vibrational diagnostics, footbridge, sensitivity analysis, rotation sensors

Anna BANAŚ[*], *Krzysztof WILDE*[**]

VIBRATION DIAGNOSTICS OF FOOTBRIDGE WITH USE OF ROTATION SENSOR

Abstract

The benefits of the additional measurement of rotational degrees of freedom on the performance of the vibration diagnosis of bridges are studied in this paper. The common vibrational diagnostics that uses translational degrees of freedom is extended by measurements of rotations. The study is curried out on a footbridge and the presence of damage as well as its location and size is determined with use of FEM updating procedure. The results showed that rotational degrees of freedom significantly improve the effectiveness of the vibrational method.

1. INTRODUCTION

In Poland evaluation of technical condition of new bridges with use of in situ measurements is a standard procedure. The bridge capacity tests are the last step before putting the structure into service. The evaluation always consists of static tests during which the displacements of selected points at the bridge deck, settlements of abutments and bridge piers are measured. As a dead load for testing, depending on the type of the structure, locomotives, trucks, road plates or containers filled with water are used. For all railway bridges, road bridges of span more than 20 m and most of the footbridges, dynamic tests are also conducted. The measured values are usually displacements and accelerations at the selected locations of the deck [1]. Bridge oscillations, in the case of road bridges, can by excited by trucks moving with different velocities and passing over the smooth road or over a road with obstacles. It is assumed that the moving truck should fall down from the obstacle of height of 10 cm. For footbridges, which are usually more flaccid, dynamic excitation may be generated by a group of pedestrians marching or running synchronously as well as performing squats or jumps at certain points of the deck [2]. The dynamic force can be also generated by a dynamic actuator or a set of actuators in case of large bridge structures.

[*] Gdańsk University of Technology, G. Narutowicza 11/12, 80-233 Gdańsk, +48 58 348 6154
anna.banas@wilis.pg.gda.pl
[**] Gdańsk University of Technology, G. Narutowicza 11/12, 80-233 Gdańsk, +48 58 347 2497
krzysztof.wilde @wilis.pg.gda.pl

Vibrational diagnostics of engineering structures is usually performed with use of acceleration measurements of translational degrees of freedom [2-6]. Measurements are performed using accelerometers and the signals are used for identifying the natural frequency and mode shapes of the structure. One possible way to evaluate the technical condition of the bridge is to derive a Finite Element Model of the bridge and to update its parameters by using the information stored in the measured natural frequencies and mode shapes [7,8]. The updated FE model contains complete data on bridge stiffness and mass in a form of coefficients of the stiffness and mass matrices. The reduction of the stiffness in the FE element indicates the presence of the damage. Analysis of the element location in FE mesh and its stiffness changes is used to estimate the lo-cation and extend of the damage [9].

The aim of the study is to examine the effectiveness of the vibrational bridge diagnostics with use of FEM updating technique based on measurements of both translational and rotational degrees of freedom [10]. In practice, measurements of rotations are performed using gyro sensors which measure the rate of change in the angle of rotation at the specific points of the bridge deck.

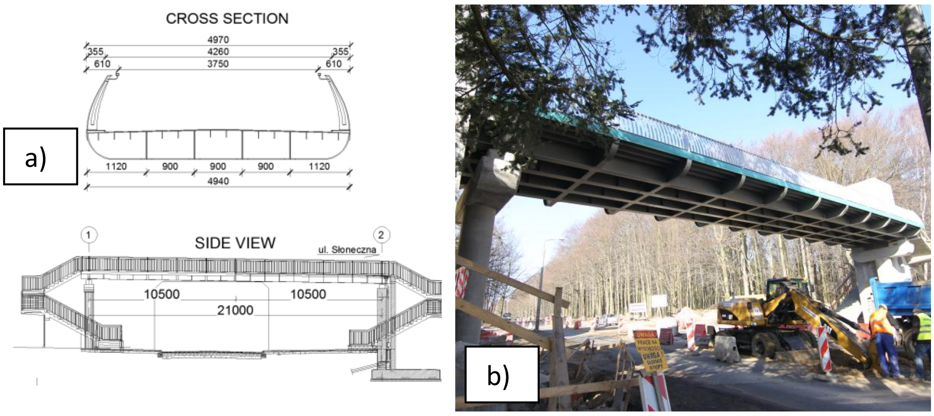


Fig. 1. a) Cross section and side view of a footbridge over the Chwarznieńska street in Gdynia b) Photography of footbridge over the Chwarznieńska street [source: own study]

