

32. Study of the structure size interval of incomplete geometrically similitude model of the elastic thin plates

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Abstract. To design the similitude test model having the same nature characteristics with the prototype of the elastic thin plates, an innovative method of confirm the structure size applicable interval of distorted (incomplete geometrically) similar model was proposed. Firstly, the complete scaling laws and the distortion scaling laws between the model and its prototype were established by using the governing equation analysis. Then, under the study of the structure size applicable interval which kept the same first-order nature characteristic (nature frequency and vibration mode), through the research of the vibration mode control interval (each order of the mode keep the same) and the predict interval of frequency, the method of structure size applicable interval of distorted similar model was obtained, which they were in different orders with the same nature characteristic. Finally, the proposed calculation method is verified by experiment.

Keywords: elastic thin plate, inherent characteristic, distorted similitude model, interval.

Nomenclature

| | |
|--------------------|---|
| a, b, h | Width, length and thickness of the elastic thin plate |
| C_{min}, C_{max} | Boundary value of the acceptable intervals |
| D | Flexural stiffness |
| E | Young's modulus |
| m, n | The number of half waves in the plate length, width direction |
| M_x, M_y | The bending moments |
| t | Time |
| w | Deflection |
| $W(x, y)$ | Modal function of w |
| ∇^2 | Laplace operator |
| η | Discrepancy of the predict value |
| λ_j | Scale factors of parameter j |
| μ | Poisson's ratio |
| ρ | Mass area ratio |
| ω | Frequency |
| ω_M | Frequency of the model |
| ω_P | Frequency of the prototype |
| ω_{Pr} | Frequency of the prediction |
| Γ | The ratio of length and width of the plate |

1. Introduction

Elastic thin plate is a common component in engineering, generally the plate whose minimum width is 5-80 times than its thickness is called thin plate [1], which can bear plane tension and bending moments, it has been widely used in aeronautics, ship, ocean platform and other engineering field. Vibration analysis is one of the important research contents in the dynamic analysis of thin plate construction, for example, the thin plate construction used in engine, ship

and ocean platform will be greatly destroyed by severe vibration which is produced by the effect of airflow or ocean waves [2, 3], however, the experimental evaluation of the prototypes is costly and time consuming because of the large volume, complexity, and high production cost, etc. Consequently, a dynamic scaled down model is necessary. The scaling laws of complete geometric similarity is rarely used in designing a similar model because of the complexity and size parameter limitations of the prototype structures, for example, for some thin plate structure, if the thickness is scaled down in the same ratio with its length and width, the structure will be too thin to process. So the incomplete geometric similar model, namely distorted model, is more widely used. However, not all distorted models can be used to predict the vibration characteristics of the prototype, especially for nature characteristic test, it is a basic requirement that there are the proportional nature frequencies and the same mode shapes between the model and the prototype.

Test models of plate structure have been studied extensively by many researchers. Li Shirong [4] studied the similitude transformation between the functionally graded and homogenous circular plates by axisymmetric bending, bucking and free vibration solutions based on the classical plate theory. Zhang Zhenhua [5] studied distorted similitude of dynamic response of the clamped and stiffened plate under uniform impact load, and established modified model that can predict impact response of the prototype. Ungbhakorn [6] established the scaling laws of distorted models by governing equations, and studied prediction accuracy of the scaling laws under the conditions of different distortion. Simitses [7] established the scaling laws of distorted models by applying the solution of critical load and nature frequency of laminated plates, and analyzed the distorted model prediction discrepancy of different materials. Rezaeepazhand [8] explained the importance of the range of distorted model similarity through applying scaled-down models for predicting vibration response of laminated plates. To sum up, it is rare that studying design methods and geometric intervals of distorted models for high order nature characteristic prediction of plates.

In this paper, by governing equation, establishing the scaling laws between the model and the prototype, and analyzing structure size factors of nature characteristic prediction discrepancy of simply supported plates and mode shape control intervals of that. A general determination method of the structure size intervals of distorted model tests of the elastic thin plates was studied, which can remain the same nature characteristics of different order between the model and the prototype.

2. Establishment of scaling laws and discrepancy analysis

2.1. Establishment of scaling laws of the four-side simply supported elastic plate

The simply supported elastic thin plate is shown in Fig. 1. In the figure, a is the length, b is the width, h is the thickness, E is the Young's modulus, μ is the Poisson's ratio and M_x, M_y are the bending moments.

