z_n} + \tilde{I}_P \frac{v_1}{v_2}} > 0.5\tilde{I}_P \min(\frac{u_n}{z_n}, v)$, we can readily obtain the upper bound of outage probability and its asymptotic expression. The asymptotic expression from the upper bound also reveals that the system diversity order is $N+1$. Therefore, we conclude that the obtained diversity order from the lower bound is the actual diversity order by applying the squeeze theorem. Moreover, we use the lower bound of the outage probability to evaluate the performance in this work, since the lower bound is tight over a wide range of SNRs, while the upper bound is not.

multiuser cognitive relay network without direct links has the diversity order of one, for any number of N [12]. In contrast, the proposed system performance improves much rapidly thanks to the direct links. Moreover, the system diversity order is independent of the number of primary destinations. Of course, more primary destinations will degrade the system performance, since this imposes a more strict constraint on the transmit power of secondary users and relay.

IV. OUTAGE PROBABILITY OF DF RELAYING

A. Analytical Outage Probability

The outage probability of the considered system with DF relaying is expressed by

$$
\begin{aligned}
\mathcal{P}_{\text{out}}^{\text{DF}} &= \Pr(\text{SNR}_{n^*}^{\text{DF}} < \gamma_{th}) \\
&= \underbrace{\Pr\left(\max_{1 \leq n \leq N} \frac{w_n}{z_n} < \frac{\gamma_{th}}{\tilde{I}_P}, |\Omega| = 0\right)}_{I_1} \\
&\quad + \underbrace{\Pr\left[\left(\max_{n \in \Omega} \frac{w_n}{z_n} + \frac{v_1}{v_2}\right) < \frac{\gamma_{th}}{\tilde{I}_P}, |\Omega| > 0\right]}_{I_2}.
\end{aligned} \tag{32}
$$

Here I_1 denotes the outage probability when the relay cannot correctly decode the message from any secondary user, while I_2 represents the outage probability when the relay can correctly decode the message from some k secondary users, where $k \in \{1, 2, \cdots, N\}$ is the candidate number of Ω. We can readily solve I_1 as $\frac{w_n}{z_n}$ with N secondary users are independent of each other. As to I_2, $\left(\frac{w_n}{z_n} + \frac{v_1}{v_2}\right)$ with k secondary users are no longer independent of each other, because of two common RVs v_1 and v_2. Similar to the derivation in AF relaying, we need to first solve the CDF of $\left(\frac{w_n}{z_n} + \frac{v_1}{v_2}\right)$ conditioned on a given v_1/v_2. Then by statistically averaging the k-th power of the conditional CDF of $\left(\frac{w_n}{z_n} + \frac{v_1}{v_2}\right)$ with respect to v_1/v_2, we can get the analytical expression of I_2. From the expressions of I_1 and I_2, we can obtain the analytical outage probability of the considered system with DF relaying.

The derivation process is detailed as follows. Firstly, as $|\Omega| = 0$ indicates that $\max_{1 \leq n \leq N} \frac{u_n}{z_n} < \frac{\gamma_{th}}{\tilde{I}_P}$, we can compute I_1 as

$$
\begin{aligned}
I_1 &= \Pr\left(\max_{1 \leq n \leq N} \frac{w_n}{z_n} < \frac{\gamma_{th}}{\tilde{I}_P}, \max_{1 \leq n \leq N} \frac{u_n}{z_n} < \frac{\gamma_{th}}{\tilde{I}_P}\right) \\
&= \left[\Pr\left(\frac{w_1}{z_1} < \frac{\gamma_{th}}{\tilde{I}_P}, \frac{u_1}{z_1} < \frac{\gamma_{th}}{\tilde{I}_P}\right)\right]^N \\
&= \left[\int_0^\infty f_{z_1}(z_1) \int_0^{\frac{\gamma_{th} z_1}{\tilde{I}_P}} f_{w_1}(w_1) dw_1 \right. \\
&\quad \left. \times \int_0^{\frac{\gamma_{th} z_1}{\tilde{I}_P}} f_{u_1}(u_1) du_1 dz_1\right]^N.
\end{aligned} \tag{33}
$$

---

[12]If without direct links, the received SNR at D is $\frac{\tilde{I}_P^2 \frac{u_n}{z_n} \cdot \frac{v_1}{v_2}}{1 + \tilde{I}_P \frac{u_n}{z_n} + \tilde{I}_P \frac{v_1}{v_2}}$, upper bounded by $\min(\tilde{I}_P \frac{u_n}{z_n}, \tilde{I}_P \frac{v_1}{v_2})$. Then the single variable $\tilde{I}_P \frac{v_1}{v_2}$ will limit the system diversity order to unit.

Applying $f_{z_1}(z_1) = \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \frac{m}{\eta} e^{-\frac{mz_1}{\eta}}$, $f_{w_1}(w_1) = \frac{1}{\varepsilon} e^{-\frac{w_1}{\varepsilon}}$ and $f_{u_1}(u_1) = \frac{1}{\alpha} e^{-\frac{u_1}{\alpha}}$ into the above equation solves I_1 as

$$
\begin{aligned}
I_1 = \Bigg\{ & \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \left[1 - \left(1 + \frac{\eta \gamma_{th}}{m \tilde{I}_P \varepsilon}\right)^{-1}\right. \\
& \left. - \left(1 + \frac{\eta \gamma_{th}}{m \tilde{I}_P \alpha}\right)^{-1} + \left(1 + \frac{\eta \gamma_{th}}{m \tilde{I}_P \alpha} + \frac{\eta \gamma_{th}}{m \tilde{I}_P \varepsilon}\right)^{-1}\right]\Bigg\}^N.
\end{aligned} \tag{34}
$$