It is assumed that there is only one damage zone in the bridge. The updating procedure is based on first few flexural vibration frequencies and mode shapes that can be determined in real in situ measurements. The research is carried out on an example of the existing footbridge located in Gdynia, Poland. However, due to lack of the measurement data the research is done in a form of “numerical experiment” where the “measurement data” for testing the damage detection technique is generated numerically and is modified by an added measurement noise.

2. DESCRIPTION OF FOOTBRIDGE

The footbridge is located in Gdynia city and is crossing the Chwarznieńska Street. The superstructure consists of four longitudinal girders and eleven traverses, in which the upper belt is the steel horizontal slab. The slab is additionally reinforced with open, longitudinal ribs. The bridge deck is mounted on the pillars through elastomeric bearings. Major supports are in the shape of the letter "T" with the pillar cross-section of a flattened circle. The basic geometric parameters of the footbridge superstructure are: theoretical span length L_{tp} = 21.00 m, total width of the deck B_c = 4,968 m, height of superstructure H_k = 0,530 ÷ 0,555 m (Fig. 1).

3. DESCRIPTION OF FOOTBRDIGE FEM MODEL

The analysis is carried out with use of two FE models of the footbridge superstructure developed in the commercial program SOFiSTiK. The detailed 3D beam-shell model (Fig. 2) consisted of 5551 nodes, 5632 four node shell elements and 3190 beam elements. The boundary conditions are modelled by setting to zero four vertical, one longitudinal and two transvers displacements at the nodes corresponding to the location of the bridge bearings.

The simplified model consists of only beam elements and has 81 nodes and 80 beam elements with 5 boundary constraints i.e. two vertical, one longitudinal and two transvers displacements are locked.

The first four flexural mode shapes computed by the detailed beam-shell model are shown in Fig. 4. The corresponding first four natural frequencies are respectively 2.98 Hz, 11.75 Hz, 25.56 Hz and 40.33 Hz. Due to symmetry of the bridge deck cross section (Fig. 3) there is a negligible coupling between the vertical, horizontal and torsional motions. Therefore, these four mode shapes are vertical bending modes. Since the number of the stiffening ribs is relatively large the mode shapes have no features characteristic for plate dynamics (Fig. 4).

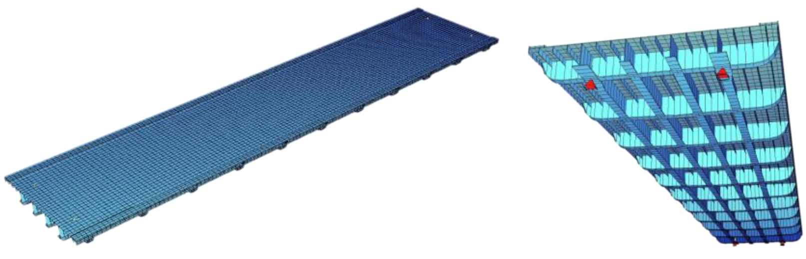


Fig. 2. Discretisation mesh of beam-shell FEM model of footbridge deck [source: own study]

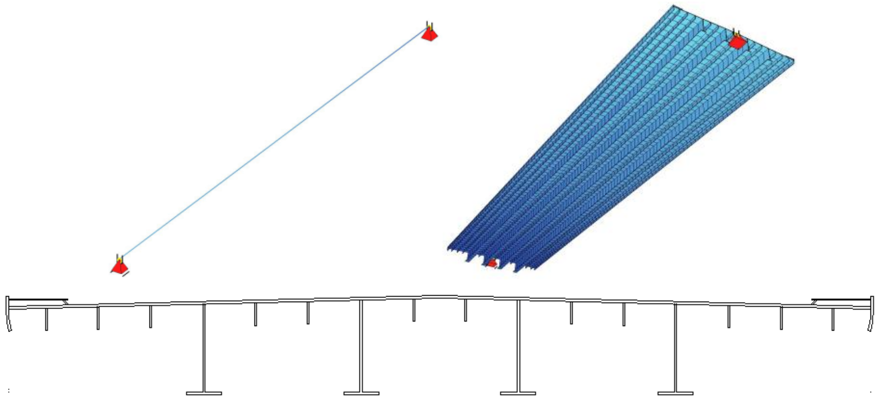


Fig. 3. Static scheme and mesh of beam FEM model and the cross-section of beam element [source: own study]