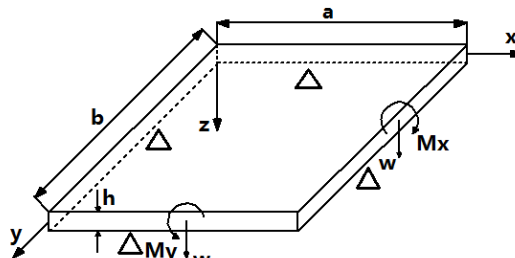


Fig. 1. Simply supported plate

According to the Kirchhoff's assumption, the governing equation of the thin plate can be written as [9]:

$$D\nabla^2\nabla^2w = \rho \frac{\partial^2 w}{\partial t^2}, \quad (1)$$

where, $D = \frac{Eh^3}{12(1-\mu^2)}$, ρ is the mass area ratio, w is the deflection, ∇^2 is the Laplace operator, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

For the four-side simply supported plate, the boundary conditions of the Hoff theory are:

$$x = 0, a: \begin{cases} w = 0, \\ \frac{\partial^2 w}{\partial x^2} = 0, \end{cases} \quad (2)$$

$$y = 0, b: \begin{cases} w = 0, \\ \frac{\partial^2 w}{\partial y^2} = 0. \end{cases} \quad (3)$$

Set vibration mode function of each point as $W(x, y)$, the deflection of the plate is defined as:

$$w = (A \cos \omega t + B \sin \omega t)W(x, y). \quad (4)$$

Substitute Eq. (4) into Eq. (1), then the free vibration characteristics are governed by:

$$D\nabla^2\nabla^2W(x, y) = \rho\omega^2W(x, y). \quad (5)$$

The scaling factor is defined as λ_j , where, j is the symbol of physical quantities, such as $j = a, b, E$, etc. Assuming that the model and prototype have the same materials, $\lambda_E = 1$ then the governing equations of the model and prototype can be written as:

$$D_P \frac{\partial^4 W_P}{\partial x_P^4} + 2D_P \frac{\partial^4 W_P}{\partial x_P^2 \partial y_P^2} + D_P \frac{\partial^4 W_P}{\partial y_P^4} = \rho_P \omega_P^2 W_P, \quad (6)$$

$$D_M \frac{\partial^4 W_M}{\partial x_M^4} + 2D_M \frac{\partial^4 W_M}{\partial x_M^2 \partial y_M^2} + D_M \frac{\partial^4 W_M}{\partial y_M^4} = \rho_M \omega_M^2 W_M. \quad (7)$$

Eq. (6) can be written as:

$$\lambda_D \frac{\lambda_W}{\lambda_a^4} D_M \frac{\partial^4 W_M}{\partial x_M^4} + 2\lambda_D \frac{\lambda_W}{\lambda_a^2 \lambda_b^2} D_M \frac{\partial^4 W_M}{\partial x_M^2 \partial y_M^2} + \lambda_D \frac{\lambda_W}{\lambda_b^4} D_M \frac{\partial^4 W_M}{\partial y_M^4} = \lambda_\rho \lambda_\omega^2 \lambda_W \rho_M \omega_M^2 W_M. \quad (8)$$

Compared with Eq. (7), for the equal corresponding parameters of the equations, the following relationship can be achieved:

$$\lambda_D \frac{\lambda_W}{\lambda_a^4} = \lambda_D \frac{\lambda_W}{\lambda_a^2 \lambda_b^2} = \lambda_D \frac{\lambda_W}{\lambda_b^4} = \lambda_\rho \lambda_\omega^2 \lambda_W. \quad (9)$$

According to $D = \frac{Eh^3}{12(1-\mu^2)}$, the following relationship can be obtained:

$$\lambda_D = \lambda_h^3. \quad (10)$$

Substitute Eq. (10) into Eq. (9), then complete geometric similarity scaling laws of the nature frequency of the four-side simply supported elastic plates are simplified as:

$$\lambda_a = \lambda_b = \lambda_p = \lambda_h, \quad \lambda_\omega = \frac{1}{\lambda_b}. \tag{11}$$

For incomplete geometric similar models, $\lambda_a \neq \lambda_b \neq \lambda_h$, then $\lambda_\omega \neq \lambda_b^{-1}$, so the new scaling laws must be established. It is assumed that $\lambda_a = C\lambda_b$, where C is a constant, then the scaling laws can be written as [10]:

$$\lambda_\omega = \frac{\lambda_h}{C^2 \lambda_b^2}, \tag{12a}$$

$$\lambda_\omega = \frac{\lambda_h}{C \lambda_b^2}, \tag{12b}$$

$$\lambda_\omega = \frac{\lambda_h}{\lambda_b^2}. \tag{12c}$$

The parameters of the prototype plate are shown in Table 1.