We now turn to compute I_2 in (32). As the set Ω may contain k $(k = 1, \cdots, N)$ candidates, we can calculate I_2 as

$$
\begin{aligned}
I_2 &= \sum_{k=1}^N \binom{N}{k} \Pr\left[\left(\max_{1 \leq n \leq k} \frac{w_n}{z_n} + v\right) < \frac{\gamma_{th}}{\tilde{I}_P},\right. \\
& \left. \frac{u_1}{z_1} \geq \frac{\gamma_{th}}{\tilde{I}_P}, \cdots, \frac{u_k}{z_k} \geq \frac{\gamma_{th}}{\tilde{I}_P}, \frac{u_{k+1}}{z_{k+1}} < \frac{\gamma_{th}}{\tilde{I}_P}, \cdots, \frac{u_N}{z_N} < \frac{\gamma_{th}}{\tilde{I}_P}\right] \\
&= \sum_{k=1}^N \binom{N}{k} \underbrace{\Pr\left(\frac{u_{k+1}}{z_{k+1}} < \frac{\gamma_{th}}{\tilde{I}_P}, \cdots, \frac{u_N}{z_N} < \frac{\gamma_{th}}{\tilde{I}_P}\right)}_{I_{21}} \\
& \times \underbrace{\Pr\left[\left(\max_{1 \leq n \leq k} \frac{w_n}{z_n} + v\right) < \frac{\gamma_{th}}{\tilde{I}_P}, \frac{u_1}{z_1} \geq \frac{\gamma_{th}}{\tilde{I}_P}, \cdots, \frac{u_k}{z_k} \geq \frac{\gamma_{th}}{\tilde{I}_P}\right]}_{I_{22}}.
\end{aligned} \tag{35}
$$

where I_{22} represents the outage probability when the relay can correctly decode the message from the k secondary users, while I_{21} denotes the probability that the relay cannot correctly decode the message from the residual $(N - k)$ secondary users. Due to identically distributed fading channels, I_{21} can be written as

$$
\begin{aligned}
I_{21} &= \left[\Pr\left(\frac{u_{k+1}}{z_{k+1}} < \frac{\gamma_{th}}{\tilde{I}_P}\right)\right]^{N-k} \\
&= \left[\sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \frac{\gamma_{th} \eta}{\gamma_{th} \eta + m \alpha \tilde{I}_P}\right]^{N-k}.
\end{aligned} \tag{36}
$$

To solve I_{22} in (35), we define $\phi(x|v)$ as

$$
\phi(x|v) = \Pr\left(\frac{w_1}{z_1} + v < x, \frac{u_1}{z_1} > x \Big| v\right), \tag{37}
$$

where $\phi(\frac{\gamma_{th}}{\tilde{I}_P}|v)$ denotes the conditional outage probability when the relay can correctly decode the message from a single secondary user. The analytical solution of $\phi(x|v)$ is given by

$$
\phi(x|v) = \begin{cases} 0, & \text{If } x \leq v \\ d_0(x) + \sum_{m=1}^M \frac{d_m}{v - b_m(x)}, & \text{If } x > v \end{cases}, \tag{38}
$$

where

$$
d_0(x) = \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \left(1 + \frac{\eta x}{m \alpha}\right)^{-1}
$$

$$
d_m = (-1)^{m-1} \binom{M}{m} \frac{m \varepsilon}{\eta}
$$

$$
b_m(x) = \left(1 + \frac{\varepsilon}{\alpha}\right) x + \frac{m \varepsilon}{\eta}.
$$

$$\mathcal{P}_{\text{out}}^{\text{DF}} = \left[\sum_{m=1}^{M}(-1)^{m-1}\binom{M}{m}\left(1 - (1 + \frac{\eta\gamma_{th}}{m\tilde{I}_P\varepsilon})^{-1} - (1 + \frac{\eta\gamma_{th}}{m\tilde{I}_P\alpha})^{-1} + (1 + \frac{\eta\gamma_{th}}{m\tilde{I}_P\alpha} + \frac{\eta\gamma_{th}}{m\tilde{I}_P\varepsilon})^{-1}\right)\right]^N$$
$$+ \sum_{k=1}^{N}\binom{N}{k}\left\{\left(\sum_{m=1}^{M}(-1)^{m-1}\binom{M}{m}\frac{\gamma_{th}\eta}{\gamma_{th}\eta + m\alpha\tilde{I}_P}\right)^{N-k}\right.$$
$$\left.\times\sum_{m=1}^{M}(-1)^{m-1}\binom{M}{m}m\lambda\Xi\left[k, m, d_0\left(\frac{\gamma_{th}}{\tilde{I}_P}\right), d_1, \cdots, d_M, -b_1\left(\frac{\gamma_{th}}{\tilde{I}_P}\right), \cdots, -b_M\left(\frac{\gamma_{th}}{\tilde{I}_P}\right)\right]\right\}. \tag{40}$$

The proof of (38) is provided in Appendix III. Note that $b_m\left(\frac{\gamma_{th}}{\tilde{I}_P}\right) = \frac{\gamma_{th}}{\tilde{I}_P} + \left(\frac{\varepsilon\gamma_{th}}{\alpha\tilde{I}_P} + \frac{m\varepsilon}{\eta}\right) > \frac{\gamma_{th}}{\tilde{I}_P}$. We can compute I_{22} from (38) as

$$I_{22} = \int_0^\infty \left[\phi\left(\frac{\gamma_{th}}{\tilde{I}_P}|v\right)\right]^k f_v(v)dv$$
$$= \int_0^{\frac{\gamma_{th}}{\tilde{I}_P}}\left[d_0\left(\frac{\gamma_{th}}{\tilde{I}_P}\right) + \sum_{m=1}^{M}\frac{d_m}{v - b_m\left(\frac{\gamma_{th}}{\tilde{I}_P}\right)}\right]^k f_v(v)dv$$
$$= \sum_{m=1}^{M}(-1)^{m-1}\binom{M}{m}\lambda\Xi\left[k, m, d_0\left(\frac{\gamma_{th}}{\tilde{I}_P}\right),\right.$$
$$\left.d_1, \cdots, d_M, -b_1\left(\frac{\gamma_{th}}{\tilde{I}_P}\right), \cdots, -b_M\left(\frac{\gamma_{th}}{\tilde{I}_P}\right)\right]. \tag{39}$$

Combining (34), (36) and (39) yields the closed-form expression for the outage probability of DF relaying, shown in eq. (40) at the top of this page.

