Vendor Managed Inventory of a Supply Chain under Stochastic Demands

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Abstract

In this research, an integrated inventory problem is formulated for a single-vendor multiple-retailer supply chain that works according to the vendor managed inventory policy. The model is derived based on the economic order quantity in which shortages with penalty costs at the retailers` level is permitted. As predicting customer demand is the most important problem in inventory systems and there are difficulties to estimate it, a probabilistic demand is considered to model the problem. In addition, all retailers are assumed to share a unique number of replenishments where their demands during lead-time follow a uniform distribution. Moreover, there is a vendor-related budget constraint dedicated to each retailer. The aim is to determine the near optimal or optimal order quantity of the retailers, the order points, and the number of replenishments so that the total inventory cost of the system is minimized. The proposed model is an integer nonlinear programming problem (NILP); hence, a meta-heuristic namely genetic algorithm (GA) is employed to solve it. As there is no benchmark available in the literature to validate the results obtained, another meta-heuristic called firefly algorithm (FA) is used for validation and verification. To achieve better solutions, the parameters of both meta-heuristics are calibrated using the Taguchi method. Several numerical examples are solved at the end to demonstrate the applicability of the proposed methodology and to compare the performance of the solution approaches.
Keywords: Supply chain management, Vendor managed inventory, Probabilistic demand, Genetic algorithm, Firefly-algorithm, Taguchi method.

1. Introduction

All parts of a supply chain are coordinated with economic, information, production, and service flows. The aim of maximizing profit for each part of a supply chain needs effective management of such service flows via information sharing and coordinated decision making (Ramanathan 2013). Then, achievement to the better performance of a supply chain by aligning all data and motivations to backing global system targets would be possible (Sahin & Robinson 2002). Furthermore, retailers understand increasingly, that their supply chain plural performance determines their competitiveness (Brown et al. 2005). Supply chain integration and collaboration happens when someone uses some industrial practices such as the vendor managed inventory (VMI) policy. VMI is a well-known practice for supply chain collaboration, in which the vendor manages inventory at the retailer and decides when and how much to replenish. Under a VMI policy, the vendor determines the interval time and quantity of replenishment by accessing the retailer's inventory and demand data (Darwish & Odah 2010). A VMI system that is designed well can reduce inventory levels and raise supply chain integration, through

reducing system costs (Achabal et al. 2000; Angulo et al. 2004; Cetinkaya & Lee 2000).

Under the VMI conditions, the retailer is required to share its customer demands information and inventory levels with the vendor who is responsible for specifying the proper inventory and replenishment policies for retailer`s distribution center or all supply chain echelons. As firms, suppliers, and vendors found out that adjacent collaboration and integration is beneficial, there has been an increasing interest in research on VMI to sustain its eminence performance in recent years. In 1980`s, when the Walmart and Procter and Gamble started their partnership under the VMI contractual agreements, many retailers such as K-mart, Home Depot, and JC Penny employed the VMI policy (Yao et al. 2007).

In a traditional inventory supply chain, each member attempts to minimize its cost function. However, when they employ the VMI policy, they aim to show that partnership is a way to reach coordination that helps members to align their decisions and reach to the minimum total cost of the supply chain (Cachon & Fisher 2000). Another flow of research focuses on operational

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profits as unifying shipments (Cheung & Lee 2002) and adjusting recent delivery rate (Chaouch 2001). The concentration of the research performed in this paper is the second aspect, however under probabilistic demands.

In this paper, a single-vendor multiple-retailer supply chain problem is investigated in which the VMI policy is employed. The problem is modeled based on the economic order quantity (EOQ). As predicting customer demand is the most important decision in inventory systems, a probabilistic demand is considered. Besides, the replenishment cycle is assumed the same for all retailers. Moreover, there is a budget constraint for the vendor to spend it for retailers. We show that the proposed model is an integer nonlinear programming problem; hence, a genetic algorithm (GA) is employed to solve it. While there is no benchmark available in the literature, another meta-heuristic called firefly algorithm (FA) is used for validation and verification. In addition, the parameters of both algorithms are tuned using the Taguchi method.

The remainder of the paper is organized as follows. In Section 2, a review of the literature is provided. In Section 3, the notations and assumptions are stated. In Section 4, the VMI problem with stochastic demands and a unique replenishment cycle is formulated. In Section 5, the solution methodologies are expressed. In Section 6, the Taguchi method is applied to calibrate the parameters of the meta-heuristics. In Section 7, the performance of the solution methods are compared. Finally, conclusion and future research recommendations are provided in Section 8.

2. Literature Review

In the supply chain literature, VMI is a mechanism that integrates the practical components of a supply chain in the field of inventory management, transportation planning, and pricing policies. Cetinkaya & Lee (2000) and Zavanella & Zanoni (2009) examined the benefits of employing VMI in coordinating the shipments from vendor to its retailers in a two-echelon supply chain. Fry et al. (2001) showed how the VMI policy could be beneficial in coordination between production and delivery of a supply chain. The advantages of using VMI in reducing a supply chain cost was investigated by Yao et al. (2007). Zhang et al. (2007) presented a single-vendor multi-retailer supply chain model under the VMI contract, in which the demand rate was assumed constant and the buyer`s ordering cycles were different. Liao et al. (2011) developed a multi-objective model for a location–inventory problem (MOLIP) under the VMI policy in a single-vendor multi-retailer supply chain and investigated the possibility of using a multi-objective evolutionary algorithm based on the non-dominated sorting genetic algorithm (NSGA-II) to solve it. Coelho et al. (2012) examined the benefits of VMI in consistency requirements of a vehicle routing problem. They analyzed

the effect of different inventory policies, routing decisions, and delivery sizes. Disney & Towill (2002) studied a supply chain under VMI, where vendor satisfies the retailer`s orders and controls retailer`s inventory by defining the order quantity and order time of the retailer. Yao & Dresner (2008) examined the benefits realized for manufacturers and retailers under VMI and compared the distribution of profit between manufacturers and retailers. They showed that the distribution of benefits would depend on the replenishment frequency and the inventory holding cost parameters.

