Journal of Engineering Science and Technology
Vol. 13, No. 10 (2018) 3277 - 3296

A SURVEY ON CONTROL TECHNIQUES OF A BENCHMARKED CONTINUOUS STIRRED TANK REACTOR

OBAID ALSHAMMARI[1], MUHAMMAD NASIRUDDIN MAHYUDDIN[2,*], HOUSSEM JERBI[1]

School of Electrical & Electronics Engineering,
Universiti Sains Malaysia, Pulau Pinang, Malaysia
Engineering College, IED, University of Hail, Saudi Arabia
*Corresponding Author: nasiruddin@usm.my

Abstract

The study carried out in this paper unveils a survey on issues related to modelling problems control strategies of a Continuous Stirred Tank Reactor (CSTR), a highly nonlinear plant containing numbers of stable and unstable operating points is considered. The issues discussed are categorised into regulation, feedback linearization, flatness, observation and estimation as well as challenges related to equilibrium points concerning CSTR. In this study, the limited capability of a conventional PID controller is discussed based on preliminary description and a dynamic modelling of the nonlinear plant. Moreover, the limitations of the conventional PID is illustrated through a simulation using nonlinear model of CSTR carried out under input constraint and the presence of bounded disturbances. The result shows that a fixed PID will not guarantee consistent performance throughout operating set points. The feedback linearization formalism is presented to prove that only regulation in the neighbourhood of operating point is possible. Non-minimum phase property exhibited by a CSTR is investigated as well. Flatness control is demonstrated as one of the possible linearization control technique achieving the objective of the trajectory tracking.

Keywords: CSTR models, Feedback linearization, Flatness, PID control, Tracking.

1. Introduction

A severe control issue could be created by chemical reactors due to their non-linearity properties as well as due to the existence of various stable and unstable operating points [1-3]. Moreover, the uncertainty impacting the kinetic parameters cannot be overlooked [2, 4, 5]. As such, it is an uphill task to model kinetic reactions. Consequently, considerable disparity could also arise within the model. Utilization of too much information by the design of the controller could result in severe deterioration of the control execution [3]. Moreover, most of the nonlinear control techniques are likely to assume accurate measurement or estimation of all state variables [6-8]. However, in the past decade, considerable interest is being shown by many industries towards studying appropriate control strategies so that these could be employed in the continuous reactor group [1, 7, 9]. In this survey paper, issues related regulation and stabilization, feedback linearization, observation and challenges associated with equilibrium points in a Continuously Stirred Tank Reactor or CSTR will be reviewed. In Section 2, the reactor's comprehensive description is given, which also accounts for the mathematical model with parameters. Section 3 elaborates the control problem and the specific operating point. Section 4 presents a study of the feedback linearization formalism. In section 5, an examination of the conventional control techniques' weakness is performed. As an important property, section 6 presents the CSTR model's minimum phase characteristic.

1.1. Regulation and stabilization

A proportional regulator has been employed to examine the temperature stabilization for a perfectly continuous stirred tank reactor (CSTR) that involves an exothermic reaction 1^{st} order [10]. The temperature of chemical reactors was regulated by employing conventional controllers known as the Proportional Integral Derivative (PID) [11]. Since their design is partly independent of the system model, these mostly turn out to be robust against model uncertainties. In particular, the nonlinear geometric control tools were employed for stabilisation of unstable equilibrium states in CSTR. The behaviour of nonlinear regulators is also being considered in newer studies, especially in the case of 1^{st} order exothermic reaction [12].

Amongst others, these studies have shown features such as practical stabilisation of the chemical reactor and the global convergence of trajectories when it is being driven by an input–output linearization as well as during the case of parametric uncertainty [10, 11]. Recently, two sequential reactions were employed in an attempt to consider non-linearities of the process within an adaptive regulator. First, the dynamic of zero is analysed. Moreover, an adaptive regulator of the reference model was developed to control the highly nonlinear CSTR model [13]. On a reactor with 1^{st} order exothermic reaction, robust control techniques were employed [14]. In terms of parametric uncertainty, input-state linearization was employed to achieve overall stabilisation on the reaction kinetics [3-5]. Until recently, for robust global stabilisation, different significant results were attained under input constraints. A state feedback control was put forward by authors in [12, 15-18], which helped to stabilise the temperature given in an arbitrary set of set points, even though there were uncertainties in the kinetics of the reaction. This can be executed at the set point of the reactor even if it does not possess any information regarding concentrations. It found in [19-20] have already achieved notable results in this space: a first-order compensator has already been developed for modelling errors. It has been shown that the chemical reactor's temperature can also be stabilised [13, 21]. Finally, a brief discussion regarding stabilisation of chemical reactors with a Proportional Integral (PI) type regulator was also presented [4-8].

1.2. Feedback linearization

Linearized modelling tools and linear systems were analysed in brief to customarily control and examine chemical processes [5, 8, 10], or those nonlinear methods with a base in linearization techniques [15]. Should the chemical process to be analysed turns out to be highly non-linear, then there is limited use for linear techniques. There are control techniques which employ direct adaptive control to control a nonlinear chemical plant such as CSTR and coupled-tank liquid system [22--25] in which the chemical plant under study belongs to a class of nonaffine nonlinear systems and contains an unknown parameter that enters the model nonlinearly. A study was done by [1, 3] regarding the dynamic behaviour of the reactor. They employed the linearization method to show that local asymptotic stability can be achieved with the open loop's unsteady equilibrium point if placed within a closed loop [26-28], whereby techniques catering to feedback linearization were employed. For use in a reactor that undergoes an exothermic reaction, a full state feedback linearization solution was specifically proposed [4, 8, 10]. To show the observed overall state feedback stabilization numerically, an input variable as coolant temperature was employed [29].

An analytical approach would be the actual input–output linearization control. The objective of employing this control is to mitigate the initial nonlinear control problem and turn it to less complicated linear control problem [26]. A strategy was put forward for input–output linearization, which can also be employed in nonlinear constraint processes and at the same time bring in the benefits of feedback linearization and predictive control [30-32]. However, the uncertainty in the robustness of the modelling cannot be ensured since the exact knowledge about the process model has to be used for techniques in feedback linearization. Despite of the challenges, great success was achieved with the development of adaptive linearization techniques that have their basis in the Lyapunov theory [15, 32-35].

