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α-TOPOLOGICAL VECTOR SPACES

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ABSTRACT:
The main objective of this paper is to present the study of α-topological vector spaces. α-topological vector spaces are defined by using α-open sets and α-irresolute mappings. Notions of convex, balanced and bounded set are introduced and studied for α-topological vector spaces. Along with other results, it is proved that every α-open subspace of an α-topological vector space is an α-topological vector space. A homomorphism between α-topological vector spaces is α-irresolute if it is α-irresolute at the identity element. In α-topological vector spaces, the scalar multiple of α-compact set is α-compact and αCl(C) as well as αInt(C) is convex if C is convex. And also, in α-topological vector spaces, αCl(E) is balanced (resp. bounded) if E is balanced (resp. bounded), but αInt(E) is balanced if E is balanced and 0 ∈ αInt(E).

KEYWORDS: α-Topological vector space, α-open set, α-irresolute mapping, left (right) translation, α-homeomorphism.

1. INTRODCUTION

Topology is an umbrella term that includes several fields of study including point set topology, algebraic topology, and differential topology. Because of this it is difficult to credit a single mathematician with introducing topology. In 1965, Njastad initiated and explored a new class of generalized open sets in a topological space called α-open sets and proved that the collection of all α-open sets in (X, τ) is a topology on X, finer that τ. If a set is endowed with algebraic and topological structures, then it is always fascinating to probe relationship between these two structures. The most formal way for such a study is to require algebraic operations to be continuous. This is the case we are investigating here for algebraic and topological structures on a set X, where algebraic operations (addition and scalar multiplication mappings) fail to be continuous. We join these two structures through weaker form of continuity.

A topological vector space (A. Grothendieck, and A. P. Robertson and W. J. Robertson) is a basic structure in topology in which a vector space X over a topological field F (R or C) is endowed with a topology τ such that:

(1) The vector addition mapping m : X × X → X defined by m((x, y)) = x + y, and
(2) Scalar multiplication mapping M : F × X → X defined by M((λ, x)) = λ · x for all λ ∈ F and x, y ∈ X are continuous with respect to τ. Equivalently, (X(F), τ) is a topological vector space if:

(1) For each x, y ∈ X and for each open neighbourhood W of x + y in X, there exist open neighbourhoods U of x and V of y in X such that U + V ⊆ W, and
(2) For each x ∈ X, λ ∈ F and for each open neighbourhood W in X containing λ · x, there exist open neighbourhoods U of λ in F and V of x in X such that U · V ⊆ W.
In this paper, several new facts concerning topologies of α-topological vector spaces are established.

2. PRELIMINARIES

Throughout in this paper X and Y are always topological spaces with no separation axioms considered until otherwise mentioned. If A ⊆ X, then Cl(A) and Int(A) denote the closure and interior of A in (X, τ), respectively. A subset A of a topological space (X, τ) is called α-open [O. Njastad] if A ⊆ Int(Cl(Int(A))). The complement of an α-open set is called an α-closed set. A subset A of a space X is called semi-open [N. Levine] if A ⊆ Cl(Int(A)). The complement of semi-open set is called semi-closed set. The intersection of all α-closed sets containing A is called the α-closure of A and is denoted by αCl(A). The α-interior of A is defined as the union of all α-open sets contained in A and is denoted by αInt(A). The family of all α-open (resp. α-closed) subsets of X is denoted by αO(X) (resp. αC(X)). For each x ∈ X, the family of all α-open sets containing x is denoted by αO(X, x). It is known that x ∈ αCl(A) if and only if, for any α-open set U containing x, U ∩ A is non-empty.
If X (F) is a vector space, then 0 denotes its identity element, and for a fixed x ∈ X, T_x: X → X; y → x + y and T_x: X → X; y → y + x, denote the left and the right translation by x, respectively. And, for every 0 ≠ λ ∈ F, $M_λ$: X → X; y → λ · y, denote multiplication operator.

Definition 2.1 (D. Jangkovic) A space X is said to be α-compact if every α-open cover of X has a finite subcover.