B. Asymptotic Outage Probability

We now derive the asymptotic outage probability of DF relaying in the high SNR region. By applying the Taylor's series expansion of $(1 + x)^{-1} \simeq 1 - x + x^2$ with small value of $|x|$ [26] into (34) and (38), we can obtain the asymptotic expressions of I_1 and $\phi(x|v)$ as

$$I_1 \approx \left(\frac{2A_{2M}\eta^2}{\alpha\varepsilon}\frac{\gamma_{th}^2}{\tilde{I}_P^2}\right)^N, \tag{41}$$

$$\phi(x|v) \approx \frac{A_{1M}\eta}{\varepsilon}(x - v) \text{ for } x > v. \tag{42}$$

Similarly, we can obtain the asymptotic expression of I_{21} from (36) as

$$I_{21} \approx \left(\frac{A_{1M}\eta\gamma_{th}}{\tilde{I}_P\alpha}\right)^{N-k}. \tag{43}$$

By pulling everything together, we can achieve the asymptotic outage probability as

$$\mathcal{P}_{\text{out}}^{\text{DF}} \approx \left(\frac{2A_{1M}\eta^2}{\alpha\varepsilon}\frac{\gamma_{th}^2}{\tilde{I}_P^2}\right)^N$$
$$+ \left[\frac{A_{1M}^{N+1}\eta^N\zeta}{\alpha^N\beta}\sum_{k=1}^{N}\binom{N}{k}\left(\frac{\alpha}{\varepsilon}\right)^k\frac{1}{k+1}\right]\left(\frac{\gamma_{th}}{\tilde{I}_P}\right)^{N+1}. \tag{44}$$

For different values of N, we can further specify the asymptotic outage probability with DF relaying as

$$\mathcal{P}_{\text{out}}^{\text{DF}} \approx \begin{cases} \left(\frac{2A_{1M}\eta^2}{\alpha\varepsilon} + \frac{A_{1M}^2\eta\zeta}{2\beta\varepsilon}\right)\left(\frac{\gamma_{th}}{\tilde{I}_P}\right)^2, & \text{If } N = 1 \\ \frac{A_{1M}^{N+1}\eta^N\zeta}{\beta}\left[\sum_{k=1}^{N}\binom{N}{k}\frac{1}{(k+1)\alpha^{N-k}\varepsilon^k}\right] \\ \quad \times\left(\frac{\gamma_{th}}{\tilde{I}_P}\right)^{N+1}, & \text{If } N \geq 2 \end{cases} \tag{45}$$

From the above asymptotic outage probability, we can find that the system diversity order is also $N + 1$ for DF relaying. Similar to AF relaying, we note that the multiuser relay network without direct links has the diversity order of one, for any number of N. As such, the performance of the considered system improves much rapidly with the effect of direct links. Moreover, the number of primary destinations does not affect the system diversity order.

In further, one can readily find that AF relaying outperforms DF relaying in asymptotic outage probability when $N = 1$. This relationship also holds for $N \geq 2$, as

$$\frac{\mathcal{P}_{\text{out}}^{\text{DF}}}{\mathcal{P}_{\text{out}}^{\text{AF}}} \approx \frac{\varepsilon^N(N+1)}{\alpha^N}\sum_{k=1}^{N}\binom{N}{k}\left(\frac{\alpha}{\varepsilon}\right)^k\frac{1}{k+1}, \tag{46}$$

$$> \frac{\varepsilon^N}{\alpha^N}\sum_{k=1}^{N}\binom{N}{k}\left(\frac{\alpha}{\varepsilon}\right)^k \tag{47}$$

$$> 1. \tag{48}$$

Hence, we can conclude that the asymptotic outage probability of AF relaying is better than that of DF relaying.

