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Optical Solitons Possessing Beta Derivative of the Chen-Lee-Liu Equation in Optical Fibers

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This research obtains some new optical soliton solutions with beta derivative for Chen-Lee-Liu equation (CLL) in optical fibers. Three integration schemes which are Ricatti-Bernoulli (RB) sub-ODE, generalized Bernoulli (GB) sub-ODE and generalized tanh (GT) methods are applied to reach such solutions. The constraints conditions for the existence of soliton solutions are reported. The solutions are obtained using newly introduced fractional derivative called beta derivative. Numerical simulations of some of the obtained solutions are illustrated.

Keywords: CLL, RB sub-ODE, GB sub-ODE, GT methods, beta derivative

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1. INTRODUCTION

Nonlinearity has been very attractive area of study whose vitality have been thought of by considering a heavy-amplitude wave motions determined in several areas starting from fluids and plasmas to solid state, chemical biological systems among others. Owing to this, solitons have been one of the most mesmerizing viewpoint in nonlinear phisical aspect. A philosophical balance of nonlinearity and dispersion are the major essence for the presence of solitonic concept [1]. Several studies on soliton and other results for the multiple traveling wave solutions of nonlinear partial differential equations can be seen in Miller and Ross [2], Podlubny [3], Oldham and Spanier [4], and Kiryakova [5]. Monopulse water wave is the first soliton reported in El-Sayed and Gaber [6]. Optical solitons has also brought about mathematical insight and innovation of the various mechanism for their analytical and numerical solutions [7–23].

2. BETA DERIVATIVE

The idea of the effect of memory has been an issue for quite a long time in the community of modeling. Naturally, the classical models are not convenient to admit this memory [24–26]. A lot of authors have proposed that the effect of the memory could be fully explained by fractional derivatives [27–30]. In Khalil et al. [31], Khalil introduced a new definition of derivative called "conformable derivative," this derivative satisfied some conventional characteristics, for instance, the chain rule. Atangana in Atangana et al. [32] analyzed some characteristics of this derivative, thereby proving some related theorems and proposing a new definitions. An exciting research that has a great relationship with this operator are stated in Cenesiz et al. [33], He et al. [34], Abdeljawad [35], Chung [36], and Cenesiz and Kurt [37]. Recently, Atangana in [38] introduced the "beta-derivative." The newly introduced derivatives satisfies a lot of characteristics that have been considered as limitation for the fractional derivatives and is used to model some physical problems. These derivatives may not be seen as fractional derivative but can be considered to be a natural extension of the classical derivative [31]. The beta-derivative is defined as Atangana et al. [38].

$$ {}_0^A D_x^\alpha(f(x)) = \lim_{\epsilon \to 0} \frac{f\left(x + \epsilon(x + \frac{1}{\Gamma(\alpha)})\right) - f(x)}{\epsilon}. \tag{1} $$

Beta derivative has the following properties

1.

$$ {}_0^A D_x^\alpha(af(x) + bg(x)) = a {}_0^A D_x^\alpha f(x) + b {}_0^A D_x^\alpha g(x) {}_0^A \tag{2} $$

2.

$$ D_x^\alpha(c) = 0, \tag{3} $$

for any constant c,

3.

$$ {}_0^A D_x^\alpha(f(x).g(x)) = g(x) {}_0^A D_x^\alpha f(x) + f(x) {}_0^A D_x^\alpha g(x) \tag{4} $$

4.

$$ {}_0^A D_x^\alpha \left(\frac{f(x)}{g(x)}\right) = \frac{g(x) {}_0^A D_x^\alpha f(x) - f(x) {}_0^A D_x^\alpha g(x)}{g^2(x)}. \tag{5} $$

Considering $\epsilon = \left(x + \frac{1}{\Gamma(\alpha)}\right)^{\alpha - 1} h$, $h \to 0$ when $\epsilon \to 0$, therefore we have

$$ {}_0^A D_x^\alpha f(x) = \left(x + \frac{1}{\Gamma(\alpha)}\right)^{1-\alpha} \frac{df(x)}{dx}, \tag{6} $$

with

$$ \xi = \frac{l}{\alpha}\left(x + \frac{1}{\Gamma(\alpha)}\right)^\alpha \tag{7} $$

where l is a constant.

