# IDENTIFICATION OF A BACKLASH ZONE IN AN ELECTROMECHANICAL SYSTEM CONTAINING CHANGES OF A MASS INERTIA MOMENT BASED ON A WAVELET-NEURAL METHOD 


#### Abstract

In this article a new method of identification of a backlash zone width in a structure of an electromechanical system has been presented. The results of many simulations in a tested model of a complex electromechanical system have been taken while changing a value of a reduced masses inertia moment on a shaft of an induction motor drive. A wavelet analysis of tested signals and analysis of weights that have been obtained during a neural network supervised learning - have been applied in a diagnostic algorithm. The proposed algorithm of detection of backlash zone width, represents effective diagnostic method of a system at changing dynamic conditions, occurring also as a result of mass inertia moment changes.


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## 1. INTRODUCTION

In this paper it has been assumed, that information has been available by measuring signals, and this most often is the base of the diagnostic reasoning methods in electromechanical systems. The process of simultaneous classification of signals in the field of time and frequency is possible due to the use of transformation methods, which allow to test these signals spectral properties. Tested signal can be presented as linear combination of certain base orthogonal signals. This method minimizes the signal model. For the purpose of the minimization of the set of important coefficients of a distribution it is necessary to fit shapes of base functions to the analyzed signal (Zając, 2009).

A progress in technology of signal processing has caused that during the last few years a big number of induction motor drives diagnostics methods by means of wavelet analysis have been studied and presented in numerous papers. Within the last two years more and more scientific works have occurred that have been using time-frequency methods and neural networks in fault diagnostics of industrial structures.

During the last few years there have been published some works that have been using a combination of a wavelet transformation with an artificial intelligence method, and which have been presented as faults detection techniques. It is worthy to mention some of them, like the following ones:

- using a combination of a neural network and a wavelet transformation for diagnostics purposes for the induction motor drive (Kowalski, 2005),
- analysis of descriptions of a current of a motor (MCSA) for a stator current with the use of a wavelet analysis in faults detection of the broken rotor's barrier in the transitional area (Douglas, Pillay \& Ziarani, 2003),
- presentation of methods of detection of rotor's breakage in the cage induction motor on the basis of analysis of the stator's current by means of using a Fourier transformation and a discrete wavelet transformation (Da Costa, Kashiwagi \& Mathias, 2015),
- analysis of stator's current using the combination of a discrete wavelet transformation and a feedforward neural network and a neural network with radial activation functions (Sridhar, Uma Rao \& Jade, 2016), as well as analysis of vibration - for the purpose of faults diagnostics of the induction motor drive using a wavelet decomposition and methods of clustering data FCM based on fuzzy logic (Chandralekha \& Yayanthi, 2016),
- presentation of a possibility of a mechanical estimation of state's variables of a two-mass drive with the use of a perceptron neural network MLP (Multi Layer Perceptron) (Orłowska-Kowalska \& Szabat, 2007),
- diagnostics of the induction motor using a packet wavelet transformation for detection of faults of a stator's current (Annamalai \& Swaminathan, 2016).


## 2. PRESENTATION OF THE METHODOLOGY AND RESEARCH ON FAULT IDENTIFICATION ALGORITHM

Diagnostic tests have been carried out using a working machine in the form of a dynamic mass-absorbing-resilient element connected to the load of the induction motor. There have been accepted nominal conditions of the induction motor, and this model has been built in a stationary coordinate system related to the stator (model $\alpha, \beta, 0$ ). The tested backlash zone width occurs between the rod of the induction motor drive and a working machine drive wheel.

A simplified form of the connection of the induction motor with a working machine has been shown on a diagram in Fig. 1.


Fig. 1. A diagram showing the connection of the induction motor drive with the dynamic mass-absorbing-resilient element by means of a clutch, in which a backlash zone occurs and gradually increases

The following parameters of the induction motor have been applied in the conducted simulation tests (parameters of a substitute circuit are expressed in relative units): circuit stator relative resistance $r_{s}=0.059$ [ $\Omega$ ], circuit rotor relative resistance $r_{w}=0.048$ [ $\Omega$ ], relative reactance of the dispersion circuit stator $x_{s}=1.92[\Omega]$, relative reactance of the dispersion circuit rotor $x_{w}=1.92[\Omega]$, relative reactance of the dispersed circuit $x_{m}=1.82[\Omega]$, mechanical time constant $T_{m}=0.86[\mathrm{~s}]$.

All tests have been carried out within a MATLAB / Simulink R2017a environment.