Yao et al. (2007) proposed an analytical model for a single vendor-single retailer supply chain based on EOQ and showed VMI would reduce the total cost. Dong & Xu (2002) modeled a retailer`s inventory system with deterministic demands using EOQ. Darwish & Odah (2010) presented a one vendor-multiple retailer supply chain model under VMI. In this model, a penalty cost on exceeded inventory the vendor sends to the retailer was assumed, where the retailers determined the upper bound. They solved the problem using heuristic algorithm to reduce computational efforts. Pasandideh at el. (2011) extended the Yao et al.'s (2007) model for several products, while the number of orders and the warehouse space were constrained. They solved the problem using a GA. Rad et al. (2014) presented a supply chain model with one vendor and two retailers. They first compared the vendor`s ordering cost under RMI and VMI policies. Then, they showed that using VMI can reduce the total cost of the supply chain. Sadeghi et al. (2013) investigated a multi-vendor multi-retailer single-warehouse supply chain operating based on the VMI policy. In addition, to suite real-world inventory problems, Sadeghi et al. (2014) hybridized an inventory problem with a redundancy-allocation optimization problem. Diabat (2014) presented a two-echelon single-vendor multiple-buyer supply chain under VMI. They tried to find the optimal sales quantity by maximizing profit. He developed a hybrid genetic-simulated annealing algorithm to solve the problem. Verma et al. (2014) proposed an alternative replenishment scheme in the supply chain in which a single vendor supplies a group of retailers under VMI. This scheme allows for different replenishment cycles for each retailer.

In stochastic inventory environments, Taha (2006) presented an inventory model with shortage in a traditional inventory system. Under the VMI policy, Fry et al. (2001) presented a (z, Z) contract where the retailer would determine the minimum and the maximum inventory levels z and Z, respectively, and the suppliers would pay penalties if these limits are violated. They maximized the service level using a Markov decision method. Song & Dinwoodie (2008), Bichescu & Fry (2009), and Zhao & Cheng (2009) proposed different order quantity policies with uncertain demand and lead-time. They examined two different states: VMI as a function of inventory levels or a function of channel potent (potent retailer, potent supplier, and equally potent

retailer and supplier). They concluded with an aspect seldom behaved in modeling papers: the value of VMI in strategic and operational terms is less. Bichescu & Fry (2009) examined decentralized supply chains that follow the (Q, R) inventory policies under VMI agreements. Within the VMI scenario, they examined the effect of divisions of channel power on supply chain and individual operator performance by examining different game theoretic models. Song & Dinwoodi (2008) modeled a supply chain problem with uncertain replenishment lead times and uncertain demands. They used dynamic stochastic programming and heuristic method in their study. Zhao & Cheng (2009) studied a two-level VMI system containing a distributer center and a retailer, both of which follow the order-up-to-level replenishment policy to maximize their overall system profit. They showed the benefits of VMI implementation at both strategic and operational levels.

This research presents a vendor managed inventory model in a one-vendor multiple-retailer supply chain, in which the demand is considered stochastic. More specifically, the probability distribution function of the demand during the lead time is assumed a uniform distribution and that the unfilled demand during the lead time is a backlogged shortage. The model is derived based on the economic order quantity. The idea in this research is an extension to the work of Darwish & Odah (2010) to suite real-world inventory problem by taking advantage of the work in Taha (2006).

3. The Assumptions and Notations

The assumptions involved to model the problem are:
1. A common replenishment cycle is assumed for all retailers. This common period eliminates the influence of the variation of the replenishment cycle.
2. The price of the goods are fixed (no discount is assumed).
3. All goods received by the retailers are sold to the customers. Hence, the annual demand of the retailers is equal to the one of the vendor.
4. The vendor is responsible for the ordering cost of the retailers and determines their economic order quantities.
5. Similar to Pasandideh et al. (2011) and Ben-Daya & Hariga (2004), the vendor's order point is considered zero.
6. The demands are assumed stochastic and follow a uniform distribution.
7. Backlogging shortages are assumed to meet retailers' demands.
8. The on-hand budget of the vendor dedicated to supply retailers' replenishments is limited.

The indices, parameters, and the decision variables are:

i :　　Index for retailers ($i = 1, 2, ..., n$)

n :　　Number of retailers

x_i :　　ith retailer`s demand during lead time; ($x_i \sim u(a_i, b_i)$)

$f(x_i)$:　The probability density function of the demand during lead time (a uniform distribution with parameters a_i and b_i during the lead time)

p_i :　　ith retailer`s shortage cost per unit inventory

D :　　Vendor`s expected demand rate

d_i :　　ith retailer`s expected demand rate

k_i :　　Ordering cost of retailer i

K :　　Ordering cost for the vendor

Y :　　Vendor's total order quantity

y_i :　　ith retailer`s order quantity (a decision variable)

y :　　Total order quantity dispatched from the vendor to all retailers in a replenishment cycle time ($y = \sum_{i=1}^{n} y_i$)

m :　　Number of replenishments of a retailer by the vendor (a decision variable)

I_V :　　Vendor's average inventory per unit time

R_i :　　Order point of retailer i (a decision variable)

c :　　Retailers' constant purchasing cost of each item

H :　　The unit holding cost of the vendor per unit time

h_i :　　The unit holding cost of ith retailer per unit time

C :　　Vendor`s maximum on-hand budget

TBC :　The expected total purchasing cost of the vendor

THC_R :　The expected total holding cost of retailers

TSC_R :　The expected total shortage cost of retailers

TIC_R :　The expected total inventory cost of the retailers

TIC_V :　The expected total inventory cost of the vendor

TOC_V :　The expected total ordering cost of the vendor

TIC_{VMI} :　The expected total inventory cost under VMI

As stated above, a vendor is assumed to supply several retailers in order to meet their customer`s demand. As all goods received by the retailers are sold to end customers, the annual demand of retailers is equal to the one of the vendor, i.e. $D = \sum_{i=1}^{n} d_i$. Moreover, the vendor defines a unique number of replenishments for the retailers. This is a logical assumption when there is VMI agreements between the vendor and retailers and the vendor makes decision about retailers replenishment cycle time. In other words, $\dfrac{q_i}{d_i} = \dfrac{q_1}{d_1}$. In addition, as the demand is stochastic, the vendor's prediction of product quantity replenished to retailers may not be enough, hence

shortage may occur at retailers' level. In this case, in addition to other inventory costs, a penalty cost is used in the integrated inventory system.