5.

$$ {}_0^A D_x^\alpha \left(\frac{f(\xi)}{g(x)}\right) = l \frac{df(\xi)}{d\xi}. \tag{8} $$

The proofs of the above beta properties were plainly presented in [29].

3. GOVERNING EQUATION

Here, we consider the evolution of a slowly varying envelope u as modeled by a family of the CLL equation of the form [30]:

$$ i {}_0^A \mathcal{D}_t^\alpha u + a {}_0^A \mathcal{D}_x^{2\alpha} u + ib(|u|^2) {}_0^A \mathcal{D}_x^\alpha u = 0, \tag{9} $$

where $u(x,t)$ is the normalized electric-field envelope, ${}_o^A\mathcal{D}_t^\alpha$ and ${}_o^A\mathcal{D}_x^\alpha$ are beta derivatives [29]. The coefficient of the constant a is group velocity dispersion, the coefficient of b is the Bohm potential that is explored in chiral solitons with quantum Hall effect.

It is imperative to know that many equations in nonlinear sciences contain an empirical parameters. These parameters can be investigated through establishing an exact solutions thereby designing an experiments to generate a convenient conditions that could determine these parameters. Thus, generating an exact traveling wave solutions is becoming more mesmerizing in nonlinear sciences [31–37].

The aim of current work is to establish optical soliton solutions by via three different analytical methods which are RB method [35], GB method [36] and GT method [37].

4. MATHEMATICAL ANALYSIS

To solve Equation (9), the starting step is

$$ u(x,t) = u(\xi) e^{i\phi(x,t)}, \tag{10} $$

$u(x,t)$ represent the shape of the pulse so that

$$ \xi = \frac{1}{\alpha}\left(x + \frac{1}{\Gamma(\alpha)}\right)^\alpha - \frac{\upsilon}{\alpha}\left(t + \frac{1}{\Gamma(\alpha)}\right)^\alpha \tag{11} $$

and the phase component is given by

$$ \phi(x,t) = -\frac{k}{\alpha}\left(x + \frac{1}{\Gamma(\alpha)}\right)^\alpha + \frac{w}{\alpha}\left(t + \frac{1}{\Gamma(\alpha)}\right)^\alpha + \theta_0(\xi), \tag{12} $$

where k denotes the soliton frequency, w is the wave number of the soliton, $\theta_0(\xi)$ is an extra phase function depending on the variable ξ, υ indicates the speed of the soliton. Substituting (10) into (9), and isolating the real and imaginary parts, we obtain the following

$$ -wu + \upsilon u\theta' + au'' - au\theta'^2 - ak^2u + 2aku\theta' - bu^3\theta' + bku^3 = 0, \tag{13} $$

and

$$ a(u\theta'' + 2u'\theta') - \upsilon u' - 2aku' + bu^2u' = 0, \tag{14} $$

where $u' = \frac{du}{d\xi}$, $u'' = \frac{d^2u}{d\xi^2}$, $\theta' = \frac{d\theta}{d\xi}$, and $\theta'' = \frac{d^2\theta}{d\xi^2}$. In order to solve the equation above, we use the ansatz of the form

$$ \theta' = z_1u^2 + z_2, \tag{15} $$

where z_1, z_2 are the nonlinear and constant chirp parameters, respectively to be found. Using Equation (15) in Equation (14), we obtain two algebraic equations that define the chirp parameters

$$ z_1 = -\frac{b}{4a},\ z_2 = k + \frac{\upsilon}{2a}. \tag{16} $$

Inserting Equation (16) along with Equation (15) into Equation (14) gives

$$ u'' + B_1u + B_2u^3 + B_3u^5, \tag{17} $$

where

$$ B_1 = \frac{\upsilon^2}{4a^2} + \frac{\upsilon k}{a} - \frac{w}{a},\ B_2 = -\frac{b\upsilon}{2a^2},\ B_3 = \frac{3b^2}{16a^2}. \tag{18} $$

Applying the balancing principle in Equation (17) gives $n = \frac{1}{2}$ which is not closed form. In order to obtain closed form solutions, we use the transformation

$$ u = U^{\frac{1}{2}}, \tag{19} $$

in Equation (17) to obtain

$$4B_1U^2 + 4B_2U^3 + 4B_3U^4 + 2UU'' - U'^2 = 0. \tag{20}$$

Applying the balancing principle in Equation (20) gives $n = 1$.