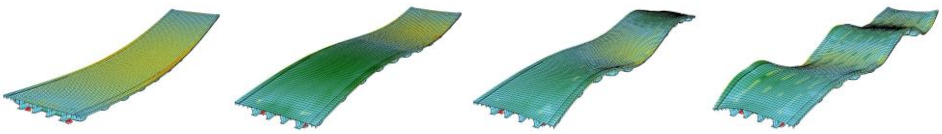


Fig. 4. First four flexural mode shapes of beam-shell model [source: own study]

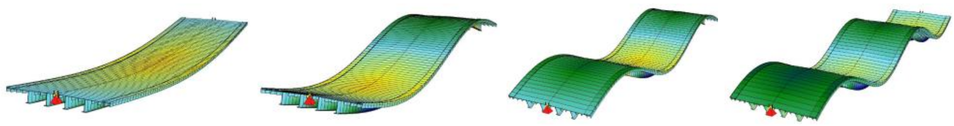


Fig. 5. First four flexural mode shapes of beam model [source: own study]

The mode shapes computed from the simple beam model (Fig. 3) are shown in Fig. 5. The corresponding natural frequencies are respectively 3.07 Hz, 11.66 Hz, 23.35 Hz and 39.71 Hz. The mode shapes form the beam model are vertical bending modes according to the classical beam theory.

The dynamic characteristics of the both FEM models are consistent. The differences in the first four frequencies are respectively 5,86%, 0,77%, 9,46% and 1,56%. The first four mode shapes computed by the detailed and simplified beam model are in a very good agreement. For the implementation of the "numerical experiment", the required accuracy of calculations using the simplified beam model is sufficient. The simulations shown below are carried out by the simplified beam model of the footbridge.

4. ITERATIVE METHOD OF UPDATING FEM MODEL PARAMETERS

The updating procedure used in this study is an iterative optimization technique defined with use of sensitivity matrix. The design parameters $\boldsymbol{\theta}$, in step j +1, are updated through the sensitivity matrix $\mathbf{S}_j$ [11-13]:

$$\begin{aligned} \boldsymbol{\theta}_{j+1} &= \boldsymbol{\theta}_j + \left[\mathbf{S}_j^T\mathbf{S}_j\right]^{-1}\mathbf{S}_j^T \;\; \mathbf{z}_m - \mathbf{z}_{aj} \qquad \text{for } N_w \geq N_\mathrm{p}, \\ \boldsymbol{\theta}_{j+1} &= \boldsymbol{\theta}_j + \mathbf{S}_j^T\left[\mathbf{S}_j\mathbf{S}_j^T\right]^{-1} \;\; \mathbf{z}_m - \mathbf{z}_{aj} \qquad \text{for } N_w \leq N_\mathrm{p}. \end{aligned} \tag{1}$$

where N_p is a number of unknown updated parameters θ_j, N_w denotes number of measured data, $\mathbf{z}_{aj}$ is describes analytical modal pairs and $\mathbf{z}_{mj}$ denoted “measured” modal pairs obtained, in this studies, by simulations with added measurement noise.

The sensitivity matrix can be expressed as:

$$\underset{(N_\mathrm{c}\, x N_\mathrm{p})}{\mathbf{S}} = \begin{bmatrix} \dfrac{\partial\lambda_{\mathrm{a}1}}{\partial\theta_1} & \cdots & \dfrac{\partial\lambda_{\mathrm{a}1}}{\partial\theta_{N_\mathrm{p}}} \\ \dfrac{\partial\boldsymbol{\phi}_{\mathrm{a}1}}{\partial\theta_1} & \cdots & \dfrac{\partial\boldsymbol{\phi}_{\mathrm{a}1}}{\partial\theta_{N_\mathrm{p}}} \\ \vdots & & \vdots \\ \dfrac{\partial\lambda_{\mathrm{a}p}}{\partial\theta_1} & \cdots & \dfrac{\partial\lambda_{\mathrm{a}p}}{\partial\theta_{N_\mathrm{p}}} \\ \dfrac{\partial\boldsymbol{\phi}_{\mathrm{a}p}}{\partial\theta_1} & \cdots & \dfrac{\partial\boldsymbol{\phi}_{\mathrm{a}p}}{\partial\theta_{N_\mathrm{p}}} \end{bmatrix} \tag{2}$$

