# Layout of Cellular Manufacturing System in Dynamic Condition 

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#### Abstract

Cellular manufacturing system (CMS) is highly important in modern manufacturing methods. Given the ever increasing market competition in terms of time and cost of manufacturing, we need models to decrease the cost and time of manufacturing. In this study, CMS is considered in condition of dynamic demand in each period. The model is developed for facing dynamic demand that increases the cost of material flow. This model generates the cells and location facilities at the same time and it can move the machine(s) from one cell to another cell and can generate the new cells for each period. Cell formation is NP-Complete and when this problem is considered in dynamic condition, surly, it is strongly NP- Complete. In this study, genetic algorithm (GA) is used as a meta-heuristic algorithm for solving problems and evaluating the proposed algorithm, Branch and Bound ( $B$ \& $B$ ) is used as a deterministic method for solving problems. Ultimately, the time and final solution of both algorithms are compared.


Key Words: Cellular manufacturing system; Genetic algorithm; Dynamic layout; Branch and bound.

## 1. Introduction

### 1.1. Preface

As time passes and with increasing competition in market, manufacturers try to decrease the cost of their products. One such significant cost is the cost of production which causes manufacturers to use the modern manufacturing systems more and more.
(Job shop, flow shop, cellular manufacturing system (CMS), virtual CMS)

Flow shop is a kind of layout that machinery are located according to their operations on products; this kind of layout is used when the variety of products is low and the volume of products is high. Job shop is a kind of layout that the same or similar machineries are located in the same group. This system is used when the volume of product is low and variety is high.

Another method of manufacturing is cellular manufacturing system (CMS); in this method, machineries are located according to similarity of operation and size and kind of production in 2 or more groups. CMS is a manufacturing system that can produce medium volume / medium variety part types more economically than other
types of manufacturing systems [31].
The ideal goal of CMS is to process all the operations of each product, only in one cell.

Some advantages of CMS: 1-facilitation of the programming 2-reduction of the material transfer volume in work place 3-better management and control of the system 4 -reduction of the number of workers 5 -work-in-process (WIP) and reduction of finished goods inventories [19] 6reduction of set up time [32] 7-reduction of tools requirement [19] 8-a reduction of required space [25].

The rate of changes in volume and variety of demands is currently high in manufacturing companies and demands can change from one period to another. Therefore, producers change their strategy from 'make to stock' (MTS) to 'engineering to order' (ETO). In dynamic conditions, period of programming is separated to shorter periods and maybe we have no demand in some periods from some products or the volume of demand is changed. In dynamic conditions, the flow of material and work-inprocess in each period are different and if there is not a proper layout, cost of material flow is increased. To reduce the material flow, reconfiguration of the facilities according to volume and variety of demand is called dynamic facility layout problem (DFLP).

The change of facility layout is divided into 2 parts:

[^0]1-Move the machinery in the cell (from one machine location (ML) to another ML)

2-Move one or more machine(s) from one cell to another cell (a new cell is generated)

This layout changing, reduces cost of material flow, but makes a new cost called 'cost of reconfiguration facilities'.

Thus, there may be some layouts that can insignificantly reduce cost of material flow but increase the cost of reconfiguration facilities very much. Obviously, these are not good layouts [23].

In this study, a mathematical model is presented.
Some advantages of this model include:
1-It focuses inside cells layout and outside cells layout at the same time.

2-It can move one or more machine(s) from one cell to another cell(s),(i.e. ability of generating the new cell)

3-It combines the generating of the cells and layout of facilities.

### 1.2. Use of heuristic and meta-heuristic algorithms

Layout of facilities in CMS is a NP-Complete problem [7,12,13,14,22].

If the layout of facilities and generation of the cells are combined and the problem is considered in dynamic condition, problem will be more complicated and surly, it is strongly NP-Complete where the necessity of use of the heuristic and Meta heuristic is increased. In large size problems, use of exact (optimum) algorithms takes a long time and in some cases it is impossible. For this reason, near optimum (not exact) algorithms are used more than past. These methods do not necessarily result in optimum solutions, but can function in shorter time than exact algorithms. In this study, genetic algorithm (GA) is used as a meta-heuristic algorithm to solve the problems and branch and bound ( $\mathrm{B} \& \mathrm{~B}$ ) algorithm is used as an exact algorithm according to the model to solve the problem. Finally, the times and solutions of 2 algorithms are compared.

## 2. Literature review

### 2.1. Preface

Nowadays, in many modern factories worldwide for which volume and variety of products are important, CMS is used. The majority of previous CMS researchers focused on generation of the cells; only a few of them addressed the layout of facilities in this decade. However, in some past decades, many researches were done on CMS, most of which considered CMS in constant conditions, and a few considered it in dynamic and fuzzy conditions.

### 2.2. Design of cellular manufacturing system

In 1990, Seifoddini [30] presented a probabilistic cell formation model to deal with uncertainty of the product mixture for a single period. He suggested that the best way to handle the uncertainty in the product mixture is to predict it and to incorporate it into the design process.

Chen [2] developed a mathematical programming model for system reconfiguration in a dynamic cellular manufacturing environment. A mixed integer programming model is developed to minimize inter cell material and machine costs as well as reconfiguration cost in a dynamic cellular manufacturing environment with anticipated changes of demand or production process for multiple time periods.