1.3. Observation and estimation

In practice, estimation of unknown parameters and unmeasured state variables involves major issues and is not easy. In most cases, readily measured parameters are temperatures and flows only. Even though advanced methods have been developed to accurately measure concentrations within a chemical reactor, these involve high operating costs and are therefore not employed on a large scale in industrial plants [3, 5, 14]. The overall feedback stabilisation of the reactor, undergoing exothermic reaction, was shown in [17]. Later, the same case was examined by considering the case where a state observer was present [27]. Finally, the reactor's overall stabilisation with the complex kinetics was examined [5-7], when a state observer is present. For a reactor that has two sequential exothermic reactions, in the presence of a state observer, an accurate execution of the control was achieved [18].

In this area, several studies were conducted by Alvarez and others [3, 5, 28, 31]. An observer of a robust state was employed to address the issue of partial state measurement. Later, feed concentration was deemed as an additional input to allow extending these outcomes to an exothermic chemical reactor [9, 30, 32]. However, to efficiently apply all the mentioned control laws earlier, a need for in-line measurement for the states of the system arises. This is denoted with a set of temperatures and concentrations sensors. In actual most industrial practice, measuring temperatures is not that difficult. However, it is tough to measure concentrations in general. The issues associated with state estimation or variable measurement can be tackled by selecting a feedback [21] to stabilise and adjust the reactor's overall temperature.

1.4. Challenges associated with equilibrium points in CSTR

For economic reasons and chemical engineering, the aim here would be to operate chemical reactors to maximize the production of a desired product. For this, a reactor is considered to be at an operating point that gives the desired product in optimal yield. However, the control system design at this point could significantly complicate the overall operation as well as the motivation to propose a benchmark problem for designing the nonlinear control system, which is as per a specific (CSTR) defined in [33]. A number of interesting features can be found with the benchmark problem. At the operating point, the steady state gain alters its sign. Thus, this reactor cannot be stabilised as well as accomplish satisfying performance through linear controllers (with integral action) [11]. At this operating point, the stability feature of zero dynamics changes. Therefore, the CSTR's qualitative behaviour varies substantially for various disturbances and set points.

A "real world" background is associated with the problem. A full set of listing performance objectives is given, including the description for uncertainty. Work in [30] provides a discussion on the implications and the reasons regarding where the first two points can be found. Moreover, based on a differential algebra technique, a solution is presented to address this benchmark problem in terms of control. This type of control scheme is also referred as nonlinear feedback linearization model controllers [34]. System description and the dynamic model of a CSTR is to be described next.

2. Preliminaries and Dynamic Modelling of A CSTR

A typical example of a reaction system would be perfectly mixed chemical reactors. These reactors are constructed with a tank that holds a reaction liquid. The liquid is generally a homogeneous permanent mixture that includes an appropriate agitation system as well. To the reactor, various reactants can be fed, which could either be in liquid or gaseous form. The formation of a reaction's products occurs in the solution within the reaction environment. Figure 1 depicts a schematic portrayal of a chemical reactor.

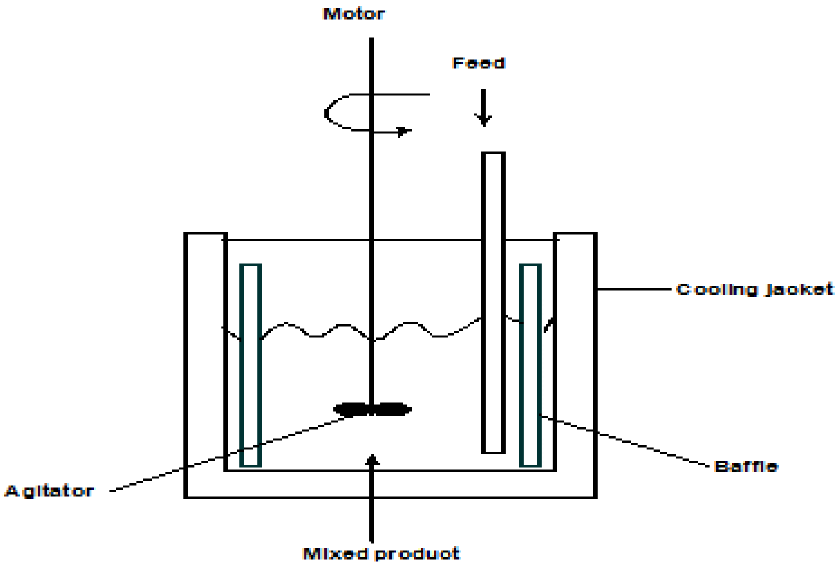

Fig. 1. Continuous flow stirred-tank reactor (CSTR).

The functioning of a perfectly mixed reactor or a homogeneous reactor is generally continuous, where the adjustment of the flow rates for the supply and withdrawal are done in that manner to maintain a constant volume in the reaction environment [1]. This is also referred to as a perfectly mixed continuous reactor [3]. In this particular reactor, the reaction mixture stays in an instantaneous and composition state, which is perfectly uniform even when there is a supply of volume. This condition is satisfied only if there is a combination of the added reagents within an infinitely short time frame and if the time to perform internal recirculation, the time taken by one molecule to travel from one point to another in the reactor, is considerably smaller than the passage time. Therefore, the same instantaneous composition will be present for the product stream taken out from the reactor to that of the reaction mixture. The reactor being studied here is shown in Fig. 2. It is the core of an exothermic reaction and a subject of high interest in many books and papers [4-6]. Often, this reactor is deemed a perfectly mixed reactor. It consists of a tank with an inbuilt liquid reaction room. It is permanently mixed by a suitable agitation system, making it a homogenous composition.

It is also deemed the heart of a heat reaction with an order *n*. A double envelope (index j) binds this particular reactor with a constant volume being traversed by a fluid that has a variable inlet temperature and a constant rate. In fact, the location of a three-way valve controls this temperature, which is also responsible for guiding the coolant to the exchangers. The circulation of the flow within the double jacket ensures the heating and cooling processes in a reactor. The flow regulation is manipulated with regards to the position to regulate the reaction environment’s temperature. While the reaction occurs, it is essential to keep the reaction environment’s temperature as near to the selected value as possible.

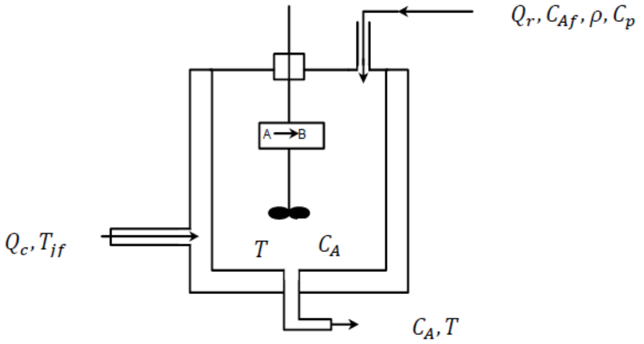

Fig. 2. Functional diagram for a second order CSTR model.