Table 1. The parameters of prototype plate

| Rectangular plate $a > b$ | | | | | |
|---------------------------|---------|---------|-----------|----------------------|-------|
| a / m | b / m | h / m | E / GPa | $\rho_{mn} / g/cm^3$ | μ |
| 15 | 10 | 0.5 | 200 | 7.8 | 0.3 |

In order to determine geometric intervals of Eq. (12a) to Eq. (12c), it is assumed that the model and the prototype have the same materials, according to the analysis of the model shown in Table 2, to determine the approximate range of geometric intervals, namely the approximate range of C .

Table 2. Size parameters of the model

| Model | a / m | λ_a | b / m | C | h / m |
|-------|---------|-------------|---------|-----|---------|
| M1 | 3 | 5 | 0.2 | 0.1 | 0.01 |
| M2 | 0.6 | 25 | 0.2 | 0.5 | 0.01 |
| M3 | 0.3 | 50 | 0.2 | 1 | 0.01 |
| M4 | 0.2 | 75 | 0.2 | 1.5 | 0.01 |
| M5 | 0.075 | 200 | 0.2 | 4 | 0.01 |

The first nature frequency of the prototype is 17.369 Hz. Based on Eq. (12a), Eq. (12b) and Eq. (12c), we can predict nature frequencies of the prototype and calculate the predicted discrepancy by using 5 kinds of models shown in Table 2, if the discrepancy $\eta \leq 10\%$, the predicted values are available [11], the predicted values are shown in Table 3.

Table 3. The theoretical value, predicted value and errors under the three scaling laws (Eq. (12))

| | Nature frequency / Hz | Predicted by Eq. (12a) | Discrepancy η_a | Predicted by Eq. (12b) | Error η_c | Predicted by Eq. (12c) | Error η_b |
|-----|-----------------------|------------------------|----------------------|------------------------|----------------|------------------------|----------------|
| P | 17.369 | — | — | — | — | — | — |
| M1 | 442.46 | 0.0885 | 994.9 % | 8.8492 | 49.05 % | 0.8849 | 94.91 % |
| M2 | 667.72 | 3.3386 | 807.8 % | 13.3544 | 23.11 % | 6.6772 | 61.56 % |
| M3 | 868.43 | 17.369 | 0.0 % | 17.369 | 0.0 % | 17.369 | 0.0 % |
| M4 | 1202.6 | 54.117 | 211.57 % | 24.052 | 38.48 % | 36.078 | 107.71 % |
| M5 | 4874.7 | 1559.9 | 8880.97 % | 97.494 | 461.31 % | 389.976 | 2145.24 % |

Analyzing the values shown in Table 3, the values predicted by Eq. (12b) is more accurate than the other method, so Eq. (12b) can be defined as the incomplete scaling law of the nature frequencies of elastic thin plates, and it is known that there are applicable geometric intervals within the range of M2–M4.

2.2. Analysis of the prediction discrepancy influence factors of elastic thin plates

Assuming the same materials and boundary conditions, incomplete similarity variables of elastic thin plates are the length a , the width b and the thickness h . $\Gamma = a/b$, where Γ is the relationship of length of elastic thin plates' sides, $\Gamma > 1$ for rectangular thin plates.

The formula of nature frequency of four-side simply supported elastic thin plates can be written as [8]:

$$\omega = \left[\left(\frac{\pi m}{a} \right)^2 + \left(\frac{\pi n}{b} \right)^2 \right] \sqrt{\frac{D}{\rho_0 h}} \quad (13)$$

where, ρ_0 is the density of the plate, m and n are the numbers of half waves in the plate length and width direction respectively.

According to the relationship between the model and the prototype with the same vibration mode shape, we obtain:

$$\omega_P = \left[\left(\frac{\pi m}{\lambda_a a_M} \right)^2 + \left(\frac{\pi n}{\lambda_b b_M} \right)^2 \right] \sqrt{\frac{\lambda_h^2 D_M}{\rho_0 h_M}} \quad (14)$$

The nature frequency of the model is represented as:

$$\omega_M = \left[\left(\frac{\pi m}{a_M} \right)^2 + \left(\frac{\pi n}{b_M} \right)^2 \right] \sqrt{\frac{D_M}{\rho_0 h_M}} \quad (15)$$

Multiply Eq. (15) by Eq. (12b), we can obtain:

$$\omega_{pr} = \lambda_\omega \omega_M = \frac{\lambda_h}{C \lambda_b^2} \left[\left(\frac{C \pi m}{\Gamma b_M} \right)^2 + \left(\frac{\pi n}{b_M} \right)^2 \right] \sqrt{\frac{D_M}{\rho_0 h_M}} \quad (16)$$