5. APPLICATIONS

In this section, we apply three integration schemes to attain optical solitons for the underlying equation.

5.1. Application of RB sub-ODE Method

This section will apply RB sub-ODE method [39] to obtain soliton solutions for Equation (9). Assuming that the solution of Equation (20) is the solution of the RB equation

$$U' = a_1U^{2-M} + b_1U + c_1U^M, \tag{21}$$

where a_1, b_1, c_1 and M are constants and will be found later. Substituting Equation (21) into Equation (20) we have

$$\begin{aligned}&-2a_1b_1mU(\xi)^3 + 4a_1b_1U(\xi)^3 - 2a_1^2mU(\xi)^{4-m}\\&+3a_1^2U(\xi)^{4-m} - c_1^2U(\xi)^{3m}\\&2a_1c_1U(\xi)^{m+2} + 4B_1U(\xi)^{m+2} + 4B_2U(\xi)^{m+3} + b_1^2U(\xi)^{m+2}\\&+2c_1^2mU(\xi)^{3m}\\&+4B_3U(\xi)^{m+4} + 2b_1c_1mU(\xi)^{2m+1} = 0.\end{aligned} \tag{22}$$

Setting $m = 0$, we obtain

$$\begin{aligned}&4a_1b_1U(r)^3 + 2a_1c_1U(r)^2 + 3a_1^2U(r)^4 + 4B_1U(r)^2\\&+4B_2U(r)^3 + 4B_3U(r)^4 + b_1^2U(r)^2 - c_1^2 = 0.\end{aligned} \tag{23}$$

Setting each coefficients of $U^i (i = 0, 2, 3, 4)$ to zero, we have

$$\begin{aligned}2a_1c_1 + 4B_1 + b_1^2 &= 0,\\4\left(a_1b_1 + B_2\right) &= 0,\\3a_1^2 + 4B_3 &= 0,\\c_1^2 &= 0.\end{aligned} \tag{24}$$

Solving Equation (24), we obtain

Result 1 $a_1 = \pm 2i\sqrt{\frac{B_3}{3}}$, $b_1 = \frac{2iB_1}{B_2}\sqrt{\frac{B_3}{3}}$, $c_1 = 0$.

Case 1. When $M \neq 1$, $b_1 \neq 0$, $c_1 = 0$, we get the following algebraic solution

$$u(x,t) = \left(-\frac{B_2}{B_1} + Ce^{-\left(\frac{2iB_1}{B_2}\sqrt{\frac{B_3}{3}}\right)\xi}\right)^{-\frac{1}{2}} e^{i\phi(x,t)}. \tag{25}$$

Case 2. When $M \neq 1$, $b^2 - 4a_1c_1 < 0$, we have the following singular periodic solutions

$$u(x,t) = \left(-\frac{B_1}{2B_2} + \frac{B_1}{2iB_2}\tan\left(\frac{B_1}{B_2}\sqrt{\frac{B_3}{3}}(\xi + C)\right)\right)^{\frac{1}{2}} e^{i\phi(x,t)}, \tag{26}$$

$$u(x,t) = \left(-\frac{B_1}{2B_2} - \frac{B_1}{2iB_2}\cot\left(\frac{B_1}{B_2}\sqrt{\frac{B_3}{3}}(\xi + C)\right)\right)^{\frac{1}{2}} e^{i\phi(x,t)}, \tag{27}$$

provided that $B_3 > 0$.

Case 3. When $M \neq 1$, $b^2 - 4a_1c_1 > 0$, we obtain the following dark optical and singular optical soliton solutions, respectively

$$u(x,t) = \left(-\frac{B_1}{2B_2} + \frac{B_1}{2iB_2}\tanh\left(\frac{B_1}{B_2}\sqrt{\frac{B_3}{3}}(\xi + C)\right)\right)^{\frac{1}{2}} e^{i\phi(x,t)}, \tag{28}$$

$$u(x,t) = \left(-\frac{B_1}{2B_2} + \frac{B_1}{2iB_2}\coth\left(\frac{B_1}{B_2}\sqrt{-\frac{B_3}{3}}(\xi + C)\right)\right)^{\frac{1}{2}} e^{i\phi(x,t)}, \tag{29}$$

provided that $B_3 > 0$.