Wicks [28] proposed a multi-period formation of the part family and machine cell formation problem. The dynamic nature of production environment is addressed by considering a multi-period forecast of the product mix and resource availability during the formation of part families and machine cells.

### 2.3. Layout of cellular manufacturing system

Benjaafar and Sheikhzadeh [26] Rosenblatt and Lee [21] Rosenblatt and Kropp [29] studied layout of CMS under static demand that according to the subject of this study and considering the dynamic demand. However, they did not elaborate on the issues of concern to the present researchers.

Shorter product life-cycle, higher product variety, unpredictable demand, shorter delivery times have caused manufacturing systems to operate under dynamic and uncertain environment these days. [30]

Rosenblatt [4] discussed the general dynamic layout problem. He developed a dynamic programming approach to solve the multi-period layout selection. In each period, a number of potential static layout alternatives need to be generated. The objective is to select the sequence of layouts which minimizes the overall sum of the material flow costs and relay out costs. If all possible static layout alternatives are considered at each period, the optimal sequence can be obtained. The early studies that considered CMS in dynamic condition, tried to save the cells (i.e. all machines belonged to one cell, couldn't move to another cell(s) until the end of programming periods) and all the machines belonged to one cell, only could move from one cell location (CL) to another CL and intercellular layout couldn't change. For example, simple model of Balakrishnan in 1992 [3].

Lacksonen presented a heuristic algorithm for DFLP when sizes of cells are different [5]. As time passes, considering that cells movement cannot be enough, some researchers tried to move the ML of some machines in the cells and use heuristic and meta-heuristic algorithms in this way. Conway and venkataramanan used GA for DFLP [6].

Kaku and mazzola used tabu search for DFLP [7]. Baykasoglu and Gindy used SA for DFLP [8]. Erel and Ghosh and Simnon combined SA and dynamic programming. They presented some heuristic algorithms [9].Tzeng presented a model to solve the facility layout problem in one cell system [10]. Solimanpur and Vrat and Shankar used ant colony algorithm for their problem [11].

Balakrishnan and cheng did some studies on CMS which are briefly presented here in this study. Balakrishnan and Jacobs presented a simple model about layout of CMS in 1992[3]. They also improved GA for layout of CMS in 2000 in 2 papers [12],[1].

They reviewed the literature on CMS and used GA for large size problems of DFLP. In this paper, they improved 1992's model of Balakrishnan and Jacobs [13]. In 2005, they studied about CMS programming in multi-period programming [14]. They compared SA and GA for solving of DFLP [15]. In 2007, they reviewed studies about layout of CMS in uncertain and multi-period condition [24]. Their last study tried to calculate the efficiency of some algorithms in uncertain condition and predicted the uncertain demand [27].

Xamber and vilarinho in 2003 tried to reduce the material flow by presenting a model and they used SA for solving [16]. Wang used linear assignment to reduce time delivery in CMS [17]. Design of CMS depends on many things, Defersha and Chen presented a mathematical model for design of CMS. Their method, initially, tried to organize the parts family and machines family (cells) [18]. Suresh and Satoglu in 2008 reviewed studies CMS and presented their goal-programming model for the design of hybrid cellular manufacturing (HCM) systems in a dual resource constrained environment, considering many realworld application issues [20].

## 3. Mathematical Model

### 3.1. Preface

In this study, product demands have been considered dynamically. Inter cell and intra cell layouts have been noticed simultaneously as well as the issue of cell formation and its layout. Most of the manufacturing parameters are dynamic in real conditions. In this study, we considered the demand from one period to another, as a changeable factor and tried to approach a real condition. We presented a mathematical model for inter cell and intra cell layout at the same time.

### 3.2. Model presentation

We present the theories in which the model has been submitted and also the objective of the model and its variables.

### 3.2.1. Assumption

1-The number of CL and ML in each cell is constant in programming period.
2-The distance between CL and ML are determined.
3 -The product demands from one period to another period are changeable and determined.
4-The number of periods is determined.
5 -all the machines can locate in each ml .
6-The cost of machinery moving is constant in each distance unit.
7-The cost of material moving is determined and constant in inter cell as well as its cost in intra cell.
8-The size of batch material for inter cell and intra cell moving are different but determined and constant.

### 3.2.2. Model Objectives

1-Minimizing the cost of material flow in inter cell and intra cell.
2- Minimizing the cost of the changing of layout.
The cost of material flow includes: A) the cost of inter cell material moving at the time of manufacturing $B$ ) the cost of intra cell material moving at the time of manufacturing.
The cost of reconfiguration includes 2 parts A) inter cell cost for machine moving B) intra cell cost for machine moving from one cell to another.
Thus, the objective function is presented as follows:
MIN Z= the cost of inter cell material flow

+ The cost of intra cell material flow
+ The cost of inter cell reconfiguration (inter cell machine moving)
+ The cost of intra cell reconfiguration (intra cell machine moving)