The predicted discrepancy of incomplete geometric similarity can be defined as:

$$\eta = \frac{|\omega_{pr} - \omega_P|}{\omega_P} \quad (17)$$

Substitute Eq. (14), Eq. (16) into Eq. (17), which can be simplified as:

$$\eta = \frac{\left| (C^2 - C) \left(\frac{m}{\Gamma} \right)^2 - (C - 1) n^2 \right|}{C \left[\left(\frac{m}{\Gamma} \right)^2 + (n)^2 \right]} \quad (18)$$

According to Eq. (18), the following conclusions can be obtained:

(1) λ_h is independent, namely λ_h has no effect on the predicted discrepancy of incomplete geometric similarity;

(2) λ_b has no effect on the predicted discrepancy of incomplete geometric similarity, but $C = \lambda_a/\lambda_b$ does;

(3) The size parameters of prototype a, b has no effect on incomplete geometric similarity applicable intervals, $\Gamma = a/b$ has effect on the intervals and mode shapes.

From above conclusions, we know that the geometric applicable intervals of simply supported elastic thin plates is the range of C under different Γ by using Eq. (12b) as the scaling law.

3. Example

3.1. Structure size intervals of the first-order nature characteristics

3.1.1. Applicable geometric intervals of distorted plate models

The geometrical parameters of the prototype plate: $a = \Gamma b$, $b = 10 \text{ m}$, $h = 0.5 \text{ m}$, $\lambda_b = \lambda_h = 20$, its material parameters are shown in Table 1. From the three above conclusions, we know that the analysis has general applicability by using the prototype.

Assuming $\Gamma = a/b = 1.05$, according to ANSYS calculation, the first-order nature frequency is $\omega_p = 23.092 \text{ Hz}$.

The first-order nature frequency ω_{pr}^1 is respectively predicted as six discrete values $C = [0.6, 0.7, 0.95, 1.3, 1.65, 1.95]$, then the relationship between the first-order natural frequencies and C is obtained by fitting a fourth order polynomial:

$$\omega_{pr}^1 = 6.34C^4 - 39.26C^3 + 93.5C^2 - 95.68. \tag{19}$$

Verify the curve by other discrete values in the range of C , the step size of which is 0.05, and they fit the curve well, as shown in Fig. 2.

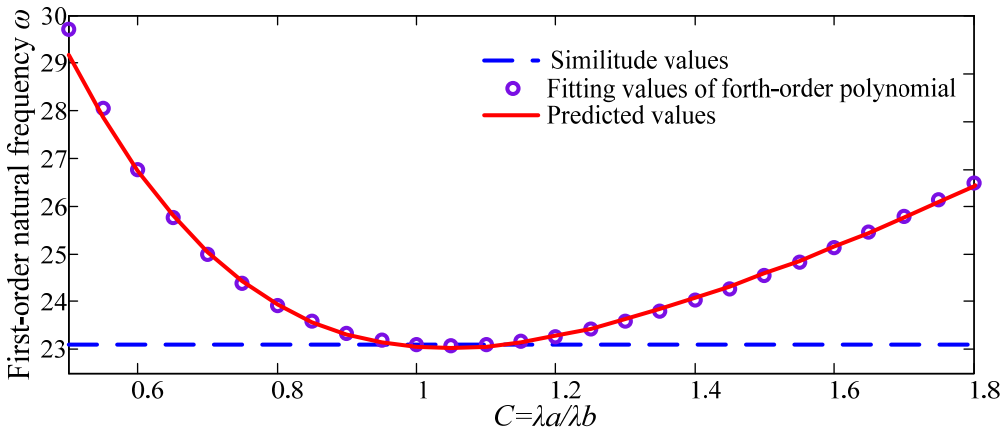


Fig. 2. Verification of fourth-order polynomial fitting results

From Fig. 2, we know that Eq. (19) has accurate fitting effects in the range of $C \in [0.6, 1.95]$. By introducing a $\pm 10\%$ discrepancy of η , the acceptable range of C are calculated as:

$$\eta = \frac{|(6.34C^4 - 39.26C^3 + 93.5C^2 - 95.68C + 58.14 - 23.09)|}{23.09} = 0.1. \tag{20}$$

The roots of Eq. (20) under the intervals $C \in [0.6, 1.95]$ are $C_{min} = 0.675$ and $C_{max} = 1.646$. So, when $\Gamma = a/b = 1.05$, the applicable geometric interval is $C \in [0.675, 1.646]$.

3.1.2. Boundary function

Calculating values of C_{min} and C_{max} under the corresponding $\Gamma = [1, 1.1, 1.15, 1.225]$ by

using the above method, as shown in Table 4, the fitting polynomial curves are shown from Fig. 3 to Fig. 6. The maximum value of 1.225 is based on the control intervals of shape modes in higher-order frequencies' prediction below.