Definition 2.2 (M. Khan and B. Ahmad) A space X is said to be P-regular, if for each semi-closed set F and x ∉ F, there exist disjoint open sets U and V such that x ∈ U and F ⊆ V.

Definition 2.3 (S. N. Maheshwari and S. S. Thakur) A space X is said to be α-T_2, if for any two distinct points x, y ∈ X, there exist two α-open sets U and V containing x and y, respectively, such that U ∩ V = φ.

Definition 2.4 (S. N. Maheshwari and S. S. Thakur) A function f: (X, τ) → (Y, σ) is said to be α-irresolute, if the inverse image of every α-open set in Y is an α-open set in X.

3. α-TOPOLOGICAL VECTOR SPACES

We denote by F a scalar field. In practice this is either R or C, the set of real or complex numbers.

Definition 3.1 Let X be a vector space. The pair $(X_{(F)}, \tau)$ is said to be an α-topological vector space over the field F (R or C) with a topology τ defined on $X_{(F)}$ and standard topology on F if the following two conditions are satisfied:

(1) For each x, y ∈ X and for each α-open set W of X containing x + y, there exist α-open sets U and V in X containing x and y respectively, such that U + V ⊆ W.

(2) For each x ∈ X, λ ∈ F and for each α-open set W of X containing λ· x, there exist α-open sets U in F containing λ and V in X containing x, such that U · V ⊆ W.

Remark 3.2 Every vector space X over F endowed with the trivial topology is an α-topological vector spaces.

Theorem 3.3 In α-topological vector spaces $(X_{(F)}, \tau)$, for any α-open set U containing 0, there exists an α-open set V containing 0 such that V + V ⊆ U.

Proof. Let U be any α-open set such that 0 = 0 + 0 ∈ U. Since $(X_{(F)}, \tau)$ is α-topological vector spaces, then there are α-open sets A and B with 0 ∈ A, 0 ∈ B and A + B ⊆ U. Let V = A ∩ B, then V is α-open, 0 ∈ V and V + V ⊆ A + B ⊆ U.

Theorem 3.4 If $(X_{(F)}, \tau)$ is an α-topological vector space. Then:

(1) The (left) right translation T_x: X → X defined by $T_x(y)$ = y + x, for all x, y ∈ X is α-irresolute.

(2) The translation M_λ: X → X defined by $M_\lambda(x)$ = λ · x, for all x ∈ X is α-irresolute.

Proof. (1) Let W be an α-open set containing $T_x(y)$ = y + x. Then by definition, there exist α-open sets U and V in X containing y and x respectively, such that U + V ⊆ W. So, $T_x(U)$ = U + x ⊆ U + V ⊆ W. This proves that, T_x: X → X is α-irresolute mapping.

(2) Let x ∈ X, λ ∈ F, then $M_\lambda(x)$ = λ · x. Let W be any α-open set of X containing λ · x, then by definition, there exist α-open sets U in F containing λ and V in X containing x, such that U · V ⊆ W. This gives that $M_\lambda(V)$ = λ · V ⊆ U · V ⊆ W. This proves that M_λ is an α-irresolute mapping.

Theorem 3.5 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. If A ∈ αO(X), then:

(1) A + y ∈ αO(X) for every y ∈ X.

(2) λ · A ∈ αO(X) for every non zero λ ∈ F.

Proof. (1) Let y ∈ X and z ∈ A + y, then we have to prove that z is an α-interior point of A + y. Now, z = x + y, where x is some point in A. We can write x ∈ A + y + (−y) = A. By the right translation T_{-y}: X → X, we have $T_{-y}(z)$ = z + (−y) = x. By Theorem 3.4 (1), T_{-y} is α-irresolute for z ∈ X. Thus, for the α-open set A containing x = $T_{-y}(z)$, there exists α-open set M_z of X containing z such that $T_{-y}(M_z) = M_z$ + (−y) ⊆ A, this implies M_z ⊆ A + y. This shows that z is an α-interior point of A + y. Hence A + y ∈ αO(X).