V. NUMERICAL RESULTS

In this section, we present some numerical and simulation results to validate the proposed studies. All links in the system undergo the Rayleigh flat fading. We consider a two dimensional network topology. The secondary nodes are placed along x axis, where the distance between the secondary users and secondary destination is fixed to one. The relay is between the secondary users and secondary destination, and the normalized distance between the relay and secondary users is denoted by D. In addition, the primary destination is above the secondary destination with the same x coordinate and unit distance in y axis. We assume the path loss factor of four,
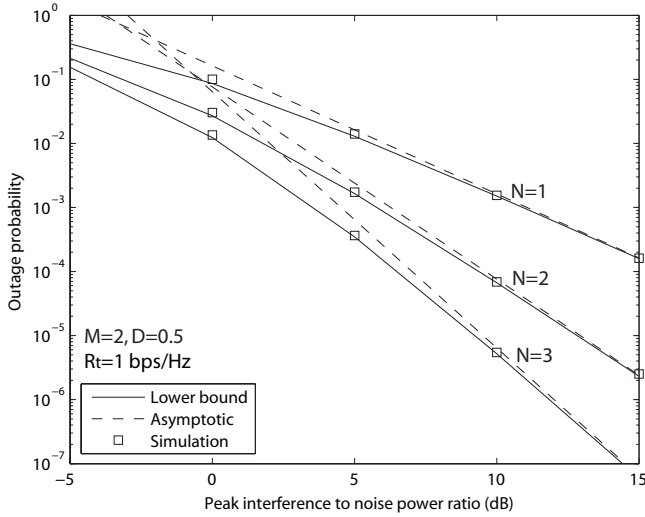
Fig. 2. Effect of N on the outage probability of multiuser cognitive relay networks with AF relaying versus $\tilde{I}_P$.
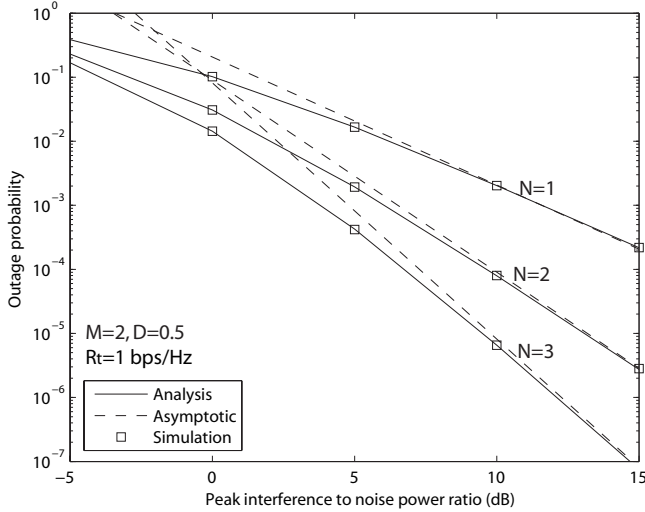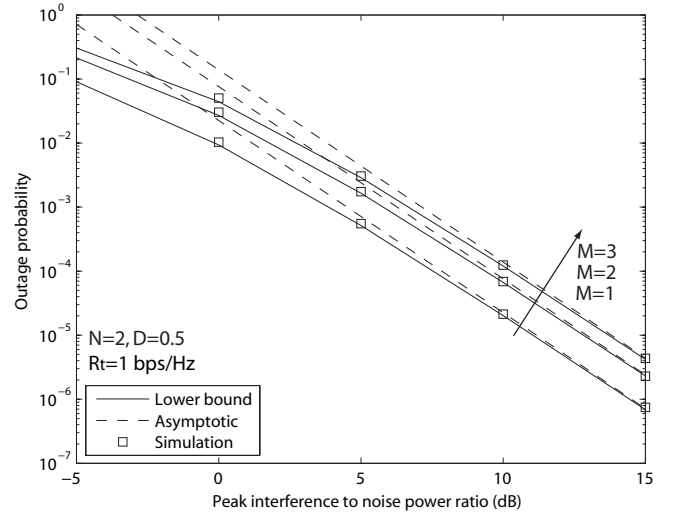

Fig. 4. Effect of M on the outage probability of multiuser cognitive relay networks with AF relaying versus $\tilde{I}_P$.

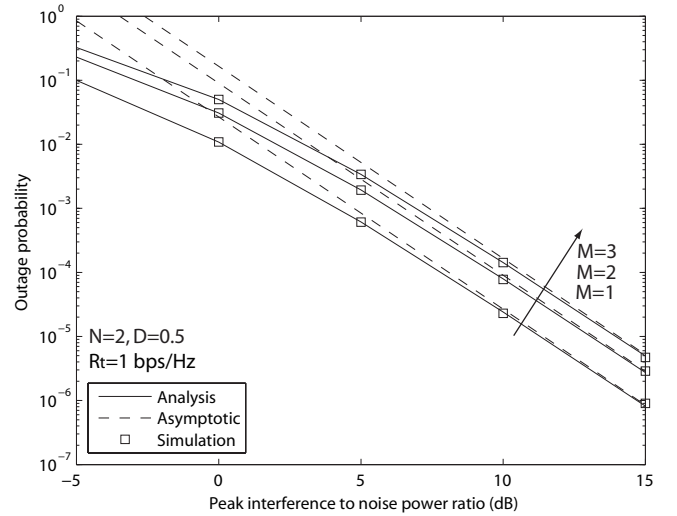Fig. 3. Effect of N on the outage probability of multiuser cognitive relay networks with DF relaying versus $\tilde{I}_P$.

Fig. 5. Effect of M on the outage probability of multiuser cognitive relay networks with DF relaying versus $\tilde{I}_P$.

so that $\alpha = D^{-4}$, $\beta = (1-D)^{-4}$, $\varepsilon = 1$, $\eta = 0.25$ and $\zeta = (1 + (1-D)^2)^{-2}$. The target data rate is R_t, and the associated SNR threshold γ_{th} is set to $2^{2R_t} - 1$ [13].

Figs. 2 and 3 show the impact of number of the secondary users on the system outage probability versus $\tilde{I}_P$, where $M = 2$, $D = 0.5$, $R_t = 1$bps/Hz and N varies from 1 to 3. Fig. 2 and Fig. 3 correspond to the AF and DF relaying, respectively. We can observe from these two figures that for different number of users, the analytical result is close to the simulation result, and the asymptotic result converges to the exact one with large $\tilde{I}_P$. This validates the derived analytical and asymptotic expressions for both AF and DF relaying. Moreover, the system performance improves rapidly with larger number of N, and the slope of the curve is proportional to $N + 1$ in the high SNR region. This verifies

[13]Due to the two-phase data transmission, the relationship between R_t and γ_{th} is $R_t = \frac{1}{2} \log_2(1 + \gamma_{th})$, resulting in $\gamma_{th} = 2^{2R_t} - 1$.

the theoretical observation that the system diversity is $N + 1$ for both AF and DF relaying.

