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Abstract

We study the impact of switching costs on the long run outcome in 2×2 coordination games played in the circular city model of local interactions. For low levels of switching costs the predictions are in line with the previous literature and the risk dominant convention is the unique long run equilibrium. For intermediate levels of switching costs the set of long run equilibria still contains the risk dominant convention but may also contain conventions that are not risk dominant. For high levels of switching costs also non-monomorphic states will be included in the set of long run equilibria. Finally, we reconcile our result with a recent paper by Norman (2009) by considering the case of large interaction neighborhoods in large populations.

Keywords: Switching Costs, Local Interactions, Learning, Coordination Games.

JEL Classification Numbers: C72, D83.

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1 Introduction

It is often costly to switch to a different technology or adopt a new social norm. For instance, switching from Windows to Apple requires not only getting familiarized to the new system but also moving files from one computer to the other. Further examples of switching costs include communicating one's new telephone number when switching providers in telecommunication, buying new tools when switching from inch screws to metric screws, or getting used to driving on the wrong side of the road when moving from a left hand traffic country to a right hand traffic country or vice versa.

In the present paper we wish to understand the role of switching costs on long run technology choice and the emergence of conventions. We will focus on coordination games as a metaphor for the choice of technology or the adaptation of social norms. A wide range of models, starting with the seminal works of Kandori, Mailath, and Rob (1993) and Young (1993), have analyzed settings where a population of boundedly rational players following decide on their actions using simple heuristics.[1] The message that emerges from these discussions is that when players use best response learning risk dominant strategies - that perform well against mixed strategy profiles - will emerge in the long run, even in the presence of payoff dominant strategies.

Norman (2009) has already analyzed the role of switching cost in global interactions setting where everybody interacts with everybody else. In the global setting switching costs turned out to influence the speed at which the population approaches the long run equilibrium. The long run prediction remain unaffected, though. Quite frequently interactions are, however, local in nature, with interaction partners corresponding to family members, friends, or work colleagues. We capture such local interactions by considering a model akin to the one proposed by Ellison's (1993) where the agents arranged around a circle and interact with their neighbors only. We focus on a setting where one strategy is risk dominant and the other strategy may or may not be payoff dominant. This allows us to analyze circumstances under which strategies that are neither payoff- no risk- dominant are selected. When determining which strategy to use the players play a best response to the distribution of play in their neighborhood in the previous period taking into account that switching strategies incurs a cost.

We find that low levels of switching costs do not change the predictions of the model as compared to the standard model without switching costs. The risk dominant strategy is still able to spread contagiously, starting from a small cluster and eventually taking over the whole population. However, for larger switching costs risk dominant strategies may no longer spread

[1] See Weidenholzer (2010) for a survey of the literature.

contagiously and non-monomorphic states where different strategies coexist become absorbing. The reason is that a player at the boundary of a risk dominant cluster will not switch under sufficiently high switching costs. It is possible to move among all of these non monomorphic absorbing states via a chain of single mutations. Transitions from different states to each others are, thus, characterized by step-by-step evolution as outlined in Ellison (2000). The question which state will be long run equilibrium essentially boils down to how difficult the set of non-monomorphic states is to access from the two monomorphic states. Interestingly, if agents only interact with a couple of neighbors there may exist a range of parameters where alongside the risk dominant convention also non-risk dominant conventions are stochastically stable. Thus, switching costs may lead to the model's prediction no longer being unique. The reason behind this phenomenon is that the number of mutations required to move from a convention to the set of non-monomorphic absorbing states is measured in integers. Especially if agents only interact with a few neighbors it may happen that the number of mistakes required to access the set of non-monomorphic absorbing states from the risk dominant convention equals the number of mistakes required to access this set from the non-risk dominant convention. Perhaps even more interestingly, also owing to the fact the mutations are measured in integers, the prediction might be non-monotonic in the level of switching costs. That is, the prediction that the risk dominant convention is selected and the prediction that both conventions are selected alternate as switching costs increase. If, however, the interaction neighborhoods are sufficiently large the risk dominant convention remains unique long run equilibrium. Finally, for very high levels of switching costs no player will switch in the absence of noise even if all neighbors choose the other strategy. Thus, all states are absorbing and we can connect them via a chain of single mutations. Consequently, all absorbing states turn out to be long run equilibria.

The implications of these observations are that the local interaction model may loose traction in the presence of switching costs as it can no longer give a clear cut prediction. This is expressed by the non-uniqueness of the long run prediction but even more aggravated by the non monotonicity of the prediction. While the risk dominant convention ceases to be unique long run equilibrium for high enough switching costs it might be again unique long run equilibrium for even higher switching costs. This is bad news since the circular city model of local interactions has some otherwise nice features as compared to the global model: i) it was observed by Ellison (1993) that in contrast to the global interaction model of Kandori, Mailath, and Rob (1993) it features a high speed of convergence. ii) Lee, Szeidl, and Valentinyi (2003) have shown that it is immune towards the Bergin and Lipman (1996) critique iii) Weidenholzer (2012) has show that it is robust to the addition and deletion of dominated strategies, a test which Kim and Wong (2010) have shown the global model fails.

The paper closest related to our work is Norman (2009) who studies switching costs in the context of a global interactions model. As already observed by Kandori, Mailath, and Rob (1993) a major drawback of the global interactions model lies in its low speed of convergence. Under global interactions number of mistakes required to move from one convention to another turns out to depended on the population size. Thus, in large populations it is questionable whether the long run limit will be observed within any reasonable time horizon.[2] Norman (2009) shows how switching costs might speed up convergence to a particular norm. As in the present paper, the presence of switching costs implies that non-monomorphic states where agents use different actions become absorbing. This enables a transition from one convention to another by first accessing the class of non-monomorphic states and then moving through this class via a chain of single mutations to the other convention. Under switching cost the step from one convention to the set of non-monomorphic states is typically smaller than the direct step from that convention to the other. Consequently, switching costs may speed up the convergence to the long run prediction.