(2) Let λ ∈ F, λ ≠ 0 and z ∈ λ · A, this means z = λ · x, for some x ∈ A. We have to show that z is an α-interior point of λ · A. By Theorem 3.4 (2), the multiplication mapping $M_{\lambda^{-1}}$: X → X is α-irresolute. Thus, for the α-open set A containing $M_{\lambda^{-1}}(z) = \lambda^{-1}$ · z = x, there exists α-open set U_z of X containing z such that $M_{\lambda^{-1}}(U_z) = \lambda^{-1} \cdot U_z$ ⊆ A this implies U_z ⊆ λ · A. This shows that z is an α-interior point of λ · A. Hence λ · A ∈ αO(X)

Corollary 3.6 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. If A ∈ αO(X), then for all u ∈ A, there exists an α-open set V containing 0 such that u + V ⊆ A.

Proof. The proof is follow by taking V = A − u.

Theorem 3.7 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space and μ_0 is a collection of all α-open sets containing 0. Then, for each U ∈ μ_0, there exists V ∈ μ_0 such that αCl(V) ⊆ U.

Proof. Let U ∈ μ_0. Then by Theorem 3.3, there exists V ∈ μ_0 such that V + V ⊆ U. Let x ∈ αCl(V). Since x − V is α-open containing x, so (x − V) ∩ V ≠ φ. Choose, y ∈ (x − V) ∩ V, then y = x − v_1 = v_2, where v_1, v_2 ∈ V. Thus, x = $v_2 + v_1$ ∈ V + V ⊆ U. Therefore, αCl(V) ⊆ U.

Theorem 3.8 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. If A ∈ αO(X) and B is any subset of X, then A + B ∈ αO(X).

Proof. Suppose A ∈ αO(X) and B ⊆ X. Then, by Theorem 3.5 (1), for each x_i ∈ B we have A + x_i ∈ αO(X). Now, for each x_i ∈ B we have A + B = A + {x_1, x_2, ...} = $\cup_{x_i \in B}$ (A + x_i). Since union of any number of α-open sets is α-open, therefore A + B is α-open in X.

Corollary 3.9 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. If A ∈ αO(X), then the set U = $\bigcup_{n=1}^{\infty}$ nA is an α-open set in X.

Proof. Let A be α-open in X. Then, by Theorem 3.8, A + A = 2A ∈ αO(X) and 2A + A = 3A ∈ αO(X). Similarly, we can prove that each set 4A, 5A, ... is α-open in X. Thus the set U = $\bigcup_{n=1}^{\infty}$ nA is α-open in X.

Theorem 3.10 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. Then, λ · (αInt(B)) = αInt(λ · B), where λ ∈ F.

Proof. Let λ · x ∈ λ · (αInt(B)) such that x ∈ αInt(B), then there exists an α-open set U such that x ∈ U ⊆ B. Now, λ · x ∈ λ · U ⊆ λ· B. As λ · U is α-open by Theorem 3.5 (2). So, λ · x ∈ αInt(λ · B). Therefore, λ · (αInt(B)) ⊆ αInt(λ · B).

Conversely, let y ∈ αInt(λ · B), where define y = λ · x for some x ∈ B, then there exists an α-open set V such that λ · x ∈ V ⊆ λ · B. Since $(X_{(F)}, \tau)$ is an α-topological vector space, then there exist α-open sets U in F containing λ and W in X containing x, such that λ · x ∈ λ · W ⊆ U · W ⊆ V ⊆ λ · B. Then, x ∈ W ⊆ B implies that x ∈ αInt(B) and so λ · x ∈ λ · (αInt(B)). Therefore, αInt(λ · B) ⊆ λ · (αInt(B)). Hence, λ · (αInt(B)) = αInt(λ · B).

Theorem 3.11 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. Then, M: F × X → X is an α-irresolute mapping.

Proof. Let λ ∈ F and x ∈ X and since M((λ, x)) = λ · x. Let W be an α-open set of X containing λ · x. Since $(X_{(F)}, \tau)$ is an α-topological vector space, therefore there exist α-open sets U in F containing λ and V in X containing x, such that U ·V ⊆ W implies that M((U, V)) = M(U ×V) = U ·V ⊆ W. Since, U ∈ αO(F, λ) and V ∈ αO(X, x), therefore, U × V ∈ αO(F × X, λ · x). This proves that M: F × X → X is an α-irresolute mapping.