## 3. IDENTIFICATION TESTS OF THE BACKLASH ZONE WIDTH IN AN ELECTROMECHANICAL SYSTEM CONTAINING A VISCOUS FRICTION

The tests have been carried out in four test groups, with the following four different values of apparent viscosity coefficient $c: 0.8[\mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}], 1.0[\mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}], 1.25[\mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}]$ and $1.4[\mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}]$. Each test group has contained seven cases with different inertia moment $J_{l}$ values. Results of the simulations for all these physical quantities have been written in the matrix $N_{[7,2048]}$. Elements of the matrix $N$ have been written for each coefficient value of the viscosity friction $c$. The value of an inertia moment $J_{l}$ has been determined as the percentage in relation to its nominal value $J_{l}$, down with value A\% and up with value C\%. Formal changes of the inertia moment $J_{l}$ have been written in matrix $K_{l}$ in the following order: $K_{1}=$ [nominal value of the inertia moment $\left(J_{l}=0.87\left[\mathrm{~kg}^{*} \mathrm{~m}^{2}\right]\right), \mathrm{A}=2.5 \%\left(J_{l}=0.8482\left[\mathrm{~kg}^{*} \mathrm{~m}^{2}\right]\right)$, $\mathrm{A}=5 \%\left(J_{l}=0.8265\left[\mathrm{~kg}^{*} \mathrm{~m}^{2}\right]\right), \mathrm{A}=7.5 \%\left(J_{l}=0.8047\left[\mathrm{~kg}^{*} \mathrm{~m}^{2}\right]\right), \mathrm{A}=12.5 \%$ $\left(J_{l}=0.7612\left[\mathrm{~kg}^{*} \mathrm{~m}^{2}\right]\right), \mathrm{C}=2.5 \%\left(J_{l}=0.8917\left[\mathrm{~kg}^{*} \mathrm{~m}^{2}\right]\right), \mathrm{C}=5 \%\left(J_{l}=0.9135\right.$ $\left.\left.\left[\mathrm{kg} * \mathrm{~m}^{2}\right]\right)\right]$. In all executed simulation tests the inertia moment $J_{2}$ of the drive wheel of a working machine has been assumed to have value equal to $0.348\left[\mathrm{~kg}^{*} \mathrm{~m}^{2}\right]$.

For each of the seven cases of changes of an inertia moment $J_{l}$ there have been carried out simulation tests for six backlash zone widths. The backlash zone width values have been taken in sequence from the epsilon matrix, in the following order: $K=[0.0025,0.00375,0.005,0.0075,0.009,0.01]$. All tests have been carried out for the discontinuity in zero value $c_{1}=0.2$.

The wavelet type and its order they have been selected in such a way that the shape of the basic wavelet approximately would be adequate to the character of the transient course of the tested physical quantity that would be obtained as a result of a simulation for the case of the smallest backlash value. Based on the carried out tests the following selections of wavelets have been made for individual physical variables, with decomposition level 10:
a) a linear acceleration of the induction motor drive $a_{l}$ - wavelet function symlet of the order 5 ,
b) an electromagnetic moment of the induction motor drive $m_{e l}$ - wavelet function daubechies of the order 6 ,
c) an angular speed of the induction motor drive rotor $\omega_{l}$ - wavelet function symlet of the order 5 ,
d) a linear acceleration of a mass $a_{2}$ - wavelet function daubechies of the order 6,
e) a linear speed of a mass $v_{2}$ - wavelet function symlet of the order 5 .

In these simulation tests, it has been assumed that the process of the electromechanical system dynamics testing within the backlash zone starts when the expression specified in the left part of the following inequality (1) is smaller than its right part:

$$
\begin{equation*}
\left|\alpha_{1}-\alpha_{2}\right|<\frac{K_{(i)}}{r} ; \quad i \in\{1,2, \ldots .6\} \tag{1}
\end{equation*}
$$

where: $\quad r$ - a radius of the drive wheel of a working machine [m],
$K_{(i)}$ - value that has been taken sequentially from the matrix $K$, corresponding to the given backlash value of its mechanical connection,
$i$ - index number within the matrix $K$.

The location change angle $\alpha_{1}$ for rod masses of the induction motor drive [rad] has been calculated using the formula: $\alpha_{1}=\int \omega_{1} \cdot d t$, and the location change angle $\alpha_{2}$ of the drive wheel of the working machine [rad], has been calculated using the formula: $\alpha_{2}=\int \omega_{2} \cdot d t$, where $\omega_{1}$ is the angular speed, done by rod masses of the induction motor drive $[\mathrm{rad} / \mathrm{s}]$, and $\omega_{2}$ is the angular speed done by a drive wheel of the working machine [rad/s].

After satisfying the condition determined by the inequality described by formula (1) the electromagnetic moment of the induction motor drive in the tested electromechanical system is set to zero value.

While conducting these simulations it has been assumed that specified earlier matrix $K$ would contain 2000 samples of a signal, chosen in a sequence starting from the time obtaining in the tested model of an electromechanical system a backlash zone and also 48 samples preceding this point in time. This assumption has been presented in Fig. 2.


Fig. 2. Testing dynamics in a backlash zone for all physical quantities carried out by means of selected time range choices (samples of the tested signal)

### 3.1. Description of the learning process of the three-layer backpropagation neural network

During simulations discussed in the Section 3.2 the value of the apparent viscosity coefficient $c$ has been set up to 1.0 , the value of the backlash zone width has been allowed to change and the value of the inertia moment $J_{l}$ has been set up, and all these assumptions have had impact on all values of variables and matrices, that have been calculated or obtained during the conducted testing.