and its coefficients can be calculated as derivatives of natural frequencies $\lambda_{\mathrm{a}i}$ and mode shapes $\boldsymbol{\phi}_{\mathrm{a}i}$:

$$\frac{\partial\lambda_{\mathrm{a}i}}{\partial\theta_j} = \boldsymbol{\phi}_{\mathrm{a}i}^\mathrm{T}\frac{\partial\mathbf{K}}{\partial\theta_j}\boldsymbol{\phi}_{\mathrm{m}i} - \lambda_{\mathrm{m}i}\boldsymbol{\phi}_{\mathrm{a}i}^\mathrm{T}\frac{\partial\mathbf{M}}{\partial\theta_j}\boldsymbol{\phi}_{\mathrm{m}i} \tag{3}$$

$$\frac{\partial\boldsymbol{\phi}_{\mathrm{a}i}}{\partial\theta_j} = \sum_{k=1;k\neq i}^{N}\frac{\boldsymbol{\phi}_{\mathrm{a}k}\boldsymbol{\phi}_{\mathrm{a}k}^\mathrm{T}}{\lambda_{\mathrm{m}i}-\lambda_{\mathrm{a}k}}\left[\frac{\partial\mathbf{K}}{\partial\theta_j} - \lambda_{\mathrm{m}i}\frac{\partial\mathbf{M}}{\partial\theta_j}\right]\boldsymbol{\phi}_{\mathrm{m}i} - \frac{1}{2}\boldsymbol{\phi}_{\mathrm{a}i}\boldsymbol{\phi}_{\mathrm{a}i}^\mathrm{T}\frac{\partial\mathbf{M}}{\partial\theta_j}\boldsymbol{\phi}_{\mathrm{m}i} \tag{4}$$

where $\mathbf{K}$ is a stiffness matrix, $\mathbf{M}$ denotes mass matrix, ϕ_{ai} or ϕ_{aj} are *i*-th or *j*-th analytical mode shapes and ϕ_{mi} or ϕ_{mj} are *i*-th or *j*-th mode shapes obtained from the “numerical experiment”.

Comparison of compliance between the measured and calculated mode shapes is made by the criterion of certainty MAC (Modal Assurance Criterion) (Eq (5)) [14,15] and by the standardized coefficient NMD (Normalized Difference Modal) (Eq (6)) [14,16].

$$\mathrm{MAC}_{ij} = \frac{\left|\phi_{mi}^{\mathrm{T}}\phi_{aj}\right|^2}{\phi_{aj}^{\mathrm{T}}\phi_{aj} \quad \phi_{mi}^{\mathrm{T}}\phi_{mi}}. \tag{5}$$

$$\mathrm{NMD}_{ij} = \sqrt{\frac{1-\mathrm{MAC}_{ij}}{\mathrm{MAC}_{ij}}}. \tag{6}$$

Furthermore, criteria for assessing the quality of the obtained results are based on two indexes:

$$I_{k\,\max} = \frac{k_{dam}}{k_{\max udam}} \geq 1.5 \tag{7}$$

$$I_{k\,\mathrm{mean}} = \frac{k_{dam}}{k_{mean\,udam}} \geq 4.0 \tag{8}$$

where k_{dam} denotes the change in stiffness of the updated damaged element, $k_{\max udam}$ is the maximum change in stiffness of undamaged elements and $k_{\mathrm{mean}\,udam}$ denotes the average change in stiffness of undamaged elements.