### 3.2.3. Introduction of Variables

C: cell counter
M: machine and ML counter
P: period
K: product counter
$\mathrm{D}_{\text {kp }}$ : Demand of part k in period p
$\mathrm{B}^{\mathrm{in}}$ : The size of product batch for intra cell moving
$B^{\text {out. }}$ : The size of product batch for inter cell moving
$\mathrm{C}^{\text {out. }}$ : The cost of inter cell moving for each batch
$\mathrm{C}^{\text {in }}$ : The cost of intra cell moving for each batch
$\mathrm{A}_{\mathrm{i}}$ : The cost of machine moving at each time (in distance unit)
$\mathrm{N}_{\mathrm{ki}}$ : The number of machine i processing on product k
$\mathrm{d}_{\mathrm{xy}}$ : The distance of $\mathrm{x}, \mathrm{y} \mathrm{ml}(\mathrm{s})$
$\mathrm{d}_{\mathrm{uw}}$ : The distance of $\mathrm{u}, \mathrm{w}$ cells
$\mathrm{V}_{\mathrm{ijk} \mathrm{k}}$ : The cost of product k moving in period p between machines i,j
$\mathrm{V}_{\mathrm{ijkp}}$ Is the result of each intra cell batch moving expense multiply the number of batches moving between $\mathrm{i}, \mathrm{j}$ machines.
$V_{i j k p}=\left\{\begin{array}{lc}C^{i n}\left\lceil\frac{D_{k p}}{B^{i n}}\right\rceil & \text { IF } \\ & \left|N_{k i}-N_{k j}\right|=1 \\ \text { Otherwise } & 1\end{array}\right.$
$f_{i j p}$ : The cost of material moving between machines $\mathrm{i}, \mathrm{j}$ in period p (intra cell) This cost is the result of product k moving in period $p$ between machines $i, j$ for entire products.
$f_{i j p}=\sum_{k=1}^{K} V_{i j k p}$
$V_{u w k p}^{\prime}$ : result of the product k moving in period p between cells $\mathrm{u}, \mathrm{w}$ from one ML to the same ML in another cell.
$V_{u w k p}^{\prime}$ is the result of each intra cell batch moving expense multiply the number of batches moving between u,w cells.
$V_{u w k p}^{\prime}=\left\{\begin{array}{cc}C^{\text {out }}\left[\frac{D_{k p}}{B^{\text {out }}}\right] & \text { IF } \\ \\ \text { Otherwise } & \left|N_{k i}-N_{k j}\right|=1\end{array}\right.$
$F_{\text {uwp }}^{\prime}$ : The cost of material moving between $u, w$ cells in period p (inter cell) This cost is the result of product k moving in period p between machines i.j located in cells $\mathrm{u}, \mathrm{w}$ for entire products.
$f_{u w p}^{\prime}=\sum_{k=1}^{K} V_{u w k p}^{\prime}$
Decision variables:
$\boldsymbol{e}_{\text {iup }}= \begin{cases}1 & \begin{array}{c}\text { if machine } \mathrm{i} \text { is dedicated to cell } \mathrm{u} \text { in } \\ \text { period } \mathrm{p}\end{array} \\ \boldsymbol{O}_{\mathrm{ixp}}= \begin{cases}1 & \text { Otherwise } \\ \text { if machine } \mathrm{i} \text { is dedicated to ML x in } \\ \text { period } \mathrm{p}\end{cases} \\ 0 & \text { Otherwise }\end{cases}$
According to defined variables, the presented mathematical model will be as follows:
Min Z=

$$
\begin{align*}
& \sum_{p=1}^{P} \sum_{u=1}^{C} \sum_{w=1}^{C} \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{x=1}^{M} \sum_{y=1}^{M} e_{i u p} e_{j w p} O_{i x p} O_{j y p}\left(f_{u w p}^{\prime} d_{u w}+f_{i j p} d_{x y}\right) \\
& +\sum_{p=1}^{P} \sum_{i=1}^{M} \sum_{u=1}^{C} \sum_{w=1}^{C} \sum_{x=1}^{M} \sum_{y=1}^{M} e_{i u p} e_{i w(p+1)} O_{i x p} O_{i y(p+1)}\left(d_{u w}+d_{x y}\right) A_{i}  \tag{5}\\
& \text { S.t. }  \tag{6}\\
& \sum_{x=1}^{M} o_{i x p=1}  \tag{7}\\
& \sum_{i=1}^{M} o_{i x p} \leq 1  \tag{9}\\
& o_{i x p} o_{j x p}+e_{i u p} e_{j u p} \leq 1  \tag{10}\\
& e_{i u p}, o_{i x p} \in\{0,1\}
\end{align*} \quad \forall p, x, y, i, j(i \neq j),
$$

In the above model, the formula (5) indicates the sum of the costs of intercellular and intracellular material moving. When u,w have the same amounts, it is indicated as intracellular material moving and if $\mathrm{u}, \mathrm{w}$ have different amounts it is indicated as intercellular material moving. For example when $u=1, w=1$, formula (5) calculates the cost of material moving in cell 1 and when $\mathrm{u}=1, \mathrm{w}=2$, formula (5) calculates the cost of material moving between cells 1and2.

Formula (6) indicates the sum cost of reconfiguration including inter and intra cells. When u,w have the same amount. It shows machinery intracellular moving. For example when $u=1, w=1$, formula (6) calculates the cost of intracellular reconfiguration in cell 1 and when $\mathrm{u}=1, \mathrm{w}=2$, formula (6) calculates the cost of intercellular reconfiguration between cells 1and2.

In the above model, the whole costs of equations (5) and (6) form objective function. Our goal is to minimize the objective function according to the limitation of (7) and (8) and (9) and (10) equations. Equation (7) indicates that each machine is just dedicated to one ML. equation (8) indicates that each ML can include almost one machine. (it means that one or some ML(s) might be empty in a period)

Equation (9) indicates that 2 machines can be located in 1 ML provided that the ML(s) belong to 2 different cells. For example, one machine can be located in ML 2 in cell 1 and another machine in the same ML in another cell.