Case 4. When $M \neq 1$, $b^2 - 4a_1c_1 = 0$, we acquire the following algebraic solution

$$u(x,t) = \left(-\frac{1}{2i\sqrt{\frac{B_3}{3}}\,(\xi + C)} - \frac{B_1}{2B_2}\right)^{\frac{1}{2}} e^{i\phi(x,t)} \tag{30}$$

5.2. Application for GB Sub-ODE Method

This section will apply GB Sub-ODE method to produce optical soliton solutions for Equation (9). According to GB method [40], Equation (20) has the solution given as

$$U(x,t) = a_0 + a_1\Phi(\xi), \tag{31}$$

where a_0 and a_1 are unknown constants and $\Phi(\xi)$ satisfies the Ricatti equation

$$\Phi' + \lambda\Phi = \mu\Phi^2, \tag{32}$$

where μ is a non-zero constant. Inserting Equation (31) along with Equation (32) into the Equation (20), we get

$$\begin{aligned}&8a_1a_0B_1\Phi(\xi) + 4a_0^2B_1 + 4a_0^3B_2 + 4a_0^4B_3 + 2a_1a_0\lambda^2\Phi(\xi)\\&-6a_1a_0\lambda\mu\Phi(\xi)^2\\&4a_1^2B_1\Phi(\xi)^2 + 12a_0^2a_1B_2\Phi(\xi) + 16a_0^3a_1B_3\Phi(\xi)\\&+a_1^2\lambda^2\Phi(\xi)^2 + 4a_0a_1\mu^2\Phi(\xi)^3\\&4a_1^3B_2\Phi(\xi)^3 + 12a_0a_1^2B_2\Phi(\xi)^2 + 24a_0^2a_1^2B_3\Phi(\xi)^2\\&-4a_1^2\lambda\mu\Phi(\xi)^3 + 3a_1^2\mu^2\Phi(\xi)^4\\&4a_1^4B_3\Phi(\xi)^4 + 16a_0a_1^3B_3\Phi(\xi)^3 = 0.\end{aligned} \tag{33}$$

Collecting the coefficients $\Phi^i (i = 0, 1, 2, 3, 4)$, we obtain

$$\begin{aligned}4a_0^2\left(a_0B_2 + a_0^2B_3 + B_1\right) &= 0,\\2a_0a_1\left(6a_0B_2 + 8a_0^2B_3 + 4B_1 + \lambda^2\right) &= 0,\\4a_1^4B_3 + 3a_1^2\mu^2 &= 0,\\a_1\left(a_1\left(4B_1 + \lambda^2\right) - 6a_0\left(\lambda\mu - 2a_1B_2\right) + 24a_1a_0^2B_3\right) &= 0,\\4a_1\left(a_1\left(a_1B_2 - \lambda\mu\right) + a_0\left(4a_1^2B_3 + \mu^2\right)\right) &= 0.\end{aligned} \tag{34}$$

