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A Review and Evaluation of Statistical Process Control Methods in Monitoring Process Mean and Variance Simultaneously

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Abstract:
In this paper, first the available single charting methods, which have been proposed to detect simultaneous shifts in a single process mean and variance, are reviewed. Then, by designing proper simulation studies these methods are evaluated in terms of in-control and out-of-control average run length criteria (ARL). The results of these simulation experiments show that the EWMA and EWMS methods are quite capable to detect large shifts in the means and variances. However, while the two EWMV procedures under study do not perform well, the Max-Min EWMA and Max-EWMA perform very well in all scenarios of mean and variance shifts.

Keywords: Statistical Quality Control, Control Chart, Simultaneous Control of Mean and Variance

1. Introduction

Based upon a Shewhart control chart Hotelling (1947) proposed the first multivariate quality control chart. Alt (1984) and Lowry and Montgomery (1995) also proposed multivariate control schemes with similar basis.
Since in situations where the changes are small, EWMA and CUSUM based control charts have more efficiencies than the Shewhart control charts, the multivariate cases of these methods have been investigated by many researchers. Woodall and Ncube (1985) and Runger (2004) are some of the authors who have studied the MCUSUM and Lowry et al. (1992) and Reynolds and Kim (2005) are some of the authors who have developed the MEWMA. All of these researchers considered the cases in which only the mean vector is monitored. However, Reynolds and Cho (2006) proposed a MEWMA based control chart on combination of sample means and on the sum of squared regression adjusted deviation from target to simultaneously monitor mean vector and covariance matrix.
The multivariate quality control schemes mentioned above are extensions of one-variable ones. One way to obtain a multivariate quality control scheme is to extend a high performance one-variable quality control scheme to multivariate situations.
In a transmission phase from one-variable to multivariate control schemes, some Univariate control charts focus on controlling the mean and the variance by only one control chart. In this research, some of these methods are studied in detail. In section two, we review the existing Univariate control charting methods that monitor both mean and variance simultaneously. Then in section three, we compare their performances through different simulation studies by ARL criteria. The conclusion comes in section four.

2. Some Methods in Univariate Simultaneous Monitoring

In this section, we review some statistical process control methods, which simultaneously control the mean and variance of a single quality characteristic. All of these procedures are based on the EWMA method.
Macgregor and Harris (1993) proposed EWMS and EWMV control charts. The former only monitor the shifts in variance and is not a simultaneous monitoring method. However, the latter is sensitive to shifts in both mean and variance. Another method was proposed by Amin et al. (1999) and named Max-Min EWMA. The last reviewed method is Max EWMA and proposed by Chen et al. at 2001. The performances of all studied method are calculated in term of ARL.

2.1 Assumptions and Notations

I. The sequence of random variables X_{ij} , $i=1,2,...,n_j$, $j=1,2,...$ is an independent normal distribution with mean μ and covariance σ^2. If all n_j are equal, then the index j will drop from n_j. Moreover, if $n_j=1$ for all j, then the index i will drop from X_{ij} .

II. The sample mean and the sample variance are estimated by equations (1) and (2) respectively.

$$\bar{X}_j = \frac{1}{n_j}\sum_{i=1}^{n_j} X_{ij} \quad (1)$$

$$S_j^2 = \frac{1}{n_j - 1}\sum_{i=1}^{n_j} (X_{ij} - \bar{X}_j)^2 \quad (2)$$

III. The standardized values of the random variables in **I.** are denoted by Y_{ij}.

2.2. EWMS Method

The basis of this method is similar to the EWMA method except that instead of X_j, the square deviation of X_j from the mean is used. In this procedure, the statistic to monitor the variance is given in equation (3).

$$S_j^2 = (1-\lambda)\, S_{j-1}^2 + \lambda\,(X_j - \mu)^2 \quad , \quad S_0^2 = \sigma^2 \quad (3)$$

It can be shown that:

$$E[S_j^2] = \sigma^2 \quad (4)$$

The approximate distribution of this statistic is given in equation (5).

$$S_j^2 \sim \sigma^2 \frac{\chi^2_{(\nu)}}{\nu} \quad , \quad \nu = \frac{2-\lambda}{\lambda} \quad (5)$$

So that the control limits are:

$$LCL = \sigma\sqrt{\frac{\chi^2_{(1-\frac{\alpha}{2};\nu)}}{\nu}} \quad , \quad UCL = \sigma\sqrt{\frac{\chi^2_{(\frac{\alpha}{2};\nu)}}{\nu}} \quad (6)$$

2.3. EWMV Method

The control statistic of this method is a combination of the control statistic in EWMA and EWMS methods as in equation (7).

$$V_j^2 = (1-\lambda)\, V_{j-1}^2 + \lambda\,(X_j - Z_j)^2 \quad , \quad V_0^2 = \sigma^2 \quad (7)$$

Where Z_j is given in equation (8).