## 4. Genetic algorithm

### 4.1. Preface

Genetic algorithm (GA) is a statistical method for optimization. The basic idea of this method was inspired by Darvin development theory and its function is based on natural genetic. We use GA for different purposes such as function optimization and system identification. GA starts
with complex of answers which are shown through chromosomes.

This complex is called initial population. In this algorithm the answers from one population are used for next population production. In this process, it is hoped that the new population should be better than the former. The selection of some answers out of the whole (parents) is done according to their fitness and in order to produce new and more developed population. Naturally, more suitable answers have higher chances for reproduction. This process continues to reach the stop criteria ( i.e. similar to the number of population or the limit of improvement). The general GA scheme is as follows:

Step1- Producing random population including $n$ chromosomes.
Step 2- Finding fitness function of each chromosome in population.
Step 3- Producing a new population based on repetition of the follow steps:
Step 3-1-Selection of 2 parent chromosome in one population based on their fitness.
Step 3-2-Considering a certain percentage of crossover operation.
Step 3-3-Considering the possibility of mutation and them the change of children in each step.
Step 3-4-Replacement of new children in the new population.
Step 4-The use of new population for new performances of algorithm.
Step 5-The stop of algorithm in cause of noticing the stop criteria and bringing the best answer to the present population otherwise, we return to step 2.

### 4.2. Implementation of $G A$

In GA relating, at first, the method of coding was considered. We used decimal numbers for coding. Each chromosome is formed by a chain of decimal numbers where its integer number shows ML and the decimal part shows CL (by formula 11) and the column $n$ in which the number is located shows the number of machine. In formula (11), $x$ is integer number and $c$ is the number of cells. There are $(\mathrm{P} \times \mathrm{M})$ integer numbers in each chromosome $P$ is the number of period and $M$ is the number of machines.
Cell number $=\lceil(x-\lfloor x\rfloor) \times C\rceil$
For example, if the problem includes 3 machines, 2periods, 2cells, each chromosome will include 6 integer numbers that its first 3 number show the location of machines 1 to 3 in first period and the second 3 numbers show the location of the some machines in second period. For better understanding of how to read the location of machine and cell by the Gene, we can cling to the above
example. We define it by considering the following chromosome.
[2.3 $\left.1.4 \begin{array}{lllll}2.8 & 2.6 & 1.4 & 1.2\end{array}\right]$
2.3: The first machine is located in the first period and in ML 2 and cell 1
1.4: The second machine is located in the first period and in ML 1 and cell 1
2.8: The third machine is located in the first period and in ML 2 and cell 2
2.6: The first machine is located in the second period and in ML 2 and cell 2
1.4: The second machine is located in the second period and in ML 1 and cell 1
1.2: The third machine is located in the second period and in ML 1 and cell 1

After showing the problem answer in the shape of a chromosome chain, we should wake an organized algorithm to solve the problems.

Pseudo-code for GA is brought in figure (1). First we should set the parameters effective in algorithm operation. For parameter setting, it is used 2approaches of 1-empirical setting of parameters 2-design of experiments (DOE)

Effective and important parameters in GA operation are
1-The size of population
2- The iteration of algorithm
3-The percentage use of crossover operator
4 - The percentage use of elite operator
5 - The percentage use of mutation operator
6-The method of making initial solution
7-The method of forming a new generation in crossover operation

According to the basic studies, we can hold that the parameters for population size and its iteration are more important in comparison to the other parameters because they have affect the quality of the final answer and the time more effectively.

Thus, the 2 parameters of the population size and iteration were calculated though DOE and we set the rest of the parameters empirically. We will present and discuss DOE in the relevant section more extensively. Figure (1) shows the pseudo-code of the used GA.
$98 \%$ Of Crossover, $2 \%$ of elite, $5 \%$ of mutation operators to produce the new generation are used. The algorithm, first generates initial solution randomly and calculates the fitness of each chromosome according to the input data and sorts the chromosomes according to their fitness.

That is, the chromosome having the best fitness gets at the top and finally the best chromosomes will be transmitted to the next generation as elite. The crossover operator is defining as one-point crossover.

```
Parameters setting [Maximum iteration, Population size,
    Crossover percentage, Mutation percentage, Elitism
    percentage]
Generate initial solution randomly
n=1
while n < iteration
    for i=1:population size
                evaluate each chromosomes (if the chromosome
                is not feasible, add penalty)
        end for
        find Elitism chromosome
        select mut pool from chromosome with roulette wheel
        method
        crossover operator
        mutation operator
        input elite chromosome in population
    End while
    if stop condition met then stop, print elite chromosome of
        last generation as a best solution
    End
```

Fig. 1Genetic algorithm. Pseudo-code.

The operation method is: 2 chromosomes are chosen as parents and a point is set randomly on each and finally children are generated. We use mutpool is used to choose the parents and make new generation and we select the members of the pool by Rolette wheel method and from the present generation chromosomes. In the next step, mutation will happen. For cases where more than 1 machine are located in 1 cell and 1 ML (which is not feasible), there will be penalties. The size of penalty will be considered as 0.7 of amount of chromosome fitness. Calculation will be performed based on iteration and, finally, the total time of calculation as well as the best fitted chromosome will be presented.