The rest of this paper is organized in the following way. Section 2 presents the model and discusses the main techniques used. Section 3 spells out our main results and Section 4 concludes.

2 The model

We consider a population of N agents who are located on a circle, as in Ellison (1993). A given agent i has agents $i-1$ and $i+1$ (mod m) as immediate neighbors. Each agent interacts with her k closest neighbors on the left and on the right of her. We assume that $k \leq \frac{N-1}{2}$ to ensure that no agent interacts with herself. Thus, agent i's interactions are confined to the set of players $N(i) = \{i-k, i-k+1 \ldots, i-1, i+1, \ldots, i+k, i+k\}$. We call agents in the set $N(i)$ neighbors of i.

We assume $|N|$ to be odd. This allows us to nest global interactions in our framework by setting $k = \frac{N-1}{2}$.[3]

Each agent i plays a 2×2 coordination game G with strategy set $S = \{A, B\}$ against all agents in her neighborhood $N(i)$. We denote by $u(s_i, s_j)$ the payoff agent i with strategy s_i receives when playing against agent j with strategy s_j. We follow Eshel, Samuelson, and Shaked (1998) and use

[2] Ellison (1993) pointed out that in the context of local interactions where some strategies might spread contagiously the speed of convergence is independent of the population size and might, thus, the long run equilibrium might be a reasonable predictor even in large populations.

[3] The results obtained for local interaction also hold for even populations.

the following normalization.

$$G = \begin{array}{c|c|c|} & A & B \\ \hline A & \alpha, \alpha & \beta, 0 \\ \hline B & 0, \beta & 1, 1 \\ \hline \end{array}$$

We assume that $\alpha > 0$ and $\beta < 1$, so that (A, A) and (B, B) are both strict Nash equilibria. Further, we assume that $\alpha + \beta > 1$, so that the equilibrium (A, A) is risk dominant in the sense of Harsanyi and Selten (1988), i.e. A is the unique best response to a mixed strategy profile which puts equal probability on A and B. We denote by

$$q^* = \frac{1 - \beta}{1 + \alpha - \beta}$$

the critical mass put on A in a mixed strategy equilibrium. Risk dominance of the Nash equilibrium (A, A) translates into $q^* < \frac{1}{2}$. Note if $\alpha > 1$ we have that (A, A) is payoff dominant and if $\alpha < 1$ (B, B) is payoff dominant. However, we do not make any such assumption on α at this stage.

We denote $m = \#\{i \in I | s_i = A\}$ the number of A-players in the population and by $m_i = \#\{j \in N(i) | s_j = A\}$ the number of A-players in agent i's interaction set. Accordingly, the number of B-players in the population is given by $N - m$ and the number of A-players in i's interaction set is given by $2k - m_i$.

We denote by $s_i(t)$ the strategy adopted by player i in period t, by $s(t) = (s_1(t), \ldots, s_N(t))$ the profile of strategies adopted by all players at t, and by $s_{-i}(t) = (s_{i-k}(t), \ldots, s_{i-1}(t), s_{i+1}(t), \ldots, s_{i+k}(t))$ the strategies adopted by all of player i's neighbors. Further, we denote the monomorphic states $(s, s, \ldots, s)$ where all agents adopt the same strategy s as $\overrightarrow{s}$.

The payoff for player i is given by the average payoff received when interacting with all neighbors.

$$U_i\big(s_i(t), s_{-i}(t)\big) = \frac{1}{2k} \sum_{j \in N(i)}^{k} u\big(s_i(t), s_j(t)\big).$$

We consider a myopic best response process with *switching costs*. In each period each agent receives the opportunity to revise her strategy with exogenous probability $\eta \in (0, 1)$.[4] We assume that changing strategies is costly. Whenever an agent changes her strategy she is subject to a switching cost c. In order to capture this idea we introduce the following function

$$c\left(s_i(t), s_i(t+1)\right) = \begin{cases} c & \text{if } s_i(t) \neq s_i(t+1) \\ 0 & \text{if } s_i(t) = s_i(t+1) \end{cases}$$

[4]Thus, we are considering a model of positive inertia where agents may not adjust their strategy every period.

When a revision opportunity arises an agent switches to a myopic best response, i.e. she plays a best response to the distribution of play in her neighborhood in the previous period, taking into account the switching costs. More formally, at time $t+1$ player i chooses

$$s_i(t+1) \in \arg\max_{s_i(t+1)\in S}\Big[U\big(s_i(t+1), s_{-i}(t)\big) - c\big(s_i(t), s_i(t+1)\big)\Big].$$

If a player has multiple best replies, we assume that she randomly chooses one of them with exogenously given probability. Further, with fixed probability $\epsilon > 0$, independent across agents and across time the agent ignores her prescription and chooses a strategy at random, i.e. she makes a mistake or mutates.

The process with mistakes is called *perturbed process*. Under the perturbed process any two states can be reached from each other. Thus, the only absorbing set is the entire state space, implying that the process is ergodic. We denote by $\mu(\epsilon)$ the unique invariant distribution of this process. We are interested in the *limit invariant distribution* (as the rate of experimentation tends to zero) $\mu^* = \lim_{\varepsilon\to 0}\mu(\varepsilon)$. Young (1993) or Ellison (2000)) have existence of such a distribution and that is a invariant distribution of the process without mistakes (the so called *unperturbed process*). It gives a stable prediction for the original process, in the sense that for ϵ small enough the play approximates that described by μ^* in the long run. The states in the support of μ^*, are called *Long Run Equilibria (LRE)* or stochastically stable states. We denote by $\mathcal{S} = \{\omega \in \Omega \mid \mu^*(\omega) > 0\}$ the set of LRE. We use characterization of the set of LRE due to Freidlin and Wentzell (1988).[5] Consider two absorbing sets X and Y and let $c(X,Y) > 0$ (referred to as a *transition cost*) denote the minimal number of mutations for a transition from the X to Y. A X-tree is a directed tree such that the set of nodes is the set of all absorbing sets, and the tree is directed into the root X. For a given tree one can calculate the cost as the sum of the costs of transition for each edge. A state X is a LRE if and only if it is the root of a minimum cost tree.