The procedure of searching the damage location requires the following steps:

1. selection of the number of rotations and generation of an array of all possible rotation locations;
2. computation of a vector of measurement data for each combination of “measurements”;
3. reduction of the $\mathbf{K}$ and $\mathbf{M}$ matrixes by the SEREP [17] method to eliminate unmeasured degrees of freedom;
4. computation of the natural frequencies and mode shapes for the reduced size matrices of the model;
5. normalization of the vector of measured mode shapes with respect to the analytical mass matrix;

6. computation of the sensitivity matrix **S**, the difference of vectors of modal pairs, selection of a matrix of weights and computation of the perturbations of design parameters;
7. minimization of the penalty function with respect to changes in designing parameters;
8. computation of the actual stiffness of the finite element model and validation of the completed calculations;
9. indication of the damage location and its extend.

5. PARAMETRIC ANALYSIS OF INFLUENCE OF ROTATION

Analysis of the effectiveness of additional measurements of the rotations for updating the stiffness of the FE model of the footbridge is conducted on a noisy numerical data. It is assumed that the damage is located in the 8th segment that consists of 4 beam elements. The FEM model of the footbridge (Fig. 6) consists of 20 segments and a total of 80 finite elements. The assumed extend of the damage is a 15% reduction in the flexural rigidity with respect to the undamaged bridge deck section. The numerical data, used instead of real "measurements", contains 5% white noise to add the characteristics of real in situ measurement errors. The vector of updating parameters is composed of flexural stiffness of 20 segments of the beam model. It is assumed that the measurements of the acceleration of translational degrees of freedom are performed in 5 locations that are equally spaced along the length of the span (Fig. 7). The parametric tests include the search for the location of damage without measuring the rotation and also assuming that one, two, three and four measurement signals of rotations are used. It is assumed that the rotation sensors can be placed in all the nodes of the footbridge featured in the FE model. The parametric studies are based on the first four flexural vibration frequencies and mode shapes.

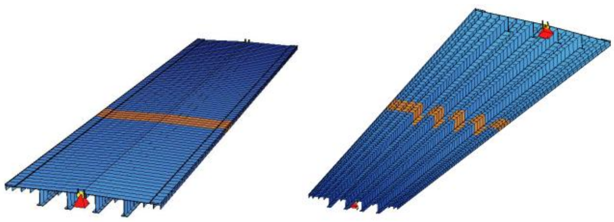


Fig. 6. Discretization mesh of beam FEM model of span with 8th element damaged [source: own study]

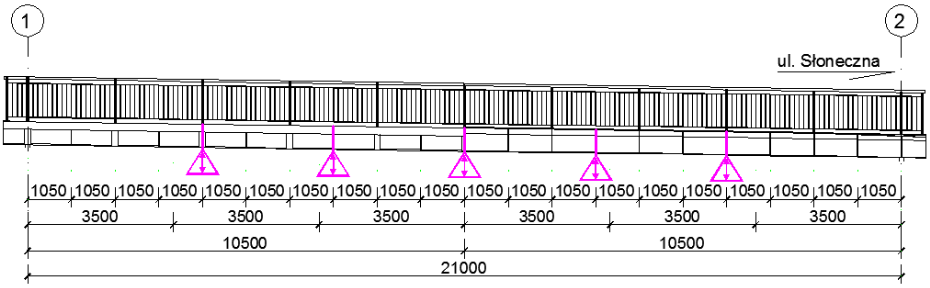


Fig. 7. Arrangement of measured accelerations points [source: own study]

The simulation results showed that updating the parameters of the FE model on data only from the translational degrees of freedom, is impossible. The results of stiffness updating of all 20 footbridge segments based only on the five acceleration signals is shown in Fig. 8. The results indicate that the damage is around the 3rd, 8th, 13th and 18th segment. The maximum change in stiffness is in the 3rd segment and should be in the 8th one.

Fig. 9-12 show results of updating procedure in case of the damage detection preformed successively on one, two, three, and four additional rotation signals. Using five translational degrees of freedom and one rotation (Fig. 9) does not allowed correct indication of the damage location. The diagram of the stiffness changes shows slight damages in several elements.