4. The Mathematical Model

According to the VMI policy, the mathematical model consists of two parts, one for the retailers and the other for the vendor. Based on the common replenishment cycle we have:

$$\frac{y_i}{d_i} = \frac{y_1}{d_1} \tag{1}$$

Assuming an equal annual demand for the vendor and all the retailers, the vendor`s order quantity is equal to the number of retailers' replenishments multiplied by their order quantities as

$$Y = m\sum_{i=1}^{n} y_i \tag{2}$$

In other words,

$$Y = m\sum_{i=1}^{n} \frac{y_1 d_i}{d_1} \tag{3}$$

The overall cost of the VMI system consists of the vendor and the retailers' inventory costs as

$$TIC_{VMI} = TIC_R + TIC_V \tag{4}$$

In what follows, TIC_R and TIC_V are derived.

4.1. Retailers' inventory cost

In an integrated inventory system, the retailers' total inventory cost includes holding and shortage costs. The total holding costs of the retailers is as follows:

$$THC_R = \sum_{i=1}^{n} h_i \left(\frac{y_i}{2} + R_i - E(x_i) \right) \tag{5}$$

where, based on the uniform distribution the expected number of items a retailer holds in its storage is

$$E(x_i) = \frac{a_i + b_i}{2} \tag{6}$$

This cost is merged into the vendor's holding cost and it will be a part of vendor`s cost. Moreover, the total shortage cost of the retailers is:

$$TSC_R = \sum_{i=1}^{n} \left(\frac{p_i d_i}{y_i} \int_{R_i}^{b_i} (x_i - R_i) f(x_i) dx_i \right) \tag{7}$$

Hence, the total annual inventory cost of the retailers is obtained using the following equation:

$$TIC_R = \left(\sum_{i=1}^{n} h_i \left(\frac{y_i}{2} + R_i - E(x_i) \right) \right) +$$
$$\left(\sum_{i=1}^{n} \left(\frac{p_i d_i}{y_i} \int_{R_i}^{b_i} (x_i - R_i) f(x_i) dx_i \right) \right) \tag{8}$$

4.2. Vendor's inventory cost

As the vendor`s ordering cost includes the retailers ordering cost and that the annual number of replenishments is $\frac{D}{Y}$, the vendor's ordering cost is obtained by

$$TOC_V = \left(K + m\sum_{i=1}^{n} (k_i) \right) \left(\frac{D}{Y} \right) \tag{9}$$

Sadeghi et al. (2013) showed that the average annual inventory of the vendor in the case of discrete orders is:

$$I_V = \left(\frac{(m+1)}{2} \right) y \tag{10}$$

in which, $y = \sum_{i=1}^{n} y_i$. Therefore, the annual holding cost of the vendor is:

$$HI_V = H \left(\frac{(m+1)}{2} \right) y \tag{11}$$

Moreover, the vendor pays the total annual purchasing cost of the products. Thus,

$$TBC = cY \tag{12}$$

Consequently, the mathematical formulation of the problem at hand becomes:

$$Min\ TIC_{VMI} = cY + \left(K + m\sum_{i=1}^{n} (k_i) \right)$$
$$\left(\frac{D}{Y} \right) + H \left(\frac{(m+1)}{2} \right) y +$$
$$\left(\sum_{i=1}^{n} h_i \left(\frac{y_i}{2} + R_i - E(x_i) \right) \right) +$$
$$\left(\sum_{i=1}^{n} \left(\frac{p_i d_i}{y_i} \int_{R}^{\infty} (x_i - R_i) f(x_i) dx_i \right) \right) \tag{13}$$

Subject to

$$cY \leq C \tag{14}$$

Inserting $y = \sum_{i=1}^{n} y_i$ and $Y = m\sum_{i=1}^{n} \frac{d_i y_1}{d_1}$ in (13), we have:

$$Min\ TIC_{VMI} = \left(cm \sum_{i=1}^{n} \frac{y_1 d_i}{d_1} \right) +$$

$$\left(K + m \sum_{i=1}^{n} (k_i) \right) \frac{d_1}{my_1} +$$

$$\left(H \left(\frac{m+1}{2d_1} y_1 \sum_{i=1}^{n} (d_i) \right) \right) +$$

$$\left(\sum_{i=1}^{n} h_i \left(\frac{y_1 d_i}{2d_1} + R_i - E\{x_i\} \right) \right) \quad (15)$$

$$+ \left(\sum_{i=1}^{n} \left(\frac{p_i d_1}{y_1} \int_{R_i}^{b_i} (x_i - R_i) f(x_i) dx_i \right) \right)$$

Subject to

$$cm \sum_{i=1}^{n} \left(\frac{d_i y_1}{d_1} \right) \le C \quad (16)$$

$$R_i \le y_i \ ; \ \forall i \in \{1, 2, ..., n\} \quad (17)$$

$$R_i, y_1, m > 0 \ and\ Integer \ ; \ \forall i \in \{1, 2, ..., n\} \quad (18)$$

Taha (2006) originally developed this model for a traditional stochastic inventory problem. However, the nonlinear integer-programming (NLIP) model presented above is derived for a single-vendor multi-retailer inventory system under the policy in which there is a capital on-hand constraint for the vendor. As a NLIP model is hard (if not impossible) to solve using an analytical approach, a GA is utilized in the next section for a near optimum solution.

5. A Solution Algorithm

Exact methods due to their time consuming computational processes are unable to solve INLP problems of large sizes. This makes one to have no choice, except using an evolutionary algorithm (EA). Nachiappan & Jawahar (2007) employed a GA to find a near-optimum solution of a single-vendor multiple buyers supply chain problem under the VMI policy. Sue-Ann et al. (2012) compared the performance of a particle swarm optimization (PSO) algorithm to the one of a hybrid GA and artificial immune system (GA–AIS). Pasandideh et al. (2011) examined a GA to solve an INLP problem in a two-echelon single-supplier single-vendor multi-product VMI inventory system. Sadeghi et al. (2013) proposed a hybrid PSO as well as a GA to solve a multi-retailer multi-vendor single warehouse VMI inventory problem.