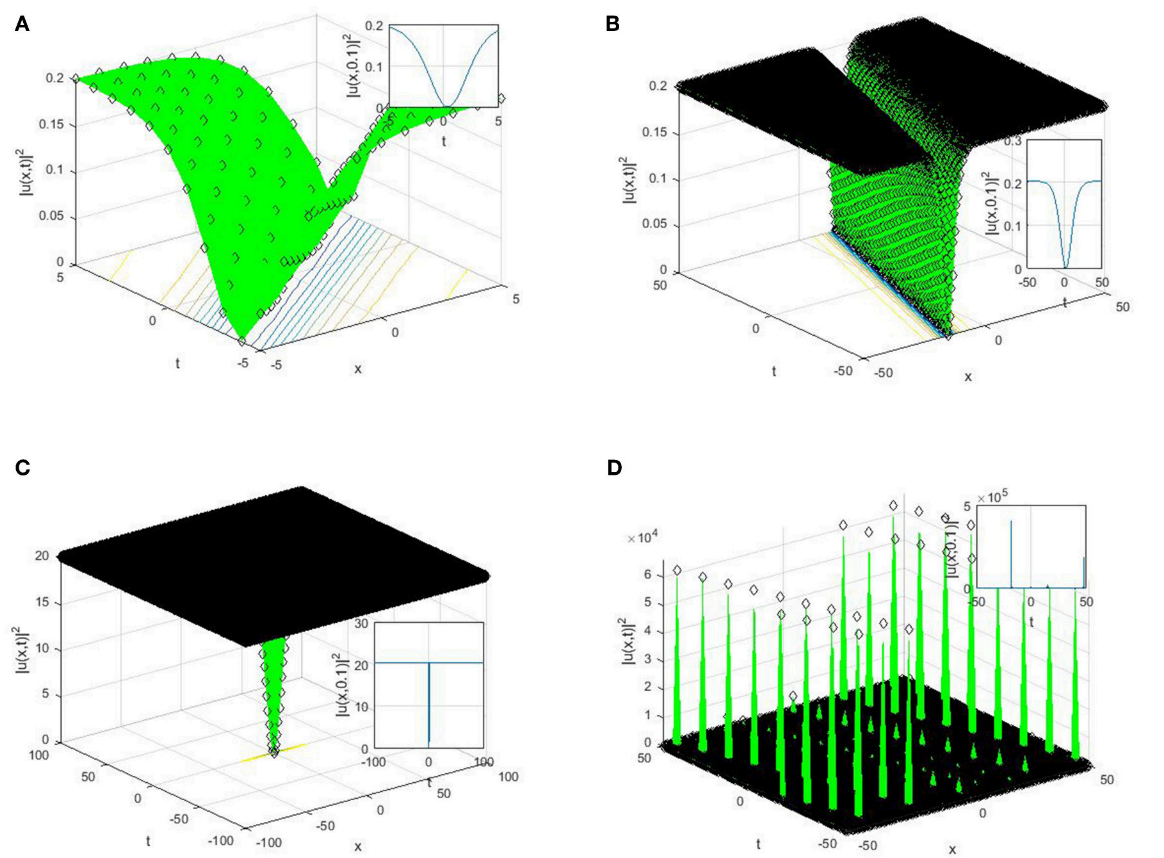

FIGURE 1 | Some physical features of the obtained solutions $a = k = w = C = 0.9$, $\mu = 1.2$, $\lambda = 1.5$, $b = 1.7$, $\alpha = 0.3$, $\upsilon = 0.3$. **(A)** 3 and 2 dimensional plots for (29). **(B)** 3 and 2 dimensional plots for (28). **(C)** 3 and 2 dimensional plots for (26). **(D)** 3 and 2 dimensional plots for (27).

Solving Equation (34), we obtain

Result 1. $\lambda\mu \neq 0$, $a_0 = \frac{\lambda^2-4B_1}{2B_2}$, $a_1 = -\frac{2(\lambda^2\mu-2B_1\mu)}{3B_2\lambda}$. This results yield the following dark optical and singular optical soliton solutions, respectively

$$u(x,t)=\left(\frac{\lambda^2-4B_1}{2B_2}-\frac{2\left(\lambda^2\mu-2B_1\mu\right)}{3B_2\lambda}\left(tanh[\frac{\lambda}{2}\xi]-1\right)\right)^{\frac{1}{2}}e^{i\phi(x,t)}, \tag{35}$$

$$u(x,t)=\left(\frac{\lambda^2-4B_1}{2B_2}-\frac{2\left(\lambda^2\mu-2B_1\mu\right)}{3B_2\lambda}\left(coth[\frac{\lambda}{2}\xi]-1\right)\right)^{\frac{1}{2}}e^{i\phi(x,t)}, \tag{36}$$