3 The role of switching costs

Essentially switching costs impede players from switching strategies. To see this point, consider an A-player. She will switch strategies with probability one if her payoff from playing B minus

[5]See Fudenberg and Levine (1998) or Samuelson (1997) for textbook treatments. Ellison (2000) provides an enhanced (and sometimes easier to apply) algorithm for identifying the set of LRE. We chose to work with the original formulation as it allows for a characterization in case of multiple LRE.

the switching cost strictly exceeds her payoff from remaining a B- player, i.e.

$$\frac{1}{2k}\Big(m_i\alpha + (2k - m_i)\beta\Big) < \frac{1}{2k}\Big(2k - m_i\Big) - c.$$

Rearranging terms yields

$$m_i < 2kq^* - \frac{2kc}{1+\alpha-\beta} := m^A(c,k)$$

An A-player will remain an A-player with certainty whenever $m_i > m^A(c,k)$ and will choose A and B with positive probability if $m_i = m^A(c,k)$. As $m^A(c,k)$ is the minimum number of neighbors such that keeping A is a unique best response, it cannot be negative.

Likewise, consider a B-player. He will switch strategies with probability one if the payoff from playing A minus the switching cost exceeds his current payoff, which yields

$$m_i > 2kq^* + \frac{2kc}{1+\alpha-\beta} := m^B(c,k).$$

A B-player will remain a B-player if $m_i < m^B(c,k)$, and will randomize between the two strategies if $m_i = m^B(c,k)$. Note that $m^B(c,k)$ is defined as the number of A-players such that a player with less than $m^B(c,k)$ A-neighbors chooses to stay at B with certainty and, thus, cannot exceed $2k$.

We remark that $m^A(0,k) = m^B(0,k) = 2kq^*$, i.e. in the absence of switching costs the thresholds are the same as in Ellison's (1993) model. For $c > 0$, we have $m^A(c,k) < m^A(0,k) = m^B(0,k) < m^B(c,k)$. Hence, in the presence of switching costs, it takes more players of the other type to induce a switch than in the absence of switching costs. Further, a B-player will require more A-opponents to switch strategies than an A-player requires to stay at her strategy. Likewise, an A-player will switch to B at a lower number of A-opponents than it takes a B-player to remain at her strategy. Thus, switching costs create regions where players with the same distribution of play in their neighborhood but with a different current strategy may behave differently. This may lead to non-monomorphic states where clusters of players with different strategies coexist becoming absorbing, which can not happen in the absence of switching costs.

In the following, we denote by $\{AB\}$ the set of non-monomorphic absorbing states, i.e.

$$\{AB\} = \{s \in S | s \neq \vec{A}, \vec{B}, m_i > m^A(c,k)\ \forall\, i \text{ with } s_i = A, \text{ and } m_j < m^B(c,k)\ \forall\, j \text{ with } s_j = B\}.$$

and by AB an element of this set. Further, we denote by $\{AB\}^\ell$ the set of set of non-monomorphic absorbing states with ℓ A-players (and $N-\ell$ B-players), i.e.

$$\{AB\}^{\ell} = \{s \in \{AB\} | m = \ell\}.$$

3.1 Two neighbor interaction

In order to build some intuition and to highlight the main mechanisms at work we start our analysis by discussing the special case where each agent only interacts with her two most immediate neighbors, i.e. $k = 1$. We analyze the case $k > 1$ in Section 3.2.

First, let us consider states where clusters of A-players and B, each of at least size two, alternate, e.g.

$$\dots BBAABBBAAAA \dots$$

Players in the middle of such a cluster only interact with players of their own kind and, hence, will never switch. Thus, let us consider the boundary between two such strings

$$\dots AABB \dots$$

Note that whenever $m^A(c, 1) < 1$ holds the boundary A- player will keep his strategy. This translates into $2c > 1 - \alpha - \beta$, which is implied by risk dominance of A. Thus, the boundary A-player will remain. Now consider the B-player. He will stay a B-player with certainty provided that $m^B(c, 1) > 1$, which translates into

$$c > \frac{\alpha + \beta - 1}{2}.$$

Thus, provided that switching costs are sufficiently high, the boundary B-player will remain too. This, in turn, implies that non-monomorphic states where clusters of B-players of at least size two and clusters of A-players of at least size two alternate are absorbing. Note that if, however, $c \leq \frac{\alpha+\beta-1}{2}$ holds a boundary B-player will switch to A with positive probability. Thus, even in the absence of mistakes, the A-cluster will grow.

Assume again $c > \frac{\alpha+\beta-1}{2}$ and consider a non-monomorphic absorbing state s with ℓ A players, i.e. $s \in \{AB\}^{\ell}$. Consider the boundary between an A-cluster and a B-cluster. If a boundary A-player (B-player) makes a mistake and switches to B (A), then we move to a new absorbing state with strictly less (more) A-players. Note that the initial switch might in fact imply that also other A-players (B-players) will switch. E.g. consider the example where the bold faced A-player mutates to B

$$\ldots BBA\mathbf{A}BBBAAAA \rightarrow BBA\mathbf{B}BBBAAAA\ldots \rightarrow BB\mathbf{BB}BBBAAAA\ldots$$

However, what is true is that it is possible to move with one mutation from a non-monomorphic absorbing state to another (possibly monomorphic) state with strictly less (more) A-players. More formally, it is possible to move from a state $\{AB\}^{\ell}$ to either a state in $\{AB\}^{a}$ or in $\{AB\}^{b}$, with $a < \ell < b$ at the cost of one mutation. Thus, under sufficiently high switching costs non-monomorphic states are absorbing and it is possible to move among states with a different number of A-players at the cost of one mutation.