Due to the familiarity of GA and its good performance to solve INLPs, it is used in this paper to find an approximate solution of the problem at hand. Beside, in order to validate the results obtained, a firefly algorithm,

the one that has never been utilized to find a near-optimum solution of an INLP under VMI, is employed. To have a better approximate solution, the parameters of both algorithms are fine-tuned using the Taguchi method.

5.1. Genetic algorithm

Holland (1962) was the first who introduced GA. Since then, interest in solution approaches based on the principles of evolution and heredity has been grown. GA is a type of evolutionary computation that mimics the principles of natural genetics. It is a random evolutionary search algorithm. In what follows the steps involved in GA are described.

5.1.1. Initial conditions

In this step, the GA parameters, i.e. the population size N_{pop}, the crossover probability P_c, the mutation probability P_m, the stopping criterion, the selection policy, the crossover operation, the mutation operation, and the number of iteration are set. Some of these parameters are tuned using the Taguchi method in Section 5.

5.1.2. Chromosomes

In GA, a chromosome is a series of genes that are possible appropriate or inappropriate solution. Chromosome demonstration is an initial part of the GA method. In this paper, a chromosome is a vector of ($n+2$) positive integer elements (genes). The first gene represents the order quantity of the first retailer, y_1, the second gene expresses the rate of replenishment, m, and the other genes indicate the order points of all retailers. For a problem with five retailers, the chromosome structure is shown in Fig. 1.

$$\begin{bmatrix} y_1 & m & R_1 & R_2 & R_3 & R_4 & R_5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 22 & 43 & 12 & 11 & 7 & 8 & 12 \end{bmatrix}$$

Fig. 1: A typical chromosome

5.1.3. Initial population and evaluation

In a GA, once a chromosome is generated a fitness value is assigned to it. In optimization problems, the fitness value is the value of the objective function. Chromosomes are usually generated randomly to construct the initial population consisting of N_{pop} chromosomes. However, some of them may be infeasible, i.e. may not satisfy the constraints. In order to generate feasible chromosomes, the death penalty approach is taken in this paper. In this method, a big value is added to the objective function value of any infeasible chromosome. In this case, the constrained optimization problem becomes a non-constraint problem.

5.1.4. Crossover

In a crossover operation, a pair of chromosomes mate to form offspring. The pair is selected randomly from the generation with probability Pc. While there are many

different types of crossover operators, a two-point crossover operator is used in this paper. An example of this operation is shown in Fig. 2 for a 4-retailers problem. Two steps are shown in this figure; (1) selecting two random points for the cut, and (2) displacing the string between the cut-off points of the two parents; leading to the creation of two offspring.
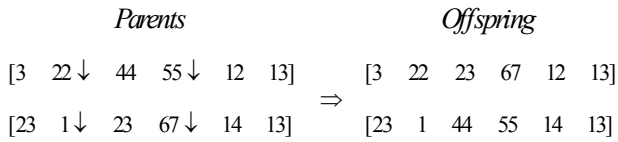
<div align="center">

Parents   Offspring

[3 22↓ 44 55↓ 12 13]   [3 22 23 67 12 13]

$\Rightarrow$

[23 1↓ 23 67↓ 14 13]   [23 1 44 55 14 13]

</div>

Fig. 2. An example of the crossover operation

5.1.5. Mutation

Mutation is the second operation in a GA to prospect new solutions. It operates on each of the chromosomes resulted from the crossover operation. In mutation, a gene is replaced with another gene randomly with probability P_m. Fig. 3 shows a representation of the mutation operator for a 4-retailer problem.
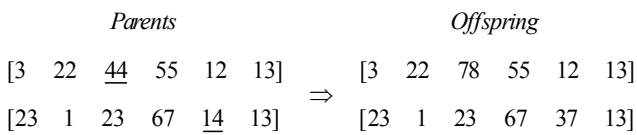
<div align="center">

Parents   Offspring

[3 22 <u>44</u> 55 12 13]   [3 22 78 55 12 13]

$\Rightarrow$

[23 1 23 67 <u>14</u> 13]   [23 1 23 67 37 13]

</div>

Fig. 3. An example of the mutation operation

5.1.6. Chromosome selection

In this step of the GA methodology, chromosomes are selected for the next generation. The selection performs with respect to the fitness value of the chromosomes. The roulette wheel selection method is used in this paper to select N_{pop} chromosomes with the best fitness values among the parents and offspring.

5.1.7. Stopping criterion

In the last step of GA, we examine whether the method has found a solution which meets the user's expectations. To do this, a set of conditions are defined such that if they are satisfied then a good solution is obtained. In this paper, we stop when we observe convergence in 100 iterations. Note that the parameters of GA are tuned using Taguchi method.

As there is no benchmark available in the literature to see how GA performs in solving an INLP of the VMI at hand and to validate the results obtained, a firefly algorithm (FA) is used in the next section. In order to have a fair comparison, the parameters of FA is calibrated using the Taguchi method as well. As pointed out previously, this algorithm has never been applied to solve a INLP problem under VMI.

5.2. Firefly algorithm

The firefly algorithm (FA), one of the most widely used EA inspired by the social behavior of fireflies, was first proposed by Yang (2008, 2009). It simulates the attraction behavior of the fireflies. Moreover, Yang (2010) proposed a FA to solve non-linear problems with some singularity and stochastic components. He used a stochastic test function along with the global optimum solution to validate FA. Farhoodnea et al. (2014) proposed a discrete FA in a multi-objective optimization problem and compared its performance to the ones of other algorithms including a continuous FA. They showed that the discrete FA was the most efficient algorithm. However, to the best knowledge of the authors no FA is used in the literature to solve a stochastic integer problem such as the one in this research.

