Nonlinear dynamic characteristics of SMA gripper under bounded noise

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Abstract. A kind of constitutive model of SMA is proposed in this paper, and the nonlinear dynamic response of a SMA gripper under bounded noise is studied. The harmonic driving signals and the random disturbance made up of bounded noise. The dynamic model of the system is established by Hamilton principle. The numerical and experimental results show that there is stochastic resonance in the system; the system's vibration amplitude reaches the most when the outside excitation is moderate.

1 Introduction

Shape memory alloy (SMA) is a type of smart material, which has shape memory effect. SMA gripper is used in medical field widely [1]. To enhance the accuracy of SMA gripper, its dynamic characteristics should be studied. Many researchers have studied SMA gripper [1–7]. Kohl et al. studied a SMA gripper's dynamic response and control [2]. Just et al. applied position control to a SMA gripper and obtained high control accuracy [3]. To SMA materials, Graesser et al. proposed a three-dimensional SMA constitutive model [4]. Ivshin et al. developed a SMA thermo-mechanical model [5]. Although many achievements have been reported, most of them focused on the constitutive model, and the results of dynamic response of SMA gripper are absent.

SMA gripper used in medicine are controlled by harmonic currents to achieve the opening and closing action. However, SMA gripper are usually under stochastic excitation in the working process. Although the stochastic excitation is weak, it will affect the gripper's motion. The harmonic control force and the stochastic excitation generate a bounded noise, which cause the different dynamic characteristics from the harmonic system.

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2 SMA constitutive model

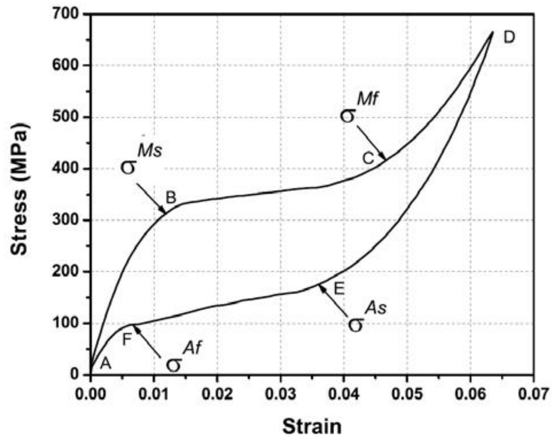


Fig. 1. Strain-stress curves of the SMA.

The experimental results of SMA are shown in Figure 1. The length of Ti-Ni SMA film is 7mm, the width is 1.5mm, and the thickness is 0.1mm. The SMA's austenite finish temperature is 34℃. Thus, the hysteretic phenomenon is induced by the superelastic behavior of SMA. Zhu et al. established a SMA's constitutive model as follows [8]:

$$\sigma = \sigma_1 + \sigma_2 = a_1\varepsilon + a_2\varepsilon^2 + a_3\varepsilon^3 + (a_4\varepsilon + a_5\varepsilon^2 + a_6\varepsilon^3 + a_7\varepsilon^4)\dot{\varepsilon} \quad (1)$$

where σ is the stress, ε is the strain. To SMA shown in Figure 1, a_1 =10000, a_2 =-32, a_3=5.7.

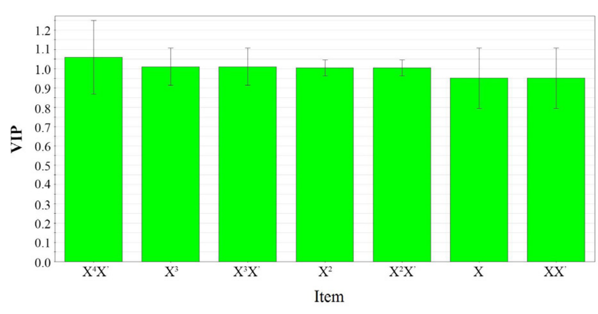


Fig. 2. Variable importance of each term.

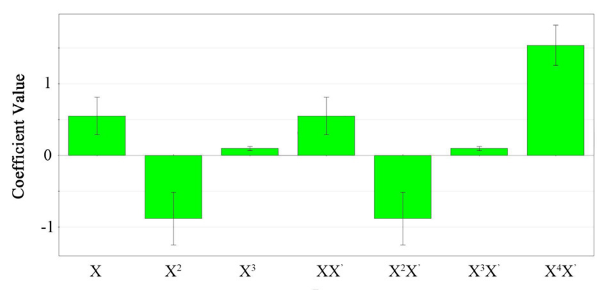


Fig. 3. Coefficient values of each term.