$$Z_j = (1-r)\, Z_{j-1} + r\, X_j \quad , \quad Z_0 = \mu \quad (8)$$

It can be proven that the approximate distribution of V_j is obtained by equation (9).

$$S_j^2 \sim \sigma^2\, g\, \chi^2_{(\nu)} \quad , g = \frac{Var[V]}{2\sigma^2 E[V]} \quad , \quad \nu = \frac{2E^2[V]}{Var[V]} \quad (9)$$

Where $E[V]$ and $Var[V]$ are calculated according to Sweet (1986). So the control limits will be:

$$LCL = \sigma[1 - \sqrt{g\chi^2_{(1-\frac{\alpha}{2};\nu)}}], \text{ and } UCL = \sigma[1 + \sqrt{g\chi^2_{(\frac{\alpha}{2};\nu)}}] \quad (10)$$

2.4. Max-Min EWMA Method

The control statistic in this method is based on applying the EWMA method on the maximum and minimum of the observations in the sample. If we define

$$M_j = \max_i\, [Y_{ij}] \quad , \quad m_j = \min_i\, [Y_{ij}] \quad (11)$$

Then we'll have:

$$L_j = (1-\lambda)\, L_{j-1} + \lambda\, m_j \quad (12)$$

$$H_j = (1-\lambda)\, H_{j-1} + \lambda\, M_j \quad (13)$$

In (12) and (13) L_0 and H_0 are selected as the expected values of m_j and M_j where process is in control.

In this method, the control limits are considered as:

$$LCL = \mu_L - K\sigma_L \quad , \quad UCL = \mu_H + K\sigma_H \quad (14)$$

Where

$$\mu_{H_j} = -\mu_{L_j} = E[\max_i [Y_{ij}]] \quad (15)$$

and

$$\sigma^2_{H_j} = \sigma^2_{L_j} = Var[\max_i [Y_{ij}]]\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2j}) \quad (16)$$

So that So that the shifts in the mean and increases in the variance are detectable, simultaneously.

2.5. Max EWMA Method

This method is based on the transformation of the usual control space (sample mean and sample variance) to a special space in which each of transformed control parameters have the same PDF, e.g. standard normal distribution. The transformed values of the sample mean and the sample variance are denoted by U and V as follow:

$$U_j = \frac{\sqrt{n_j}\,(X_j - \mu)}{\sigma} \quad (17)$$

$$V_j = \Phi^{-1}[H(\frac{(n_j - 1)S_j^2}{\sigma^2}, (n_j - 1))] \quad (18)$$

In (18), H is CDF of a chi square random variable with $(n_j - 1)$ degrees of freedom and Φ^{-1} is the inverse function of CDF of a standard normal random variable. It is clear that U_j has standard normal distribution. If we define random variable A_j as $A_j = H(\frac{(n_j - 1)S_j^2}{\sigma^2}, (n_j - 1))$, it is easy to show that A_j has continues uniform distribution. So $V_j = \Phi^{-1}[A_j]$ follows standard normal distribution, the same distribution as U_j. We note that by these transformations the effects of different rational subgroups are omitted. To detect smaller shifts in the mean and variance, a EWMA method is setup on each of U and V variables and new variables are named Y and Z, respectively as:

$$Y_j = (1-\lambda)\, Y_{j-1} + \lambda\, U_j \quad , \quad Y_0 = 0 \tag{19}$$

$$Z_j = (1-\lambda)\, Z_{j-1} + \lambda\, V_j \quad , \quad Z_0 = 0 \tag{20}$$

In the Max EWMA method, we then combine the EWMAs given in equations (19) and (20) into one chart by defining variable M as:

$$M_j = \max[\,|\,Y_j\,|, |\,Z_j\,|\,] \tag{21}$$

Since M_j is positive, only upper control limit is required and it is considered as:

$$UCL_j = E[M_j] + K\,\sqrt{Var[M_j]} \tag{22}$$

The moments of M_j is obtained by using its probability distribution as follows:

$$f(m) = 4 r_j\, \phi(r_j m)[2\,\Phi(r_j m) - 1] \tag{23}$$

Where

$$r_j = \sqrt{\frac{2-\lambda}{\lambda(1-(1-\lambda)^{2j})}} \tag{24}$$

See Chen et al. (2001) for more details.

3. Performance Evaluations of the Existing Methods

To compare the performances of the existing methods in simultaneous monitoring of the mean and variance, we have set their input parameters so that their ARL_0 for a specific value of smoothing parameter (i.e. λ), are equal. Knowing that in EWMV method there are two smoothing parameters, λ and r, in this research for a specific amount of λ, two values of r were considered ($r = 0.05, 0.10$). The comparison is done for $ARL_0 = 185, 250, 370, 500$, and $\lambda = 0.05, 0.10$.