Consider now the state where there is only one A-player.

$$\ldots BBABB\ldots$$

As we have previously seen the adjacent B-players will switch with positive probability if $m^{B}(c,1) \leq 1$, which implies $c \leq \frac{\alpha+\beta-1}{2}$. Whenever this inequality holds B-agents with an A-neighbor will switch strategies and the presence of one A-player is enough to trigger a contagious spread of strategy A, eventually covering the entire population. Thus, one mistake is enough to move from the state $\overrightarrow{B}$ to $\overrightarrow{A}$.

If, however, $c > \frac{\alpha+\beta-1}{2}$ the B-player will retain their strategy. As the A-player has no A-neighbors he will switch to B with positive provability if $m^{A}(c,1) \geq 0$ which translates into

$$c \leq 1-\beta.$$

Conversely, if $c > 1-\beta$, the A-player will keep his strategy and states with lonesome A-players are absorbing.

Likewise, consider the case when there is a lonesome B-player.

$$\ldots AABAA\ldots$$

The B-player has two A-neighbors and will switch strategies with positive probability provided that $m^{B}(c,1) \leq 2$, which can be rewritten as $c \leq \alpha$. However, whenever $c > \alpha$ a lonesome B-player will remain. Note, by risk dominance of A, $\alpha > 1-\beta$, implying that whenever lonesome B-players will keep their strategy, lonesome A-player will do the same.

Let us now identify the set of stochastically stable states. First, consider the case where

$c \leq \frac{\alpha+\beta-1}{2}$. We have seen that any state with one or more A-players lies in the basin of attraction of $\overrightarrow{A}$. This implies that i) the only absorbing states are $\overrightarrow{A}$ and $\overrightarrow{B}$ and ii) moving from $\overrightarrow{A}$ to $\overrightarrow{B}$ is possible at the cost of one mutation. Provided that $N \geq 3$ (which follows from $k = 1$ and $2k \leq N-1$) we can not exhibit a B-tree with cost smaller than 2 implying that $\overrightarrow{A}$ is the unique LRE.

Consider now the case where $c > \frac{\alpha+\beta-1}{2}$. The most important feature of this case is that now monomorphic states become absorbing. First, let us consider $c \leq 1-\beta$. Note that this case only occurs if $\frac{\alpha+\beta-1}{2} > 1-\beta$ which translates into $\alpha + 3\beta < 3$. In this case, we can move from $\overrightarrow{A}$ to a state in the set $\{A,B\}^{N-2}$ at the cost of two mutations and from $\overrightarrow{B}$ to a state in the set $\{A,B\}^2$ at the cost of two mutations. Further, we can connect all states in the set $\{A,B\}$ with each other via a chain of single mutations. We can also move from $\{A,B\}^{N-2}$ to $\overrightarrow{A}$ and from $\{A,B\}^2$ to $\overrightarrow{B}$ at the cost of one mutation. Thus, we can exhibit the following $\overrightarrow{A}$- and $\overrightarrow{B}$- trees

$$\overrightarrow{A} \overset{1}{\leftarrow} \{A,B\}^{N-2} \overset{1}{\leftarrow} \ldots \overset{1}{\leftarrow} \{A,B\}^2 \overset{2}{\leftarrow} \overrightarrow{B}$$
$$\overrightarrow{B} \overset{1}{\leftarrow} \{A,B\}^2 \overset{1}{\leftarrow} \ldots \overset{1}{\leftarrow} \{A,B\}^{N-2} \overset{2}{\leftarrow} \overrightarrow{A}$$

Let L denote the total number of AB-states. Together with the two monomorphic absorbing states $\overrightarrow{A}$ and $\overrightarrow{B}$ this makes $L+2$ absorbing states. Thus, each tree has $L+1$ arrows on it. This implies that the above trees have a cost of $L+2$ each. Now consider the set of all possible AB-trees. Each of these trees has to have an arrow going from $\overrightarrow{A}$ to and from $\overrightarrow{B}$ into the set of AB-states, the cost of which is two each. Further it has to connect all AB-states. It follows that there is no AB with cost strictly smaller than $L+3$. Consequently, $\overrightarrow{A}$ and $\overrightarrow{B}$ are the only LRE.

Now consider the case where $1-\beta < c \leq \alpha$. Now there exist non-monomorphic absorbing states with a single A-player and we can connect $\overrightarrow{B}$ to states in this set, $\{A,B\}^{N-1}$, at the cost of one mutation. Thus, we can exhibit the following A-tree

$$\overrightarrow{A} \overset{1}{\leftarrow} \{A,B\}^{N-2} \overset{1}{\leftarrow} \ldots \overset{1}{\leftarrow} \{A,B\}^1 \overset{1}{\leftarrow} \overrightarrow{B},$$

with cost $L+1$. Since, we can not leave $\overrightarrow{A}$ with only one mutation, it follows that we can not construct any other tree with cost smaller than $L+2$. Thus, $\overrightarrow{A}$ is unique LRE in this case.

We finally discuss the case where $c > \alpha$. Essentially, switching costs are now so high that no player will switch in the absence of mistakes and any distribution of strategies across players is absorbing. We can move from $\overrightarrow{A}$ and $\overrightarrow{B}$ to states in the set $\{AB\}$, can move among all these states, and can leave this set of states, all at a cost of one mutation. Thus, we can exhibit A- and

B-trees with cost $L+1$. Further, for each AB-state we can also construct a tree of cost $L+1$. Thus, for $c > \alpha$ we have that all absorbing states are LRE.