## 5. Tuned proposed algorithm

### 5.1. DOE

Design of experiments (DOE) is an experiment or a series of experiments that make changes in input variables process to calculate the amount of changes in output process. DOE is an effective and practical method to do experiments in such a way that the outputs are statistically analyzable.

In this way, we can obtain the maximum information through the minimum experiments. The process is shown in fig. 2 below.


Fig. 2 DOE diagram

### 5.2. Implementation of $D O E$

Some factors influence operation of GA. Iteration and size of problem are more important than other factors. The increase or decrease of iteration and size of population have effects on total answer. Thus, we should set them, so that at the end; we will have the best answer and time. So, DOE was used.

For performance of DOE, a problem with 12parts, 3operatins, 12machines, 2periods, and 3cells is used and other data as matrixes are put. Mutation=5\% and elite $=2 \% \times$ size of population were considered .Then, a series of 2 factors experiment is considered.

The first factor was iteration and its levels are 1000, $1500,2000,2500,3000$. Second factor was size of population and its levels are 20,40,60,80. Each experiment is done for 2times ( $n=2$ ). Then, all the experiments are done and we found the total answers and times.

An important point in this study is that the total answer is more important than time. So, we considered only total answer in our calculations. According to Dankan comparison test, every time, we did not have a big variance in our answers, we used time for our calculations and an answer with shorter time was chosen.

At the end, we found out that when the size of population is 80 and iteration is 2500 , we have the best situations to find the best answers.
Table 1
Result of tests

|  | Popsize |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 20 | 40 | 60 | 80 |
|  | 1000 | N1= | 36339 | 36995 | 31531 | 31497 |
|  | 1000 | N2= | 40039 | 36501 | 33001 | 34213 |
|  | 1500 | N1= | 27706 | 35842 | 29315 | 31718 |
|  | 1500 | N2= | 33883 | 32417 | 30765 | 35170 |
|  | 2000 | N1= | 32092 | 32817 | 31099 | 32454 |
|  | 2000 | N2= | 37032 | 33318 | 36689 | 27826 |
|  | 2500 | N1= | 32944 | 30417 | 36314 | 29776 |
|  | 2500 | N2= | 30956 | 30860 | 30889 | 26887 |
|  | 3000 | N1= | 31922 | 30109 | 30265 | 28219 |
|  | 3000 | N2= | 33614 | 32396 | 31234 | 30381 |

Table 2
Experimental design ANOVA with two factors

|  |  | SS | d.f | MS | f |
| :--- | :---: | :---: | :---: | :---: | :---: |
| p-value |  |  |  |  |  |
| treatment A | 83231186.75 | 4 | 20807796.69 | 3.8343 | 0.018 |
| treatment B | 488666863.28 | 3 | 16288954.43 | 3.16 | 0.0548 |
| Interaction | 90966940.85 | 12 | 7580578.404 | 1.3968 | 0.2458 |
| AB | 108535029.5 | 20 | 5426751.475 |  |  |
| Error | 331600020.4 | 39 | 8502564.625 |  |  |
| Total |  |  |  |  |  |

## 6. Calculation results

### 6.1. Preface

In this part, we try to solve 20 sample problems by GA and $B \& B$ as a Meta-heuristic algorithm and finally, the answers and times are compared. The problem has been classified by small, medium, large sizes. It should be mentioned that the criterion of the problem sizes is the time that is needed to solve them by $\mathrm{B} \& \mathrm{~B}$.

In case, B \& B can be solved less than in 3600 seconds (1 hour), it is classified as small problems and if $B \& B$ can just enter the solving area at the end of the mentioned time, it will be a medium sized problem. B \& B will present 2 amounts of upper bound $\left(\mathrm{Z}_{\mathrm{U}}\right)$ and lower bound $\left(\mathrm{Z}_{\mathrm{L}}\right)$ at the time of solving small and medium sized problems. In which, the optimum solution will be in between. $\mathrm{Z}_{\mathrm{U}}$ is the best amount of objective function till that moment (surly, feasible) and $\mathrm{Z}_{\mathrm{L}}$ in minimizing problems, is the lower bound for objective function (possibly, infeasible).

Gradually $\mathrm{Z}_{\mathrm{L}}$ and $\mathrm{Z}_{\mathrm{U}}$ will approach. If $\mathrm{Z}_{\mathrm{U}}=\mathrm{Z}_{\mathrm{L}}$, we have the optimal solution. The usable system for solving these is a notebook containing inter 2core 2.53 GHz processor, 3GB RAM. To solve the problems by B \& B, we used the $8^{\text {th }}$ edition of lingo software. To solve the problems by GA, we used the MATLAB software. Considering that GA has a random nature, each problem was solved 20 times and the best solution was selected. To compare the problems solution by the 2 algorithms, we define 2 criteria:

1- $G A P_{Z}$ : The relative gap between the solutions of the 2 algorithms
$G A P_{Z}=\frac{Z_{G A}-Z_{U}}{Z_{U}} \times 100$

2- $G A P_{T}$ : The relative gap between the solving times of the 2 algorithms
$G A P_{T}=\frac{T_{G A}-T_{B \& B}}{T_{B \& B}} \times 100$

### 6.2. The method of problem generation

Each problem needs some basic data, as produced in Table (3) randomly and based on uniform distribution.