To calculate sets of parameters which lead to desired values of ARL_0 with a specific λ, 100,000 runs were done (Table 1). To compute ARL_1 s of each method for different scenarios of mean and variance shifts, 10,000 replications were made in each of the above four in-control average run lengths. The results are shown in Tables 2 to 5. In these tables the "-" sign indicates cases in which their ARL_1 are too large, i.e., their corresponding mean shifts could not be detected. Moreover, each *ARL* is given along with its estimated standard error

The results of Tables two to five show that the EWMA and EWMS methods, as expected, can only detect the shifts in the mean and changes in variance with relatively large magnitudes. The performance of the two EWMV methods is not as well as expected. However, the last two methods, namely Max-Min EWMA and Max EWMA, work well. In some cases the former is the best and in others the latter is the best. Hence these two methods were selected to be generalized to the multivariate case.

Table 1
Parameter estimation of the existing Methods

Parameters		Methods					
		EWMA	EWMS	EWMV(r=0.05)	EWMV(r=0.10)	Max-Min EWMA	Max EWMA
ARL_0	λ	L	α	α	α	K	K
185	0.05	2.17829	0.03382	0.0300000	0.0192090	2.18436	2.23051
	0.10	2.42047	0.01910	0.0221194	0.0157558	2.48318	2.59557
250	0.05	2.31984	0.02363	0.021666	0.012613	2.33985	2.44787
	0.10	2.54428	0.01336	0.016141	0.011328	2.62476	2.78646
370	0.05	2.48804	0.01476	0.013637	0.007430	2.53128	2.70893
	0.10	2.70001	0.00083	0.010826	0.007453	2.79967	3.02235
500	0.05	2.61368	0.01027	0.009593	0.005000	2.66976	2.90146
	0.10	2.81400	0.00573	0.007992	0.005462	2.92450	3.19703

4. Conclusion

In order to simultaneously monitor the mean and the variance of a quality characteristic in quality control environments, in this research first we reviewed and summarized the existing methods. Then, by proper and extensive simulation experiments we compared their performances by in-control and out-of-control average run length in different scenarios of mean and variance changes. The results of these simulation experiments showed that the EWMA and EWMS methods are quite capable to detect large shifts in the mean and variances. However, while the two EWMV Procedures did not perform well, the Max-MinEWMA and Max-EWMA performed very well in all scenarios.