Nonlinear neural network's learning algorithm based on the backpropagation of errors method enables reproduction of the assumed values of errors into deeper layers of this network (to which a direct access is not available), by back propagating errors detected on this network's output. During analysis of a single hidden layer neuron - errors of all of neurons, to which this single neuron has transferred his output signal have been taken into account. Then these errors have been summed up taking also into consideration these neurons weights. Fig. 3 presents used in the simulations 3-layers backpropagation neural network.


Fig. 3. Used in the simulations 3-layers backpropagation neural network
The first layer of this network represents the given set of sample pairs of this neural network input signals $X_{l}$ and values of weights $W_{l}$. The second and third layer of this network represents the set of the output signals of the second and third neural network layer $X_{2}$ and $X_{3}$, as well as sample values of their weights (i.e. $W_{2}$ and $W_{3}$ ).

The identification procedure could be successfully carried out in conditions of changing values of the reduced masses inertia moment and the induction motor drive $J_{l}$, which has been stiffly connected to the rotor.

The results of the backpropagation neural network processing depend on the appropriate defining of the input signals in the first layer $X_{l}$ and also on the initial selection of the weight values in the first, second and the third layer (i.e. in $W_{l}$, in $W_{2}$ and in $W_{3}$ ).

Input values $X_{I}$ in the first layer of the neural network represent values of the matrix $N$ normalized to the range $\left[h_{2}, k_{2}\right]$ and have been calculated according to the formula:

$$
\begin{equation*}
X_{1(p)(j)}=\left(\frac{\left(N_{(p)(j)}-k_{1}\right)}{h_{1}-k_{1}}\right) \cdot\left(h_{2}-k_{2}\right)+k_{2} ; j \in\{1,2, \ldots, 2048\}, p \in\{1, \ldots, 7\} \tag{2}
\end{equation*}
$$

where: $\quad N$ - values of the matrix registered during the test,
$h_{l}$ and $k_{l}$ - the minimal and the maximal value of the matrix $N$ determined during the test,
$h_{2}$ and $k_{2}$ - the initial and the final value of the range, which contains normalized values of the matrix $N$ calculated during the test, $p$ - a column number of the matrix " $K_{4}$ ".

For all tested physical quantities variables $h_{2}$ and $k_{2}$ represent the minimal $\left(h_{2}=\min \left(h_{3}, h_{4}\right)\right)$ and the maximal $\left(k_{2}=\max \left(h_{3}, h_{4}\right)\right)$ value. The values of the variables $h_{3}$ and $h_{4}$ have been calculated in the following way: $h_{3}=\left(h_{5}-h_{6}\right)$ and $h_{4}=\left(h_{6}-h_{5}\right)$, where $h_{5}$ and $h_{6}$ have been calculated as follows:

$$
\begin{align*}
& h_{5}=\left(N_{1(1)}-N_{1(2)}\right)-\left(N_{1(2)}-N_{1(3)}\right)-\left(N_{1(3)}-N_{1(4)}\right)-\left(N_{1(4)}-N_{1(5)}\right),  \tag{3}\\
& h_{6}=\left(N_{2(1)}-N_{2(2)}\right)-\left(N_{2(2)}-N_{2(3)}\right)-\left(N_{2(3)}-N_{2(4)}\right)-\left(N_{2(4)}-N_{2(5)}\right) . \tag{4}
\end{align*}
$$

Values of two matrices $N_{m}$ for $m \in\{1,2\}$ represent arithmetic means of the matrix $N$ and during the test values of their rows' elements have been calculated for a given value of the inertia moment $J_{l}$ using the following formula:

$$
N_{m(p)(i)}=\frac{\sum_{j=1}^{n} N_{(p)(j)}}{n}
$$

for $i \in\{1, \ldots, 5\}, p \in\{1, \ldots, 7\}$ and for $m \in\{1,2\}$; and with the notice that for $m=1$ (i.e. for matrix $N_{l}$ ): for $i=1 n$ is equal to 100 ; for $i=2 n$ is equal to 200; for $i=3 n$ is equal to 300 ; for $i=4 n$ is equal to 400 and for $i=5 n$ is equal to 500; and for $m=2$ (i.e. for matrix $N_{2}$ ) with the notice that: for $i \in\{1, \ldots, 5\}$ and $p \in\{1, \ldots, 7\}$; and with the notice that for $i=1 n$ is equal to 256 ; for $i=2$ $n$ is equal to 512 ; for $i=3 n$ is equal to 768 ; for $i=4 n$ is equal to 1024 and for $i=5 n$ is equal to 1280 .

The best result of calculations has been obtained while using the two cases of determining the values of variables $h_{3}$ and $h_{4}$ for simulations with apparent viscosity coefficient coefficient $c=1.0$.

