POWER ENGINEERING AND ELECTRICAL ENGINEERING VOLUME: 17 | NUMBER: 1 | 2019 | MARCH

LOAD FLOW ANALYSIS IN POWER SYSTEM NETWORK INCORPORATING STATCOM: A COMPARISON OF THE DIRECT AND INDIRECT ALGORITHM OF THE NEWTON-RAPHSON METHOD

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DOI: 10.15598/aeee.v17i1.3054

Abstract. *This paper presents load flow analysis and the mathematical steady-state modeling of Static Synchronous Compensator (STATCOM) to study its effect on the power system network. More precisely, we propose a new approach method so-called direct algorithm and then compare it with the indirect algorithm in testing cases: IEEE 5-bus, IEEE 14-bus, and IEEE 30-bus systems. We compare the accuracy, the number of iterations and the computational time. The simulation results show that the direct algorithm is effective in terms of accuracy, speed of computation and various practical applications.*

Keywords

FACTS, iteration, load flow, Newton-Raphson, power flow, power system analysis, STATCOM.

1. Nomenclature

P_{Li}: Active power of load at i^{th} bus in pu (per unit system).

Q_{Li}: Reactive power of load at i^{th} bus in pu.

P_{Si}: Active power obtained from STATCOM at i^{th} bus in pu.

Q_{Si}: Reactive power obtained from STATCOM at i^{th} bus in pu.

V_i: System bus voltage magnitude at i^{th} bus in pu.

δ_i: Phase angle of bus voltage at i^{th} bus.

V_{Si}: STATCOM output voltage at i^{th} bus in pu.

δ_{Si}: Phase angle of STATCOM output voltage at i^{th} bus.

V_i^{con}: Bus voltage control reference in pu.

I_{Si}: STATCOM current in pu.

α_{Si}: Phase angle of STATCOM current.

R_{ij}, X_{ij}: Resistance and reactance of branch ij in pu.

Admittance of branch ij in pu:
$1/Z_{ij} = 1/(R_{ij} + jX_{ij}) = |Y_{ij}|\angle\theta_{ij} = G_{ij} + jB_{ij}$.

R_{Si}, X_{Si}: Resistance and reactance of STATCOM in pu.

Admittance of STATCOM in pu:
$1/Z_{Si} = 1/(R_{Si} + jX_{Si}) = g_{Si} + jb_{Si}$.

$V_{Si\,\min}$: The minimum of STATCOM output voltage in pu.

$V_{Si\,\max}$: The maximum of STATCOM output voltage in pu.

$Q_{Si\,\min}$: The minimum value of reactive power of STATCOM in pu.

$Q_{Si\,\max}$: The maximum value of reactive power of STATCOM in pu.

2. Introduction

Together with the development of power electronics, the Flexible AC Transmission system (FACTs) devices have been proposed and used in the electrical power system to improve power quality. Various FACTs (e.g., Static Synchronous Compensator (STATCOM), Static VAR Compensator (VSC), Thyristor Controlled Series Compensator (TCSC), Static Synchronous Series Compensator (SSSC) and Unified Power Flow Controller (UPFC)) are used to control the bus voltage magnitude and the power flow along the transmission lines. Among these devices, STATCOM is one of the most useful ones in power systems because it can regulate the voltage very fast, improve transient stability and compensate variable reactive power [1], [2], [3], [4], [5], [6], [7], [8], [9], [10] and [11].

Load flow (or power flow) is a solution for the steady state of the power systems network. The studies of load flow provide methods for calculating various bus voltage magnitudes, phase angles, active and reactive power flowing through the line of the power system. Load flow analysis of power systems incorporating STATCOM is an important tool to further determine the inject power of STATCOM to regulate voltage under steady state conditions. The power system embedded with STATCOM in load flow requires an accurate method for computation and controlling of the bus voltage magnitudes to determine the steady states and power planning purposes of the power system.

Many different researches have been proposed for the load flow incorporating STATCOM [1], [2], [3], [4], [5], [6], [7], [8], [9], [10] [11], [14], [15], [16] and [17]. In general, there are two main categories of load flow techniques: Current Injection model (CI) and Power Injection model (PI) or Voltage Source model. In the CI model, the STATCOM is represented as the current source connected in shunt at the bus for controlling of voltage magnitudes. In the PI model, it is represented as shunt voltage source behind an equivalent impedance, and controls bus voltage magnitudes by adjusting its voltage magnitude and phase angle. Techniques in both categories have advantages and disadvantages. However, the PI proved its effectiveness in term of computation speed and accuracy [2] and [17].