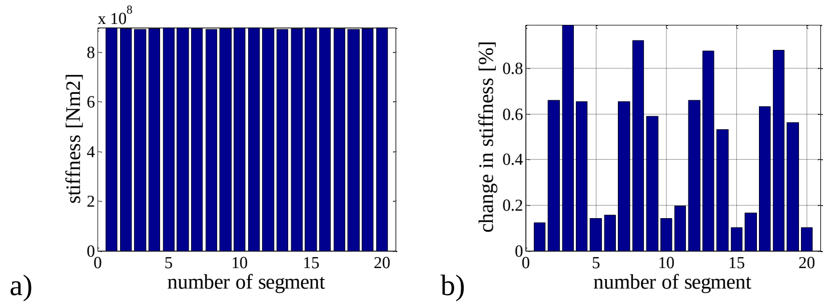
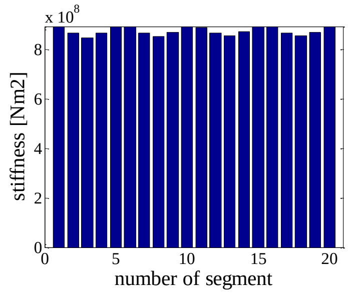


Fig. 8. Calculated change in the stiffness of the segments for the updating on 5 translations; a) change from baseline; b) the change in the stiffness in each segment [source: own study]

From the results shown in Fig. 10-12 it can be concluded that an updating on 5 translations enriched with at least two rotations allows the correct identification of the damage. The maximum value of the stiffness change occurs in segment 8 and it clearly indicates the place of the largest reduction in the beam rigidity. With two additional measurements of rotation change in stiffness of the damaged element is 21%, with three rotations 27%, and 35% with four additional rotations.



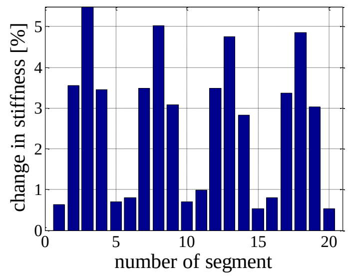
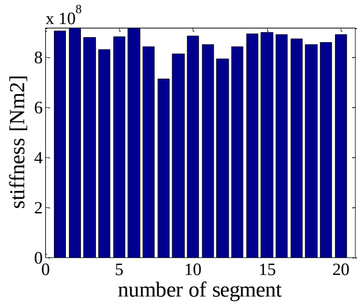


Fig. 9. Calculated change in the stiffness of the segments for an updating on the basis of: 5 translation and 1 rotation; a) change from baseline; b) the percentage change in the stiffness of the element [source: own study]



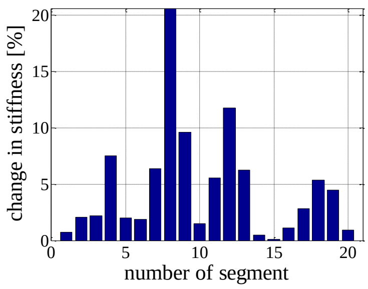
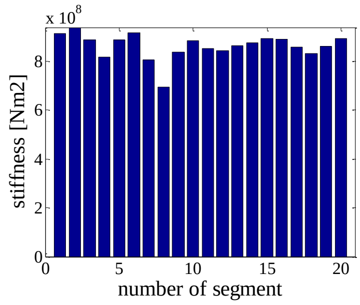


Fig. 10. Calculated change in the stiffness of the segments for an updating on the basis of: 5 translation and 2 rotations; a) change from baseline; b) the percentage change in the stiffness of the element [source: own study]



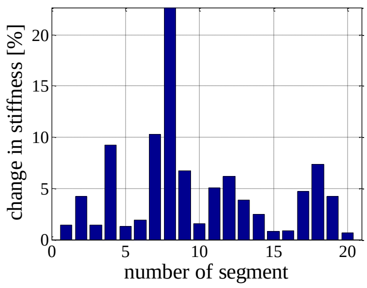


Fig. 11. Calculated change in the stiffness of the segments for an updating on the basis of: 5 translation and 3 rotations; a) change from baseline; b) the percentage change in the stiffness of the element [source: own study]