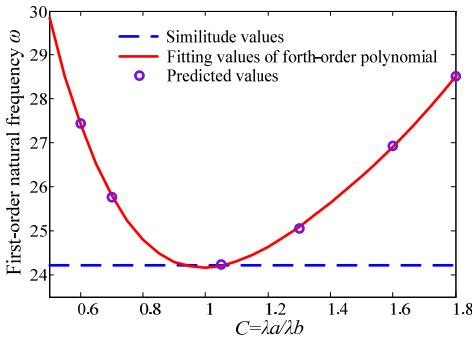


Fig. 3. Fitting curve of $\Gamma = 1.0$

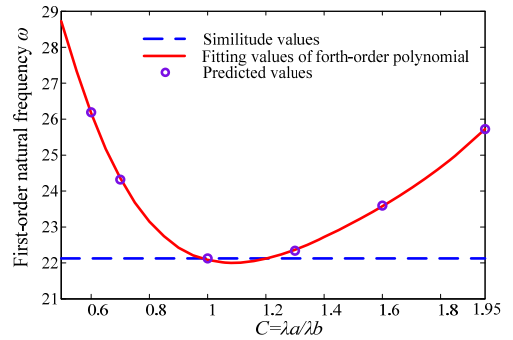


Fig. 4. Fitting curve of $\Gamma = 1.1$

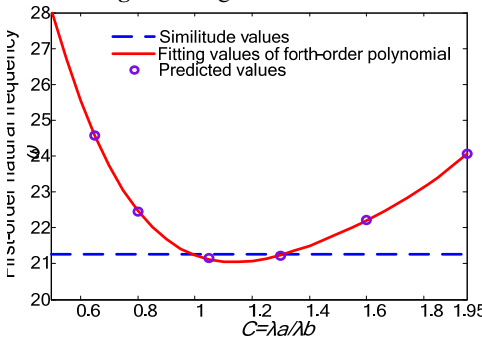


Fig. 5. Fitting curve of $\Gamma = 1.15$

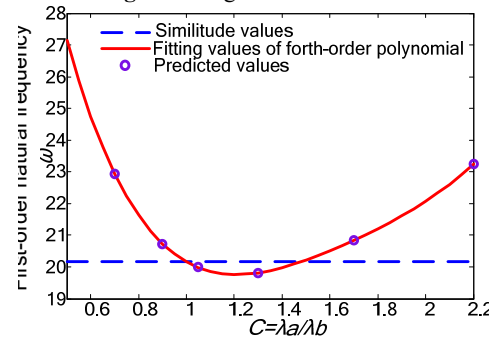


Fig. 6. Fitting curve of $\Gamma = 1.225$

Table 4. Interval boundary values with different Γ

| | $\Gamma = 1.0$ | $\Gamma = 1.1$ | $\Gamma = 1.15$ | $\Gamma = 1.225$ |
|-----------------------------------|---|--|--|--|
| Fourth order polynomial equations | $8.04C^4 - 47.24C^3 + 107C^2 - 104.2C + 60.6$ | $6.69C^4 - 41.02C^3 + 96.61C^2 - 98.93C + 58.73$ | $5.84C^4 - 36.44C^3 + 87.64C^2 - 92.34C + 56.52$ | $3.85C^4 - 26.19C^3 + 68.59C^2 - 78.24C + 52.16$ |
| Frequencies of the prototype / Hz | 24.22 | 22.12 | 21.27 | 20.18 |
| Boundary values | $C_{min} = 0.644,$ $C_{max} = 1.55$ | $C_{min} = 0.702,$ $C_{max} = 1.741$ | $C_{min} = 0.723,$ $C_{max} = 1.855$ | $C_{min} = 0.753,$ $C_{max} = 2.028$ |

From the boundary values of $\Gamma = 1.05$ and the values shown in Table 4, we know:

$$\begin{cases} \Gamma = [1.0, 1.05, 1.1, 1.15, 1.225], \\ C_{min} = [0.644, 0.675, 0.702, 0.723, 0.753], \\ C_{max} = [1.55, 1.646, 1.741, 1.855, 2.028]. \end{cases} \quad (21)$$

By using fitting third-order polynomial, C_{min} and C_{max} can be written as:

$$C_{min} = 2.38\Gamma^3 - 8.62\Gamma^2 + 10.81\Gamma - 3.93, \quad (22)$$

$$C_{max} = -0.96\Gamma^3 + 4.65\Gamma^2 - 4.664\Gamma + 2.52. \quad (23)$$

The functional curves of Eq. (22) and Eq. (23) are shown in Fig. 7.