As per the general mole balance or mass conservation principle and on the basis of the energy balance, one can write [4]:

$$\frac{dC_A}{dt} = \frac{Q_r}{V_r}\left(C_{Af} - C_A\right) - k_0 e^{\frac{E_a}{RT}} C_A$$

$$\frac{dT}{dt} = \frac{Q_r}{V_r}\left(T_f - T\right) - \frac{\Delta H}{\rho C_p} k_0 e^{\frac{E_a}{RT}} C_A + \frac{U_A}{\rho C_p V_r}\left(T_j - T\right) \tag{1}$$

$$\frac{dT_j}{dt} = \frac{U_A}{\rho C_p V_j}\left(T - T_j\right) + \frac{Q_c}{V_j}\left(T_{jf} - T_j\right)$$

where V_r = Reactor volume, Q_r = Feed rate, k_0 = Time constant, R = Perfect gas constant, E_a =Activation energy, ΔH = Enthalpy of reaction, T_j = Temperature of the double envelope/jacket, T = Reaction temperature, Qc = Input rate, T_f = Supply temperature, C_{Af} = Feeding Concentration of A, c_p = Specific heat capacity at constant pressure, C_A = Concentration of component A in the reactor, ρ = Density of solution, U_A = Thermal transfer coefficient, V_j = Volume of the double envelope, and T_{jf} = Coolant temperature feeding into the envelope

Let us define the following state variables:

$$X_1 = C_A,\ X_2 = T, \text{ and } X_3 = T_j \tag{2}$$

Control input is

$$U = Q_c \tag{3}$$

Under the standard form, it can be written as:

$$\dot{X} = F(X) + G(X)U \text{ and } y = h(X) \tag{4}$$

where:

$$F(X) = \begin{bmatrix} \frac{Q_r}{V_r}(C_{Af} - X_1) - k_0 \exp\left(\frac{E_a}{RX_2}\right)X_1 \\ \frac{Q_r}{V_r}(T_f - X_2) - \frac{\Delta H}{pC_p} k_0 \exp\left(\frac{E_a}{RX_2}\right)X_1 + \frac{UA}{pC_pV_r}(X_3 - X_2) \\ \frac{UA}{pC_pV_j}(X_2 - X_3) \end{bmatrix} \tag{5}$$

$$G(X)=\begin{bmatrix}0\\0\\\frac{U}{V_j}(T_{jf}-X_3)\end{bmatrix} \tag{6}$$

and

$$y=h(X)=X_2 \tag{7}$$

Consider the state variable change stated below:

$$\begin{aligned}x_1&=X_1-X_{1n}\\x_2&=X_2-X_{2n}\\x_3&=X_3-X_{3n}\\u&=U-U_n\end{aligned} \tag{8}$$

where $x=[x_1\quad x_2\quad x_3]$ is the variation model around an operating point with $X_{1n}=\frac{C_{Af}}{1+\frac{V_r k_0}{Q_r}e^{-\frac{E_a}{RX_{2n}}}}$, $X_{3n}=\frac{U_A X_{2n}+\rho C_p T_{jf} U_n}{U_A+\rho C_p T_{jf} U_n}$. For instance, for U_n = 0.1 m^3/min and solving Eq. (9) for various values of X_{2n},

$$Q_{ge} = -Q_{ev} \tag{9}$$

where Q_{ge} is the generated energy and Q_{ev} is the evacuated energy, we attain the solutions at S_1, U and S_2 (point of interest) as shown in Fig. 3. 0

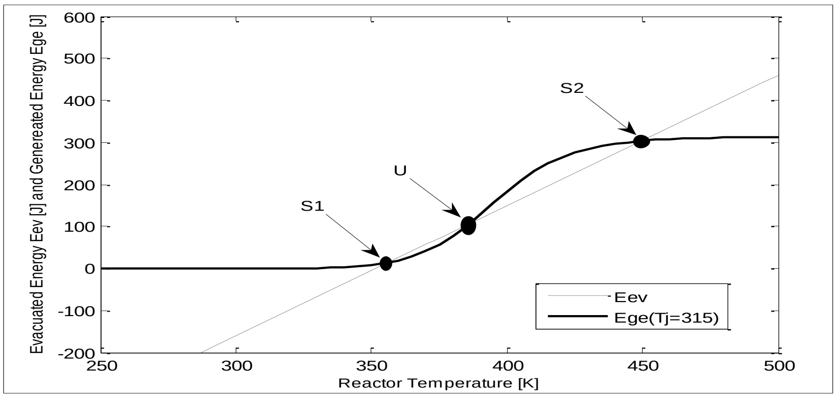

Fig. 3. Multiple steady states of a CSTR.

3. Feedback Linearization Formalism

The feedback linearization theory comprises finding nonlinear transformation known as diffeomorphism [10, 11, 26]:

$$Z = \phi(x) \tag{10}$$

Then, the Byrnes-Isidori normal form can be realised:

$$\dot{Z} = \begin{bmatrix} 0 & 1 & 0 & \dots & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \dots & \dots & 0 \end{bmatrix} Z + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix} v \tag{11}$$

The feedback control cab be attained as:

$$u = \alpha(x) + \beta(x)v = u = \frac{v - L_f^y h(x)}{L_g L_f^{y-1} h(x)} = \alpha(x) + \beta(x)v \tag{12}$$

In this aspect, calculating the studied system's relative degree is important. To determine an input-output relation, it is crucial to derive the reactor model's output signified by Eq. (3) and the number of times the output requires derivation corresponding to the system's relative degree. When deriving the output, one gets:

$$\begin{cases} \dot{y} = L_F h(X) + (L_G h(X))U \\ \ddot{y} = L_F^2 h(X) + (L_G L_F h(X))U \end{cases} \tag{13}$$

where:

$$L_F h(X) = \frac{Q_r}{V_r}(T_f - X_2) - \frac{\Delta H}{pC_p} k_0 \exp\left(-\frac{Ea}{RX_2}\right) X_1 + \frac{U_A}{pC_p V_r}(X_3 - X_2)$$

$$L_F^2 h(X) = \begin{bmatrix} \frac{\partial \dot{X}_2}{\partial X_1} & \frac{\partial \dot{X}_2}{\partial X_2} & \frac{\partial \dot{X}_2}{\partial X_3} \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix}$$

$$L_G h(X) = 0$$

and $$L_G L_F h(X) = \frac{U_A (T_{jf} - X_3)}{V_j \; pC_p V_r}$$

For the output's second derivation, the control input emerges. The system's relative degree thus becomes equal to two, which is lower when compared with the system order (n = 3). The system's dynamics (3) thus gets decomposed into an unobservable internal part as well as an external input–output part. By employing the change in coordinates in Eq. (8), transformation of the reactor model into the normal form occurs, which is based on Eq. (9) given as

$$\Phi(X)=\begin{bmatrix} X_2 \\ \frac{Q_r}{V_r}(T_f-X_2)-\frac{\Delta H}{pC_p}k_0\exp\left(-\frac{E_a}{RX_2}\right)X_1+\frac{U_A}{pC_pV_r}\left(X_3-X_2\right) \\ q_1(X) \end{bmatrix} \tag{14}$$