Table 2
Out-of-control Average Run Length for $\mathbf{ARL}_0 = 185$

λ	μ	σ	EWMA		EWMS		EWMV(r = 0.05)		EWMV(r = 0.10)		Max-Min EWMA		Max EWMA	
			ARL	error	ARL	error	ARL	error	ARL	error	ARL	error	ARL	error
0.05	0.0	0.5	-	0.00	16.43	0.03	15.01	0.02	15.96	0.02	-	0.00	6.49	0.01
		0.75	1054.7	10.49	37.93	0.19	30.14	0.14	30.28	0.13	-	0.00	16.67	0.07
		1	186.63	1.77	184.6	1.76	184.2	1.71	184.56	1.65	184.4	1.78	183.1	1.71
		1.5	43.69	0.39	13.29	0.1	17.89	0.13	25.32	0.19	7.36	0.03	8.16	0.03
		2	21.69	0.19	6.06	0.04	7.35	0.05	9.26	0.06	3.8	0.01	4.21	0.01
	0.5	0.5	23.38	0.08	31.47	0.13	18.14	0.03	17.54	0.03	-	0.00	6.42	0.01
		0.75	22.67	0.11	136.5	1.17	35.73	0.15	32.69	0.13	47.37	0.31	8.98	0.02
		1	21.39	0.12	73.44	0.67	188.9	1.69	186.61	1.61	13.09	0.06	9.34	0.03
		1.5	18.98	0.14	10.9	0.08	15.75	0.12	23.87	0.18	5.51	0.02	6.66	0.02
		2	15.22	0.12	5.69	0.04	7.02	0.05	8.88	0.06	3.49	0.01	4.03	0.01
	1.0	0.5	9.03	0.02	164.1	1.46	27.41	0.04	21.85	0.03	16.69	0.05	4.3	0.01
		0.75	9.09	0.03	32.49	0.24	48.33	0.16	38.64	0.13	8.55	0.02	4.35	0.01
		1	9.22	0.04	16.2	0.11	172.2	1.71	197.0	1.66	5.94	0.02	4.42	0.01
		1.5	9.24	0.05	7.44	0.05	11.6	0.1	20.46	0.18	3.84	0.01	4.34	0.01
		2	9.06	0.06	4.77	0.03	5.9	0.04	7.76	0.06	2.89	0.01	3.49	0.01
	1.5	0.5	5.71	0.01	9.58	0.04	38.34	0.04	27.59	0.03	6.31	0.01	3.00	0.00
		0.75	5.8	0.01	8.2	0.04	50.71	0.25	45.55	0.13	4.81	0.01	3.00	0.00
		1	5.88	0.02	6.84	0.04	80.75	1.39	191.54	1.7	3.92	0.01	3.01	0.01
		1.5	5.97	0.03	4.87	0.03	7.18	0.06	14.46	0.15	2.93	0.01	3.03	0.01
		2	6.15	0.03	3.83	0.03	4.74	0.03	6.31	0.05	2.42	0.01	2.87	0.01
	2.0	0.5	4.28	0.01	4.28	0.01	19.26	0.19	33.44	0.03	4.05	0.01	2.12	0.00
		0.75	4.31	0.01	4.1	0.02	13.23	0.2	47.25	0.19	3.42	0.01	2.22	0.00
		1	4.37	0.01	3.88	0.02	9.98	0.36	127.55	1.64	2.99	0.01	2.28	0.00