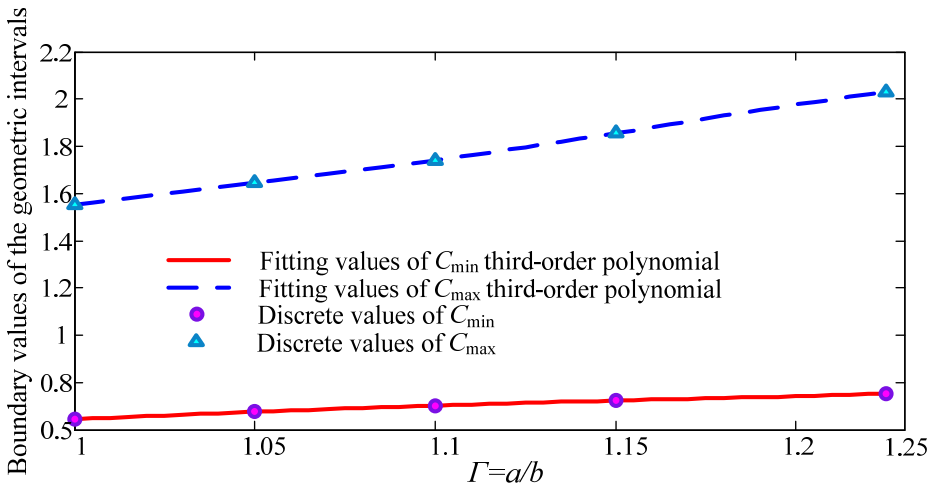


Fig. 7. Boundary values of structure size applicable intervals

In Fig. 7, the area among the two curves is the applicable geometric intervals of first-order nature frequency of simply supported plates in the range of $\Gamma \in (1, 1.225)$.

3.2. Structure size intervals of the higher-order nature characteristics

The geometric size distortion will cause the jump of thin plate models' mode shapes, for the prototype in this paper, it is assumed that the range of size which can keep the same mode shapes between the model and the prototype is $\Gamma \in (1, \Gamma_j]$, as shown in Table 5.

Table 5. Size intervals of each order

| Order j | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----------------------------|---|---------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $\Gamma \in (1, \Gamma_j]$ | - | $(1, \infty)$ | $(1, 1.633]$ | $(1, 1.291]$ | $(1, 1.291]$ | $(1, 1.265]$ | $(1, 1.265]$ | $(1, 1.225]$ |

According to $\lambda_b = C\lambda_a$, the ratio of the length to the width of the plate model is:

$$\Gamma_m = \frac{a_m}{b_m} = \frac{\Gamma}{C} \tag{24}$$

For satisfying the prediction accuracy of higher-order nature frequencies of the plate model, and keeping the same mode shapes between the model and the prototype, so the model size is satisfied as:

$$\begin{cases} C \in [C_{min}^j(\Gamma), C_{max}^j(\Gamma)], \\ \Gamma_m \in (1, \Gamma_j], \end{cases} \quad j = 1, 2, 3, \dots, \tag{25}$$

where, j represents the order number of nature frequency.

According to Eq. (24) and Eq. (25), the applicable geometric intervals of higher-order nature frequencies of the distorted model is:

$$C \in [C_{min}^j(\Gamma), C_{max}^j(\Gamma)] \cap [\Gamma/\Gamma_j, \Gamma], \quad j = 1, 2, 3, \dots, \tag{26}$$

where, $[C_{min}^j(\Gamma), C_{max}^j(\Gamma)]$ is the predicted intervals of higher-order nature frequencies, $[\Gamma/\Gamma_j, \Gamma]$ is the mode shape control intervals of the prototype in this paper, their intersection is the applicable

geometric intervals of higher-order nature frequencies in $\Gamma \in (1, \Gamma_j]$.

According to actual requirements, the range of Γ is chosen based on different principles, so the range of each size is different, in this paper, choosing the mode shape control intervals based on the same first 8 modes, so $\Gamma_8 = 1.225$.

It is different from the first-order nature frequency that the mode shapes of higher-order nature frequencies will change along with the change of C , and it is difficult that $C_{min}^j(\Gamma)$ and $C_{max}^j(\Gamma)$ are obtained by using low-order polynomial fitting, considering the boundary values of applicable geometric intervals are the intersection of the predicted intervals of higher-order nature frequencies and the mode shape control intervals, so when analyzing the geometric intervals of higher-order nature frequencies by numerical method, the C value of the plate model satisfies $C \in [\Gamma/\Gamma_j, \Gamma)$, solving the applicable geometric intervals of the fifth-order nature frequency of the prototype, as shown in 4.1.1.