The dynamic compensator can be attained by resolving Eq. (15):

$$\frac{\partial(q_1(X))}{\partial X_3}\left(\frac{T_{jf}-X_3}{V_j}\right)=0 \tag{15}$$

A solution of Eq. (15) can be:

$$q_1(X)=X_1 \tag{16}$$

As mentioned earlier, this paragraph's aim is to abide with the reaction's desired trajectory in terms of the temperature of the chemical reactor studied. All states must be bounded to address this issue of pursuit. According to the solution presented by Eq. (16), this hypothesis was not satisfied. Therefore, regulation is possible only in an operating point's neighbourhood.

4. A Conventional Control Technique

Around the point of linearization on the process, fixed controllers like Proportional Integral Derivative controllers could function [6-8]. Therefore, this section's objective is to demonstrate the fixed PID controller's limitation when placed under control signal constraint for managing the feeding coolant's temperature in the nonlinear CSTR plant [3]. Attempts have been made for PID controllers in such processes [7, 8, 14].

A block diagram of the CSTR controlled system shows in Fig. 4, where the reference control input is represented by $r(t)$, the plant output is $y(t)$ and the load disturbance by $d(t)$. In this section, the employed PID controller scheme in accordance to the parallel structure is given as follows:

$$G_C(s)=\frac{u(s)}{e(s)}=K_p\left(1+\frac{K_i}{s}+K_d s\right)\left(\frac{1}{k_f s+1}\right) \tag{17}$$

where $u(s)$ is the control signal, $e(s)$ is the error defined by: $e(s)=r(s)-y(s)$, the proportional controller gain is K_p, the integral controller gain is K_i, the derivative controller gain is K_d and the filter coefficient is k_f suitably designed to reduce derivative's gain noise amplification. It is assumed that the CSTR model is exposed to feed temperature as well as composition uncertainties.

Let $T_{f0}=10\,'C$ and $C_{Af0}=100$ mol/m^3 be the nominal values, a time varying measurable disturbance $d(t)$ is stated as below:

$$d(t)=10.1\sin(10t) \tag{18}$$

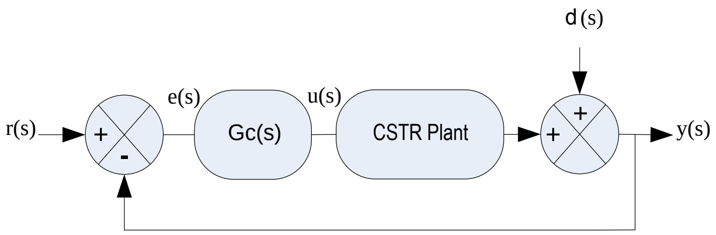

Fig. 4. PID conventional control of a CSTR.

The CSTR system with the control law, Eq. (17), is simulated by employing the Simulink tool solver to execute in the MATLAB. The first simulation study was performed with a nominal, undisturbed system as shown in Fig. 5. Based on the following, the performance index, Integral Absolut Error (IAE), is computed:

$$IAE = \int_0^t |e|\,dt \tag{19}$$

Subsequently, the Integral Square Error (ISE) is also computed as below:

$$ISE = \int_0^t |e|\,dt \tag{20}$$

In this simulation, we put a constraint on the control input, i.e., the coolant's temperature being fed into the jacket. This enables a realistic simulated process. 0 'C is selected as the lower saturation period while 77 'C as the upper saturation period. In a practical process, these values can be viewed further based on the substance type used as coolant.

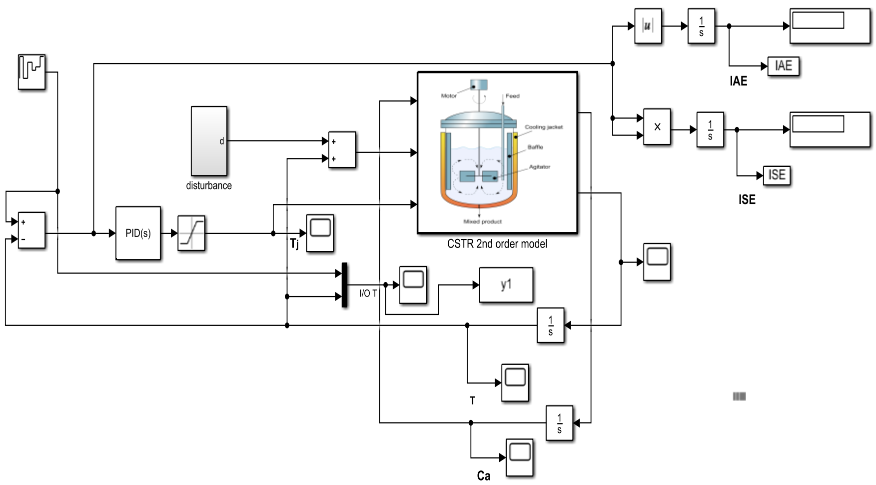

Fig. 5. Simulink block diagram of the PID conventional control of a CSTR.

The initial conditions $T_{f0} = 10\,'C$,and $C_{Af0} = 100\ \mathrm{mol/m^3}$ were employed for simulations. Since all constraints are fulfilled by these initial conditions, these are deemed suitable for examining PID performance [5]. The controller parameters were set to $K_p = 77, K_i = 8$, and $K_d = 0.005$. Table 1 shows the numerical values of CSTR parameters used in the simulation. To exemplify the limitation of the conventional PID in brevity, we have assumed a second order model of CSTR which can be expressed as

$$\begin{aligned} \dot{X}_1 &= \frac{Q_r}{V_r}\left(C_{Af} - X_1\right) - k_0 e^{\frac{E_a}{RX_2}} X_1 \\ \dot{X}_2 &= \frac{Q_r}{V_r}\left(T_f - X_2\right) - \frac{\Delta H}{\rho C_p} k_0 e^{\frac{E_a}{RX_2}} X_1 + \frac{U_A}{\rho C_p V_r}(u - X_2) \end{aligned} \tag{21}$$

$u(s)$ is the control signal from the controller in Eq. (17).