The firefly with more brightness attracts the others, therefore there is an efficient explore in the search space. In FA, the attractiveness caused by the brightness pattern of the firefly species is simulated by a mathematical formula. Similar to the chromosomes in GA, the fireflies in FA are considered solution candidates. Moreover, the brightness function in FA acts similarly to the fitness function in GA. Three rules are used in the implementation of a FA (Yang 2008, 2009, 2010);

1) All fireflies are unisexual (all fireflies attracts each other regardless of their gender.)
2) The attractiveness of the fireflies is proportional to their brightness, where the brighter one attracts the less-brighter one. If the distance between them increases, the brightness and attractiveness will decrease. A particular firefly who does not find the more brightness one will move randomly over the entire search space.
3) The brightness of a firefly is determined by the objective function of the problem.

Similar to GA, the initial responses or fireflies in FA are first generated randomly. Each firefly has a location and a light attractiveness, equal to the response and the cost function of a given problem, respectively. Thus, in the second step, the light intensity of each firefly as a response candidate is determined using the cost function (Eq. 15). In the third step, the new position of the fireflies is obtained with respect to each other using Eq. (19) explained in the next subsection. In order to find the fireflies with the best light intensity in the initial population, they are ranked in the fourth step. These steps are repeated until a stopping criterion is met. The stopping criteria of this paper are a pre-determined number of iterations along with the convergence of the response in 100 iterations.

5.2.1. Updating firefly location

Denoting the current location of the ith firefly by x_i, the new position (x') is defined based on pair wise comparisons of the flies using the following equation:

$$x'_i = x_i + \beta_0 e^{-\gamma r^m} (x_j - x_i) + \alpha \varepsilon_i \qquad (19)$$

In Eq. (19), β_0 is the light attractiveness, γ is the light absorption coefficient, r (or r_{ij}) is the distance between the two specified fireflies i and j, and m defines the type of the light source taking a value between zero and two. Moreover, ε_i is a random vector for deviation of the movement of the firefly i to firefly j, which is controlled by α. One can assume α as the mutation coefficient of the movement, where it is a factor of randomness supplied by the user. While in this paper $r = 0$, $\gamma = 1$, and $m = 2$ are chosen, the mutation coefficient (α), the light attractiveness coefficient (β), the initial population of fireflies, the number of iterations (It), all are tuned using the Taguchi method in the next section.

6. Parameter Tuning

As the parameters of a meta-heuristic algorithm play an important role on the quality of the solution obtained, they must be tuned using a calibration method such as response surface methodology (RSM) or Taguchi (Taguchi et al. 2005). The parameters to be calibrated act as controllable factors in the design of experiments (DOE) (Montgomery 2005). As the required number of experiments in the Taguchi method is less than the one in RSM, the first is utilized in this paper. Many studies employed the Taguchi approach to tune the parameters of meta-heuristic algorithms. A few recent and relevant are: Naderi et al. (2009), Rahmati et al. (2013), and Fazel Zarandi et al. (2013).

6.1. The Taguchi method

Taguchi (1993) improved a family of fractional factorial matrices in experiments to reduce the number of experiments required to determine the optimal levels of the factors that significantly affect a response. Taguchi uses orthogonal arrays to study a large number of decision variables by use of a small number of experiments. Taguchi categorized the factors into two main classes: 1) controllable factors, and 2) noise factors. While omitting the noise factors are impossible, Taguchi attempts to minimize the effects of the noise factors and to determine the optimal level of the significant controllable factors. Taguchi changes the repetitive data to the values which are the variation`s measure of the results. Taguchi et al. (2005) aims to maximize the ratio of the signal or controllable factors to noise (S/N). This ratio computes the variation of the response. According to the type of the problems, there are three standards values of this ratio, (S/N), including;

(1) Nominal is the best; the aim is to reduce the amount of variability around the specific objective value. In this case the S/N is defined as

$$SN_T = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}(\bar{y} - y_i)^2\right) \qquad (20)$$

(2) Smaller is the better; it is using for experiments whose objective function is the minimization type. In this case the S/N is defined as

$$SN_S = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}y_i^2\right) \qquad (21)$$

(3) Larger is the better; it is using for experiments whose objective function is the maximization type. The S/N is defined here as

$$SN_L = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right) \qquad (22)$$

In Eqs. (20)-(22), n denotes the number of iterations, y_i represents the obtained response in ith iteration, and $\bar{y}$ is the average response in all iterations. Note that as the objective function of the problem at hand is of a minimization type, the smaller is the better-type in Eq. (21) is used in this research.

6.2. Taguchi method implementation

The Taguchi implementation is taken place in 5 steps. First, the parameters that affect the response significantly are defined. Second, we the levels of the parameters are determined via a trial and error process. Third, in this step the smallest orthogonal array is chosen to minimize the experimentation time. Fourth, the obtained design is used to find a solution. Finally, the results are analyzed based on the (S/N) strategy.

The GA parameters that affect the solution significantly are the population size (N_{pop}), the maximum number of iterations (It), the mutation probability (P_m), and the crossover probability (P_c). In Table 1, the three levels of these parameters that are obtained using a trial and error procedure are shown. Similarly, the significant parameters and their levels in FA are shown in Table 2. The parameters are the attractiveness coefficient β_0, the mutation coefficient α, the number of the fireflies (N_{pop}), and the maximum number of iterations (It).

Table 1
GA parameters and their levels

Variable	Level		
It	1000	1250	1500
N_{pop}	30	40	50
P_c	0.6	0.7	0.8
P_m	0.1	0.15	0.2

Table 2
FA parameters and their levels

Variable	Level		
It	1000	1250	1500
N_{pop}	30	40	50
α	0.1	0.15	0.2
β_0	1	1.5	2

With reference to the Taguchi standard arrays table, the L_9 orthogonal arrays, as the most suitable design, is used to tune the FA and GA parameters. Table 3 contains the input data for a 5-retailers problem as an example. For this example, the experimental results of five replications along with their *(S/N)* ratio are shown in Tables 4 and 5 for GA and FA, respectively. In these tables, the values 1, 2, and 3 correspond to the three levels of each parameter.