During the test calculations of values of variables $h_{3}$ and $h_{4}$ have been carried out for the specified earlier value of inertia moment $J_{1}$ in the following way:

$$
\begin{gather*}
h_{3}=\left\{\begin{array}{l}
\left(h_{5}-h_{6}\right), \text { for } h_{7}<=h_{8} \\
\left\{\frac{\left(m_{1}\right)}{\left(h_{11}\right)},\right. \\
\text { for } h_{9}<=h_{10} \text { and } h_{7}>h_{8}, \\
h_{12}, \\
\text { for } h_{9}>h_{10}
\end{array}\right. \\
h_{4}=\left\{\begin{array}{l}
\left(h_{6}-h_{5}\right) ; \text { for } h_{7}<=h_{8} \\
\begin{cases}\left(m_{1}\right) \cdot\left(h_{11}\right) ; & \text { for } h_{9}<=h_{10} \text { and } h_{7}>h_{8}, \\
m_{2} ; & \text { for } h_{9}>h_{10}\end{cases}
\end{array}\right. \tag{5}
\end{gather*}
$$

where: $\quad h_{7}, h_{9}-$ values of the variables representing the result of calculations executed with using specific, appropriate variables during the test, $h_{8}$ the minimal value of the matrix $N_{3}$ determined during the test, $h_{10}, h_{11}, h_{b 12}$ - values of these variables have been obtained in the test, $m_{1}, m_{2}-$ arithmetic means calculated during the test.

Value of the variable $h_{7}$ has been calculated in the following way: $h_{7}=h_{13}+\left(m_{3}-h_{8}\right)$. The variable $h_{13}$ carries the maximum value of the matrix $N_{3}$ (i.e. $h_{13}=\max \left(N_{3(p)(i)}\right)$, for $i \in\{1, \ldots, 6\}$ and $p \in\{1, \ldots, 7\}$ ). The variable $h_{8}$ carries the minimum value of the matrix $N_{3}$ (i.e. $h_{8}=\max \left(N_{3(p)(i)}\right)$ for $i \in\{1, \ldots, 6\}$ and $p \in\{1, \ldots, 7\}$ ). The variable $m_{3}$ is the arithmetic mean calculated for the set up number of elements of the matrix $N_{3}$ during the test - i.e.

$$
m_{3}=\frac{\sum_{i=1}^{3} N_{3(p)(i)}}{\int 3}
$$

for $i \in\{1, \ldots, 6\}$ and $p \in\{1, \ldots, 7\}$ ).

Elements of the matrix $N_{3}$ - carry the results of the cyclic summations of values of variable $h_{16}$ calculated for the tests: $N_{3(p)(i)}=\sum h_{16}$ for $N_{(p)(i, j)}>\partial_{1(p)(i)}$, for $i \in\{1, \ldots, 6\}, \mathrm{j} \in\{1, \ldots, 2048\}$, and for $\mathrm{p} \in\{1, \ldots, 7\}$.
$\partial_{1(p)(i)}$ is the standard deviation of elements of matrix $N i$-th row: $\partial_{I(p)(i)}=\sqrt{\frac{\left(N_{(p)(i, j)}-m_{\notin(i)}\right)^{2}}{2048}}$, for $i \in\{1, \ldots, 6\}, j \in\{1, \ldots, 2048\}$, and $p \in\{1, \ldots, 7\}$.

The value of the arithmetic mean $m_{4}$ has been calculated using the formula:
$m_{4(p)(i)}=\frac{\sum_{j=1}^{2048} N_{(p)(i, j)}}{2048}$, for $i \in\{1, \ldots, 6\}, j \in\{1, \ldots, 2048\}$, and $p \in\{1, \ldots, 7\}$.
The variable $h_{16}$ stores the biggest value of the matrix $N$, which has been registered during simulations (i.e. $h_{16}=\max \left(N_{3(p)(i, j)}\right)$, for $i \in\{1, \ldots, 6\}$, $j \in\{1, \ldots, 2048\}$ and $p \in\{1, \ldots, 7\})$.

The values of variables $h_{b 9}, h_{b 10}, h_{14}$ and $h_{15}$ have been calculated as follows:
$h_{9}=\left|h_{14}-h_{18}\right|$ and $h_{10}=\max \left(h_{15}, h_{16}\right), h_{14}=\sum h_{16}$, for $N_{(p)(j)}>m_{2(p)}$, for $j \in\{1, \ldots, 2048\}$, and $p \in\{1, \ldots, 7\} ; h_{15}=\left|h_{17}-h_{18}\right|$. The value of the variable $m_{2}$, which is arithmetic mean of the tested matrix $N$, has been calculated as: $m_{2(\mathrm{p})}=\frac{\sum_{j=1}^{2048} N_{(p)(j)}}{2048}$, for $j \in\{1, \ldots, 2048\}$, and $p \in\{1, \ldots, 7\}$. The variables $h_{15}$ and $h_{16}$ are obtained and used during the test. The variable $h_{17}$ and $h_{18}$ store the biggest and the smallest value of the matrix $N_{4}$ (i.e. $h_{17}=\max \left(N_{4(p)(i)}\right)$ and $h_{18}=\min \left(N_{4(p)(i)}\right)$, for $i \in\{3,4, \ldots, 6\}$ and $\left.p \in\{1, \ldots, 7\}\right)$.