Table 1. Numerical values of CSTR parameters used in the simulation.

	Parameters	Values
Volumetric flowrate	Q_r	0.2 m^3/min.
Volume of Reactor	V_r	2 m^3
Time constant related to coefficient of discharge	k_o	3.5×10^6 min^{-1}
Activation energy	E_a	49.884 kJ/mol
Molar Gas constant	R	8.33×10^{-6} kJ/mol.°C
Enthalpy of reaction	$-\Delta H$	500 kJ/mol
Concentration of the feed	C_{Af}	100 mol/m^3
Feed temperature	T_f	30 °C
Specific heat capacity at constant pressure	C_p	4.2 kJ/kg.°C
Density of the reactant	ρ	100 kg/m^3
Thermal transfer coefficient	U_A	252 kJ/min.
Volume of the envelope	V_f	0.4 m^3
Temperature of jacket	T_{jf}	10 °C

This work's main aim is to examine the impact of state constraints on the system variables, especially on the reaction temperature. The outcomes are depicted in Figs. 6-8.

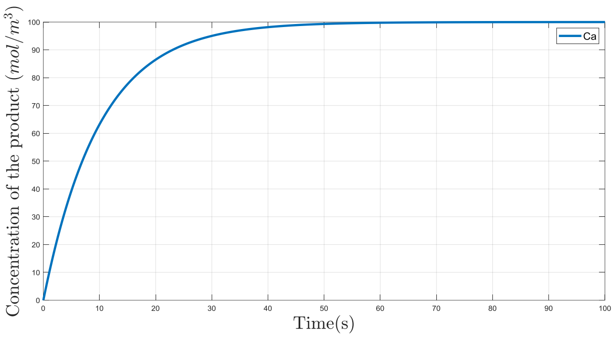

Fig. 6. Concentration for the undisturbed CSTR around the nominal conditions.

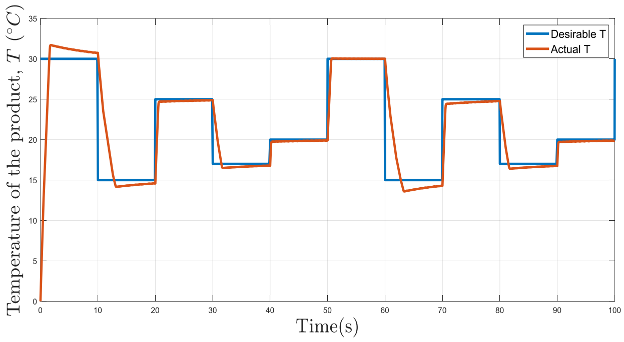

Fig. 7. Temperature for the undisturbed CSTR around the nominal conditions.

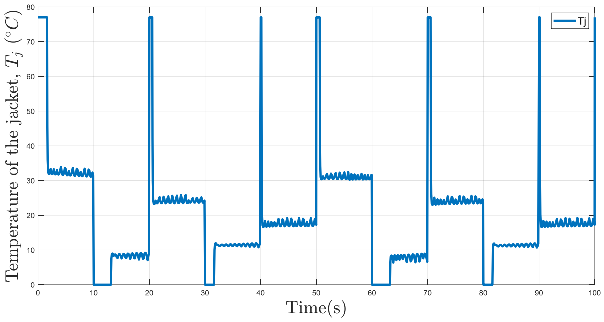

Fig. 8. Control input signal.

According to these figures, the stability of the reaction temperature can be guaranteed for the entire trajectory tracking objective employing the PID control scheme. However, it suffers from inaccuracy. When employing the PID controller, the later performance cannot be guaranteed, in which careful parameter tuning was executed for the obtained results. The CSTR's resulting time trajectories are presented in Figs. 9 and 10. The large sinusoidal disturbance is responsible for the oscillating behaviour. Nevertheless, as observed in Fig. 11, the control input was found to be unsatisfactory. More certainly, there is amplification of oscillatory behaviour. This is undesirable in practice. This concludes regarding the PID control's weakness to maintain high performance quality even when disturbances occur. Table 2 shows the Integral square error and integral absolute error of PID controller.

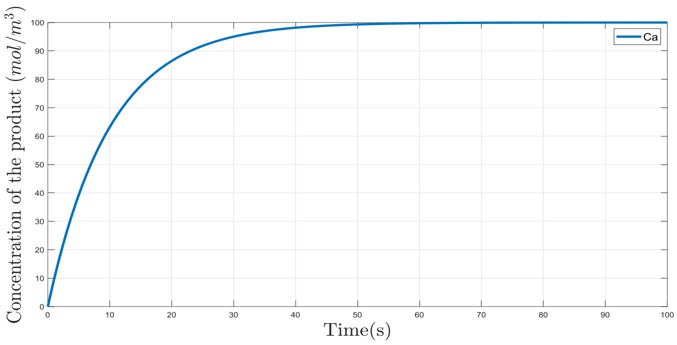

Fig. 9. Concentration for the disturbed CSTR around the nominal conditions.

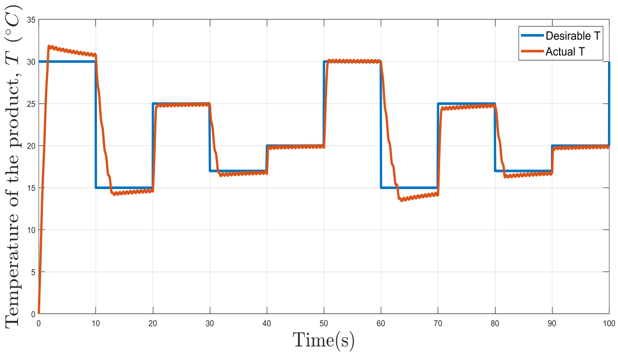

Fig. 10. Temperature for the disturbed CSTR around the nominal conditions.

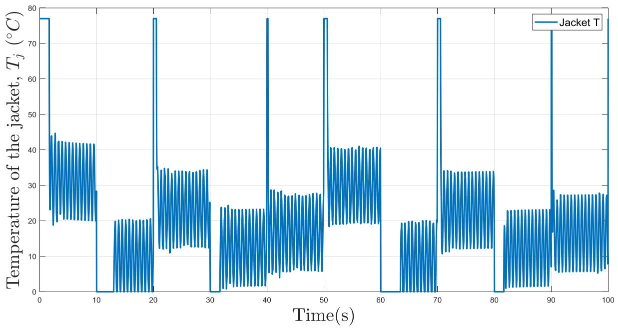

Fig. 11. Control input signal.