5.3. Application for GT Method

This section will apply GB Sub-ODE method to produce optical soliton solutions for Equation (9). According to GB method [41], Equation (20) has the solution given as

$$U(x,t) = a_0 + a_1\Phi(\xi), \tag{37}$$

where a_0 and a_1 are unknown constants and $\Phi(\xi)$ satisfies the Ricatti equation

$$\Phi' = C + \Phi^2, \tag{38}$$

where μ is a non-zero constant. Inserting Equation (37) along with Equation (38) into the Equation (20), we get

$$\begin{aligned}
&8a_1a_0B_1\Phi(\xi) + 4a_0^2B_1 + 4a_0^3B_2 + 4a_0^4B_3 - a_1^2C^2 + 4a_1a_0C\Phi(\xi),\\
&4a_1^2B_1\Phi(\xi)^2 + 12a_1a_0^2B_2\Phi(\xi) + 16a_1a_0^3B_3\Phi(\xi) + 2a_1^2C\Phi(\xi)^2,\\
&4a_1^3B_2\Phi(\xi)^3 + 12a_0a_1^2B_2\Phi(\xi)^2 + 24a_0^2a_1^2B_3\Phi(\xi)^2 + 4a_0a_1\Phi(\xi)^3,\\
&4a_1^4B_3\Phi(\xi)^4 + 16a_0a_1^3B_3\Phi(\xi)^3 + 3a_1^2\Phi(\xi)^4 = 0.
\end{aligned} \tag{39}$$

Collecting the coefficients $\Phi^i (i = 0, 1, 2, 3, 4)$, we obtain

$$\begin{aligned}
4a_0^2B_1 + 4a_0^3B_2 + 4a_0^4B_3 - a_1^2C^2 &= 0,\\
4a_0a_1\left(3a_0B_2 + 4a_0^2B_3 + 2B_1 + C\right) &= 0,\\
2a_1^2\left(6a_0B_2 + 12a_0^2B_3 + 2B_1 + C\right) &= 0,\\
4a_1\left(a_1^2B_2 + a_0\left(4a_1^2B_3 + 1\right)\right) &= 0,\\
a_1^2\left(4a_1^2B_3 + 3\right) &= 0.
\end{aligned} \tag{40}$$

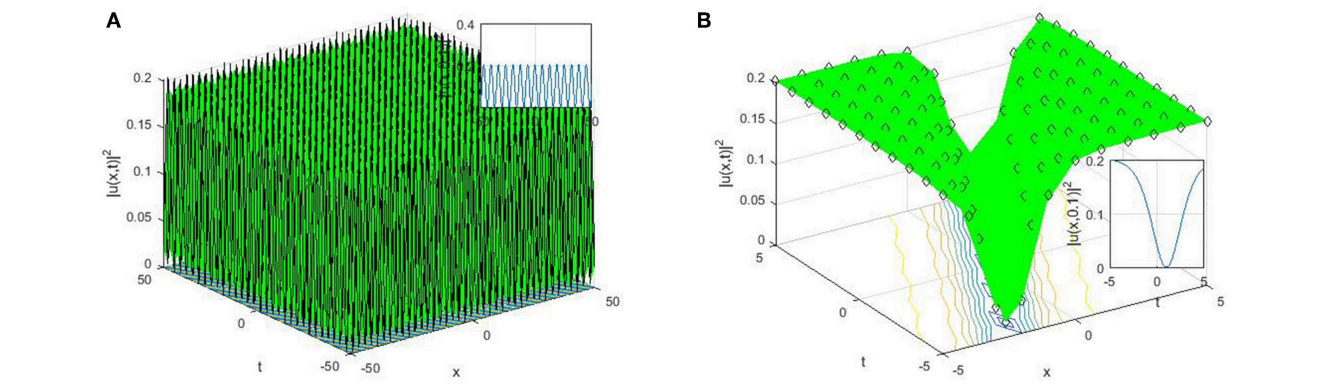

FIGURE 2 | Some physical features of the obtained solutions with the parameter values $a = k = w = C = 1.9$, $\mu = 1.2$, $\lambda = 1.5$, $b = 1.7$, $\alpha = 0.3$, $\upsilon = 0.3$. **(A)** 3 and 2 dimensional plots for (36). **(B)** 3 and 2 dimensional plots for (35).