		1.5	4.52	0.02	3.51	0.02	4.65	0.03	8.88	0.10	2.44	0.01	2.37	0.01
		2	4.58	0.02	3.01	0.02	3.61	0.02	4.75	0.04	2.09	0.01	2.34	0.01
0.10	0.0	0.5	-	0.00	15.49	0.04	13.56	0.03	13.32	0.03	-	0.00	5.61	0.01
		0.75	1696.1	17.01	47.45	0.33	33.34	0.21	29.55	0.17	-	0.00	16.65	0.09
		1	182.6	1.76	187.1	1.81	185.3	1.77	183.87	1.71	186.82	1.83	188.5	1.80
		1.5	34.22	0.3	12.65	0.1	15.75	0.13	22.98	0.19	6.4	0.03	6.9	0.03
		2	15.73	0.14	5.59	0.04	6.42	0.05	8.17	0.06	3.2	0.01	3.46	0.01
	0.5	0.5	33.39	0.2	40.29	0.25	17.15	0.04	15.03	0.03	-	0.00	5.50	0.01
		0.75	26.37	0.17	208.3	1.91	39.97	0.22	32.7	0.18	77.06	0.65	8.00	0.02
		1	21.97	0.15	72.96	0.68	183.9	1.72	190.27	1.75	12.88	0.07	8.31	0.03
		1.5	16.99	0.13	10.62	0.08	14.26	0.12	22.02	0.19	4.91	0.02	5.57	0.02
		2	12.11	0.1	5.32	0.04	6.13	0.05	7.84	0.06	2.98	0.01	3.31	0.01
	1.0	0.5	8.38	0.02	355.6	3.47	27.04	0.05	19.56	0.03	20.35	0.09	3.57	0.01
		0.75	8.44	0.03	37.53	0.31	51.91	0.23	38.35	0.17	8.12	0.03	3.67	0.01
		1	8.35	0.04	16.19	0.13	154.2	1.73	191.11	1.75	5.32	0.02	3.71	0.01
		1.5	8.16	0.05	7	0.05	10.1	0.1	18.29	0.18	3.33	0.01	3.59	0.01
		2	7.61	0.05	4.43	0.03	5.17	0.04	6.75	0.05	2.49	0.01	2.84	0.01
	1.5	0.5	4.97	0.01	9.64	0.05	35.85	0.06	24.67	0.03	5.70	0.01	2.38	0.00
		0.75	5.08	0.01	7.9	0.05	47.34	0.31	43.39	0.18	4.23	0.01	2.43	0.01
		1	5.14	0.02	6.44	0.04	67.38	1.34	170.89	1.72	3.43	0.01	2.48	0.01
		1.5	5.22	0.03	4.59	0.03	6.3	0.06	12.74	0.15	2.56	0.01	2.55	0.01
		2	5.25	0.03	3.57	0.03	4.12	0.03	5.51	0.05	2.10	0.01	2.34	0.01
	2.0	0.5	3.64	0.01	3.91	0.01	15.32	0.17	28.52	0.05	3.50	0.01	2.00	0.00
		0.75	3.68	0.01	3.72	0.02	11.07	0.2	39.26	0.24	2.95	0.01	2.00	0.00
		1	3.74	0.01	3.59	0.02	9.52	0.4	102.03	1.56	2.57	0.01	2.00	0.00
		1.5	3.87	0.02	3.21	0.02	3.99	0.03	7.46	0.1	2.11	0.01	2.00	0.01
		2	3.95	0.02	2.87	0.02	3.27	0.02	4.29	0.04	1.82	0.01	1.95	0.01