Table 2. Integral square error and integral absolute error of PID controller.

	Model No. 1	Model No. 2
PID Controller	ISE	IAE
Without Disturbance	874.4	112.7
With Disturbance	880.1	113.4

For fixed operation conditions, the design of the controller should be in accordance to the presence of potentially large uncertainty. Due to highly uncertain and nonlinear dynamics, a PID that was designed just for working efficiently under fixed operation conditions is likely to degrade significantly in terms of its performance should there be a change in conditions [3-7]. Such illustrated degradation in tracking and regulation performance affirms the limited capability of a fixed control such as PID when controlling nonlinear dynamical system such that of a CSTR in the presence of control input saturations.

5. Investigation of the Non-Minimum Phase Property of the CSTR Model

The following coordinate transformation is employed to derive this system, known as the Byrnes-Isidori normal form:

$$\begin{bmatrix} \zeta \\ \eta \end{bmatrix} = T(x) = \begin{bmatrix} h(x) \\ t_1(x) \end{bmatrix} \tag{22}$$

As a solution for the partial differential equation, the $t_1(x)$ function can be obtained as,

$$L_g t_1(x) = \begin{bmatrix} \dfrac{\partial(t_1(x))}{\alpha_1} & \dfrac{\partial(t_1(x))}{\alpha_2} \end{bmatrix} \begin{bmatrix} 0 \\ \dfrac{UA}{pC_pV_r} \end{bmatrix} = 0 \tag{23}$$

Equation (25) provides a possible solution for this issue, given by $t_1(x) = x_1$. The change of coordinates in Eq. (23) yields:

$$\begin{bmatrix} \zeta \\ \eta \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} \tag{24}$$

and it's inverse:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \eta \\ \zeta \end{bmatrix} \tag{25}$$

can be deployed to realise the Byrnes-Isidori normal form:

$$\begin{cases} \dot{\xi} = \alpha(\xi,\eta) + \beta(\xi,\eta) \\ \dot{\eta} = q(\xi,\eta) \\ y = \xi \end{cases} \tag{26}$$

where:

$$\alpha(\xi,\eta) = -\left(\frac{Q_r}{V_r} + \frac{UA}{pC_pV_r}\right)\xi + \frac{-\Delta H}{pC_p} k_0 \begin{pmatrix} \exp\left(\dfrac{E}{R(\xi + X_{2n})}\right)(\eta + X_{1n}) \\ -\left(\dfrac{E}{RX_{2n}}\right)X_{1n} \end{pmatrix}$$

$$\beta(\xi,\eta) = \frac{UA}{pC_pV_r}$$

$$q(\xi,\eta) = -\left(\frac{Q_r}{V_r} + \exp\left(-\frac{E}{R(\xi + X_{2n})}\right)\right)\eta + \frac{k_0\left(-\exp\left(-\dfrac{E}{R(\xi + X_{2n})}\right) + \exp\left(-\dfrac{E}{RX_{2n}}\right)\right)C_{Af}}{1 + \dfrac{V_r k_0}{Q_r}\exp\left(-\dfrac{E}{RX_{2n}}\right)}$$

Thus, the corresponding zero dynamic expression by setting $\dot{\eta} = q(\xi = 0, \eta)$ is

$$\dot{\eta} = \left(\frac{Q_r}{V_r} + \exp\left(\frac{E}{R(X_{2n})} \right) \right) \eta \tag{27}$$

Therefore, it was found that globally asymptotic stability (always converges to 0) is associated with the zero dynamics, and globally minimum phase is its CSTR.

6. Flatness Control Technique

Let us consider again the 2nd order CSTR model in Eq. (21). Flatness as described in [20] is the CSTR model's basic property that is employed here [36-40]. The term flatness used here signifies that there is a flat output $Y = (Y_1, \ldots, Y_m)$. The components of Y can be computed from the state variable vector X, the control variable U, and a defined number of the control input derivatives. Moreover, there are equations of the type $Y_i = \left(X, U, \ldots, U^{(\alpha)}\right), i = 1, \ldots, m$, which justify the fact that the Y components are independent and not related by a differential equation:

$$Q(Y, \dot{Y} \ldots, Y^{(\beta)}) = 0$$

It is demonstrated below that the CSTR model is an example of a flat system. Here, Y=X_2 is depicted to form a fat output [38, 40, 41-43]. To attain the Fliess canonical form, one differentiates the output a couple of times:

$$\dot{Y} = \frac{Q_r}{V_r}(T_f - X_2) + \frac{(-\Delta H)}{pC_p} k_0 \exp\left(-\frac{E}{RX_2} \right) X_1 + \frac{UA}{pC_p V_r}(U - X_2)$$

$$\ddot{Y} = \frac{Q_r}{V_r}(T_f - \dot{X}_2) + \frac{(-\Delta H)}{pC_p} k_0 \exp\left(-\frac{E}{RX_2} \right) \left(\dot{X} + \frac{\dot{E}}{RX_2^2} X_1 \right) + \frac{UA}{pC_p V_r}(\dot{U} - \dot{X}_2) \tag{28}$$

Employing the inverse transformation:

$$X_1 = \frac{\dot{Y} - \frac{Q_r}{V_r}(T_f - Y) - \frac{UA}{pC_p V_r}(U - Y)}{\frac{(-\Delta H)}{pC_p} k_0 \exp\left(-\frac{E}{RY} \right)} \tag{29}$$

$$X_2 = Y$$

For the CSTR problem, one can attain an external differential representation:

$$\ddot{Y} = \frac{Q_r}{V_r}(T_f - \dot{X}_2) + \frac{(-\Delta H)}{pC_p} k_0 \exp\left(-\frac{E}{RX_2}\right)\left(\dot{X}_1 + \frac{E}{RX_2^2} X_1\right) + \frac{UA}{pC_pV_r}(\dot{U} - \dot{X}_2) \tag{30}$$

Lastly, for computing U :

$$U = \frac{\dot{Y}\,\frac{Q_r}{V_r}(T_f - X_2)\frac{(-\Delta H)}{pC_p} k_0 \exp\left(-\frac{E}{RX_2}\right) X_1 + \frac{UA}{pC_pV_r} X_2}{\frac{UA}{pC_pV_r}} \tag{31}$$

A simulation of the controlled CSTR model was carried out and the outcomes are depicted in Figs. 12 and 13. Figure 12 shows the output trajectory, whereas Fig. 13 shows the flatness control input.