Table 3
Data-base for making problems

| 10 | $B^{\text {in }}$ | $\mathrm{U}\left(200 \_2000\right)$ | $D_{K P}$ |
| :---: | :---: | :---: | :---: |
| 70 | $B^{\text {out }}$ | $\mathrm{U}\left(1 \_5\right)$ | $d_{m}$ |
| 4 | $C^{\text {in }}$ | $\mathrm{U}\left(5 \_20\right)$ | $d_{c}$ |
| 7 | $C^{\text {out }}$ | $\mathrm{U}\left(10 \_20\right)$ | $A_{i}$ |

It should be mentioned that for several reasons including the possibility of moving the most amounts in inter cell in comparison to intra cell according to movement machinery, the following will be obvious. $\left.\frac{C^{\text {in }}}{B^{\text {in }}}\right\rangle \frac{C^{\text {out }}}{B^{\text {out }}}$

### 6.3. Results of the study

According to table (3), 20 problems were generated with several sizes which were solved through GA and B \& B. The results are presented in Table (4). It is to mention that, in Table (4), the numbers of parts are shown by $K$, operations by OP, machines by M , cells by C and periods by P have been shown.

As Table (4) indicates, problems 1 to 9 are small sized, in which $\mathrm{Z}_{\mathrm{U}}=\mathrm{Z}_{\mathrm{L}}$ and there is no gap between $\mathrm{Z}_{\mathrm{U}}, \mathrm{Z}_{\mathrm{L}}, \mathrm{Z}_{\mathrm{GA}}$ and $G A P_{Z}=0$.

Problems 10 to 16 are medium sized in which there is a gap between $\mathrm{Z}_{\mathrm{U}}, \mathrm{Z}_{\mathrm{L}}$. In these problems, in some case $\mathrm{Z}_{\mathrm{GA}}$ is between $\mathrm{Z}_{\mathrm{U}}, \mathrm{Z}_{\mathrm{L}}$ that indicates finding a feasible better solution in which $G A P_{Z}$ is negative. But in cases that $\mathrm{Z}_{\mathrm{GA}}$ is out of the distance of $\left[\mathrm{Z}_{\mathrm{L}}, \mathrm{Z}_{\mathrm{U}}\right] G A P_{\mathrm{Z}}$ will be positive. Problems 17 to 20 are large sized in which B \& B algorithm problem could not enter the solution area at the end of 3600 seconds. But we notice that the related GA has solved the problems in a short period of time, and this shows the high ability of this algorithm.

Table 4
Proposed algorithm and mathematical model solved with Branch and Bound method results

| Problem number | dimension |  |  |  |  | B\&B |  |  | GA |  | GAP\% |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | K | OP | M | P | C | $\mathrm{Z}_{\mathrm{L}}$ | $\mathrm{Z}_{\mathrm{U}}$ | $\mathrm{T}_{\mathrm{B} \& \mathrm{~B}}$ | $\mathrm{Z}_{\mathrm{GA}}$ | $\mathrm{T}_{\mathrm{GA}}$ | $\mathrm{GAP}_{\mathrm{Z}}$ | $\mathrm{GAP}_{\mathrm{T}}$ |
| 1 | 4 | 3 | 3 | 2 | 2 | 9038 | 9038 | 62 | 9038 | 7 | 0.00 | -88.7 |
| 2 | 6 | 2 | 6 | 2 | 3 | 9832 | 9832 | 149 | 9832 | 10.1 | 0.00 | -93.2 |
| 3 | 12 | 3 | 12 | 2 | 3 | 26887 | 26887 | 311 | 26887 | 13.8 | 0.00 | -95.5 |
| 4 | 8 | 3 | 6 | 3 | 2 | 33366 | 33366 | 714 | 33366 | 14.4 | 0.00 | -97.9 |
| 5 | 8 | 4 | 8 | 2 | 3 | 29993 | 29993 | 1134 | 29993 | 13.1 | 0.00 | -98.8 |
| 6 | 10 | 4 | 10 | 2 | 2 | 36303 | 36303 | 1925 | 36303 | 16.9 | 0.00 | -99.1 |
| 7 | 10 | 4 | 10 | 2 | 3 | 33905 | 33905 | 2433 | 33905 | 16 | 0.00 | -99.3 |
| 8 | 10 | 5 | 10 | 2 | 4 | 46243 | 46243 | 2804 | 46243 | 16.6 | 0.00 | -99.4 |
| 9 | 12 | 5 | 10 | 2 | 4 | 75759 | 75759 | 3112 | 75759 | 17.2 | 0.00 | -99.4 |
| 10 | 12 | 5 | 12 | 2 | 4 | 71345 | 74880 | >3600 | 74230 | 19.3 | -0.86 | - |
| 11 | 12 | 6 | 12 | 2 | 4 | 110223 | 117811 | $>3600$ | 119617 | 20.3 | 1.53 | - |
| 12 | 12 | 6 | 12 | 2 | 5 | 119750 | 125662 | $>3600$ | 128251 | 20 | 2.06 | - |
| 13 | 12 | 6 | 12 | 3 | 5 | 159156 | 166722 | $>3600$ | 162143 | 28 | -2.74 | - |
| 14 | 14 | 6 | 12 | 3 | 5 | 287118 | 299102 | $>3600$ | 288182 | 29.3 | -3.65 | - |
| 15 | 16 | 6 | 14 | 3 | 5 | 264883 | 274128 | $>3600$ | 270872 | 32.6 | -1.18 | - |
| 16 | 16 | 7 | 14 | 3 | 5 | 296452 | 301734 | >3600 | 299272 | 33.9 | -0.81 | - |
| 17 | 14 | 7 | 14 | 4 | 6 | - | - | $>3600$ | 428260 | 43.3 | - | - |
| 18 | 15 | 8 | 14 | 4 | 6 | - | - | >3600 | 552953 | 45.9 | - | - |
| 19 | 15 | 8 | 16 | 5 | 6 | - | - | $>3600$ | 809107 | 63.2 | - | - |
| 20 | 16 | 9 | 16 | 5 | 7 | - | - | >3600 | 1040899 | 67.5 | - | - |