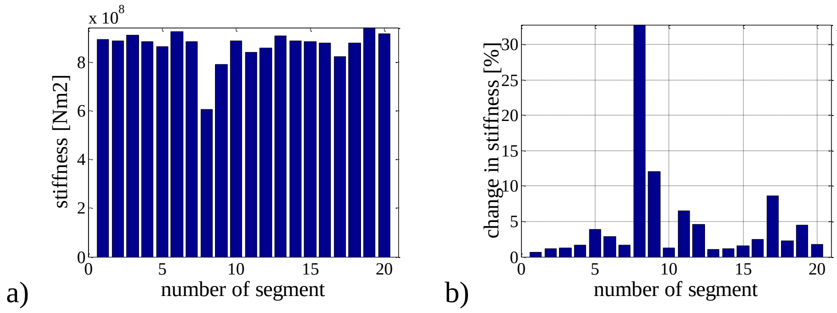


Fig. 12. Calculated change in the stiffness of the segments for an updating on the basis of: 5 translation and 4 rotations; a) change from baseline; b) the percentage change in the stiffness of the element [source: own study]

In addition, to achieve the correct updating results the rotational measurements must be performed in the precisely defined locations (Fig. 13). With two rotations only two combinations of the measurement points allow proper diagnostics. When three measurements of rotations are used only three locations of rotation sensors permits the correct detection of a damaged segment. The correct identification of the damage in the 8th segment is possible for the 24 sensor location patterns, if four measurements of the angles of rotation are available.

Table 1 summarizes the tests results with NMD criterion depending on the number of additional rotations. Table 2 lists the corresponding values of the quality indexes. In both cases, the results are presented for the most appropriate location of the rotational sensors. The results presented in Table 2 show that if only one rotation is used, the quality indexes have values of less than 1% and therefore, the procedure incorrectly indicates the defective segment. By using at least two rotation, values of quality indices increase significantly and exceeds the level of 8% for $I_{k\,\mathrm{mean}}$ and 3% for $I_{k\,\max}$, which allows the proper identification of the damage location.

Tab. 1 NMD criterion value depending on the number of rotations

Frequency	NMD			
	1 rotation [-]	2 rotations [-]	3 rotations [-]	4 rotations [-]
1	6.2986	4.7563	4.8451	4.3197
2	1.8096	1.5797	1.6239	1.5616
3	3.6048	1.5629	1.4888	1.6313
4	8.0029	3.2491	3.2455	3.2244

Tab. 2 Quality index values depending on the number of rotations

Index	Quality index values			
	1 rotation [-]	2 rotations [-]	3 rotations [-]	4 rotations [-]
$I_{k\,\mathrm{mean}}$	0.99539	8.2566	8.393	9.7005
$I_{k\,\mathrm{max}}$	0.99179	3.495	3.4561	3.385

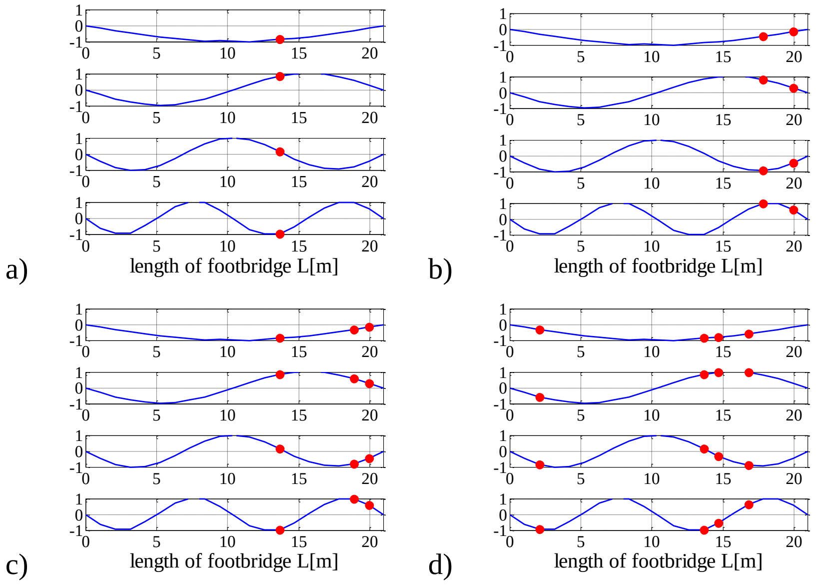


Fig. 13. Location of the best additional measurement points of rotations for a) one signal b) two signals c) three signals d) four signals [source: own study]

6. SUMMARY

The paper presents the parametric study of the vibrational damage identification method based on additional measurements of the rotational degrees of freedom. Analysis of the tests conducted on the numerical data with added noise for the steel footbridge showed that that the additional information from rotations improve the performance of the method. The correct detection of the damage for assumed damage extent corresponding to 15% stiffness reduction can be obtained if at least two additional rotations are measured. The effectiveness of the method depends also on the location of the rotation sensors. If only two sensors are used only two patterns of they locations are available. In case when five rotations are used they can by placed in 24 combinations of sensor locations.