Theorem 3.1. Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. Then, m: X × X → X is an α-irresolute mapping.

Proof. Let x, y ∈ X and m((x, y)) = x + y. Let W be an α-open set of X containing x + y. Since $(X_{(F)}, \tau)$ is an α-topological vector space, therefore there exist α-open sets U containing x and V containing y in X, such that U + V ⊆ W implies that m((U, V)) = m(U × V) = U + V ⊆ W. Since, U ∈ αO(X, x) and V ∈ αO(X, y), therefore, U × V ∈ αO(X × X, x × y). This proves that m: X × X → X is an α-irresolute mapping.

Definition 3.13 A bijective mapping f from a topological space to itself is called α-homeomorphism if it is α-irresolute and for every α-open set A of X, the set f(A) is α-open in Y .

Theorem 3.14. Suppose that ($X_{(F)}$, τ) is an α-topological vector space. For given y ∈ X and λ ∈ F with λ ≠ 0, each translation mapping T_y: x → x + y and multiplication mapping M_λ: x → λ · x, where x ∈ X is α-homeomorphism onto itself.
Proof. First, we show that T_y: x → x + y is an α-homeomorphism. It is obviously bijective. By Theorem 3.4 (1), T_y is α-irresolute. Moreover, by Theorem 3.5 (1), for any α-open set U, we have $T_y(U)$ = U + y is α-open. Similarly, we can prove that M_λ: x → λ · x is an α-homeomorphism.

Definition 3.15 An α-topological vector space ($X_{(F)}$, τ) is said to be α-homogenous space if for each x, y ∈ X, there is an α-homeomorphism f of the space X onto itself such that f(x) = y.
Theorem 3.16 Every α-topological vector space ($X_{(F)}$, τ) is an α-homogenous space.
Proof. Take any x, y ∈ X and put z = (−x) + y. Then, by Theorem 3.14, T_z: X → X is an α-homeomorphism and $T_z(x)$ = x + z = y. Therefore, ($X_{(F)}$, τ) is an α-homogenous space.

Theorem 3.17 Let f : ($X_{(F)}$, τ_X) → ($Y_{(F)}$, τ_Y) be a homomorphism of α-topological vector spaces. If f is α-irresolute at 0 ∈ X, then f is α-irresolute on X.
Proof. Let x ∈ X. Suppose that W is an α-open set in Y containing y = f(x). Since T_y: Y → Y is α-irresolute, therefore there is an α-open set V containing 0 such that $T_y(V)$ = V + y ⊆ W. Now from α-irresolute of f at 0 of X, there exists α-open U in X containing 0 such that f(U) ⊆ V. Since T_x: X→ X is α-homeomorphism, therefore the set U + x is α-open set containing x. Thus, f(U + x) = f(U) + f(x) = f(U) + y ⊆ V + y ⊆ W. Therefore, f is α-irresolute at x of X, and hence on X.

Theorem 3.18 Suppose ($X_{(F)}$, τ) is an α-topological vector space and S is a subspace of X. If S contains a non-empty α-open subset of X, then S is α-open in ($X_{(F)}$, τ).
Proof. Suppose U is a non-empty α-open subset in X, such that U ⊆ S. For any y ∈ S, the set $T_y(U)$ = U + y is α-open in X and U + y ⊆ S. Therefore, the subspace S = $\cup_{y\in S}$ (U + y) is α-open in X as the union of α-open sets.

Theorem 3.19 Suppose that ($X_{(F)}$, τ) is an α-topological vector space. Then every α-open subspace of X is α-closed in X.
Proof. Let S be an α-open subspace of X. As right translation T_x: X→X is α-homeomorphism, therefore, S + x is α-open in X. Then Y = $\cup_{x\in X\setminus S}$ (S + x) is also α-open. Now S = X\Y is α-closed.