Fig (3) shows the time for solving GA and B\&B small sized problems. It is obvious that the solution time for GA is much less than $B \& B$.

Fig (4) shows the solutions of the 2 algorithms for small and medium problems. Obviously, the solution curved for the small problem completely coincide and they get farther when starting $\mathrm{Z}_{\mathrm{U}} \quad, \mathrm{Z}_{\mathrm{L}}, \mathrm{Z}_{\mathrm{GA}} \quad$ medium problems.


Fig. 3. comparison of solving time


Fig. 4. comparison of final solutions
Fig (5) shows the gap percentage between GA and $\mathrm{B} \& \mathrm{~B}$ solutions for small and medium problems. As it is shown in fig(5), $G A P_{Z}=0$ for small sized problems(1 to 9) and it is not the same for medium size problem in a way that $G A P_{Z}$ is negative for problems $10,13,14,15,16$ which indicates finding a better solution by GA in comparison to B\&B.

## 6-4- The examination of a problem

Problem No. 4 was considered, Demand of products is shown in Table (5).


Fig. 5. Gap between B\&B and GA

Table 5
Demand of products

| $D_{\text {KP }}$ | K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P1 | 250 | 285 | 490 | 880 | 212 | 1150 | 2000 | 1520 |
| P2 | 1670 | 290 | 886 | 1850 | 1230 | 2000 | 1843 | 580 |
| P3 | 912 | 842 | 1722 | 250 | 1510 | 2000 | 593 | 918 |

The cost of machine movement is shown in Table (6).
Table 6
Cost of machine movement

| Cost of machine movement |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| machine | $M_{1}$ | $M_{2}$ | $M_{3}$ | $M_{4}$ | $M_{5}$ | $M_{6}$ |
| $A_{i}$ | 14 | 11 | 10 | 18 | 12 | 13 |

The distance among the cells is shown in Table (7) and the distance among the machines is depicted in Table (8).

Table 7
Cells distance

| $d_{C}$ | $C_{1}$ | $C_{2}$ |
| :---: | :---: | :---: |
| $C_{1}$ | 0 | 7 |
| $C_{2}$ | 7 | 0 |

Table 8
Machines distance

| $d_{M}$ | $M_{1}$ | $M_{2}$ | $M_{3}$ | $M_{4}$ | $M_{5}$ | $M_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{1}$ | 0 | 2 | 4 | 2 | 3 | 5 |
| $M_{2}$ | 2 | 0 | 2 | 3 | 2 | 3 |
| $M_{3}$ | 4 | 2 | 0 | 5 | 3 | 2 |
| $M_{4}$ | 2 | 3 | 5 | 0 | 2 | 4 |
| $M_{5}$ | 3 | 2 | 3 | 2 | 0 | 2 |
| $M_{6}$ | 5 | 3 | 2 | 4 | 2 | 0 |

Table 9
Sequence of operations for each product

| $K-O P$ | $O P_{1}$ | $O P_{2}$ | $O P_{3}$ |
| :---: | :---: | :---: | :---: |
| $K_{1}$ | 4 | 6 | 6 |
| $K_{2}$ | 1 | 4 | 1 |
| $K_{3}$ | 2 | 6 | 2 |
| $K_{4}$ | 4 | 3 | 1 |
| $K_{5}$ | 6 | 5 | 3 |
| $K_{6}$ | 3 | 1 | 5 |
| $K_{7}$ | 5 | 2 | 6 |
| $K_{8}$ | 4 | 3 | 2 |

The method for processing each part follows Table (9). The figures in Table (9) are the number of the machines in which the part enters to be process, in every period.

According to table (1), $\quad C^{\text {in }}=4, \quad C^{\text {out }}=7, \quad B^{\text {in }}$ $=10, B^{\text {out }}=70$.After solving the problem, the optimum solution is calculated as 33366\$.
$32911 \$$ is for material flow cost and $455 \$$ is for machines movement cost in 3 periods. Considering the final solution, the chart for cell configuration (fig.6) is shown in 3 periods.

As Fig (6) reveals, machines 1,3 and 6 have inter cell movement in period 1and 2 . Machines 1 and 3 moved from cell 1 to cell 2 and machine 6 moved from cell 2 to cell 1.

But there is no inter cell machine movement in between period 2 and period 3 and machine movement in this condition was of intra cell kind as in Fig (6).

Fig (7) shows the locations of machines in 3 periods.
Fig (7) obviously shows that machine movements between periods 1 and 2 are both inter cell and intra cell.