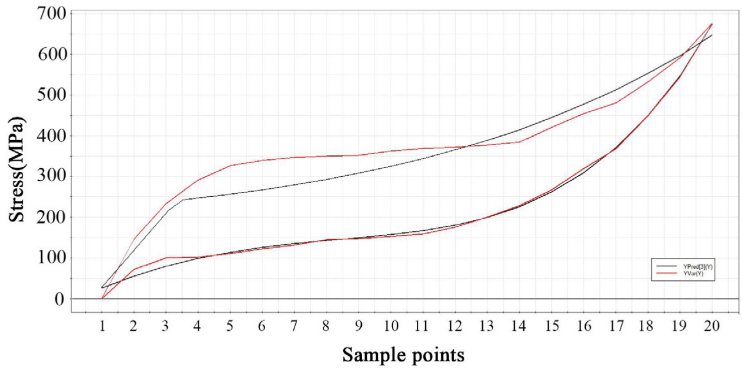


Fig. 4. Results of forecast test for the fitting effect of Eq. (1) on strain-stress data of SMA.

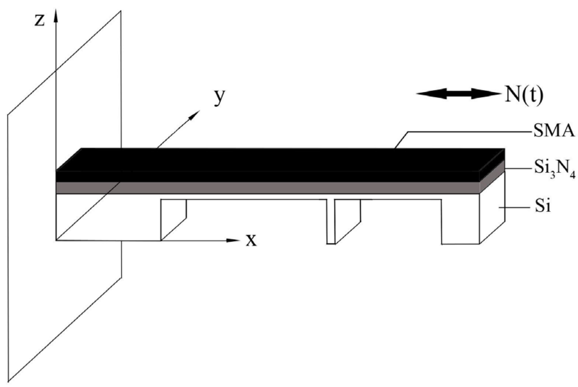


Fig. 5. Results of forecast test for the fitting effect of Eq. (1) on strain-stress data of SMA.

The results of prediction test to Eq. (1) are shown in Figure 4, and the mechanical model of a SMA gripper under bounded noise is shown in Figure 5, where $N(t) = F\sin(\Omega t + \chi + \sigma B(t))$ is the bounded noise. The Hamilton's variational principle is:

$$\delta S = \int_{t_1}^{t_2} \delta(T_1 + T_2 - U_1 - U_2 + W_d + W)dt = 0 \tag{3}$$

where $T_1 = \frac{1}{2}\int_0^L \rho_1 A_1 (\frac{\partial u}{\partial t})^2 dx$, $T_2 = \frac{1}{2}\int_0^L \rho_2 A_2 (\frac{\partial u}{\partial t})^2 dx$, $U_1 = \int_0^L \frac{E_1 I_2}{2}(\frac{\partial^2 u}{\partial x^2})^2 dx + \frac{E_1 A_1}{8L}[\int_0^L (\frac{\partial u}{\partial x})^2 dx]^2$, $U_2 = \frac{1}{2}E_2 A_2 \int_0^L (\frac{\partial u}{\partial x})^2 dx$, $W_d = -\int_0^L cu\frac{\partial u}{\partial t}dx$, $W = \int_0^L \delta u N dx$.

Thus, the dynamic model of a SMA gripper is:

$$m\frac{\partial^2 u}{\partial t^2}+[c+\int_0^L E_1A_1(a_4u+a_5u^2+a_6u^3+a_7u^4)dx]\frac{\partial u}{\partial t}+b_1\frac{\partial^2 u}{\partial x^2}$$
$$+b_2\frac{\partial^4 u}{\partial x^4}-b_3\frac{\partial^2 u}{\partial x^2}\int_0^L(\frac{\partial u}{\partial x})^2dx=N+\frac{1}{2}(E_2A_2\frac{\partial^2 u}{\partial x^2}+\rho_2A_2\frac{\partial^2 u}{\partial t^2}) \tag{4}$$

where $m=\frac{\rho_1A_1l}{L}+\rho_2A_2$, $b_1=\frac{a_1E_2A_2}{3}+\frac{\varepsilon_{33}^sE_3A_1}{2}$, $b_2=a_2E_2I_2$, $b_3=\frac{E_1A_1}{8L}$.