Choosing $\Gamma = [1, 1.05, 1.1, 1.15, 1.225]$, calculating the boundary values of the applicable geometric intervals by interpolation method, as shown in Table 6.

Table 6. Interval boundary values of discrete Γ

| Γ | Mode shape control interval | Discrete C values | Fourth-order polynomial equations | The solution in control intervals ($\eta < 10\%$) | Boundary values of C |
|----------|-----------------------------|------------------------------|---|---|--|
| 1 | [0.775, 1) | [0.8, 0.85, 0.9, 0.95, 1] | $-994C^4 + 3573.07C^3 - 4789.09C^2 + 2930.07C - 599.07$ | $C_1 = 0.872$ | $C_{min} = 0.872$ $C_{max} = 1$ |
| 1.05 | [0.813, 1.05) | [0.85, 0.9, 0.95, 1, 1.05] | $327.25C^4 - 1254.44C^3 + 1812.67C^2 - 1084.85C + 310.21$ | $C_1 = 0.869$ | $C_{min} = 0.869$ $C_{max} = 1.05$ |
| 1.1 | [0.852, 1.1) | [0.9, 0.95, 1, 1.05, 1.1] | $702.59C^4 - 2759.11C^3 + 4066.09C^2 - 2588.08C + 680.59$ | $C_1 = 0.865$ | $C_{min} = 0.865$ $C_{max} = 1.1$ |
| 1.15 | [0.891, 1.15) | [0.9, 0.95, 1.05, 1.1, 1.15] | $-9.34C^4 + 14.76C^3 + 24.46C^2 + 14.5C + 50.03$ | $C_1 = 0.897,$ $C_2 = 1.132$ | $C_{min} = 0.897$ $C_{max} = 1.132$ |
| 1.225 | [0.949, 1.225) | [0.95, 1, 1.05, 1.1, 1.225] | $186.27C^4 - 807.22C^3 + 1318.14C^2 - 899.06C + 286.54$ | $C_2 = 1.137$ | $C_{min} = 0.949$ $C_{max} = 1.137$ |

According to the data shown in Table 6, we obtain:

$$\begin{cases} \Gamma = [1.0, 1.05, 1.1, 1.15, 1.225], \\ C_{min} = [0.872, 0.869, 0.865, 0.897, 0.94], \\ C_{max} = [1, 1.05, 1.1, 1.132, 1.137], \end{cases} \quad (27)$$

By using fitting third-order polynomial, C_{min} and C_{max} can be written as:

$$C_{min} = -2.23\Gamma^3 + 10.26\Gamma^2 - 14.18\Gamma + 7.02, \tag{28}$$

$$C_{max} = -13.6\Gamma^3 + 42.16\Gamma^2 - 42.51\Gamma + 14.96. \tag{29}$$

The functional images of Eq. (22) and Eq. (23) are shown in Fig. 8.

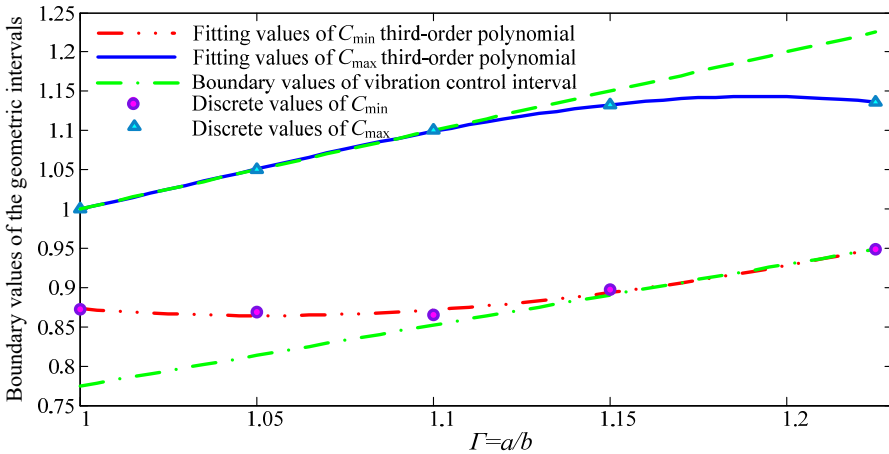


Fig. 8. The fifth-order boundary values of structure size applicable intervals

4. Determination method and verification of the applicable geometric intervals

Choosing the simply supported plate under a certain mode shape as the example above, we obtain the geometric intervals boundary values C_{min} , C_{max} under different orders, as shown in Eq. (22), Eq. (23), Eq. (28) and Eq. (29), then the flow of the geometric intervals determination method is summarized and verified by a test.