Figures 4 and 5 depict the average *(S/N)* ratios obtained in Tables 4 and 5, respectively. As lesser values of *(S/N)* ratio is desired, then based on Fig. 4 the optimal values of GA parameters are obtained as: *It=1500; Npop=30; P_c=0.8, P_m=0.2.* Similarly, according to Fig. 5, the optimal values of the FA parameters are determined as: *It=1500; N_{pop}=30; α=0.2, β=2.*

Consequently, the GA solution based on its tuned parameters is shown in Table 6.

Table 3
Input data

K	k_i	d_i	h_i	H	E_i	P_i	N
467	386	827	7	4	41	8	5
	377	864	9		41	10	
	115	983	9		50	9	
	322	825	8		41	6	
	181	995	9		40	7	

Table 4
The result of the experiments using GA

It	N_{pop}	P_c	P_m	y_1	y_2	y_3	y_4	y_5	Mean	S/N
1	1	1	1	2665811	263575	2687814	2719583	3070514	2755894	-888198
1	2	2	2	2617221	2620681	2635078	262421	2724388	2644316	-884473
1	3	3	3	2611465	261983	2623587	3019879	2619876	2698927	-886392
2	1	2	3	2618148	2720891	28613	3010232	2733002	2788715	-889182
2	2	3	1	2678594	2652474	3015537	2720477	264793	2743002	-887756
2	3	1	2	2718853	2621761	2615101	2613584	2869312	2687722	-885936
3	1	3	2	2865286	273949	2862794	2651224	288676	2801111	-889512
3	2	1	3	2620594	3151842	2618402	2617201	2633491	2728306	-88744
3	3	2	1	2722148	2614263	2885894	2624702	2737274	2716856	-88687

Table 5
The result of the experiments using FA

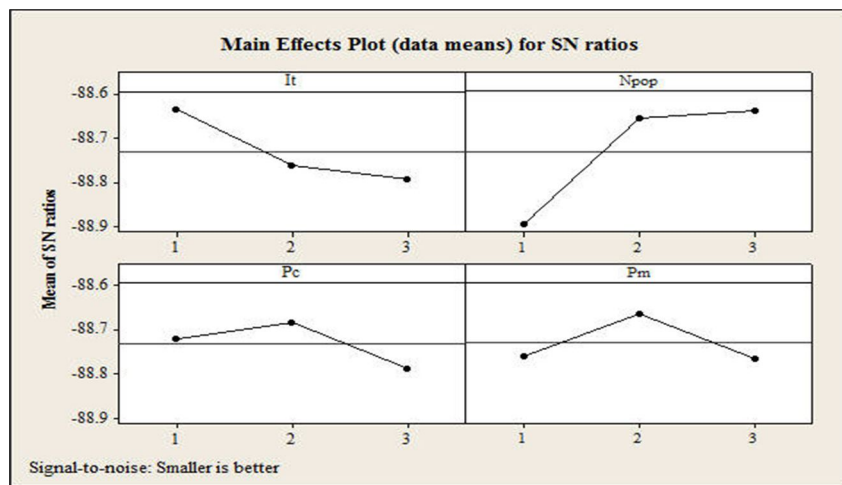| It	N_{pop}	α	$β_0$	y_1	y_2	y_3	y_4	y_5	Mean	S/N
1	1	1	1	35060496	3519668	39016553	34037816	35036054	36390378	-91228571
1	2	2	2	33217334	35700333	34569874	37642106	35296791	3456443	-90775436
1	3	3	3	32479609	3239399	33261783	33961788	32246373	32868709	-90337352
2	1	2	3	34537297	34442518	34067877	35004651	34029403	34416349	-9073576
2	2	3	1	34548896	34148599	36386571	34752204	3365086	34697426	-90809028
2	3	1	2	34652564	35339062	33711428	35983244	35134409	34964141	-90874499
3	1	3	2	3430755	36752477	35831313	35080564	35633705	35521122	-91011999
3	2	1	3	34991837	34853412	32583577	34800727	34925054	34430922	-90742113
3	3	2	1	37054262	35232055	35084181	35297718	34707404	35475124	-91000774

Fig. 4. The average S/N ratio for GA vs. different values of its parameters

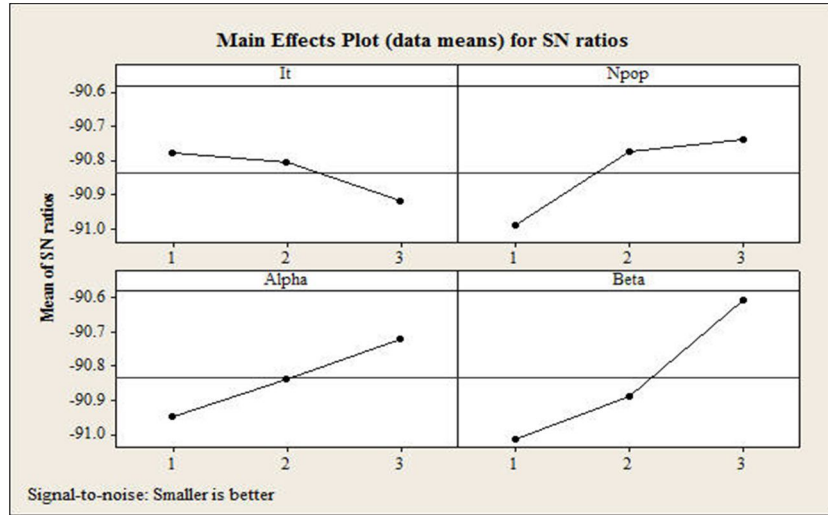Fig. 5. The average S/N ratio for FA vs. different values of its parameters

Table 6
Solutions using Tune-parameter GA

Y_1	R_1	R_2	R_3	R_4	R_5	m
186	37	41	36	42	38	1
116	47	52	58	43	60	4
112	43	45	34	44	45	6
155	42	48	34	47	41	2
178	38	43	33	44	35	1