The equation of the system's response are:

$$\begin{cases}\ddot{u}_1+b_1u_1+b_2u_1^3+\gamma u_1u_2^2+(2\eta+b_3u_1^2+b_4u_1^4)\dot{u}_1=e\sin(\Omega t+\theta+\sigma B(t))\\ \ddot{u}_2+c_1u_1+c_2u_1^3+\gamma u_1u_2^2+(2\eta+c_3u_1^2+c_4u_1^4)\dot{u}_1=g\sin(\Omega t+\theta+\sigma B(t))\end{cases} \tag{5}$$

where η is the damping coefficient, $b_1=\frac{a_1\pi^2(\frac{9}{a^2}+\frac{1}{b^2})^2-\frac{\pi^2}{b^2}N_0}{\rho h}$, $c_1=\frac{a_1\pi^2(\frac{1}{a^2}+\frac{9}{b^2})^2-\frac{\pi^2}{a^2}N_0}{\rho h}$, $b_2=\frac{a_3\pi^4}{4\rho hab}(\frac{27a}{b^3}+\frac{b}{a^3})$, $c_2=\frac{a_3\pi^4}{4\rho hab}(\frac{a}{b^3}+\frac{27b}{a^3})$, $b_3=\frac{a_5\pi^6}{16\rho hab}(\frac{81a}{b^4}+\frac{b}{a^4})$, $c_3=\frac{a_5\pi^6}{16\rho hab}(\frac{a}{b^4}+\frac{81b}{a^4})$, $b_4=\frac{a_7\pi^8}{64\rho hab}(\frac{243a}{b^4}+\frac{b}{a^4})$, $c_4=\frac{a_7\pi^8}{64\rho hab}(\frac{a}{b^4}+\frac{243b}{a^4})$, $e=\frac{F\pi^2(\frac{9}{a^2}+\frac{1}{b^2})^2}{\rho h}$, $g=\frac{F\pi^2(\frac{1}{a^2}+\frac{9}{b^2})^2}{\rho h}$; γ is the coupling coefficient.

Let $u_1=q$, the first equation of Eqs.5 can be shown as follows:

$$\ddot{q}+b_1q+b_2q^3+(2\eta+b_3q^2+b_4q^4)\dot{q}=e\sin(\Omega t+\theta+\sigma B(t)) \tag{6}$$

3 Nonlinear dynamic characteristics of a SMA gripper

When the noise intensity $\sigma=0$, the outside excitation becomes harmonic excitation, and the dynamic model can be shown as follows:

$$\ddot{q}+b_1q+b_2q^3+(2\eta+b_3q^2+b_4q^4)\dot{q}=e\sin(\Omega t+\theta) \tag{7}$$

The solution of Eq. (7) is:

$$q=q_0\cos(\omega t+\theta)+q_1\cos(\omega t+\theta)+q_2\cos3(\omega t+\theta)+$$
$$q_3\sin(\omega t+\theta)+q_4\sin3(\omega t+\theta)+q_5\cos5(\omega t+\theta) \tag{8}$$

where, $q_0=\frac{e}{\overline{\omega}}b_1$, $q_1=\frac{3e^3}{4\overline{\omega}}b_2$, $q_2=-\frac{e^3}{4\overline{\omega}}b_2$, $q_3=\frac{3e^3}{4\overline{\omega}}b_3\omega$, $q_4=\frac{e^3}{4\overline{\omega}}b_3\omega$, $q_5=\frac{e^5}{16\overline{\omega}}b_4\omega$, $\omega=\sqrt{b_1}$, $\overline{\omega}=\sqrt{(\Omega^2-\omega^2)^2+(2\eta\omega)^2}$.

When the noise intensity $\sigma\neq0$, the outside excitation becomes bounded noise, and Eq. (6) becomes a stochastic nonlinear differential equation. The averaged Ito equation of Eq. (6) are:

$$\begin{cases} dA = m_1(A,\Delta')dt \\ d\Delta' = m_2(A,\Delta')dt + \sigma dB(t) \end{cases} \tag{9}$$

where

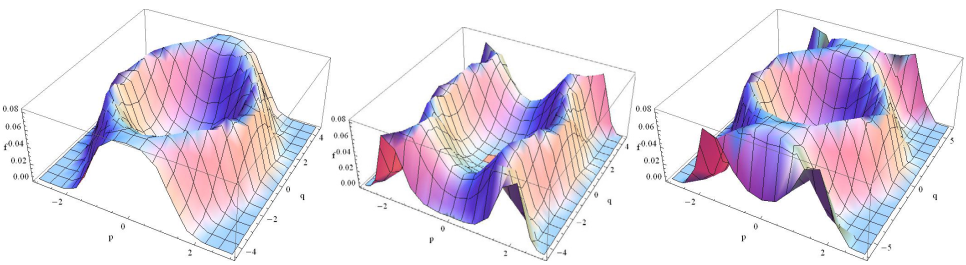
$\Delta' = \Omega t + \sigma B(t) + \chi - \Theta$