We summarize the above discussion next proposition.

Proposition 1. *In the two neighbor interaction model*

a) if $c \leq \frac{\alpha+\beta-1}{2}$ *we have* $\mathcal{S} = \{\overrightarrow{A}\}$

b) if $\frac{\alpha+\beta-1}{2} < c \leq \alpha$ *and*

- *i) if* $c \leq 1-\beta$ *we have* $\mathcal{S} = \{\overrightarrow{A}, \overrightarrow{B}\}$
- *ii) if* $c > 1-\beta$ *we have* $\mathcal{S} = \{\overrightarrow{A}\}$*,*

c) if $c > \alpha$ *we have* $\mathcal{S} = \{\overrightarrow{A}, \overrightarrow{B}, \{A, B\}\}$*.*

Thus, the presence of switching costs may imply that the prediction for the long run is altered. The essential mechanism that underlies this result is that switching costs may stop the contagious spread of the risk dominant strategy. Without contagion the question which equilibrium will emerge in the long run boils down to how difficult it is to access the set of non-monomorphic states from the two conventions. Risk dominance only implies that it can never be easier to move out of the risk dominant convention than to move out of the non-risk dominant convention. This, in turn, implies that the risk dominant convention is always contained in the set of LRE. However, it might not be the unique prediction. In particular, there exists a parameter range where both conventions can be left with two mutations and, thus, are both LRE. If switching costs increase even further both conventions may be left with one mutations and all states (including the non-monomorphic ones) can be accessed from each other via a chain of single mutations. Thus, all absorbing states are LRE.

Whether it is actually possible that a non risk dominant convention is LRE does not only depend on the level of switching costs but also on the parameters of the underlying game. To see this point note that case bi) in the previous proposition only occurs if $\frac{\alpha+\beta-1}{2} > 1-\beta$ This translates into $\alpha + 3\beta < 3$. This condition is fulfilled if the advantage of strategy A over B is not too large, but per se is not related to payoff dominance or risk dominance. We illustrate the set of LRE depending on the level of switching cost in this case in Figure 1. If, strategy A is sufficiently advantageous compared to B ($\alpha + 3\beta > 3$) it will be uniquely selected (up to the point where $c > \alpha$ and all absorbing states are selected).

It is also interesting to note that the prediction is "non-monotonic" in the level of switching costs. With increasing switching costs the prediction switches from $\overrightarrow{A}$ to $\overrightarrow{A}$,$\overrightarrow{B}$ back to $\overrightarrow{A}$ and finally to $\overrightarrow{A}$,$\overrightarrow{B}$, $\{A, B\}$ in games with $\alpha + 3\beta < 3$. The reason behind this is that transitions are measured in rounded up values of functions which are decreasing in c. As c increases the required numbers of transitions jump, and happen to be the same for certain parameter ranges.

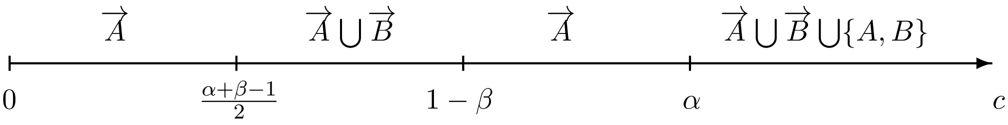

Figure 1: Long run equilibria under two player interaction with switching costs and $\alpha + 3\beta < 3$.

3.2 $2k$-player interaction and global interactions

We will now generalize the insights of the two player interaction model to $2k$ neighbor interaction. We show that we can expect similar phenomena as in the simple two neighbor model for small interaction neighborhoods. However, as the the size of the interaction neighborhood,k, increases switching costs do no longer influence the prediction, with the exception being very high levels of switching costs, where in the absence of noise no player would switch regardless the distribution of strategies in her neighborhood. The following lemma provides a characterization of the set of absorbing states.

Lemma 2.
For positive switching costs, $c > 0$,

i) there are no non-singleton absorbing sets.

ii) the only absorbing states are $\overrightarrow{A}$, $\overrightarrow{B}$, and $\{A, B\}$.

Proof. To prove the first part consider an absorbing set W. Consider a state $\tilde{s} \in W$ where the number of A-players is maximal. Let $\tilde{m}$ be the number of A-players at this state. It follows that at this state there does not exist a B-player who, when given revision opportunity, switches to A with positive probability. Thus, $m_i < m^B$ for all i with $s_i = B$. If it is the case that $m_j > m^A$ for all j with $s_j = A$ then $\tilde{s}$ is the only state in W. If $m_j \leq m^A$ for some players j with $s_j = A$ we proceed in the following manner. With positive probability, one of these agents receives revision opportunity and switches to B. We reach a new state s'. At this new state there are strictly fewer A-players. Provided that $c > 0$ for the new B-player we have $m_j \leq m^A < m^B$, implying that he will not switch back. For all old B-players it is still true that $m_i < m^B$, implying that none of them will switch. If there is no A-player with $m_j \geq m^A$ left the state s' is absorbing (contradicting that $\tilde{s} \in W$). If there are still such A-players left we iterate the procedure until we reach an absorbing state, eventually contradicting the assumption $\tilde{s} \in W$.

The second part follows from the definition of $\overrightarrow{A}$, $\overrightarrow{B}$, and $\{A, B\}$. $\square$

With the help of this lemma we are able to provide the following result.