The variable $h_{l}$ stores the smallest value determined for selected elements of the matrix $N_{4}$.

The matrix $N_{4}$ has been calculated using the formula: $N_{4(\mathcal{p})(i)}=\sum h_{16}$, for $N_{(p)(i, j)}>m_{4(p)(i)}$, for indexes $i \in\{1,2, \ldots, 6\}, j \in\{1,2, \ldots, 2048\}$ and $p \in\{1, \ldots, 7\}$. The variable $m_{l(p)}$ is the arithmetic mean calculated for the specific row of the matrix $N$ (i.e. $m_{1(p)}=\frac{\sum_{j=1}^{2048} N_{(p)(1, j)}}{2048}$, for $j \in\{1, \ldots, 2048\}$ and $p \in\{1, \ldots, 7\}$ ), where $N$ is the matrix registered during the test, while the backlash zone width has its smallest value.

The value of the variable $h_{11}$ has been calculated in the test using the following formula: $h_{11}=\left(h_{19}-h_{21}\right)-\left(h_{20}-h_{21}\right)-\left(m_{5}-h_{21}\right)$,
where: $\quad h_{19}, h_{20}-$ variables storing the smallest and the biggest value of the matrix $N_{5}$,
$h_{21}$ - a variable used in the test,
$m_{5}$ - the value of the arithmetic mean of the matrix $N_{5}$.
The values of the matrix $N_{5}$ have been calculated during the test in the following way:

$$
\begin{gathered}
N_{5(p)(i)}=\left(N_{2(p)(i, 1)}-N_{2(p)(i, 2)}\right)-\left(N_{2(p)(i, 2)}-N_{2(p)(i, 3)}\right)-\left(N_{2(p)(i, 3)}-N_{2(p)(i, 4)}\right) \\
-\left(N_{2(p)(i, 4)}-N_{2(p)(i, 5)}\right),
\end{gathered}
$$

for $i \in\{1,2, \ldots, 6\}$ and $p \in\{1, \ldots, 7\}$.
The variables $h_{19}$ and $h_{20}$ store the smallest and the biggest values of the matrix $N_{5}$ (i.e. $h_{19}=\min \left(N_{5(p)(i)}\right)$ and $h_{20}=\max \left(N_{5(p)(i)}\right)$, for $i \in\{1, \ldots, 6\}$ and $p \in\{1, \ldots, 7\})$. The value of the arithmetic mean $m_{5}$ has been calculated using the formula: $m_{5}=\frac{\sum_{i=1}^{6} N_{5(p)(i)}}{6}$ for $p \in\{1, \ldots, 7\}$. The value of the variable $h_{21}$ has been calculated in the following way:

$$
h_{21}=\left(N_{1(1)}-N_{1(2)}\right)+\left(N_{1(2)}-N_{1(3)}\right)+\left(N_{1(3)}-N_{1(4)}\right)+\left(N_{1(4)}-N_{1(5)}\right) .
$$

The value of the variable $h_{12}$ has been calculated according to the following summation: $h_{12}=\sum h_{16}$, for $N_{(p)(j)}>\partial_{2(p)}$, for $j \in\{1,2, \ldots, 2048\}$ and $p \in\{1, \ldots, 7\}$, and $\partial_{2(p)}$ is the standard deviation calculated as: $\partial_{2(\mathrm{p})}=\sqrt{\frac{\left(N_{(p)(j)}-m_{2}\right)^{2}}{2048}}$, for $j \in\{1, \ldots, 2048\}$ and $p \in\{1, \ldots, 7\}$. The variable $m_{2}$ is the arithmetic mean of the tested matrix $N$.

Initial values of the weights in all layers of the backpropagation neural network have been calculated using the arithmetic mean $m_{l}$ described earlier (i.e. $W_{(d)(i, j)}=m_{1(\mathrm{i})}$, with the notice that for $d=1 i \in\{1, \ldots, 15\}$ and $j \in\{1, \ldots, 2048\}$; for $d=2 i \in\{1, \ldots, 15\}$ and $j \in\{1, \ldots, 15\}$; for $d=3 i \in\{1, \ldots, 6\}$ and $j \in\{1, \ldots, 15\}$, and where: $m_{1}-$ is the value of the arithmetic mean calculated in the determined group of analysis, $d$ - the layer number of the neural network.