Table 3

Out-of-control Average Run Lengths for $\mathbf{ARL}_0 = 250$

λ	μ	σ	EWMA		EWMS		EWMV(r = 0.05)		EWMV(r = 0.10)		Max-Min EWMA		Max EWMA	
			ARL	error	ARL	error	ARL	error	ARL	error	ARL	error	ARL	error
0.05	0.0	0.5	-	0	17.8	0.03	16.27	0.03	17.36	0.03	-	0	6.88	0.01
		0.75	1806.79	18.01	42.66	0.23	33.83	0.16	33.99	0.15	-	0	17.85	0.07
		1	247.35	2.34	253.14	2.43	248.46	2.33	246.45	2.26	250.98	2.44	254.03	2.43
		1.5	52.03	0.46	14.48	0.11	19.37	0.14	27.87	0.2	8	0.04	8.62	0.04
		2	24.74	0.21	6.36	0.04	7.75	0.05	9.83	0.07	4.07	0.02	4.43	0.02
	0.5	0.5	26.14	0.09	35.2	0.15	19.47	0.03	18.93	0.03	-	0	6.8	0.01
		0.75	25.02	0.12	177.51	1.55	39.5	0.17	36.56	0.15	56.04	0.38	9.54	0.02
		1	23.4	0.13	85.41	0.78	255.26	2.33	253.91	2.28	14.33	0.07	9.93	0.03
		1.5	20.74	0.15	11.93	0.09	17.4	0.13	26.56	0.2	5.91	0.02	7.06	0.03
		2	16.71	0.13	6	0.04	7.36	0.05	9.49	0.07	3.73	0.01	4.24	0.01
	1.0	0.5	9.72	0.02	231.75	2.14	28.86	0.04	23.19	0.03	18.48	0.05	4.55	0.01
		0.75	9.87	0.03	37.8	0.27	52.14	0.17	42.28	0.15	9.3	0.03	4.6	0.01
		1	9.88	0.04	17.8	0.12	238.82	2.38	261.39	2.27	6.38	0.02	4.64	0.01
		1.5	9.88	0.05	7.94	0.05	12.57	0.1	22.48	0.19	4.06	0.01	4.57	0.01
		2	9.85	0.06	5.05	0.03	6.24	0.04	8.3	0.06	3.07	0.01	3.69	0.01
	1.5	0.5	6.12	0.01	10.59	0.04	39.8	0.04	28.93	0.03	6.81	0.01	3.07	0
		0.75	6.2	0.01	8.84	0.04	57.1	0.25	49.51	0.15	5.14	0.01	3.12	0
		1	6.27	0.02	7.27	0.04	124.24	2.04	255.2	2.3	4.18	0.01	3.15	0.01
		1.5	6.44	0.03	5.29	0.03	7.93	0.07	16.8	0.17	3.15	0.01	3.19	0.01
		2	6.5	0.04	3.99	0.03	4.91	0.03	6.69	0.05	2.56	0.01	3	0.01
	2.0	0.5	4.54	0.01	4.59	0.01	25	0.21	34.88	0.03	4.33	0.01	2.29	0
		0.75	4.59	0.01	4.38	0.02	17.69	0.26	53.35	0.19	3.64	0.01	2.36	0
		1	4.63	0.01	4.11	0.02	16.14	0.68	188.89	2.23	3.16	0.01	2.39	0
		1.5	4.76	0.02	3.64	0.02	4.92	0.04	9.94	0.12	2.57	0.01	2.46	0.01
		2	4.89	0.02	3.24	0.02	3.92	0.03	5.27	0.04	2.22	0.01	2.46	0.01
0.10	0.0	0.5	-	0	16.83	0.04	14.57	0.03	14.3	0.03	-	0	5.92	0.01
		0.75	2821.17	28.46	57.77	0.42	38.43	0.25	33.89	0.2	-	0	18.39	0.1
		1	252.56	2.45	252.51	2.47	251.72	2.39	251.09	2.41	251.74	2.46	248.71	2.44
		1.5	39.54	0.35	13.64	0.11	17.03	0.14	25.16	0.21	6.92	0.03	7.39	0.04
		2	17.95	0.16	5.89	0.05	6.76	0.05	8.57	0.06	3.39	0.01	3.62	0.01
	0.5	0.5	40.38	0.25	48.03	0.31	18.35	0.04	16.06	0.03	-	0	5.77	0.01
		0.75	29.72	0.2	292.68	2.75	45.49	0.26	36.88	0.2	98.92	0.86	8.51	0.03
		1	24.31	0.17	85.55	0.81	251.42	2.43	253.45	2.35	14.19	0.08	8.84	0.04
		1.5	17.97	0.14	11.17	0.09	15.09	0.13	23.79	0.21	5.15	0.02	5.83	0.02
		2	13.28	0.11	5.45	0.04	6.32	0.05	8.16	0.06	3.11	0.01	3.43	0.01
	1.0	0.5	9.02	0.02	573.69	5.62	28.41	0.05	20.64	0.04	23.54	0.11	3.73	0.01
		0.75	9.04	0.03	43.65	0.38	57.47	0.26	42.76	0.2	8.74	0.03	3.82	0.01
		1	9.02	0.04	17.82	0.14	217.37	2.43	252.03	2.36	5.67	0.02	3.87	0.01
		1.5	8.78	0.05	7.35	0.06	10.95	0.11	20.13	0.2	3.52	0.01	3.75	0.01
		2	8.15	0.06	4.69	0.03	5.45	0.04	7.25	0.06	2.61	0.01	2.97	0.01
	1.5	0.5	5.27	0.01	10.75	0.05	37.28	0.06	25.68	0.03	6.11	0.01	2.54	0
		0.75	5.33	0.01	8.4	0.05	53.55	0.34	47.82	0.2	4.47	0.01	2.54	0.01
		1	5.41	0.02	6.73	0.04	98.49	1.85	235.63	2.34	3.58	0.01	2.56	0.01
		1.5	5.53	0.03	4.85	0.03	6.73	0.06	14.53	0.17	2.7	0.01	2.63	0.01
		2	5.53	0.03	3.72	0.03	4.35	0.03	5.82	0.05	2.19	0.01	2.45	0.01
	2.0	0.5	3.83	0.01	4.17	0.01	19.19	0.19	29.81	0.04	3.71	0.01	2	0
		0.75	3.87	0.01	3.97	0.02	14.17	0.24	45.62	0.26	3.12	0.01	2.02	0
		1	3.9	0.01	3.73	0.02	12.44	0.54	151.38	2.25	2.7	0.01	2.04	0
		1.5	4.05	0.02	3.34	0.02	4.19	0.03	8.32	0.11	2.21	0.01	2.07	0.01
		2	4.13	0.02	2.95	0.02	3.39	0.02	4.46	0.04	1.89	0.01	2.03	0.01