Theorem 3.20 Every α-open subspace S of an α-topological vector space ($X_{(F)}$, τ) is also an α-topological vector space (called α-topological subspace of X).
Proof. Let x, y ∈ S and W be an α-open set of S containing x + y. This gives W is an α-open set of X containing x + y. Hence, there exist α-open sets U and V in X containing x and y respectively, such that U + V ⊆ W. Now, the sets A = U ∩ S and B = V ∩ S are α-open sets in S containing x and y respectively and also A + B ⊆ U + V ⊆ W. Again, let λ ∈ F and x ∈ S. Let W be an α-open set of S containing λ · x. Since S is α-open in X, therefore W is α-open set of X containing λ·x. Hence, there exist α-open sets U ⊆ F containing λ and V ⊆ X containing y such that U · V ⊆ W. Now, the set A = U ∩ F is α-open set of F containing λ and the set B = V ∩ S is α-open set of S containing y and also A · B ⊆ U · V ⊆ W. This proves that S is an α-topological vector space.

Theorem 3.21 Let A and B be subsets of an α-topological vector space ($X_{(F)}$, τ). Then αCl(A) + αCl(B) ⊆ αCl(A + B).
Proof. Suppose that x ∈ αCl(A) and y ∈ αCl(B). Let W be an α-open set containing x + y. Then there are α-open sets U and V containing x and y respectively, such that U + V ⊆ W. Since x ∈ αCl(A) and y ∈ αCl(B), there are a ∈ A ∩ U and b ∈ B ∩ V . Then a + b ∈ (A + B) ∩(U + V) ⊆ (A + B) ∩ W. This means x + y ∈ αCl(A + B), that is αCl(A) + αCl(B) ⊆ αCl(A + B).

Theorem 3.22 Suppose ($X_{(F)}$, τ) is an α-topological vector space and A, B are subsets of X. If B is α-open, then for any set A, we have A + B = αCl(A) + B.
Proof. As we know that A ⊆ αCl(A), so A + B ⊆ αCl(A) + B. Conversely, let y ∈ αCl(A) + B and write y = x + b where x ∈ αCl(A) and b ∈ B. There exists an α-open set V containing zero such that $T_b(V)$ = V + b ⊆ B. Now, V is α-open in X containing 0, this gives that −V is also α-open in X containing 0. Since, x ∈ αCl(A), so, a ∈ A ∩ (x − V). We know that y = x + b = a − a + x + b ∈ a + V + b ⊆ A + B. Therefore, αCl(A) + B ⊆ A + B. Hence, A + B = αCl(A) + B.

Theorem 3.23 Suppose that ($X_{(F)}$, τ) is an α-topological vector space, then for any A ⊆ X, αCl(A) = ∩{A + U: U ∈ αO(X, 0)}.
Proof. Let x ∈ αCl(A), this implies that for every U ∈ αO(X, 0), we have x + U ∈ αO(X, x) and (x + U) ∩ A ≠ φ. Let a ∈ (x + U) and a ∈ A. Hence a = x + u_1 for some u_1 ∈ U. This gives x = a − u_1 ∈ a − U ⊆ A − U. Thus,
x ∈ ∩{A − U: U ∈ αO(X, 0)} and so x ∈ ∩{A + U: U ∈ αO(X, 0)}.
Conversely, assume that x ∉ αCl(A). Then, there exists U ∈ αO(X, 0) such that (−U + x) ∩ A = φ, that is, x ∉ A + U, hence x ∉ ∩{A + U: U ∈ αO(X, 0)}. This shows that ∩{A + U: U ∈ αO(X, 0)} ⊆ αCl(A). Therefore, we have αCl(A) = ∩{A + U: U ∈ αO(X, 0)}.

Theorem 3.24 Suppose ($X_{(F)}$, τ) is an α-topological vector space. Then the scalar multiple of α-closed set is α-closed.
Proof. Let B ∈ αC(X), then X\B ∈ αO(X) and M_λ(X\B) = λ · (X\B) = λ · X\ λ · B = X \ λ · B ∈ αO(X). Therefore, λ · B ∈ αC(X).