The output signals $Z_{l}$ in the first layer of the neural network have been calculated using the formula: $Z_{1(i)}=\sum_{j=1}^{2048} X_{1(j)} \cdot W_{1(i, j)}$, with the notice that for $d \in\{1,2\}$ the index $i$ belongs to the set $\{1, \ldots, 15\}$; and for $d=3$ the index $i$ belongs to the set $\{1, \ldots, 6\}$. The output signals of each layer have been used to calculate the value of the activation function of neurons according to the formula: $Y_{(d)(i)}=\frac{1}{1+\exp ^{\left(-z_{(d)(i)}\right)}}$, with the same notice as mentioned before, i.e. for $d \in\{1,2\}$ the index $i$ belongs to the set $\{1, \ldots, 15\}$; and for $d=3$ the index $i$ belongs to the set $\{1, \ldots, 6\}$. The input signals in the hidden layer, as well as in the output layer of the neural network have been calculated using the formula: $X_{(d)(j)}=Y_{(d-1)(i)}$, for $d \in\{2,3\}, i \in\{1, \ldots 15\}$, and $j \in\{1, \ldots 15\}$. The output signals $Z_{2}$ and $Z_{3}$ from neurons within the hidden layer and the output layer of the neural network have been calculated using the formula: $Z_{(d)(i)}=\sum_{j=1}^{15} X_{(d)(j)} \cdot W_{(d)(i, j)}$, with the notice that for $d=2$ the index $i$ belongs to the set $\{1, \ldots, 15\}$; and for $d=3$ the index $i$ belongs to the set $\{1, \ldots, 6\}$. The general formula of the output of the exponential activation function for the given input to the neural network $T$ is the following: $T_{(i)}=\exp \left(-h_{2}\right)$, for $i \in\{1, \ldots 6\}\left(h_{2}-\right.$ was explained at the bottom of the 6th page of this article). The index $i$ shows the row number within the matrix $T$. Values of errors $E$ on neurons in the output layer have been calculated using the formula: $E_{(i)}=T_{(i)}-Y_{3(i)}$, for $i \in\{1, \ldots 6\}$. The value of the root-meansquare error $R M S_{I}$ has been determined according to the formula:

$$
R M S_{I}=\sqrt{\frac{\sum_{i=1}^{6}\left(E_{(i)}\right)^{2}}{6}}
$$

It has been assumed that the process of learning of this neural network would end, when the following condition would be satisfied: $R M S_{I}<\delta_{1}$, where $\delta_{I}$ is the experimentally assumed value during conducted simulations for the necessity of stopping the neural network learning process.

The correction of weights, carried out in the respective layers of this neural network has been calculated according to its general formula (Osowski, 1996): $W_{(d)(i, j)}=W_{(d)(i, j)}+l \cdot \sigma_{(d)(i)} \cdot X_{b(d)(j)}$, with the notice that for $d=1 i \in\{1, \ldots, 15\}$ the index $j \in\{1, \ldots, 2048\}$; for $d=2$ the index $i \in\{1, \ldots, 15\}$ and the index $j \in\{1, \ldots, 15\}$; for $d=3$ the index $i \in\{1, \ldots, 6\}$ and the index $j \in\{1, \ldots, 15\}$, where: $l$ - the learning coefficient of the neural network, $l \in[0, \ldots, 1]$,
$\sigma_{(d)(i)}$ - the error value calculated in a given layer using the formula:
$\sigma_{(d)(i)}=E_{1(d)(i)} \cdot \sum_{i=1}^{15} W_{(d+1)(i, j)} \sigma_{(d+1)(i)}$, with the notice that for $d=1$ the indexes $i$ and $j$ belong to the set $\{1, \ldots, 15\}$; and for $d=2$ the index $i$ belongs to the set $\{1, \ldots, 15\}$, but the index $j$ belongs to the set $\{1, \ldots, 6\}$. For $d=3$ and for the index $i$ belonging to the set $\{1, \ldots, 6\}$ the error value in the third layer has been calculated using the formula: $\sigma_{(d)(i)}=E_{1(d)(i)} *\left(T_{(\mathrm{d})(i)}-Y_{3(\mathrm{~d})(i)}\right) . E_{(d)(i)}-$ is the value of the derivative of the sigmoidal activation function, calculated on the $i$-th neuron within the layer $d$ of the neural network $T$, using the formula: $E_{1(d)(i)}=Y_{(d)(i)} \cdot\left(1-Y_{(d)(i)}\right)$, with the notice that for $d \in\{1,2\}$ the index $i$ belongs to the set $\{1, \ldots, 15\}$; and for $d=3$ the index $i$ belongs to the set $\{1, \ldots, 6\}$.

Pattern matrices $W_{w}$ have been created for all tested physical quantities for the given group of tests. It has been noticed that creation of pattern matrices and tested matrices $W_{t}$ with using the values of the $W_{3}$ neural network weights matrix has allowed to obtain much better identification of the backlash zone width, compared to using weights matrix from other layers of this neural network. This has been noticed for all tested physical quantities for the given group of tests.