$m_1(A,\Delta') = -\pi(b_3A^2 + \frac{1}{4}b_4A^4) - \frac{eA}{2\eta}\cos\Delta'$

$m_2(A,\Delta') = 2\pi\Omega - \frac{1}{\eta}(\frac{3}{4}\pi b_2A^3 - \frac{e}{2}\sin\Delta')$

The system's averaged equation is:

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial A}(m_1 f) - \frac{\partial}{\partial \Delta'}(m_2 f) + \frac{\sigma^2}{2}\frac{\partial^2 f}{\partial \Delta'^2} \tag{10}$$



(a) $\sigma = 0.1$ (b) $\sigma = 0.5$ (c) $\sigma = 0.8$

Fig. 6. Stationary probability density of the system's response.

The numerical results of the system's response are presented in Figure 6, and the experimental results of SMA gripper under bounded noise are shown in Figures 7-9, where the frequency Ω =30Hz. Ti–Ni alloy is chosen as SMA film. The length of the micro gripper is 10cm, and its width is 1cm. The length of SMA thin film is 3cm, its width is 1cm, and its thickness is 0.4 mm. The stochastic resonance phenomenon occurs in the process.

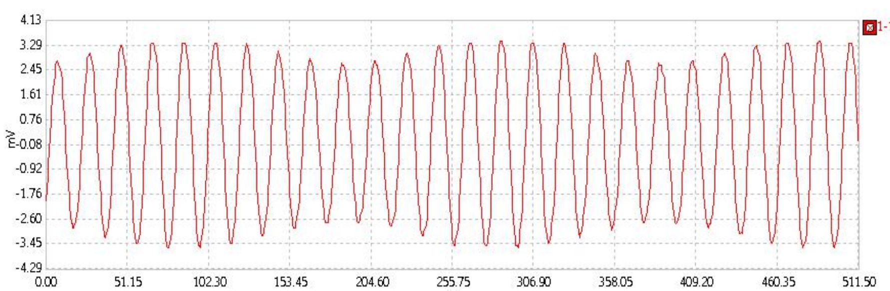


Fig. 7. Response of SMA gripper when $\sigma = 0.1$.

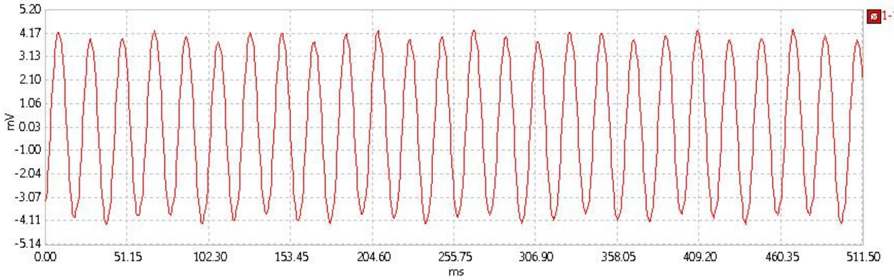


Fig. 7. Response of SMA gripper when $\sigma = 0.5$.

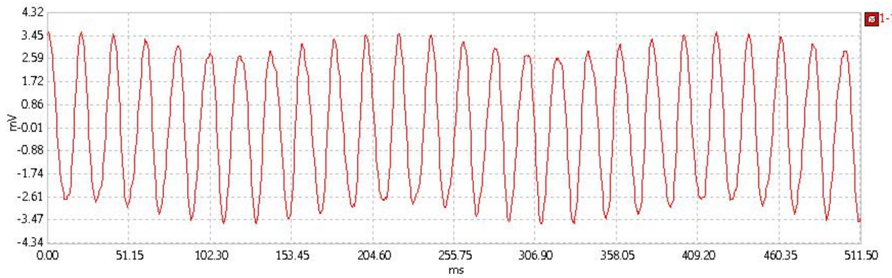


Fig. 7. Response of SMA gripper when $\sigma = 0.8$.

4 Conclusion

A kind of constitutive model of SMA is proposed in this paper, and the nonlinear dynamic response of a SMA gripper under bounded noise is studied. The harmonic driving signals and the random disturbance made up of bounded noise. The dynamic model of the system is established by Hamilton principle. The numerical and experimental results show that there is stochastic resonance in the system; the system's vibration amplitude reaches the most when the outside excitation is moderate.

Acknowledgements

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