Theorem 3.25 Suppose ($X_{(F)}$, τ) is an α-topological vector space. Then scalar multiple of α-compact set is α-compact.
Proof. Let A be an α-compact subsets of X. Let {U_i: i ∈ I} be an α- open cover of λ · A for some non-zero λ ∈ F, then λ · A ⊆ $\cup_{i\in I}$ U_i. This gives A ⊆ 1/λ · $\cup_{i\in I}$ U_i = $\cup_{i\in I}$ 1/λ · U_i. Since, U_i ∈ αO(X) and ($X_{(F)}$, τ) is an α-topological vector space, therefore, 1/λ · U_i ∈ αO(X) for each i ∈ I. Since, A is α-compact therefore, there exist a finite subset I_0 of I such that A ⊆ $\cup_{i\in I_0}$ 1/λ · U_i. This implies that λ · A ⊆ $\cup_{i\in I_0}$ U_i. Hence λ · A is α-compact in X.

Theorem 3.26 Suppose ($X_{(F)}$, τ) is a P-regular and α-topological vector space. Then the algebraic sum of an α-compact set A and α-closed set B is α-closed.
Proof. Let x ∉ A+B, then for some a ∈ A, x∉ a + B. Since, the translation mapping is α-homeomorphism, so $T_a(B)$ = a + B, where a + B is α-closed. Since X is P-regular space, therefore, there exist open sets U_a and V_a such that x ∈ U_a, a + B ⊆ V_a and U_a ∩ V_a = φ. Also V_a − B = $\cup_{b\in B}$ (V_a − b) is α-open and contains a. Hence, A ⊆ $\cup_{a\in A}$ (V_a − B). Since, A is α-compact, therefore there exists a finite subset {a_1, a_2, a_3, ..., a_n} of elements of A, such that A ⊆ $\bigcup_{i=1}^{n}$ (V_{ai} − B). Let U = $\bigcup_{i=1}^{n}$ U_{ai}, then U is an α-open set containing x. We claim that U ∩ (A + B) = φ. If not, then y = a + b ∈ U ∩ (A + B), then y ∈ V_{ai} for some i and y ∈ U_{ai}, which is contradiction to the fact that U_a ∩ V_a = φ.

Theorem 3.27 Suppose that ($X_{(F)}$, τ) is an α-topological vector space. If H ⊆ X is linear subspace, then so is αCl(H).

Proof. Let H be a linear subspace of X, which means that, H + H ⊆ H and for all λ ∈ F, λ · H ⊆ H. By Theorem 3.21, αCl(H) + αCl(H) ⊆ αCl(H + H) ⊆ αCl(H). Since, scalar multiplication is an α-homeomorphism it maps the α-closure of a set into the α-closure of its image, namely, for every λ ∈ F, λ · (αCl(H)) = αCl(λ·H) ⊆ αCl(H). Therefore, αCl(H) is linear subspace.

Definition 3.28 A subset E of an α-topological vector space $(X_{(F)}, \tau)$ is said to be balanced if for all λ ∈ F, |λ| ≤ 1, λ · E ⊆ E.

Theorem 3.29 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. For every B ⊆ X:
(1) If B is balanced so is αCl(B).
(2) If B is balanced and 0 ∈ αInt(B), then αInt(B) is balanced.
Proof. (1) Since multiplication by a (non-zero) scalar is an α-homeomorphism, thus for every λ ∈ F, λ · (αCl(B)) = αCl(λ·B). If B is balanced, then for |λ| ≤ 1, λ · (αCl(B)) = αCl(λ · B) ⊆ αCl(B), hence αCl(B) is balanced.
(2) Let B be balanced subset of X. By Theorem 3.10, for every 0 < |λ| ≤ 1, λ · (αInt(B)) = αInt(λ · B). Since, B is balanced, therefore λ · B ⊆ B, |λ| ≤ 1. Also, λ · (αInt(B)) = αInt(λ · B) ⊆ αInt(B). Since for λ = 0, λ · (αInt(B)) = {0}, we must require 0 ∈ αInt(B) for the latter to be balanced.

Theorem 3.30 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space, then for every U ∈ μ_0, there exists a balanced V ∈ μ_0 such that V ⊆ U.
Proof. The proof is clear.
Definition 3.31. A set C is said to be convex if for t ∈ [0, 1], t C + (1 − t) C ⊆ C.