Table 4

Out-of-control Average Run Length for $\mathbf{ARL}_0 = 370$

λ	μ	σ	**EWMA**		**EWMS**		**EWMV(r = 0.05)**		**EWMV(r = 0.10)**		**Max-Min EWMA**		**Max EWMA**	
			ARL	error	ARL	error	ARL	error	ARL	error	ARL	error	ARL	error
0.05	0.0	0.5	-	0	19.45	0.03	17.78	0.03	19.02	0.03	-	0	7.32	0.01
		0.75	3581.4	35.58	49.98	0.28	38.68	0.19	39.07	0.17	-	0	19.58	0.08
		1	365.83	3.56	370.36	3.51	366.3	3.43	366.42	3.42	366.46	3.5	368.49	3.53
		1.5	65.07	0.58	15.57	0.12	20.9	0.15	30.7	0.23	8.82	0.04	9.2	0.04
		2	29.55	0.25	6.78	0.05	8.26	0.06	10.53	0.07	4.42	0.02	4.68	0.02
	0.5	0.5	29.94	0.11	40.64	0.18	21.14	0.03	20.59	0.03	-	0	7.24	0.01
		0.75	28.05	0.14	249.01	2.24	44.66	0.2	41.52	0.17	69.69	0.49	10.27	0.03
		1	26.42	0.15	106.85	0.99	374.66	3.5	374.58	3.41	15.82	0.07	10.58	0.04
		1.5	22.95	0.16	12.97	0.09	19.23	0.15	30.03	0.23	6.44	0.03	7.59	0.03
		2	18.83	0.15	6.37	0.04	7.85	0.05	10.13	0.07	3.98	0.01	4.47	0.01
	1.0	0.5	10.57	0.02	356.18	3.3	30.73	0.04	24.99	0.03	20.94	0.06	4.82	0.01
		0.75	10.67	0.03	44.34	0.33	57.68	0.2	47.53	0.18	10.18	0.03	4.86	0.01
		1	10.7	0.04	19.66	0.14	354.33	3.54	384.79	3.44	6.95	0.02	4.93	0.01
		1.5	10.83	0.06	8.64	0.06	14.25	0.12	26.17	0.22	4.4	0.01	4.88	0.01
		2	10.51	0.07	5.32	0.04	6.65	0.05	8.99	0.06	3.27	0.01	3.88	0.01
	1.5	0.5	6.61	0.01	11.84	0.05	41.6	0.04	30.6	0.03	7.43	0.01	3.2	0
		0.75	6.68	0.01	9.74	0.05	64.28	0.26	54.5	0.18	5.57	0.01	3.27	0
		1	6.77	0.02	7.97	0.04	207.51	3.14	379.3	3.49	4.51	0.01	3.31	0.01
		1.5	6.86	0.03	5.59	0.04	8.7	0.07	19.16	0.2	3.35	0.01	3.36	0.01
		2	6.97	0.04	4.24	0.03	5.25	0.04	7.36	0.06	2.73	0.01	3.17	0.01
	2.0	0.5	4.87	0.01	4.97	0.01	32.76	0.22	36.61	0.03	4.67	0.01	2.58	0
		0.75	4.92	0.01	4.75	0.02	24.13	0.32	59.4	0.2	3.93	0.01	2.55	0
		1	4.97	0.01	4.45	0.02	28.27	1.19	301.71	3.48	3.4	0.01	2.55	0.01
		1.5	5.1	0.02	3.88	0.02	5.32	0.04	11.7	0.14	2.74	0.01	2.58	0.01
		2	5.19	0.02	3.37	0.02	4.12	0.03	5.65	0.04	2.34	0.01	2.58	0.01
0.10	0.0	0.5	-	0	18.6	0.05	15.88	0.04	15.54	0.03	-	0	6.27	0.02
		0.75	5803.97	58.34	74.25	0.55	46.46	0.32	40.05	0.24	-	0	20.38	0.11
		1	370.81	3.66	370.83	3.62	367.51	3.61	361.42	3.42	374.56	3.72	365.81	3.53
		1.5	48.92	0.45	15.05	0.12	18.78	0.15	28.19	0.23	7.61	0.04	7.89	0.04
		2	20.68	0.18	6.28	0.05	7.18	0.05	9.14	0.07	3.65	0.02	3.81	0.01
	0.5	0.5	52.86	0.36	60.98	0.42	19.99	0.05	17.43	0.04	-	0	6.16	0.01
		0.75	35.6	0.24	480.06	4.65	53.33	0.31	43.29	0.25	138.94	1.26	9.14	0.03
		1	27.98	0.2	109.89	1.04	371.15	3.59	381.61	3.6	15.83	0.09	9.4	0.04
		1.5	20.32	0.16	12.32	0.1	16.88	0.15	27.25	0.24	5.52	0.03	6.19	0.03
		2	14.74	0.12	5.95	0.04	6.89	0.05	8.92	0.07	3.35	0.01	3.65	0.01
	1.0	0.5	9.84	0.03	1066.5	10.6	30.13	0.05	22	0.04	28.62	0.15	3.93	0.01
		0.75	9.81	0.04	53.83	0.46	65.37	0.32	48.73	0.25	9.6	0.03	4	0.01
		1	9.67	0.04	19.89	0.16	329.38	3.62	384.1	3.64	6.09	0.02	4.05	0.01
		1.5	9.39	0.06	8.07	0.06	12.14	0.12	23.06	0.23	3.73	0.01	3.93	0.01
		2	8.78	0.06	4.96	0.04	5.77	0.04	7.69	0.06	2.77	0.01	3.12	0.01
	1.5	0.5	5.66	0.01	12.21	0.06	39.16	0.06	27.01	0.04	6.62	0.01	2.73	0
		0.75	5.74	0.02	9.41	0.05	64.27	0.38	54.74	0.26	4.81	0.01	2.67	0.01
		1	5.81	0.02	7.38	0.05	171.38	3.02	350.94	3.62	3.83	0.01	2.68	0.01
		1.5	5.84	0.03	5.18	0.04	7.35	0.07	16.24	0.19	2.82	0.01	2.73	0.01
		2	5.9	0.04	3.92	0.03	4.56	0.03	6.24	0.05	2.31	0.01	2.57	0.01
	2.0	0.5	4.06	0.01	4.53	0.02	24.6	0.21	31.22	0.04	3.97	0.01	2	0
		0.75	4.11	0.01	4.29	0.02	20.39	0.33	53.24	0.29	3.31	0.01	2.04	0
		1	4.17	0.01	4.01	0.02	23.11	1.05	246.78	3.4	2.87	0.01	2.09	0
		1.5	4.25	0.02	3.5	0.02	4.46	0.04	9.91	0.15	2.32	0.01	2.13	0.01
		2	4.39	0.02	3.09	0.02	3.53	0.03	4.72	0.04	2	0.01	2.12	0.01