M1 which is in cell 1 and ML3 in period 1 moved to the same ML in cell2. As well M2 which is in cell 2 in period 1 remains in the same cell in period 2 and it just moved from ML6 to ML5.

M3 moved from cell 1 and ML6 in period 1 to cell 2 and ML6 in period 2. M4 has remained in cell 1 in period 1 and 2 moved from ML6 to ML5 by intra cell movement. M5 movement between period 1 and 2 has been of intra cell kind and remained in cell 2 but it just moved from ML3 to ML2.

M6 movement has been of inter cell kind and it moved from cell 2 and ML5 in period 1 to the same ML in cell 1 in period 2. Between period 2 and 3 there has been no inter cell machine movement and all movements have been intra cell kind. In a way that M5, M3 and M2, M1 have remained in cell 2 and M1 moved from ML3 to ML5 and M2 from ML5 to ML2 and M3 has remained in ML6 and also M5 has remained in ML2.

M4 and M6 have remained in cell 1 between periods 2 and 3. In a way that is in ML6 without any moving but M6 moved from ML5 to ML3.

|  |  | C 1 |  |  |  |  | C 2 |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $K_{2}$ | $K_{4}$ | $K_{6}$ | $K_{8}$ | $K_{1}$ | $K_{3}$ | $K_{5}$ | $K_{7}$ |  |
| C1 | $M_{1}$ | 1,3 | 3 | 2 |  |  |  |  |  |  |
|  | $M_{3}$ |  | 2 | 1 | 2 |  |  | 3 |  |  |
|  | $M_{4}$ | 2 | 1 |  | 1 | 1 |  |  |  |  |
|  | $M_{2}$ |  |  |  | 3 |  | 1,3 |  | 2 |  |
|  | $M_{5}$ |  |  | 3 |  |  |  | 2 | 1 |  |
|  | $M_{6}$ |  |  |  |  | 2,3 | 2 | 1 | 3 |  |

A-first period

|  |  | C 1 | C 2 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C 1 |  | $K_{1}$ | $K_{2}$ | $K_{3}$ | $K_{4}$ | $K_{5}$ | $K_{6}$ | $K_{7}$ | $K_{8}$ |
|  | $M_{4}$ | 1 | 2 |  | 1 |  |  |  | 1 |
|  | $M_{6}$ | 2,3 |  | 2 |  | 1 |  | 3 |  |
| C2 | $M_{1}$ |  | 1,3 |  | 3 |  | 2 |  |  |
|  | $M_{2}$ |  |  | 1,3 |  |  |  | 2 | 3 |
|  | $M_{3}$ |  |  |  | 2 | 3 | 1 |  | 2 |
|  | $M_{5}$ |  |  |  |  | 2 | 3 | 1 |  |

B-second period

|  |  | C1 | C2 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $K_{1}$ | $K_{2}$ | $K_{3}$ | $K_{4}$ | $K_{5}$ | $K_{6}$ | $K_{7}$ | $K_{8}$ |
| C1 | $M_{4}$ | 1 | 2 |  | 1 |  |  |  | 1 |
|  | $M_{6}$ | 2,3 |  | 2 |  | 1 |  | 3 |  |
| C2 | $M_{1}$ |  | 1,3 |  | 3 |  | 2 |  |  |
|  | $M_{2}$ |  |  | 1,3 |  |  |  | 2 | 3 |
|  | $M_{3}$ |  |  |  | 2 | 3 | 1 |  | 2 |
|  | $M_{5}$ |  |  |  |  | 2 | 3 | 1 |  |

C-third period
Fig. 6.diagram of cells configuration


Fig. 7. layout for machines in each periods

## 7. Conclusion and Future Studies

In this study, after a literature review on manufacturing systems, a model for the design and layout of CMS when the demand is dynamic was presented. CMS is one of several manufacturing systems whose application has increased parallel with the rise of market competition and variety of production as well as other production limitations. When variety of product and volume of production are in a medium level, these systems have their maximum ability in comparison to other systems.

Considering the change of demands from one period to the next, to minimize the cost of WIP (work-in-process) material flow, we should make some changes in machinery layout. The objective function of the presented model contains 2 parts.

1- Material flow cost and 2-machinery movement cost (i.e. layout changes).

One of the characteristics of the presented model is the abilities of cell changes and inter cell and intra cell layout. Meanwhile, another characteristic is noticing the distance among the machines and cell in material flow cost and also batch movement of material in inter and intra cells. To study the proposed model, 20 sample problems with different sizes were generated. The proposed model was solved by branch and bound ( $\mathrm{B} \& \mathrm{~B}$ ) and final solution as
well as the solution time for GA and B \& B were compared. This comparison showed the high ability of GA large size problems.

At the end, for better understanding, one of the problems was completely studied.

For future studies, the proposed model can be examined in different dimensions.

1- Use of other Meta-heuristic algorithm such as ANT COLONY and TABU SEARCH.
2- Noticing dynamic and Fuzzy demands of part simultaneously.
3- Considering probable demands and use of simulating methods to approach the real world conditions and to solve.
4- Considering all machines and, ML(s) in different sizes that means each machine can locate in same ML(s).
5- Presentation of a model with multi-objectives. For example, we can consider other objectives such as solving time along with the cost.
6- Studying the aforementioned problem in turbulent conditions. These are conditions in which changes happen very quickly.
7- Use of flexible machinery to minimize the cost and production time.

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