Theorem 3.32 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. If C is convex, then so is αCl(C).
Proof. Convexity is a purely algebraic property, but α-closures and α-interiors are topological concepts. The convexity of C implies that for all t ∈ [0, 1], t C + (1 − t) C ⊆ C. Let t ∈ [0, 1], then t (αCl(C)) = αCl(t C) and (1 − t) (αCl(C)) = αCl((1−t)C). By Theorem 3.21, t (αCl(C)) + (1 − t) (αCl(C)) = αCl(t C) + αCl((1 − t) C) ⊆ αCl(t C + (1 − t) C) ⊆ αCl(C). Thus, αCl(C) is convex.

Theorem 3.33 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. If C is convex, then αInt(C) is convex.
Proof. Suppose that C is convex. Let x, y ∈ αInt(C). This means there exist α-open sets U and V containing 0 such that x + U ⊆ C and y + V ⊆ C. Since C is convex, so, t(x + U) + (1 − t) (y + V) = (t x + (1 − t) y) + t U + (1 − t) V ⊆ C, which proves that t x + (1 – t) y ∈ αInt(C), namely αInt(C) is convex.

Definition 3.34 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. A subset E ⊆ X is said to be bounded if for all α-open set V containing 0, there exists s ∈ R such that for all t > s, E ⊆ tV. That is, every α-open set containing zero contains after being blown up sufficiently.

Theorem 3.35 Suppose that $(X_{(F)}, \tau)$ is an α-topological vector space. If E is bounded, then αCl(E) is bounded.
Proof. Let V be an α-open set containing 0, then by Theorem 3.7, there exist W ∈ μ_0 such that αCl(W) ⊆ V . Since E is bounded, so E ⊆ tW ⊆ tαCl(W) ⊆ tV, for sufficiently large t. It follows that for large enough t, αCl(E) ⊆ tαCl(W) ⊆ tV. Thus, αCl(E) is bounded.
The following result provides a characterization for α-T_2 of α-topological vector space.

Theorem 3.36 Let $(X_{(F)}, \tau)$ be an α-topological vector space. Then the following statements are equivalent:
(1) X is α-T_2.
(2) If x ∈ X, x ≠ 0, then there exists U ∈ μ_0 such that x ∉ U.
(3) If x, y ∈ X, x ≠ y, then there exists V ∈ μ_x such that y ∉ V.
Proof. (1) ⇒ (2). Let x ∈ X, x ≠ 0 by assumption, there exist U, V ∈ αO(X) such that 0 ∈ U, x ∈ V and U ∩ V = φ. Thus, U ∈ μ_0, V ∈ μ_x and x ∉ U.
(2) ⇒ (1). Let x, y ∈ X such that x − y ≠ 0. Then there exists U ∈ μ_0 such that x − y ∉ U. By Theorem 3.3, there exists W ∈ μ_0 such that W + W ⊆ U and by Theorem 3.30, W can be assumed to be balanced. Let V_1 = x + W and V_2 = y + W and note that V_1 ∈ μ_x, V_2 ∈ μ_y and V_1 ∩V_2 = φ, since if a ∈ V_1 ∩ V_2, then −(a − x) ∈ W, as W is balanced and a − y ∈ W. It follows that x − y = (a − y) + (−(a − x)) ∈ W + W ⊆ U, which is a contradiction. So, we must have V_1 ∩ V_2 = φ. This shows that X is α-T_2.
(1) ⇒ (3). Obvious.
(3) ⇒ (2). Obvious.
The following result follows from Theorem 3.36.

Corollary 3.37 Let $(X_{(F)}, \tau)$ be an α-topological vector space. Then the following statements are equivalent:
(1) X is α-T_2.
(2) ∩{U: U ∈ μ_0} = {0}.
(3) ∩{V: V ∈ μ_x} = {x}.