Table 5

Out-of-control Average Run Length for $\mathbf{ARL}_0 = 500$

λ	μ	σ	EWMA		EWMS		EWMV(r = 0.05)		EWMV(r = 0.10)		Max-Min EWMA		Max EWMA	
			ARL	error	ARL	error	ARL	error	ARL	error	ARL	error	ARL	error
		0.5	-	0	20.75	0.04	18.93	0.03	20.25	0.03	-	0	7.67	0.01
		0.75	6160.44	60.37	56.1	0.31	42.9	0.21	43.34	0.2	-	0	20.69	0.08
	0.0	1	495.88	4.82	500.16	4.82	497.85	4.76	497.93	4.67	500.92	4.83	505.5	4.78
		1.5	75.44	0.69	16.83	0.12	22.51	0.16	33.28	0.24	9.39	0.04	9.63	0.04
		2	33.53	0.29	7.12	0.05	8.63	0.06	11.02	0.07	4.7	0.02	4.87	0.02
		0.5	33.33	0.13	44.9	0.2	22.43	0.04	21.89	0.03	-	0	7.59	0.01
		0.75	30.79	0.15	319.41	2.89	48.7	0.22	45.6	0.2	83.56	0.61	10.82	0.03
	0.5	1	28.65	0.17	123.28	1.11	510.8	4.81	510.82	4.77	17.02	0.08	11.08	0.04
		1.5	25.04	0.18	13.68	0.1	20.44	0.15	32.17	0.25	6.71	0.03	7.94	0.03
		2	20.39	0.16	6.67	0.05	8.22	0.06	10.64	0.07	4.23	0.02	4.67	0.01
		0.5	11.26	0.02	513.95	4.79	32	0.04	26.21	0.03	23	0.07	5.02	0.01
		0.75	11.32	0.03	49.64	0.37	61.89	0.22	51.52	0.2	10.85	0.03	5.07	0.01
0.05	1.0	1	11.31	0.04	21.22	0.15	487.57	4.92	515.68	4.73	7.33	0.02	5.1	0.01
		1.5	11.4	0.06	9.07	0.06	15.12	0.13	28.03	0.23	4.61	0.01	5.08	0.02
		2	11.18	0.07	5.61	0.04	7.02	0.05	9.49	0.07	3.45	0.01	4.06	0.01
		0.5	6.96	0.01	12.73	0.05	42.88	0.04	31.91	0.03	7.86	0.01	3.34	0
		0.75	7.02	0.02	10.22	0.05	69.89	0.27	58.81	0.2	5.85	0.01	3.39	0.01
	1.5	1	7.09	0.02	8.36	0.05	293.87	4.3	506.77	4.66	4.73	0.01	3.43	0.01
		1.5	7.26	0.03	5.83	0.04	9.3	0.08	21.65	0.22	3.5	0.01	3.49	0.01
		2	7.34	0.04	4.43	0.03	5.55	0.04	7.81	0.06	2.87	0.01	3.31	0.01
		0.5	5.13	0.01	5.28	0.02	37.75	0.21	37.92	0.03	4.93	0.01	2.76	0
		0.75	5.18	0.01	5.04	0.02	30.86	0.37	64.38	0.22	4.13	0.01	2.69	0
	2.0	1	5.24	0.01	4.73	0.02	50.22	2.04	430.28	4.76	3.58	0.01	2.66	0.01
		1.5	5.34	0.02	4.03	0.02	5.69	0.04	13.23	0.16	2.87	0.01	2.68	0.01
		2	5.46	0.02	3.51	0.02	4.35	0.03	6.09	0.05	2.46	0.01	2.68	0.01
		0.5	-	0	20.15	0.06	16.86	0.04	16.47	0.04	-	0	6.54	0.02
		0.75	-	0	92.15	0.72	53.63	0.37	45.89	0.29	-	0	22.17	0.12
	0.0	1	496.93	4.89	501.66	5.01	500.7	4.83	503.88	4.79	510.13	5.05	508.77	4.95
		1.5	55.98	0.52	16.46	0.13	20.22	0.16	30.47	0.25	8.13	0.04	8.25	0.04
		2	23.07	0.2	6.66	0.05	7.58	0.06	9.63	0.07	3.87	0.02	3.98	0.02
		0.5	66.64	0.49	75.56	0.55	21.24	0.05	18.44	0.04	-	0	6.46	0.01
		0.75	40.64	0.28	722.86	7.02	61.29	0.38	49.04	0.3	180.31	1.67	9.62	0.03
	0.5	1	31.58	0.23	135.48	1.3	500.54	4.93	502.33	4.76	17.25	0.1	10.01	0.04
		1.5	22.29	0.17	13.28	0.1	18.33	0.16	29.28	0.25	5.86	0.03	6.57	0.03
		2	15.9	0.13	6.18	0.05	7.05	0.05	9.19	0.07	3.49	0.01	3.76	0.01
		0.5	10.52	0.03	1829.48	18.24	31.41	0.06	23	0.04	33.25	0.18	4.09	0.01
		0.75	10.53	0.04	64.71	0.57	73.13	0.38	54.43	0.29	10.26	0.04	4.15	0.01
0.10	1.0	1	10.46	0.05	22.36	0.18	450.16	4.88	505.23	4.81	6.44	0.02	4.22	0.01
		1.5	9.92	0.06	8.52	0.06	13.1	0.13	24.95	0.24	3.9	0.01	4.09	0.01
		2	9.34	0.07	5.11	0.04	5.95	0.05	8.03	0.06	2.87	0.01	3.24	0.01
		0.5	5.95	0.01	13.53	0.07	40.5	0.06	28.02	0.04	7.02	0.01	2.83	0
		0.75	6.03	0.02	10.08	0.06	72.62	0.44	60.16	0.3	5.01	0.01	2.78	0.01
	1.5	1	6.07	0.02	7.95	0.05	257.21	4.32	479.34	4.88	4.01	0.01	2.78	0.01
		1.5	6.21	0.03	5.45	0.04	7.87	0.08	18.24	0.21	2.95	0.01	2.83	0.01
		2	6.12	0.04	4.05	0.03	4.74	0.04	6.5	0.06	2.39	0.01	2.64	0.01
		0.5	4.26	0.01	4.84	0.02	28.92	0.21	32.37	0.04	4.15	0.01	2.01	0
		0.75	4.31	0.01	4.56	0.02	24.5	0.39	59.4	0.32	3.45	0.01	2.07	0
	2.0	1	4.36	0.01	4.24	0.02	37.03	1.7	338.76	4.51	2.97	0.01	2.13	0
		1.5	4.49	0.02	3.69	0.02	4.71	0.04	10.65	0.16	2.4	0.01	2.21	0.01
		2	4.54	0.02	3.17	0.02	3.62	0.03	4.92	0.04	2.06	0.01	2.16	0.01

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