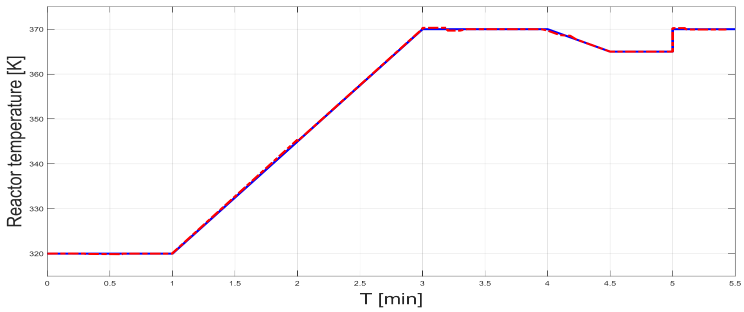

Fig. 12. Reactor temperature obtained through the flatness control technique.

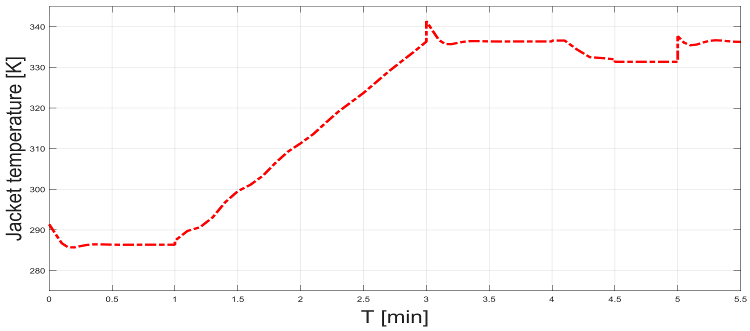

Fig. 13. Dynamics of the flatness control input

7. Conclusions

This study examined various control techniques for the instance of a benchmark CSTR model. It was indicated that the sign at the operating point is altered by the steady state gain. Thus, linear controllers (with proportional integral derivative actions) are unable to steady the reactor dynamics and deliver a convincing performance. The stability property is altered by the zero dynamics at this operating point. Thus, the CSTR's qualitative behaviour varies to a great extent for various set points as well as disturbances. Moreover, a resolution is offered for controlling this benchmark problem on the basis of a differential algebra method. These control schemes are typically labelled nonlinear feedback linearization model controllers and flatness control. The flat control input is stated as a function of particular output and a finite number of its derivatives.

The variable is not physically significant in the majority of cases. A state estimator for the flat output and its derivatives is necessary for executing the synthesized control. This problem is termed as a key challenge in confronting the differential flatness formalism's exploitation. Designing a flat output for a set of nonlinear systems is a major task. Most solutions are based on the proficiency of the user. On some occasions, simplifying assumptions and constrictions are espoused to attain a significant outcome. The nonlinear flat control's robustness too is a crucial partially solved problem. Certainly, there are no explicit criteria that validate the selection of the particular poles for the static as well as dynamic endogenous feedbacks.

For a flat nonlinear control, the trajectory tracking's accuracy and the closed loop system's stability are guaranteed. The execution of the control by differential flatness could present certain difficult problems. Ascertaining a proper poles placement for the input flat control's synthesis is extremely crucial or else a grave predicament of control saturation might surface. There is a settlement between the poles' value and the multiple performances of the flat control approach. For key values of the static as well as dynamic endogenous feedbacks, the accuracy error asymptotically converges to 0. The input is nevertheless saturated. For the feedback control poles' minor values, the tracking precision error's dynamic is not quite acceptable.

8. Future Work

The developed study in this paper proved that the PID conventional controller is conceptually simple and easy to implement. However, it was unable to achieve high performance at different operating points, in particular, external disturbances were present and input saturation is imposed. Our future works will focus on developing an advanced gain scheduling PID controller. Fuzzy sets will be exploited in a novel scheme to define the controller parameters. Furthermore, the fuzzy rules will be used to represent the human reasoning for an advanced PID gain programming. A special attention will be dedicated to bringing remedy to overcome the external unknown disturbances using the stated approach.

References

1. Klatt, K.-U.; and Engell, S. (1998). Gain-scheduling trajectory control of a continuous stirred tank reactor. *Computers & Chemical Engineering*, 22(4-5), 491-502.
2. Burger, M.; and Guay, M. (2010). Robust constraint satisfaction for continuous-time nonlinear systems in strict feedback form. *IEEE Transactions on Automatic Control*, 55(11), 2597-2601.
3. Bayer, F., Bürger, M.; Guay, M.; and Allgöwer, F. (2011). On state-constrained control of a CSTR. *IFAC Proceedings Volumes*, 44(1), 6079-6084.

4. Bingi, K., Ibrahim, R.; Karsiti, M.N.; and Hassan, S.M. (2017). Fuzzy gain scheduled set-point weighted PID controller for unstable CSTR systems. *2017 IEEE International Conference on Signal and Image Processing Applications (ICSIPA)*, 289-293.
5. Pérez-Polo, M.; and Albertos, P. *(*2007). *Nonisothermal stirred-tank reactor with irreversible exothermic reaction A→ B: 2. Nonlinear phenomena*. *In Selected Topics in Dynamics and Control of Chemical and Biological Processes*, 243-279. Springer, Berlin, Heidelberg.
6. Wu, W. (2003). Adaptive-like control methodologies for a CSTR system with dynamic actuator constraints. *Journal of Process Control*, 13(6), 525-537.
7. Åström, K.J.; and Wittenmark, B. (2013). *Adaptive control* (2nd ed.). Courier Corporation.
8. Adebekun, K.; and Schork, F.J. (1991). On the robust global stabilization of nth-order reactions. *Chemical Engineering Communications*, 101(1), 1-15.
9. Doyle III, F.J.; Packard, A.K.; and Morari, M. (1989). Robust controller design for a nonlinear CSTR. *Chemical Engineering Science*, 44(9), 1929-1947.
10. Bequette, B.W. (1991). Nonlinear control of chemical processes: A review. *Industrial & Engineering Chemistry Research*, 30(7), 1391-1413.
11. Antonelli, R.; and Astolfi, A. (2003). Continuous stirred tank reactors: easy to stabilise? *Automatica*, 39(10), 1817-1827.
12. Jemai, W.J.; Jerbi, H.; and Abdelkrim, M.N. (2010). Synthesis of an approximate feedback nonlinear control based on optimization methods. *WSEAS Transactions on Systems and Control*, 5(8), 646-655.
13. Jerbi, H. (2017). Design and performance analysis of a PV solar powered water pumping system. *International Journal of Advanced and Applied Sciences*, 4(5), 127-132.
14. Alvarez-Ramírez, J.; and Morales, A. (2001). Robust stabilization of jacketed chemical reactors by PID controllers. *Chemical Engineering Science*, 56(8), 2775-2787.
15. Gao, D., Yang, Q.; Wang, M.; and Yu, Y. (2012). Feedback Linearization Optimal Control Approach for Bilinear Systems in CSTR Chemical Reactor. *Intelligent Control and Automation*, 3(03), 274.
16. Jemai, W.J.; Jerbi, H.; and Abdelkrim, M.N. (2011). Nonlinear state feedback design for continuous polynomial systems. *International Journal of Control, Automation and Systems*, 9(3), 566-573.
17. Jerbi, H.; Braiek, N.B.; and Bacha, A.B.B. (2014). A method of estimating the domain of attraction for nonlinear discrete-time systems. *Arabian Journal for Science and Engineering*, 39(5), 3841-3849.
18. Jerbi, H. (2017). Estimations of the domains of attraction for classes of nonlinear continuous polynomial systems. *Arabian Journal for Science and Engineering*, 42(7), 2829-2837.
19. Jerbi, H.; and Jemai, W.J. (2014). Synthesis of an advanced state feedback control for continuous nonlinear polynomial systems. *In Applied Methods and Techniques for Mechatronic Systems*. Springer, Berlin, Heidelberg, 61-78.
20. Fliess, M.; Lévine, J.; Martin, P.; and Rouchon, P. (1993). On differentially flat nonlinear systems. *IFAC Proceedings Volume*, 25(13), 159-163.