Proposition 3. *In the $2k$-neighbor interaction model*

a) if $c \leq \frac{\alpha+\beta-1}{2}$ and $N > k(k+1)$ we have $\mathcal{S} = \{\overrightarrow{A}\}$

b) if $\frac{\alpha+\beta-1}{2} < c \leq 1-\beta$ and

i) if $\lfloor m^A(c,k) \rfloor = \lfloor 2k - m^B(c,k) \rfloor$ we have $\mathcal{S} = \{\overrightarrow{A}, \overrightarrow{B}\}$,

ii) if $\lfloor m^A(c,k) \rfloor < \lfloor 2k - m^B(c,k) \rfloor$ we have $\{\mathcal{S} = \overrightarrow{A}\}$,

c) if $1-\beta < c \leq \alpha$ we have $\mathcal{S} = \{\overrightarrow{A}\}$.

d) if $c > \alpha$ we have $\mathcal{S} = \{\overrightarrow{A}, \overrightarrow{B}, \{A, B\}\}$.

Proof. For part a) note if $c \leq \frac{\alpha+\beta-1}{2}$ we have $m^B(c,k) \leq k$, implying that a B-player switches to A with positive probability whenever half (or more) of his $2k$-neighbors choose A. Thus, A may spread contagiously and we are back in the model outlined by Ellison (1993), where $\mathcal{S} = \{\overrightarrow{A}\}$ if $N > k(k+1)$.[6]

We now consider the case where $c > \frac{\alpha+\beta-1}{2}$. Here we have $m^B(c,k) > k$. Thus, B-players will no longer switch if they have half of their neighbors playing A. This implies A can no longer spread out contagiously. Further, we will now have non-monomorphic absorbing states, meaning that the set $\{AB\}$ is non-empty.

We next show that it is possible to move from an absorbing state $AB \in \{AB\}^\ell$ to either a state in $\{AB\}^a$ or in $\{AB\}^b$, with $a < \ell < b$ at the cost of one mutation. We will show that there exists an A- (and a B-player) such that if he mutates to B (to A) he will not switch back and no other player will switch to A. By the definition of $\{AB\}^\ell$ we have $m_i > m^A(c,k)$ for all i with $s_i = A$ and $m_j < m^B(c,k)$ for all j with $s_j = B$. Consider now an A-player i whose adjacent neighbor j is playing B. As they are direct neighbors they have only one player who is not a joint neighbor. Call i's disjoint neighbor $\tilde{i}$ and j's disjoint neighbor $\tilde{j}$. Further j also faces i who is an A-player. It follows that j faces either the same number of A-neighbors as i (if $s_{\tilde{i}} = A$ and $s_{\tilde{j}} = B$), has one more A-neighbors than i (if $s_{\tilde{i}} = s_{\tilde{j}}$), or two more A-neighbors (if $s_{\tilde{i}} = B$ and $s_{\tilde{i}} = A$). Thus,

[6] Note that we have a model with positive inertia whereas Ellisons model features strategy adjustment in each round. See Weidenholzer (2010) for a discussion of the model with inertia.

$m_j \in \{m_i, m_i+1, m_i+2\}$. Assume that j mutates to A. Since $m_j \geq m_i > m^A(c,k)$ he will not switch back. Further, as there is now more A-players non of the old A-players will switch, showing that we will reach a state $\{AB\}^b$ with $b > \ell$. An analogous argument can be used to show that it is also possible with mutation to move to a state $\{AB\}^a$ with $a < \ell$.

Now consider $\overrightarrow{B}$. We want to find the minimum number of mutations required for a transition from $\overrightarrow{B}$ to a state in the set $\{AB\}$. Let $c(\overrightarrow{B}, AB)$ denote this number. Recall that $m^A(c,k)$ is defined such that if a player has strictly more than $m^A(c,k)$ A-neighbors she will strictly prefer to stay at A. If $m^A(c,k) < 0$ we have that an A-player remains even if she does not have an A neighbor. Thus, if $m^A(c,k) < 0$ one mutation is enough to move from $\overrightarrow{B}$ to a state in $\{AB\}^1$. Now consider $m^A(c,k) \geq 0$. First, consider the case where $m^A(c,k) \notin \mathbb{Z}$. In this case, if $\lceil m^A(c,k) \rceil + 1$ adjacent players mutate to A each of them will have $\lceil m^A(c,k) \rceil > m^A(c,k)$ players choosing B. Thus, none of them will switch and we have reached an absorbing state in the set $\{AB\}^{\lceil m^A(c,k) \rceil + 1}$. Note that if less than $\lceil m^A(c,k) \rceil + 1$ players switch to A, all of them will switch back when given revision opportunity. It follows that $c(\overrightarrow{B}, AB) = \max\{\lceil m^A(c,k) \rceil, 0\} + 1$ for $m^A(c,k) \notin \mathbb{Z}$. Now consider $m^A(c,k) \in \mathbb{Z}$. In this case for all A players to stay with probability one each of them needs strictly more than $m^A(c,k)$ A-neighbors. Thus, if $m^A(c,k) + 2$ players switch to A each of them will have $m^A(c,k) + 1$ and will not switch back with positive probability. Thus, $c(\overrightarrow{B}, AB) = \max\{m^A(c,k) + 1, 0\} + 1$ for $m^A(c,k) \in \mathbb{Z}$. Summing up, we have

$$c(\overrightarrow{B}, AB) = \begin{cases} \max\{\lceil m^A(c,k) \rceil, 0\} + 1, & \text{if } m^A(c,k) \notin \mathbb{Z} \\ \max\{m^A(c,k) + 1, 0\} + 1, & \text{if } m^A(c,k) \in \mathbb{Z} \end{cases}.$$

This can be written as $c(\overrightarrow{B}, AB) = \max\{\lfloor m^A(c,k) \rfloor + 1, 0\} + 1$.