Figs. 4 and 5 show the effect of the number of primary destinations on the system outage probability versus $\tilde{I}_P$, where $N = 2$, $D = 0.5$, $R_t = 1$bps/Hz and M varies from 1 to 3. Fig. 4 and Fig. 5 correspond to the AF and DF relaying, respectively. As expected from these two figures, the system performance degrades with larger M, as more primary destinations impose a more strict constraint on the transmit power of the secondary users and the relay. Moreover, the slopes of all the outage curves are in parallel with each another, indicating that the system diversity remains unchanged with the number of primary destinations.

Figs. 6 and 7 show the system outage probability versus the target data rate R_t, where $\tilde{I}_P = 10$dB, $D = 0.5$, $M = 2$ and $N = 1, 2, 3$. Fig. 6 and Fig. 7 correspond to the AF and DF relaying, respectively. We can observe from these two figures
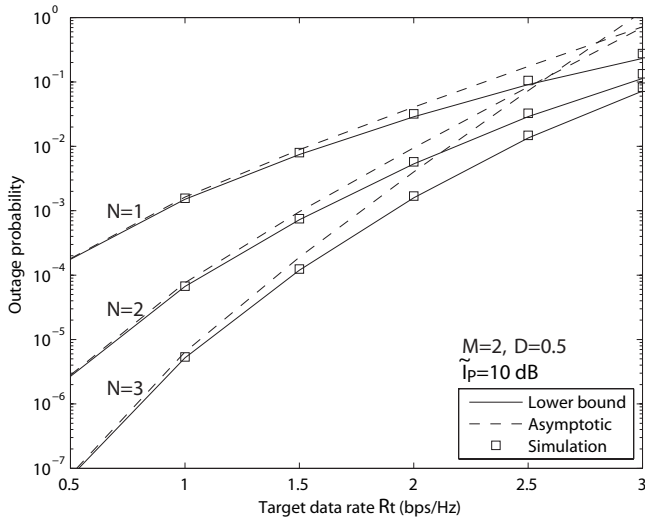
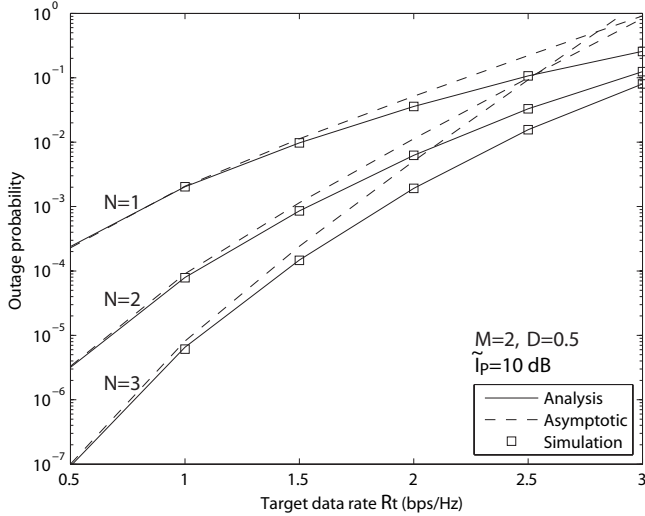Fig. 6. Outage probability of multiuser cognitive relay networks with AF relaying versus the target data rate R_t.

Fig. 8. Performance comparison between AF and DF relaying versus $\tilde{I}_P$.

Fig. 7. Outage probability of multiuser cognitive relay networks with DF relaying versus the target data rate R_t.

Fig. 9. Performance comparison between the optimal and sub-optimal selection with AF relaying versus $\tilde{I}_P$.

that the system performance improves rapidly with N for low target data rate. The asymptotic result slightly deviate from the exact value, as large R_t leads to an increase in the SNR threshold γ_{th}.

Fig. 8 demonstrates the performance comparison between AF and DF relaying versus $\tilde{I}_P$, where $M = N = 2$, $R_t = 1$bps/Hz and $D = 0.5, 0.7$. We can find that AF relaying outperforms DF relaying in outage probability, and the performance gap increases with larger D. Figs. 9 and 10 compare the proposed optimal user selection with two sub-optimal user selection methods, where one sub-optimal selection is only based on the direct links [18][14], while the other sub-optimal selection is based on the relay links only. Fig. 9 and Fig. 10 correspond to the AF and DF relaying,

Fig. 10. Performance comparison between the optimal and sub-optimal selection with DF relaying versus $\tilde{I}_P$.