Matrix $W_{4}$ represents values of weights of the first neuron of the output layer $W_{3}$, i.e.: $W_{4(j)}=W_{3(1, j)}$ for $j \in\{1, \ldots, 15\}$. The values of the pattern matrix $W_{w}$ as well as the values of the tested matrix $W_{t}$, which have been applied in the process of identification of the backlash zone width have been obtained using the following formulas: $W_{w(i, j)}=W_{4(i, j)}$, for $i \in\{1, \ldots, 6\}$ and $j \in\{1, \ldots, 15\}$; and $W_{t(j)}=W_{4(j)}$, for $j \in\{1, \ldots, 15\}$. Then it has been created the matrix $D$ in the following way: $D_{(i)}=\sum_{j=1}^{15}\left|W_{t(j)}-W_{w(i, j)}\right|$ for $i \in\{1, \ldots, 6\}$. Then, based on calculating the minimal value of the matrix $D$ it has been determined the searched index $n r_{5}$, $n r_{5}=i$, such that $D_{\left(n r_{5}\right)}=\min \left(D_{(i)}\right)$. The searched index $i$ (i.e. called $n r_{5}$ ) in the matrix $D$ determines the column number in the matrix $K$, which contains the correct backlash zone width.

### 3.2. The results of the simulations of the diagnostic algorithm applied for identification of a backlash zone width in electromechanical system

In the column named The results there have been presented assumed in the fault identification process of the backlash zones widths. In this column using bold font there have been distinguished obtained the final values of the matrix $D$, resulting from applying the identification procedure, which has allowed to obtain the fault identification of number. The tables 1 through 4 show all the values of all parameters that have been used in the tests. It could be noticed that for the neural network learning coefficient $l$ changing within the [0.6...0.9] range and for the value of the variable $\delta_{1}$, used for stopping the neural network learning process changing within the [0.07...0.08] range - for all analyzed physical quantities have been obtained satisfactory result concluded with finding the minimum value of the matrix $D$, which was determining the searched index $n r_{5}$ (i.e. the column number in the matrix $K$, which contains the correct backlash zone width).

Tab. 1. Exemplified in matrix $D$ the results of tests for linear acceleration of the induction motor drive $a_{1}$

| Applied in the test <br> parameters | The <br> results | Applied in the test <br> parameters | The results |
| :---: | :---: | :---: | :---: |
| backlash zone $=0.0075$, | 0.2736 | backlash zone $=0.0075$, | 0.2693 |
| inertia moment | 0.2727 | inertia moment | 0.2684 |
| $J_{l}=0.8917$, | 0.1933 | $J_{l}=0.8917$, | 0.1864 |
| $c=1.5$, epochs $=11$, | $\mathbf{0 . 0 0 0 6}$ | $c=1.5$, epochs $=12$, | $\mathbf{0 . 0 0 0 6}$ |
| $l=0.9, \delta_{l}=0.08$ | 0.0116 | $l=0.9, \delta_{l}=0.07$ | 0.0122 |
|  | 0.0791 |  | 0.0824 |
| backlash zone $=0.0025$, | $\mathbf{0 . 0 0 2 0}$ | backlash zone $=0.0025$, | $\mathbf{0 . 0 0 2 1}$ |
| inertia moment | 0.1912 | inertia moment | 0.1733 |
| $J_{l}=0.8482$, | 0.2786 | $J_{l}=0.8482$, | 0.3289 |
| $c=0.8$, epochs $=12$, | 0.3581 | $c=0.8$, epochs $=15$, | 0.4099 |
| $l=0.9, \delta_{l}=0.08$ | 0.2390 | $l=0.8, \delta_{l}=0.07$ | 0.2226 |
|  | 0.0238 |  | 0.0262 |

Tab. 2. Exemplified in matrix $D$ the results of tests for electromagnetic moment of the induction motor drive $m_{e l}$

| Applied in the test <br> parameters | The <br> results | Applied in the test <br> parameters | The results |
| :---: | :---: | :---: | :---: |
| backlash zone $=0.009$, | 0.0616 | backlash zone $=0.009$, | 0.0172 |
| inertia moment | 0.0106 | inertia moment | 0.0466 |
| $J_{l}=0.8265$, | 0.0081 | $J_{l}=0.8265$, | 0.0087 |
| $c=1.5$, epochs $=22$, | 0.0082 | $c=1.5$, epochs $=26$, | 0.0088 |
| $l=0.9, \delta_{l}=0.08$ | $\mathbf{0 . 0 0 0 4}$ | $l=0.9, \delta_{l}=0.07$ | $\mathbf{0 . 0 0 0 4}$ |
|  | 0.0068 |  | 0.0073 |
| backlash zone $=0.0025$, | $\mathbf{0 . 0 0 3 8}$ | backlash zone $=0.0025$, | $\mathbf{0 . 0 0 4 1}$ |
| inertia moment | 0.2669 | inertia moment | 0.3013 |
| $J_{l}=0.9135$, | 0.2659 | $J_{l}=0.9135$, | 0.3002 |
| $c=0.8$, epochs $=19$, | 0.2584 | $c=0.8$, epochs $=25$, | 0.2921 |
| $l=0.9, \delta_{l}=0.08$ | 0.2514 | $l=0.8, \delta_{l}=0.07$ | 0.2528 |
|  | 0.2501 |  | 0.2514 |

Additionally, as a result of decreasing a previously set up range of the variable $\delta_{1}$, used for stopping the neural network learning process to the range from 0.08 to 0.07 , by lowering the range of the neural network learning coefficient $l$ to the range from 0.9 to 0.6 , it has been noticed a big increase in number of epochs of the neural network learning process. All this is visible in all four tables, that present the results of the analysis.