Conversely, consider the convention $\overrightarrow{A}$. We aim to understand how many mutations to B we need so that the new B-players will keep their strategy with certainty. If $m^B(c,k) > 2k$, this would be the case even if all neighbors choose A. Thus, one mutation is enough to move from $\overrightarrow{A}$ to a state in $\{AB\}^1$ whenever $m^B(c,k) > 2k$. Assume $m^B(c,k) \leq 2k$. Now a B-player will keep her strategy whenever $m_i < m^B(c,k)$. Initially the B-players had $2k$ A-neighbors. Thus, each of them needs strictly more than $2k - m^B(c,k)$ of their neighbors to play B to keep their strategy with probability 1. Again, let us distinguish the cases $2k - m^B(c,k) \in \mathbb{Z}$ and $2k - m^B(c,k) \notin \mathbb{Z}$. In the latter case we have that with $\lceil 2k - m^B(c,k) \rceil + 1$ mutations we can move from $\overrightarrow{B}$ to a state in the set $\{AB\}^{\lceil 2k - m^B(c,k) \rceil + 1}$. Thus, $c(\overrightarrow{A}, AB) = \max\{\lceil 2k - m^B(c,k) \rceil, 0\} + 1$. If $2k - m^B(c,k) \in \mathbb{Z}$ we need $2k - m^B(c,k) + 2$ mutations to ensure that each B player has more than $2k - m^B(c,k)$ neighbors playing B. As above, we can unify the cases $2k - m^B(c,k) \in \mathbb{Z}$ and $2k - m^B(c,k) \notin \mathbb{Z}$

by using $c(\overrightarrow{A}, AB) = \max\{\lfloor 2k - m^B(c,k) \rfloor + 1, 0\} + 1$.

We will now determine the set of LRE. Let L denote the number of non monomorphic absorbing states. Thus, together with the states $\overrightarrow{A}$ and $\overrightarrow{B}$ we have $L+2$ absorbing states. We can connect all L AB states to each other and to $\overrightarrow{A}$ and $\overrightarrow{B}$ via a chain of single mutations. Further, we can move from $\overrightarrow{B}$ into the class of AB states at the cost of $c(\overrightarrow{B}, AB)$. Thus, we can exhibit minimum A-trees of cost $L+c(\overrightarrow{B}, AB)$. Likewise, the minimum B-trees have cost $L+c(\overrightarrow{A}, AB)$. Further, for each state $AB \in \{AB\}$ we can exhibit a minimum cost tree of cost $L-1+c(\overrightarrow{A}, AB)+c(\overrightarrow{B}, AB)$.

First note that if $c > \alpha$, we have $m^A(c,k) < 0$ and $m^B(c,k) > 2k$. It follows $c(\overrightarrow{A}, AB) = c(\overrightarrow{B}, AB) = 1$. Thus, the minimum cost $\overrightarrow{A}$-, the $\overrightarrow{B}$-, and all minimum cost AB-trees have cost $L+1$. Thus, $\mathcal{S} = \{\overrightarrow{A}, \overrightarrow{B}, \{A,B\}\}$. Now assume $1-\beta < c \leq \alpha$. For this range we have $m^A(c,k) < 0$ and $m^B(c,k) \leq 2k$. Thus $c(\overrightarrow{A}, AB) > 1$ and $c(\overrightarrow{B}, AB) = 1$. Thus, we can find minimum $\overrightarrow{A}$-trees of cost $L+1$ and we can not find $\overrightarrow{B}$- or AB-trees of cost smaller than $L+2$. Hence, $\mathcal{S} = \overrightarrow{A}$. Finally, consider $\frac{\alpha+\beta-1}{2} < c \leq 1-\beta$. First, observe that $\lfloor 2k - m^B \rfloor = \lfloor 2k(1-2q) + m^A(c,k) \rfloor \geq \lfloor m^A(c,k) \rfloor$. Thus, $c(\overrightarrow{A}, AB) \geq c(\overrightarrow{B}, AB)$. So, we either have $c(\overrightarrow{A}, AB) > c(\overrightarrow{B}, AB)$ in which case $\mathcal{S} = \overrightarrow{A}$ or $c(\overrightarrow{A}, AB) = c(\overrightarrow{B}, AB)$ in which case $\mathcal{S} = \overrightarrow{A} \cup \overrightarrow{B}$. □

Thus, the presence of switching costs may imply that under local interactions the risk dominant convention is no longer unique long run equilibrium. Let us provide some technical intuition for this result. First, if $c \leq \frac{\alpha+\beta-1}{2}$ the risk dominant strategy may still spread out contagiously and, thus, remains unique long run equilibrium. For $\frac{\alpha+\beta-1}{2} < c < 1-\beta$ there exist absorbing AB states. The question which convention is long run equilibrium translates from which of the two it is more difficult to move to the set of AB-states. This is measured by the numbers $c(\overrightarrow{A}, AB)$ and $c(\overrightarrow{B}, AB)$ which are in turn rounded down values of the functions $2k - m^B(c,k) + 2$ and $m^A(c,k) + 2$. Risk dominance implies that $c(\overrightarrow{B}, AB) \leq c(\overrightarrow{A}, AB)$. Thus, the risk dominant convention is always contained in the set of long run equilibria. The functions $2k - m^B(c,k) + 21$ and $m^A(c,k) + 2$ only differ by a constant and are linearly decreasing in the switching costs. It may very well be the case that the rounded down values are the same, $c(\overrightarrow{A}, AB) = c(\overrightarrow{B}, AB)$. In this case both conventions turn out to be long run equilibria. Finally, for $c > \alpha$ we have agents will not switch strategies, no matter what the distribution of strategies among their neighbors is and all absorbing states turn out to be LRE.