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[14]The difference is that a downlink communication scenario was considered in [18] while we study an uplink communication scenario in this work.
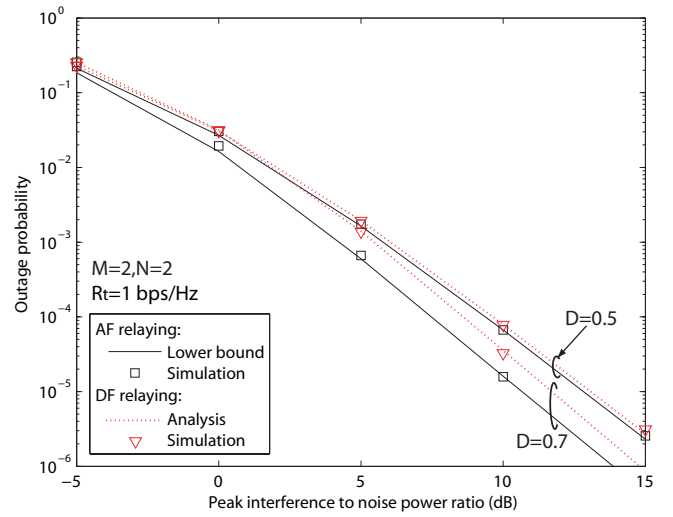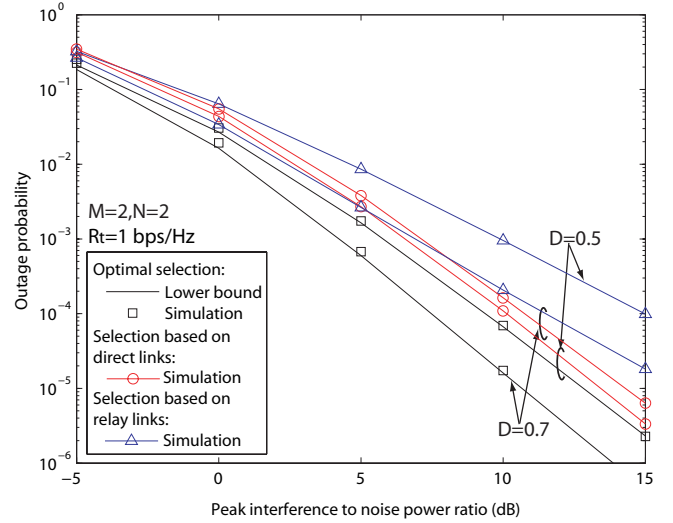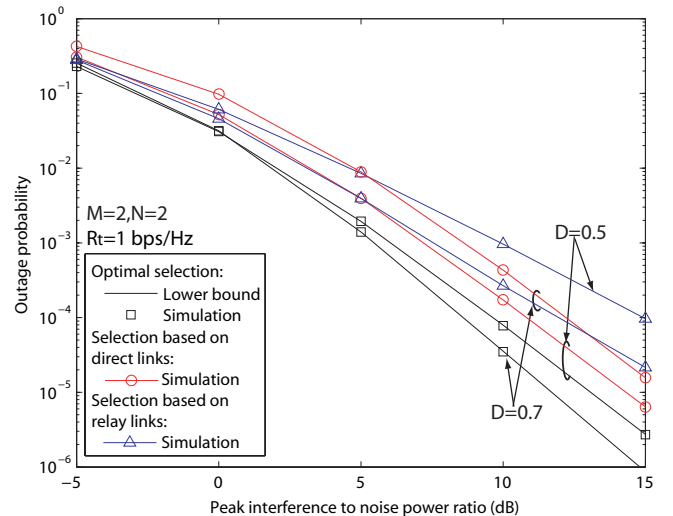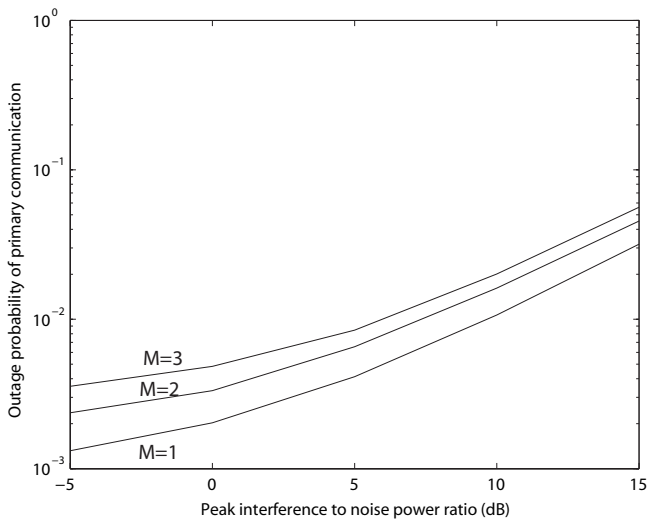
Fig. 11.   Outage probability of the primary communication versus $\tilde{I}_P$.

respectively. From these two figures, we can find that the proposed optimal selection always outperforms the three sub-optimal selection methods, as the optimal selection can efficiently exploit both direct and relay links on the user selection criterion. The performance gap between the optimal selection and the selection based on direct channels increases with larger D, which is consistent with the results in [27].

Fig. 11 illustrates the outage probability of primary communication versus $\tilde{I}_P$, where M varies from 1 to 3. For the primary communication, we consider a multicast primary network where there is one transmitter and M destinations. The primary network is in outage if any of M destinations is in outage. The channel experiences the Rayleigh flat fading with average channel gain of unit. Besides the interference from the secondary users and relay, the additive white Gaussian noise at the primary receiver is assumed to have unit variance. The transmit power of primary transmitter is set to 30dB, and the target data rate of the primary communication is set to 1 bps/Hz. As observed from Fig. 11, we can find that for different values of M, the performance of primary communication deteriorates with larger peak power of interference from the secondary transmitters.

VI. CONCLUSIONS

In this paper, we proposed a multiuser cognitive relay network with multiple secondary destinations, where multiple secondary users compete to communicate with a secondary destination assisted by a single AF or DF relay. From a practical standpoint, we considered a moderate shadow environment so as the direct links from the secondary user to the primary and secondary destinations exist. By taking into account the joint impact of the direct and the relay link on cognitive multiuser scheduling, the optimal secondary source has been selected so as to maximize the received SNR at the secondary destination using MRC. For both AF and DF relaying, we derived exact, lower bounds, and asymptotic expressions for the outage probability. We showed that the proposed cognitive

multiuser scheduling achieves the full diversity order for both AF and DF relaying.