Solving Equation (40), we obtain

Result 1. $a_0 = -\frac{3B_2}{8B_3}$, $a_1 = \frac{1}{2}i\sqrt{\frac{3}{B_3}}$. If $C < 0$, this results yield the following dark optical and singular optical soliton solutions, respectively.

$$u(x,t) = -\frac{3B_2}{8B_3} - \frac{1}{2}i\sqrt{\frac{3}{B_3}}\sqrt{-C}\left(tanh\left(\sqrt{-C}\xi\right)\right)^{\frac{1}{2}} e^{i\phi(x,t)}, \quad (41)$$

and

$$u(x,t) = -\frac{3B_2}{8B_3} - \frac{1}{2}i\sqrt{\frac{3}{B_3}}\sqrt{-C}\left(coth\left(\sqrt{-C}\xi\right)\right)^{\frac{1}{2}} e^{i\phi(x,t)}, \quad (42)$$

If $C > 0$, this results yield the following dark optical and singular optical soliton solutions, respectively.

$$u(x,t) = -\frac{3B_2}{8B_3} - \frac{1}{2}i\sqrt{\frac{3}{B_3}}\sqrt{C}\left(tan\left(\sqrt{C}\xi\right)\right)^{\frac{1}{2}} e^{i\phi(x,t)}, \quad (43)$$

and

$$u(x,t) = -\frac{3B_2}{8B_3} + \frac{1}{2}i\sqrt{\frac{3}{B_3}}\sqrt{C}\left(cot\left(\sqrt{C}\xi\right)\right)^{\frac{1}{2}} e^{i\phi(x,t)}, \quad (44)$$

6. RESULTS AND DISCUSSION

The RB sub-ODE, GB Sub-ODE and GT integration schemes are employed to establish optical and other solitons for the Chen-Lee-Liu equation in optical fibers. Dark, singular and albegraic solutions are constructed successfully. The RB sub-ODE scheme provided dark soliton (Equation 28), singular soliton (Equation 29), trigonometric solutions (Equations 26, 27), algebraic solutions (Equations 25,30). The GB Sub-ODE scheme provided dark and singular optical solitons reported in Equations (35) and (36), respectively. The GT scheme provided similar solution as RB sub-ODE that is dark, singular and trigonometric solutions reported in Equations (41–44), respectively. The GB sub-ODE scheme could not provide the algebraic solutions and trigonometric solutions in comparison with RB sub-ODE and GT schemes. Moreover, the GT schemes could not provide the algebraic solutions as provided by RB sub-ODE. The following paragraph will give some explanations for the obtained results.

Dark optical soliton explains the solitary waves with smaller intensity than the background, the singular soliton solutions depict a solitary wave possessing discontinuous derivatives; an instance of such solitary waves are compactions, which possess a finite (compact) support, and peakons, whose peaks possess a discontinuous first derivative. These kinds of solitary waves are of extreme important owing to their efficiency and of course flexibility in the long-distance optical communication.

It worth noting that optical fibers are thin long strands of ultra-pure glass or plastic such that a light can be transmitted from one end to another without much attenuation or loss. In order to have a clear vision on the affect of parameters to the transmission of solitons, we consider the following investigation:

Suppose that $\alpha \in \mathbb{C}$, then the solutions reported in Equations (28), (29), (35), (36), (41), and (42) will turn to periodic wave solutions with singularity. This shows that when $\alpha \in \mathbb{C}$, the long distance light transmission through the optical materials will automatically be affected or lost owing to the smaller attenuation. The plain understanding for the physical features and mechanisms to the reported solutions by suitable choice of the parameter values are shown through 2D and 3D. The perspective view and the propagation pattern of the wave along the x-axis of the obtained dark optical solitons appeared in (28), (35), (41), singular optical solitons appeared in Equations (29), (36), and (42), trigonometric solutions appeared in Equations (25), (26), (43), and (44) can be seen in the 3D and 2D plots in **Figures 1–3**.