Theorem 3.38 Any α-topological vector space $(X_{(F)}, \tau)$ is α-T_2.
Proof. Pick u_0, u_1 ∈ X such that $u_0 \neq u_1$. Thus V = X\{$u_1 - u_0$} is an α-open set containing zero. As 0 + 0 = 0, by $(X_{(F)}, \tau)$ is an α-topological vector space, there exist V_1 and V_2 sets containing 0 such that $V_1 + V_2$ ⊆ V . Define U = V_1 ∩ V_2 ∩ (−V_1) ∩ (−V_2), thus U = −U and U + U ⊆ V and hence u_0 + U + U ⊆ u_0 + V ⊆ X\ {u_1}, so that $u_0 + v_1 + v_2 \neq u_1$, for all v_1, v_2 ∈ U, or $u_0 + v_1 \neq u_1 - v_2$, for all v_1, v_2 ∈ U, and since U = −U, therefore (u_0 + U) ∩ (u_1 + U) = φ.

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بۆشاییا ئاراستەبرێ توپولوجی ژ جورێ α

کورتیا لێکولینێ:

مەرەم ژڤێ کاری ئەوە کو پێشکێشکرن و خویندنا بۆشاییا ئاراستەبرێ توپولوجی ژ جورێ α. بۆشاییا ئاراستەبرێ توپولوجی ژ جورێ α هاتیە پێناسەکرن برێکا کومێن ڤەکری ژ جورێ α و نەخشا دوو دلی ژ جورێ α. بیرۆکا قۆقز، هاوسەنگ و کوما سنوردار هاتینە پێشکێشکرن وخاندن بۆ بۆشاییا ئاراستەبرێ توپولوجی ژ جورێ α. بەدرێژایی دگەل ئەنجامێن دی، هاتە سەلماندن کو هە می بوشایی بەشی یا ڤەکری ژ جورێ α ژ بۆشاییا ئاراستەبرێ توپولوجی ژ جورێ α دبیتە بۆشاییا ئاراستەبرێ توپولوجی ژ جورێ α. هومۆمۆرپیسم لە نێوان بۆشاییێن ئاراستەبرێ توپولوجی ژ جورێ α دبیتە نەخشا دوو دل ژ جورێ α ئەگەر ئەو نەخشەکا دوو دل بیت لسەر توخمێ هاوئەنجام. لێ بۆشاییێن ئاراستەبرێ توپولوجی ژ جورێ α لێکدانا ژمارێ دگەل کوما پتەو ژ جورێ α دبیتە پتەو ژ جورێ α و αCl(C) هەروەسا αInt(C) دبنە قۆقز ئەگەر C قۆقز بیت. و هەروەسا، لێ بۆشاییێن ئاراستەبرێ توپولوجی ژ جورێ α، αCl(E) دبیتە هاوسەنگ (سنوردار) ئەگەر E هاوسەنگ (سنوردار) بیت، بەلام αInt(E) دبیتە هاوسەنگ ئەگەر E هاوسەنگ بیت و αInt(E) ∋0.

فضاءات متجه التوبولوجية من النمط α

خلاصة البحث:

الغرض من هذا العمل هو تقديم و دراسة فضاءات متجه التوبولوجية من النمط α. فضاء متجه التوبولوجية من النمط α عرفناها باستخدام المجموعات المفتوحة من النمط α و الدوال المتعدية من النمط α. درسنا المفاهيم محدب، توازن و المجموعة مقيد في فضاء متجه التوبولوجية من النمط α. جنبا إلى جنب مع غيرها من النتائج، أثبتنا أن كل فضاء الجزئي المفتوح من النمط α في فضاء متجه التوبولوجية من النمط α تكون فضاء متجه التوبولوجية من النمط α. مفهوم التماثل بين فضاءات متجه التوبولوجية من النمط α تكون الدالة المتعدية من النمط α اذا كانت الدالة المتعدية من النمط α على العنصر احادي. في فضاءات متجه التوبولوجية من النمط α ضرب كمية عددية غير موجهة مع مجموعة المتراص من النمط α تكون المتراص من النمط α و αCl(C) كذلك αInt(C) تكونا المحدب اذا كانت C محدب. و أيضا، في فضاءات متجه التوبولوجية من النمط α، αCl(E) تكون متوازن (مقيد) اذا كانت E متوازن (مقيد)، و لكن αInt(E) تكون متوازن اذا كانت E متوازن و αInt(E) ∋0.