7. Performance Evaluation and Comparison

In order to validate the results obtained using GA and to evaluate and compare its performance with the one of the FA, 9 different problems sizes of the example given in Taha (2006) are solved using the parameter-tuned GA and FA, each 10 times. Five different measures, namely the average solution, the required CPU-time, the best solution obtained, the relative percentage deviation *(RPD)*, and the relative deviation index *(RDI)* are used for the comparison study. The *RPD* is defined as:

$$RPD = \frac{Alg_{sol} - Best_{sol}}{Best_{sol}}100 \qquad (23)$$

in which Alg_{sol} is the objective function obtained by either GA or FA and $Best_{sol}$ is the best value of the objective function among the solutions obtained for a specific problem. Off course less value of *RPD* is desired. Moreover, *RDI* is defined as:

$$RDI = \frac{Alg_{sol} - Min_{sol}}{Max_{sol} - Min_{sol}}100 \qquad (24)$$

In which, Min_{sol} and Max_{sol} respectively represent the minimum and the maximum values of the objective function for a specific problem. An algorithm with *RDI* closer to zero is the better one (Naderi et al. 2009).
Table 7 contains the values of the six measures for GA and FA, when they are employed to solve 9 problems of different sizes.
The results in Table 7 show that GA is the better algorithm in terms of the required CPU time, the best solution obtained, and the average cost. Figures 6-10 show this conclusion graphically.

Table 7
GA and FA comparison

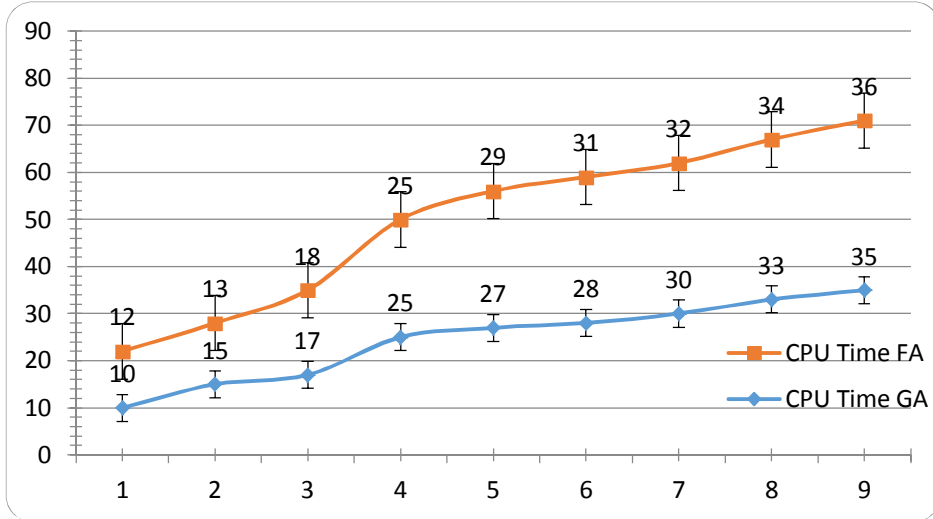|	GA						FA					
Problems	CPU Time	STDEV (standard deviation)	Best F	Ave F	$\overline{RPD}$	$\overline{RDI}$	CPU Time	STDEV (standard deviation)	Best F	Ave F	$\overline{RPD}$	$\overline{RDI}$
5	10	916	16388	17019	4	32	12	610	18921	20232	7	58
10	15	2092	26237	27803	6	23	13	584	34001	34749	2	41
15	17	6744	40351	44547	10	23	18	1693	49270	52372	6	48
20	25	6556	40390	45228	12	23	25	4809	60394	64515	7	29
25	27	6359	45081	49859	11	25	29	6299	82196	88781	8	28
30	28	16687	83788	96790	16	26	31	24810	96876	119930	24	32
35	30	27735	108313	140326	30	45	32	21762	124358	148199	19	35
40	33	11264	115789	129013	11	41	34	34367	127592	160810	26	31
45	35	41627	177141	221391	25	34	36	70427	188408	255725	36	37
Average:	24	13331	72609	85775	14	30	26	18374	86891	105035	15	38

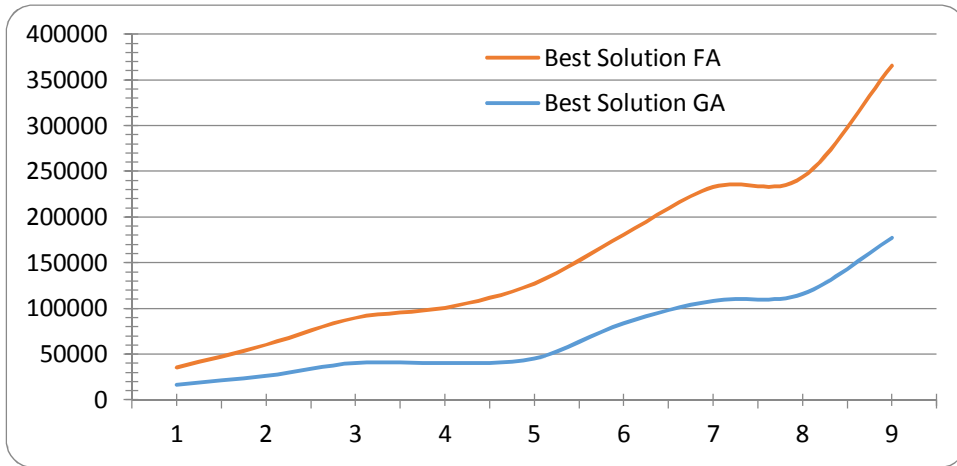Fig. 6. CPU time comparison of the two algorithms