In Figure 2 we plot the transition costs from either convention to the set of non-monomorphic states as a function of the switching costs. Whenever $c(\overrightarrow{A}, AB)$ lies above $c(\overrightarrow{B}, AB)$ the convention $\overrightarrow{A}$ is unique LRE. When $c(\overrightarrow{A}, AB)$ and $c(\overrightarrow{B}, AB)$ coincide both conventions, $\overrightarrow{A}$ and $\overrightarrow{B}$, are long run equilibrium. When the two functions are equal to one another both conventions, $\overrightarrow{A}$ and $\overrightarrow{B}$,

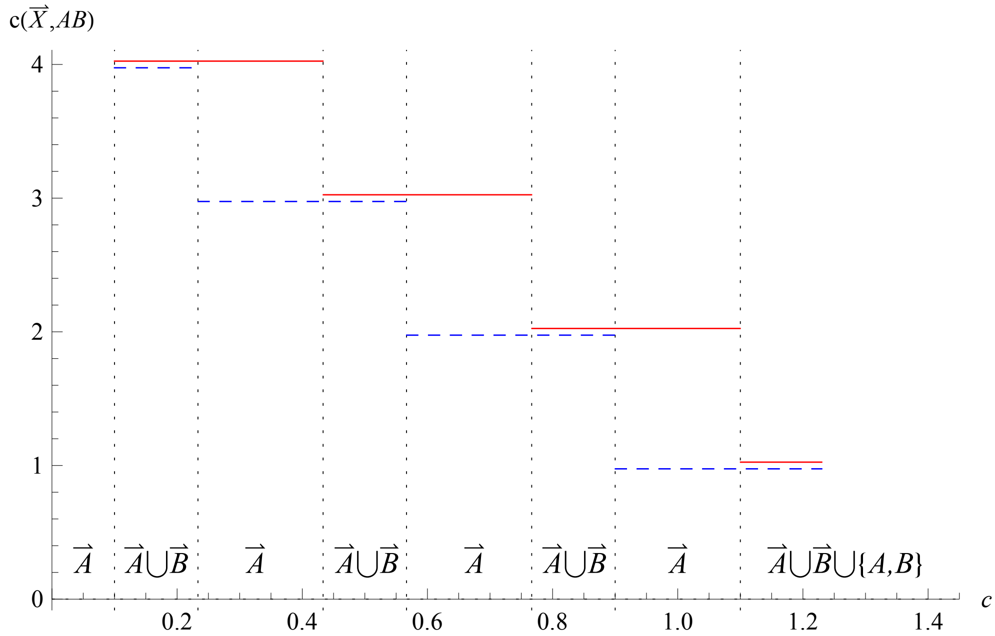

Figure 2: LRE in the game $[\alpha, \beta] = [1.1, 0.1]$ with interaction radius $k = 3$. The solid line plots the transition costs $c(\vec{A}, AB)$ and the dashed line plots the transition costs $c(\vec{B}, AB)$. Whenever $c(\vec{A}, AB)$ lies above $c(\vec{B}, AB)$ the convention $\vec{A}$ is unique LRE. When $c(\vec{A}, AB)$ and $c(\vec{B}, AB)$ coincide both conventions, $\vec{A}$ and $\vec{B}$, are LRE. When the two functions are equal to 1, both conventions, $\vec{A}$ and $\vec{B}$, and the set of non-monomorphic states $\{AB\}$ are LRE.

and the set of non-monomorphic states $\{AB\}$ are LRE. Note that as in the two player interaction case the prediction are non-monotonic in the level of switching costs. In particular, in the region the prediction that the risk dominant convention is unique long run and the prediction that both of them are long run equilibria alternate k-times with increasing switching costs. The reason is that the functions $2k - m^B(c, k) + 2$ and $m^A(c, k) + 2$ only differ by a constant. Thus, if there exists, e.g., a range of parameters for which we have $\lfloor 2k - m^B(c, k) \rfloor = \lfloor m^A(c, k) \rfloor = 1$ there also exists a range of parameters for which $\lfloor 2k - m^B(c, k) \rfloor = \lfloor m^A(c, k) \rfloor = r$ where $r \in \mathbb{Z}$.

We will now investigate the conditions under which the risk dominant convention remains unique long run equilibrium. We are able to derive the following corollary from the previous proposition.

Corollary 4. *For $\frac{\alpha+\beta-1}{2} < c < \alpha$, sufficiently large k, and a sufficiently large population $N \geq 2k + 1$ we have $\mathcal{S} = \overrightarrow{A}$*

Proof. Consider case bii) in the previous Proposition. We have $\mathcal{S} = \overrightarrow{A}$ if $\lceil m^A(c, k) \rceil < \lceil 2k - m^B(c, k) \rceil$. This can be written as $\lfloor m^A(c, k) \rfloor < \lfloor 2k(1 - 2q) + m^A(c, k) \rfloor)$. By risk dominance, we have $1 - 2q > 0$. Thus, by taking k sufficiently large $2k(1 - 2q) \geq 1$. This implies $\lfloor m^A(c, k) \rfloor < \lfloor 2k - m^B(c, k) \rfloor$. $\square$

Thus, if agents interact with sufficiently many other agents switching costs do not influence the prediction. For, in large populations the difference between $c(\overrightarrow{A}, AB)$ and $c(\overrightarrow{B}, AB)$ is strictly larger than one. Thus, it is always more difficult to move from the risk dominant convention to the set of AB-states than it is from the non-risk dominant convention, implying that the risk dominant convention is unique LRE.

We can reconcile our findings with the results of Norman (2009) by simply setting $k = \frac{N-1}{2}$, thus, obtaining a model of global interactions. For small populations switching costs may very well have an impact on the set of long run equilibria. However, in large populations, as considered by Norman (2009), the prediction is robust to switching costs. In this case, switching costs speed up convergence but do not alter the long run behavior of the population.

4 Conclusion

We have established that under local interactions the set of long run equilibria may be altered by the presence of switching costs. In particular, risk dominant conventions may no longer be unique long run equilibria. However, if agents interact with sufficiently many other agents risk dominant conventions are still uniquely selected. One may, thus, be tempted to argue that our results are a curiosity that occurs only in the small interaction or population case. However, social networks are often characterized by local interactions, where large populations are structured in a way that each individual only interacts with a few neighbors.

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