Tab. 3. Exemplified in matrix $D$ the results of tests for angular speed of the rotor of the induction motor drive $\omega_{1}$

| Applied in the test <br> parameters | The <br> results <br> $\cdot \mathbf{1 0}^{-3}$ | Applied in the test <br> parameters | The <br> results <br> $\cdot \mathbf{1 0}^{-3}$ |
| :---: | :---: | :---: | :---: |
| backlash zone $=0.0025$, | 0.3001 | backlash zone $=0.0025$, | 0.3008 |
| inertia moment | 0.1935 | inertia moment | 0.1939 |
| $\boldsymbol{J}_{l}=0.8047$, | $\mathbf{0 . 0 1 8 5}$ | $J_{l}=0.8047$, | $\mathbf{0 . 0 1 8 5}$ |
| $c=0.8$, epochs $=22$, | 0.2341 | $c=0.8$, epochs $=25$, | 0.2346 |
| $l=0.9, \delta_{l}=0.08$ | 0.4384 | $l=0.8, \delta_{l}=0.08$ | 0.4393 |
|  | 1.2073 |  | 1.2098 |
|  | 0.2754 |  | 0.2937 |
| backlash zone $=0.01$, inertia | 0.3908 | backlash zone $=0.01$, | 0.4168 |
| moment | 0.1967 | inertia moment | 0.2099 |
| $\boldsymbol{J}_{l}=0.87$, | $J_{l}=0.87$, | 0.0966 |  |
| $c=1.25$, epochs $=22$, | 0.0907 | $c=1.25$, epochs $=39$, | 0.1751 |
| $l=0.9, \delta_{l}=0.08$ | 0.1643 | $l=0.6, \delta_{l}=0.07$ | $\mathbf{0 . 0 2 6 0}$ |

Tab. 4. Exemplified in matrix $D$ the results of tests for linear acceleration of the mass $\boldsymbol{a}_{\mathbf{2}}$

| Applied in the test <br> parameters | The <br> results <br> $\cdot \mathbf{1 0}^{-3}$ | Applied in the test <br> parameters | The <br> results <br> $\cdot \mathbf{1 0}^{-3}$ |
| :---: | :---: | :---: | :---: |
|  | 0.1350 | backlash zone $=0.005$, | 0.1353 |
| backlash zone $=0.005$, | 0.0792 | inertia moment | 0.0794 |
| inertia moment | $\mathbf{0 . 0 2 5 8}$ | $J_{l}=0.7612$, | $\mathbf{0 . 0 2 5 8}$ |
| $J_{l}=0.7612$, | 0.1070 | $c=0.8$, epochs $=25$, | 0.1072 |
| $c=0.8$, epochs $=22$, | 0.1759 | $l=0.8, \delta_{l}=0.08$ | 0.1762 |
| $l=0.9, \delta_{l}=0.08$ | 0.1584 |  | 0.1588 |
|  | 0.2145 |  | 0.2337 |
|  |  | backlash zone $=0.0075$, | 0.1729 |
| backlash zone $=0.0075$, | 0.1587 | inertia moment | 0.1147 |
| inertia moment | 0.1052 | $J_{l}=0.7612$, | $\mathbf{0 . 0 3 0 0}$ |
| $J_{l}=0.7612$, | $\mathbf{0 . 0 2 7 6}$ | $c=1.5$, epochs $=27$, | 0.1050 |
| $c=1.5$, epochs $=22$, | 0.0964 | $l=0.9, \delta_{l}=0.07$ | 0.0860 |
| $l=0.9, \delta_{l}=0.08$ | 0.0790 |  |  |

## 4. CONCLUSIONS

The executed simulation tests of not established states of a complex model of an electromechanical system containing heavy nonlinearities have confirmed that using time-frequency methods with multistage decomposition of signal, or by a method of three-layer backpropagation neural network - these methods have been proved to be the effective research tools.

Using wavelet and neural network to analyze non-stationary signals of the tested electromechanical system enables effective limitation of possible losses resulting from consequences of increasing ranges of faults, which could lead to occurrence of a breakdown. Together with a simulation modeling this approach should be sufficient to enable complex verification process of the diagnostic algorithm.

Information contained in non-stationary diagnostic signals can be extracted from them both, through distributions of wavelet decomposition of coefficients as well as through values of weights of the neural network first layer for chosen parameters of a state, which describe physical quantities. This information may determine the type and place of occurrence of undesirable states of a complex nonlinear electromechanical system. Therefore, there is a possibility of obtaining correct diagnostic results concerning a system containing changing value of a viscous friction $c$ for the given value of discontinuity in zero of the absorbing characteristics $\left(c_{1}\right)$.

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