REFERENCES

[1] BIEN J., ZWOLSKI J.: *Dynamic Tests in Bridge Monitoring – Systematics and Applications.* International Modal Analysis Conference, Orlando, Florida, USA, 2007, CD (10 s.).

[2] SIWOWSKI T., JANAS L.: *Numerical and experimental assessment of the dynamic behaviour of a circular footbridge.* Proceedings of the 9th International Conference on Structural Dynamics, EURODYN 2014, Porto, Portugal, 2014.

[3] BACINSKAS D., KAMAITIS Z., JATULIS D., KILIKEVICIUS A.: *Load Testing and Model Updating of a Single Span Composite Steel-Concrete Railway Bridge.* 11th International Conference on Modern Building Materials, Structures and Techniques, MBMST 2013.

[4] GONILHA J. A., CORREIA J. R., BRANCO F. A.: *Dynamic response under pedestrian load of a GFRP–SFRSCC hybrid footbridge prototype: Experimental tests and numerical simulation.* Composite Structures 95(2013), pp.453-463.

[5] LINK M., VOLLAN A.: *Identification of Structural System Parameters from Dynamic Response Data*. Z. Flugwiss. Weltraumforsch, Vol. 2, No. 3, 1978, pp. 165-174.

[6] ZIVANOVIĆ S., PAVIC A., REYNOLDS P.: *Finite element modelling and updating of a lively footbridge The complete process.* Journal of Sound and Vibration 301 (2007), pp. 126-145.

[7] HIRSZ M., WILDE K.: *Imperfection localization in beams by FE model updating based on vibration tests.* CMM Computer Methods in Mechanics: 17th International Conference: proceedings, Lodz-Spala, Poland, 19-22 June 2007.

[8] PANDEY A. K., BISWAS M., SAMMAN M. M.: *Damage detection from changes in curvature mode shapes*. Journal of Sound and Vibration;145:321-332, 1991.

[9] CAWLEY, P., ADAMS R. D.: *The location of defects in structures from measurements natural frequencies*. Journal of Strain Analysis; 14:49-57, 1979.

[10] ZEMBATY Z., KOKOT S., BOBRA P., *Application of rotation rate sensors in an experiment of stiffness 'reconstruction'.* Smart Materials and Structures, 22 (2013) 077001 (5pp), 2013.

[11] FRISWELL M. I., MOTTERSHEAD J. E.: *Finite Element Model Updating in Structural Dynamics.* KAB Netherlands 1995.

[12] FRISWELL M. I., LINK M., MOTTERSHEAD J. E.: *The sensitivity method in finite element model updating: A tutorial.* Mechanical Systems and Signal Processing 25 (2011) pp. 2275-2296.

[13] HIRSZ M., WILDE K.: *Identification of plate dynamic parameters for structural health monitoring.* The 8th Conference "SHELL STRUCTURES: Theory and Applications", Poland, October 2005.

[14] MAIA N. M. M., SILVA J. M. M.: *Theoretical and Experimental Modal Analysis*. RSP LTD England 1998.

[15] ALLEMANG R. J., BROWN D. L.: *A Correlation Coefficient for Modal Vector Analysis.* 1th International Modal Analysis Conference (IMAC I), Orlando, Florida, November 1982, pp. 110-116.

[16] WATERS T. P.: *Finite Element Model Updating Using Measured Frequency Response Function.* Ph.D. Thesis, Department of Aerospace Engineering, University of Bristol, U.K, 1995.

[17] O'CALLAHAN J. C., AVITABILE O., RIEMER R.: *System Equivalent Reduction Expansion Process.* 7^{th} International Modal Conference, Las Vegas, January, 1989.