21. Boubaker, S.; Mekni, S.; and Jerbi, H. (2017). Assessment of Electrical Safety Beliefs and Practices: A Case Study. *Engineering, Technology & Applied Science Research*, 7(6), 2231-2235.
22. Zhang, T.; and Guay, M. (2005) Adaptive control of uncertain continuously stirred tank reactors with unknown actuator nonlinearities. *ISA Transactions*, 44(1), 55-68.
23. Mahyuddin, M.N.; Arshad, M.R.; and Mohamed, Z. (2007). Simulation of direct model reference adaptive control on a coupled-tank system using nonlinear plant model. *International Conference on Control, Instrumentation and Mechatronics Engineering (CIM07).* Johor Bahru, Johor, Malaysia, 569-576.
24. Mahyuddin, M.N.; and Arshad, M.R. (2008). Performance evaluation of direct model reference adaptive control on a coupled-tank liquid level system, *Elektrika, Journal of Electrical Engineering,* 10(2), 9-17.
25. Andoulssi, R.; Draou, A.; Jerbi, H.; Alghonamy, A.; and Khiari, B. (2013). Non-Linear control of a photovoltaic water pumping system. *Energy Procedia*, (42), 328-336.
26. Yazdi, M.B.; and Jahed-Motlagh, M. R. (2009). Stabilization of a CSTR with two arbitrarily switching modes using modal state feedback linearization. *Chemical Engineering Journal*, 155(3), 838-843.
27. Calvet, J.P.; and Arkun, Y. (1988). Feedforward and feedback linearization of nonlinear system and its implementation using internal model control (IMC). *Industrial & Engineering Chemistry Research*, 27(10), 1822-1831.
28. Bruns, D.D.; and Bailey, J.E. (1977). Nonlinear feedback control for operating a nonisothermal CSTR near an unstable steady state. *Chemical Engineering Science*, 32(3), 257-264.
29. Prakash, J.; and Srinivasan, K. (2009). Design of nonlinear PID controller and nonlinear model predictive controller for a continuous stirred tank reactor. *ISA Tansactions*, 48(3), 273-282.
30. Aslam, F.; and Kaur, G. (2011). Comparative analysis of conventional, P, PI, PID and fuzzy logic controllers for the efficient control of concentration in CSTR. *International Journal of Computer Applications*, 17(6), 12-16.
31. Upadhyay, R.; and Singla, R. (2010). Analysis of CSTR temperature control with adaptive and PID controller (a comparative study). *International Journal of Engineering and Technology*, 2(5), 453.
32. Calvet, J.P.; and Arkun, Y. (1988). Feedforward and feedback linearization of non-linear systems with disturbances. *International Journal of Control*, 48(4), 1551-1559.
33. Tang, G.Y.; Ma, H.; and Zhang, B.L. (2005). Successive-approximation approach of optimal control for bilinear discrete-time systems. *IEE Proceedings-Control Theory and Applications*, 152(6), 639-644.
34. Daoutidis, P., Soroush, M.; and Kravaris, C. (1990). Feedforward/feedback control of multivariable nonlinear processes. *AIChE Journal*, 36(10), 1471-1484.
35. Hamidi, F.; and Jerbi, H. (2009). On the estimation of a maximal Lyapunov function and domain of attraction determination via a genetic algorithm. *6th International Multi-Conference on Systems, Signals and Devices, 2009*, 1-6.

36. Belgacem, A.; Jerbi, H.; and Abdkerim, M.N. (2011). Fault parametric detection using flatness property of systems. *Eighth International Multi-Conference on Systems, Signals & Devices*, 1-4.
37. Fliess, M.; Lévine, J.; Martin, P.; and Rouchon, P. (1995). Flatness and defect of non-linear systems: introductory theory and examples. *International Journal of Control*, 61(6), 1327-1361.
38. Rothfuss, R.; Rudolph, J.; and Zeitz, M. (1996). Flatness based control of a nonlinear chemical reactor model. *Automatica*, 32(10), 1433-1439.
39. Charfeddine, S.; and Jerbi, H. (2013). A survey on non-linear gain scheduling design control for continuous and discrete time systems. *International Journal of Modelling, Identification and Control*, 19(3), 203-216.
40. Rudolph, J.; and Winkler, J. (2003). Contributions to flatness- based control of systems with input- dependent delays. *Proceedings in Applied Mathematics and Mechanics* (*PAMM*), 2(1), 104-105.
41. Rudolph, J.; and Mounier, H. (2001). Trajectory tracking for π-flat nonlinear delay systems with a motor example. *In Nonlinear control in the year 2000 Volume 2* (339-351). Springer, London.
42. Bröcker, M.; and Herrmann, L. (2017). Flatness based control and tracking control based on nonlinearity measures. *IFAC-PapersOnLine*, 50(1), 8250-8255.
43. Petit, N.; Rouchon, P.; Boueilh, J. M.; Guerin, F.; and Pinvidic, P. (2002). Control of an industrial polymerization reactor using flatness. *Journal of Process Control*, 12(5), 659-665.