Fig. 7. Comparison of the best solutions obtained by the two algorithms
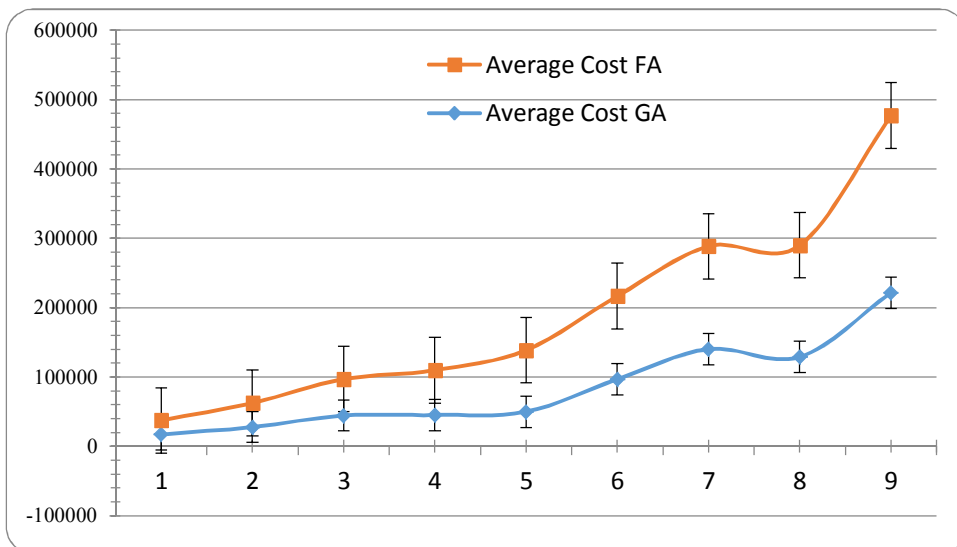

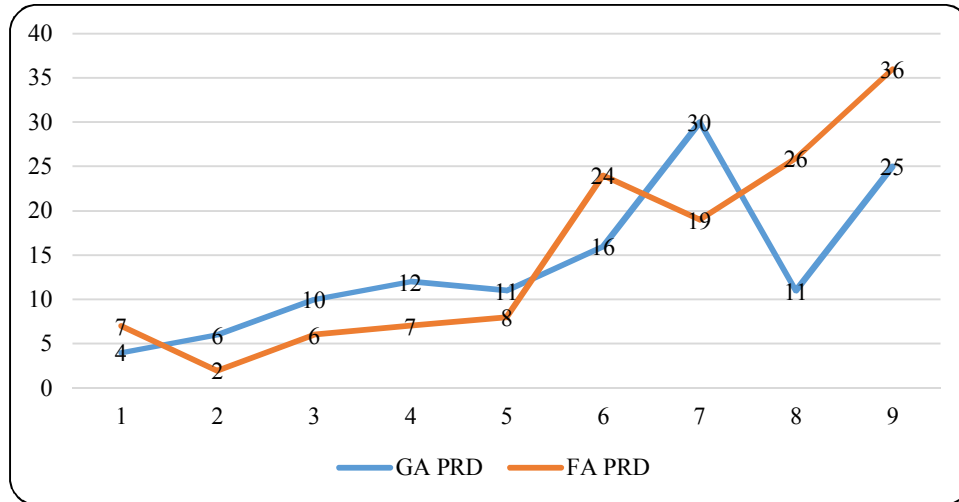Fig. 8. Comparison of the average cost obtained by the two algorithms

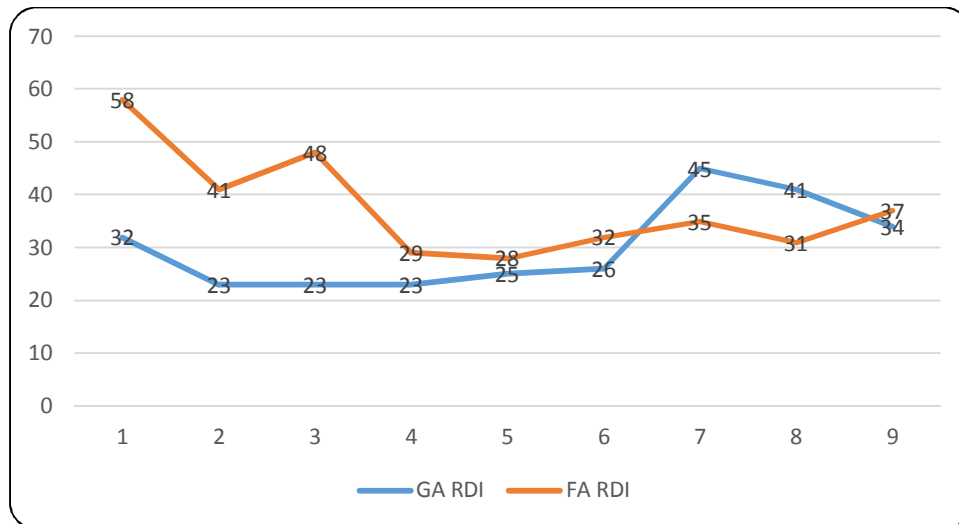Fig. 9. Comparison of the two algorithms in terms of PRD values

Fig. 10. Comparison of the two algorithms in terms of RDI values

Note that in this research all programs are coded in MATLAB 2012 and that a PC with 4 GHZ, Core i3 CPU is used to run the programs in Windows 7.

8. Conclusion and Future Research

In this paper, an integrated stochastic inventory model in a one-vendor multiple-retailer two-echelon supply chain was developed based on the vendor-managed inventory policy. In this model, retailers faced stochastic demands and there was a limitation on the vendor`s budget. The aim was to determine the time, the number of the retailers' inventory replenishments, the replenishment quantities, and the reorder points such that the total cost of the chain would be minimized. As this model shown to be a non-linear integer programming; hard to be solved using exact methods, a genetic algorithm was utilized for a near-optimum solution. Moreover, as there was no benchmark

available in the literature, a relatively new algorithm called firefly was used for validation and verification. While the parameters of both algorithms were tuned using the Taguchi method, we showed that GA was the better algorithm in terms of five measures. The main reason may be the paired comparison of the fire flies algorithm through the whole solution space that makes this algorithm slower than GA. Moreover, the firefly algorithm is a powerful local search in which the brightness should be associated with the objective function. Consequently, sometimes it may be trapped into several local optima and cannot find a global optimum solution.

For future research in this area, we recommend the followings:
  (a) Extending the model for a multi-vendor multi-retailer two echelon supply chain
  (b) Investigating the possibility of a central warehouse for the supply chain

(c) Investigating the effects of using discount or inflation

(d) Modeling the problem based on the economic production quantity (EPQ)

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