APPENDIX I
PROOF OF THEOREM 1

The CDF of $\theta_n = \min(\frac{u_n}{z_n}, v) + \frac{w_n}{z_n}$ conditioned on a given v is given by

$$F_{\theta_n}(\theta|v) = \Pr\left[\min\left(\frac{u_n}{z_n}, v\right) + \frac{w_n}{z_n} < \theta\right] \qquad (A.1)$$

$$= \underbrace{\Pr\left(v + \frac{w_n}{z_n} < \theta, u_n > vz_n\right)}_{J_1}$$

$$+ \underbrace{\Pr\left(\frac{u_n}{z_n} + \frac{w_n}{z_n} < \theta, u_n \leq vz_n\right)}_{J_2}. \qquad (A.2)$$

In the following, we derive J_1 and J_2 by differentiating two cases of $\theta < v$ and $\theta \geq v$.

A. When $\theta < v$

In this case, J_1 equals to zero as $v + \frac{w_n}{z_n} < \theta$ cannot hold. And J_2 becomes

$$J_2 = \Pr\left[w_n < \theta z_n - u_n, u_n \leq \theta z_n\right] \qquad (A.3)$$

$$= \int_0^\infty f_{z_n}(z_n) \left[\int_0^{\theta z_n} f_{u_n}(u_n) \int_0^{\theta z_n - u_n} f_{w_n}(w_n) dw_n du_n\right] dz_n, \qquad (A.4)$$

where the PDFs of z_n, u_n and w_n can be respectively presented as

$$f_{z_n}(z_n) = \sum_{m=1}^{M} (-1)^{m-1} \binom{N}{m} \frac{m}{\eta} e^{-\frac{mz_n}{\eta}},$$

$$f_{u_n}(u_n) = \frac{1}{\alpha} e^{-\frac{u_n}{\alpha}},$$

$$f_{w_n}(w_n) = \frac{1}{\varepsilon} e^{-\frac{w_n}{\varepsilon}}. \qquad (A.5)$$

Substituting (A.5) into (A.3) yields the closed-form expression for J_2 as

$$J_2 = 1 + \frac{1}{\varepsilon - \alpha} \sum_{m=1}^{M} (-1)^{m-1} \binom{M}{m} \left(\frac{m\alpha^2}{m\alpha + \eta\theta} - \frac{m\varepsilon^2}{m\varepsilon + \eta\theta}\right). \qquad (A.6)$$

B. When $\theta \geq v$

In this case, J_1 is shown as

$$J_1 = \Pr\left[w_n < (\theta - v)z_n, u_n > vz_n\right] \tag{A.7}$$

$$= \int_0^\infty f_{z_n}(z_n)\left[\int_0^{(\theta-v)z_n} f_{w_n}(w_n)dw_n\right.$$

$$\left. \times \int_{vz_n}^\infty f_{u_n}(u_n)du_n\right]dz_n \tag{A.8}$$

$$= \sum_{m=1}^M (-1)^{m-1}\binom{M}{m}\left(\frac{m\alpha}{m\alpha + \eta v}\right.$$

$$\left. - \frac{m\alpha\varepsilon}{m\alpha\varepsilon + \eta\alpha\theta + \eta(\varepsilon - \alpha)v}\right). \tag{A.9}$$

As $\theta \geq v$, J_2 becomes

$$J_2 = \Pr(w_n < \theta z_n - u_n, u_n \leq vz_n) \tag{A.10}$$

$$= \int_0^\infty f_{z_n}(z_n)\left[\int_0^{vz_n} f_{u_n}(u_n)\right.$$

$$\left. \times \int_0^{\theta z_n - u_n} f_{w_n}(w_n)dw_n du_n\right]dz_n \tag{A.11}$$

$$= 1 - \sum_{m=1}^M (-1)^{m-1}\binom{M}{m}\frac{m\alpha}{m\alpha + \eta v} + \frac{1}{\varepsilon - \alpha}\sum_{m=1}^M (-1)^{m-1}$$

$$\times \binom{M}{m}\left(\frac{m\alpha\varepsilon^2}{m\alpha\varepsilon + \eta\alpha\theta + \eta(\varepsilon - \alpha)v} - \frac{m\varepsilon^2}{m\varepsilon + \eta\theta}\right). \tag{A.12}$$

By summarizing the results of J_1 and J_2 in the two cases of $\theta < v$ and $\theta \geq v$, we arrive at $F_{\theta_n}(\theta|v)$ in Theorem 1, which finalizes the proof.

APPENDIX II
CLOSED-FORM SOLUTION OF (24)

To obtain the closed-form expression of $\Xi(n, l, c_1, c_{21}, \cdots, c_{2M}, c_{31}, \cdots, c_{3M})$, we first rewrite $c_1 + \sum_{m=1}^M \frac{c_{2m}}{v + c_{3m}}$ as

$$c_1 + \sum_{m=1}^M \frac{c_{2m}}{v + c_{3m}} = c_1 + \frac{\psi(v)}{(v + c_{31})\cdots(v + c_{3M})}, \tag{B.1}$$

where $\psi(v) = \sum_{m=1}^M c_{2m}\prod_{i=1,i\neq m}^M (v + c_{3i})$. Then by using the binomial expansion [26, eq. (1.111)], we can express $\left(c_1 + \sum_{m=1}^M \frac{c_{2m}}{v+c_{3m}}\right)^n$ as

$$\left(c_1 + \sum_{m=1}^M \frac{c_{2m}}{v + c_{3m}}\right)^n = c_1^n + \sum_{q=1}^n \binom{n}{q}c_1^{n-q}$$

$$\times \frac{\psi^q(v)}{(v + c_{31})^q \cdots (v + c_{3M})^q}. \tag{B.2}$$

From (B.2), we have

$$\Xi(n, l, c_1, c_{21}, \cdots, c_{2M}, c_{31}, \cdots, c_{3M})$$

$$= c_1^n \int_0^{\frac{\gamma_{th}}{\bar{I}_P}} \frac{1}{(v + l\lambda)^2}dv + \sum_{q=1}^n \binom{n}{q}c_1^{n-q}$$

$$\times \int_0^{\frac{\gamma_{th}}{\bar{I}_P}} \frac{\psi^q(v)}{(v + c_{31})^q \cdots (v + c_{3M})^q(v + l\lambda)^2}dv \tag{B.3}$$

$$= \frac{c_1^n \frac{\gamma_{th}}{\bar{I}_P}}{l\lambda + \frac{\gamma_{th}}{\bar{I}_P}} + \sum_{q=1}^n \binom{n}{q}c_1^{n-q}$$

$$\times \int_0^{\frac{\gamma_{th}}{\bar{I}_P}} \frac{\psi^q(v)}{(v + c_{31})^q \cdots (v + c_{3M})^q(v + l\lambda)^2}dv. \tag{B.4}$$