7. CONCLUSION

This research obtained some new optical soliton solutions with beta derivative for CLL in optical fibers. Three integration schemes which are RB sub-ODE, GB sub-ODE and GT are

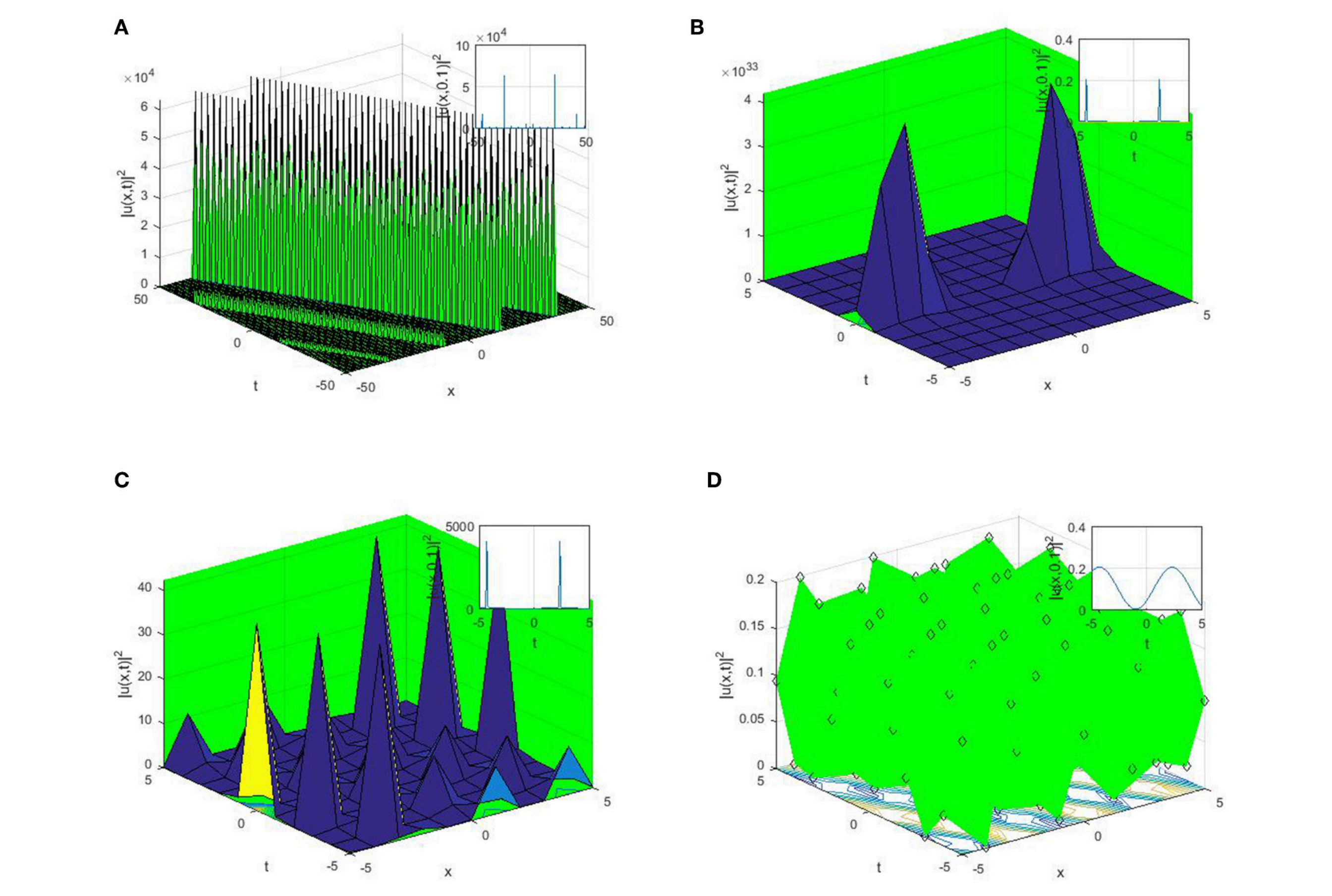

FIGURE 3 | Some physical features of the obtained solutions with the parameter values $a = k = w = 1.2$, $C = -1$, $\mu = 2.2$, $\lambda = 2.5$, $b = 1.7$, $\alpha = 0.3$, $\upsilon = 0.3$. **(A)** 3 and 2 dimensional plots for (41). **(B)** 3 and 2 dimensional plots for (42). **(C)** 3 and 2 dimensional plots for (43). **(D)** 3 and 2 dimensional plots for (44).

applied to reach such solutions. The constraints conditions for the existence of soliton solutions are reported. The solutions are obtained using newly introduced fractional derivative called beta derivative. Numerical simulations of some of the obtained solutions are illustrated.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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