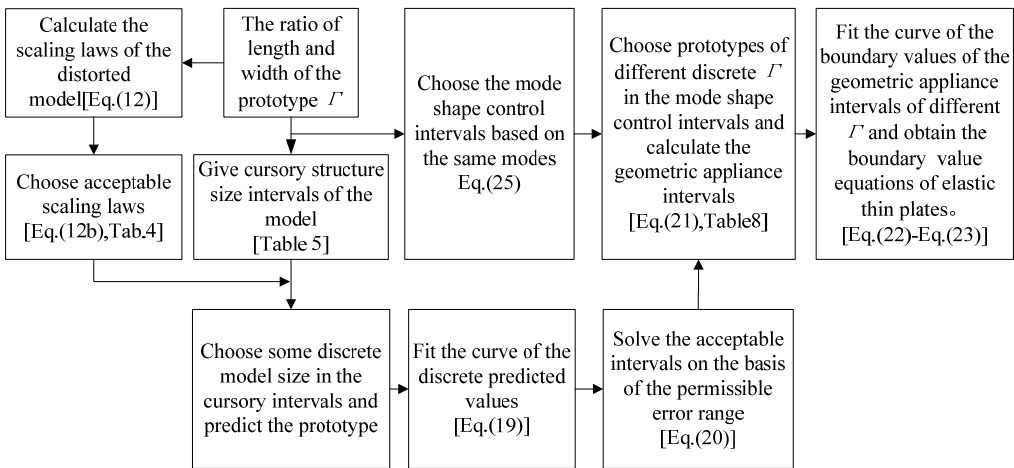


Fig. 9. Flow chart about determination of structure size applicable intervals

4.1. The flow of determination method

The flow chart about determination of geometric applicable intervals of elastic thin plates' similar models is shown in Fig. 9.

4.2. Experiment

For the accuracy of the scaling laws and the intervals determination method, choose the titanium alloy single layer plates with same materials and different sizes as the experimental models, their parameters of materials and structural sizes are shown in Table 7.

Table 7. Size of experimental plates

| | Length A / mm | Width b / mm | Thickness h / mm | Young's modulus E / GPa | Poisson's ratio μ | Density $\rho_0 / \text{kg/m}^3$ |
|---------------|---------------------------|--------------------------|------------------------------|-------------------------------------|-----------------------|-------------------------------------|
| The prototype | 125 | 110 | 1.5 | 110.32 | 0.3 | 4420 |
| The model | 90 | 80 | 1.5 | 110.32 | 0.3 | 4420 |

The ratio of length to width of the prototype is $\Gamma = 1.136$, that of the model is $\Gamma = 1.125$, $C = 1.01$, in the applicable geometric intervals, the values of nature frequencies of the prototype and the model, and the predicted discrepancy are shown in Table 8.

Table 8. Comparison of prototype and scaling model's experimental results

| Order | Value of the prototype test / Hz | Value of the model test / Hz | Predicted value of the prototype / Hz | Error between the prediction and the test / % |
|-------|----------------------------------|------------------------------|---------------------------------------|---|
| 1 | 71.362 | 127.71 | 67.685 | 5.1526 |
| 2 | 193.78 | 385.08 | 204.08 | 5.3167 |
| 3 | 432.56 | 788.74 | 418.02 | 3.3618 |
| 4 | 663.39 | 1317.0 | 698.01 | 5.2215 |
| 5 | 754.08 | 1547.1 | 819.95 | 8.7364 |
| 6 | 1210.5 | 2232.4 | 1183.14 | 2.2574 |

From Table 8, we know that the predicted values of the model are accurate in applicable geometric intervals, and it is verified that the applicable geometric intervals are valid for the design of similar models.

5. Conclusions

In this paper, the determination method of applicable geometric intervals boundary value of thin rectangular plates was studied, and the boundary value equations of the applicable intervals of nature frequencies was obtained and analyzed. Furthermore, the experiments of the titanium alloy single-layer plates with same materials and different sizes are studied; the conclusions are listed as follows:

(1) The incomplete scaling laws of the distorted model of elastic thin plates are set up, and the predicted discrepancy influencing factors from the model to its prototype were obtained.

(2) The mode shapes control interval $C \in [\Gamma/\Gamma_{max}^j, \Gamma/\Gamma_{min}^j]$ ($C \in [\Gamma/\Gamma_j, \Gamma]$ in this paper) is studied based on the same first 8 modes, the boundary value equations of applicable geometric intervals of nature frequencies are obtained by using twice interpolation method in the mode shape control intervals.

(3) It is verified by the experiment that the models which are selected based on the interval boundary value equations can predict nature characteristics of the prototype accurately.

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