Next, to solve the above integral, we decompose $\frac{\psi^q(v)}{(v+c_{31})^q\cdots(v+c_{3M})^q(v+l\lambda)^2}$ as [26, eq. (2.102)]

$$\frac{\psi^q(v)}{(v + c_{31})^q \cdots (v + c_{3M})^q(v + l\lambda)^2}$$

$$= \sum_{i=1}^2 \frac{\tau_i}{(v + l\lambda)^i} + \sum_{k=1}^M \sum_{j=1}^q \frac{\rho_{kj}}{(v + c_{3k})^j}, \tag{B.5}$$

where

$$\tau_i = \frac{1}{(2-i)!}\frac{d^{2-i}}{dv^{2-i}}\varphi(v)(v + l\lambda)^2|_{v=-l\lambda},$$

$$\rho_{kj} = \frac{1}{(q-j)!}\frac{d^{q-j}}{dv^{q-j}}\varphi(v)(v + c_{3k})^q|_{v=-c_{3k}},$$

$$\varphi(v) = \frac{\psi^q(v)}{(v + c_{31})^q \cdots (v + c_{3M})^q(v + l\lambda)^2}.$$

By plugging (B.5) into (B.4), we obtain

$$\int_0^{\frac{\gamma_{th}}{\bar{I}_P}} \frac{\psi^q(v)}{(v + c_{31})^q \cdots (v + c_{3M})^q(v + l\lambda)^2}dv$$

$$= \tau_1 \ln\left(\frac{\frac{\gamma_{th}}{\bar{I}_P} + l\lambda}{l\lambda}\right) + \sum_{k=1}^M \rho_{k1} \ln\left|\frac{\frac{\gamma_{th}}{\bar{I}_P} + c_{3k}}{c_{3k}}\right|$$

$$+ \tau_2\left(\frac{1}{l\lambda} - \frac{1}{l\lambda + \frac{\gamma_{th}}{\bar{I}_P}}\right)$$

$$+ \sum_{k=1}^M \sum_{j=2}^q \frac{\rho_{kj}}{j-1}\left(\frac{1}{c_{3k}^{j-1}} - \frac{1}{(c_{3k} + \frac{\gamma_{th}}{\bar{I}_P})^{j-1}}\right). \tag{B.6}$$

Finally, by applying the result of (B.6) into (B.4), we can obtain the closed-form expression of $\Xi(n, l, c_1, c_{21}, \cdots, c_{2M}, c_{31}, \cdots, c_{3M})$ with $c_{3m} > 0$ or $c_{3m} < -\frac{\gamma_{th}}{\bar{I}_P}$ as

$$\Xi(n, l, c_1, c_{21}, \cdots, c_{2M}, c_{31}, \cdots, c_{3M})$$

$$= \frac{c_1^n \frac{\gamma_{th}}{\bar{I}_P}}{l\lambda + \frac{\gamma_{th}}{\bar{I}_P}} + \sum_{q=1}^n \binom{n}{q}c_1^{n-q}\left[\tau_1 \ln\left(\frac{\frac{\gamma_{th}}{\bar{I}_P} + l\lambda}{l\lambda}\right)\right.$$

$$+ \sum_{k=1}^M \rho_{k1} \ln\left|\frac{\frac{\gamma_{th}}{\bar{I}_P} + c_{3k}}{c_{3k}}\right| + \tau_2\left(\frac{1}{l\lambda} - \frac{1}{l\lambda + \frac{\gamma_{th}}{\bar{I}_P}}\right)$$

$$\left. + \sum_{k=1}^M \sum_{j=2}^q \frac{\rho_{kj}}{j-1}\left(\frac{1}{c_{3k}^{j-1}} - \frac{1}{(c_{3k} + \frac{\gamma_{th}}{\bar{I}_P})^{j-1}}\right)\right]. \tag{B.7}$$

APPENDIX III

PROOF OF EQ. (38)

If $x \leq v$, we can readily find that $\phi(x|v)$ equals to zero, as $\frac{w_1}{z_1} + v < x$ cannot hold. Otherwise, $\phi(x|v)$ is derived as

$$\phi(x|v) = \Pr\left[w_1 < (x-v)z_1, u_1 > xz_1 | v\right] \qquad (\text{C.1})$$

$$= \int_0^\infty f_{z_1}(z_1) \int_0^{(x-v)z_1} f_{w_1}(w_1) dw_1$$

$$\times \int_{xz_1}^\infty f_{u_1}(u_1) du_1 dz_1. \qquad (\text{C.2})$$

Applying the PDFs of z_1, w_1 and u_1 given in (A.5) into the above equation leads to

$$\phi(x|v) = \sum_{m=1}^M (-1)^{m-1} \binom{M}{m} \left[\left(1 + \frac{\eta x}{m\alpha}\right)^{-1} + \frac{m\varepsilon}{\eta} \frac{1}{v - b_m} \right], \qquad (\text{C.3})$$

which completes the proof.

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