In the PI model, the buses with STATCOM in power system network can be solved by Eq. (1) a STATCOM represented as a PV bus and Eq. (2) a STATCOM represented as an independent variable. The parameters of Eq. (1) are calculated by the voltage at the bus where the STATCOM is placed, and the main equations of load flow and the STATCOM are solved separately (indirect algorithm) [6], [7], [8] and [9]. In Eq. (2), the Jacobian matrix in the main equation of load flow is modified (direct algorithm) [14], [15], [16] and [17]. In this paper, a MATLAB program is developed for the load flow analysis of power system network incorporating STATCOM. We provide two algorithms for this issue in Sec. 5. and Sec. 6. The load flow study is then performed in IEEE 5-bus, IEEE 14-bus and IEEE 30-bus systems in Sec. 7.

3. Newton-Raphson Load Flow in Power System without STATCOM

The Newton-Raphson method is robust load flow method used in power system. From the node-voltage equation:

$$I_{bus} = Y_{bus} \cdot V_{bus}, \tag{1}$$

where I_{bus} is the vector of the injected bus currents, V_{bus} is the vector of bus voltages and Y_{bus} is known as the bus admittance matrix.

This equation can be rewritten in form for an n-bus system:

$$I_i = \sum_{j=1}^{n} Y_{ij} \cdot V_j = \sum_{j=1}^{n} |Y_{ij}||V_j|\angle\theta_{ij} + \delta_j. \tag{2}$$

The active and reactive power at bus i is:

$$P_i - jQ_i = V_i^* \cdot I_i = V_i\angle -\delta_i \cdot (\sum_{j=1}^{n} |Y_{ij}||V_j|\angle\theta_{ij} + \delta_j). \tag{3}$$

Therefore:

$$\begin{gathered} P_i = Re\{V_i^* \cdot I_i\} = \\ = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j), \end{gathered} \tag{4}$$

$$\begin{gathered} Q_i = -Im\{V_i^* \cdot I_i\} = \\ = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j). \end{gathered} \tag{5}$$

The main equation of Newton-Raphson power flow can be expressed as follow:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{21} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}. \tag{6}$$

The elements of the Jacobian matrix are:

$$\mathbf{J}_{11} = \frac{\partial P_i}{\partial \delta_j}, \mathbf{J}_{12} = \frac{\partial P_i}{\partial |V_j|}, \mathbf{J}_{21} = \frac{\partial Q_i}{\partial \delta_j}, \mathbf{J}_{22} = \frac{\partial Q_i}{\partial |V_j|}. \tag{7}$$

The term ΔP and ΔQ are the difference between the specified and calculate values, given by:

$$\Delta P_i^{(k)} = P_i^{Spec} - P_i^{(k)}, \tag{8}$$

$$\Delta Q_i^{(k)} = Q_i^{Spec} - Q_i^{(k)}. \tag{9}$$

The new estimates for bus voltage are:

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}, \tag{10}$$

$$|V_i|^{(k+1)} = |V_i|^{(k)} + \Delta|V_i|^{(k)}. \tag{11}$$

4. Modeling of STATCOM

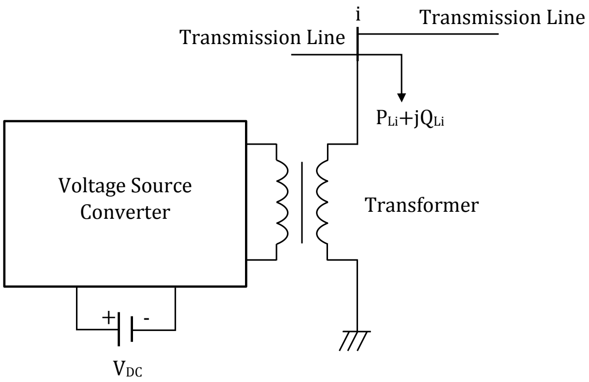

Fig. 1: Schematic diagram of STATCOM.

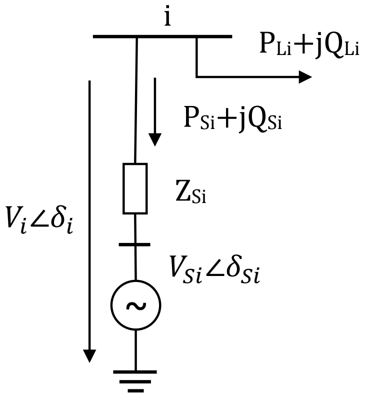

Fig. 2: Equivalent circuit of STATCOM.

Generally, a STATCOM including a coupling transformer, Voltage Source Converter (VSC) and a DC energy storage device (Fig. 1). In its simplest form, a DC capacitor is used to replace the energy storage device, thus the STATCOM is capable of only exchange reactive power with the power systems. In the 90s of previous century, the internal loss power of the transformer and the STATCOM are neglected so that the result is not accurate [6]. Then the STATCOM can be represented by an equivalent circuit as shown in Fig. 2. It is able to regulate the bus voltage magnitude by injecting or absorbing reactive power to or from the bus where it is connected.

The equation for the i^{th} bus with STATCOM can be written as follows:

$$S_i = (P_{Li} + jQ_{Li}) + (P_{Si} + jQ_{Si}), \tag{12}$$

$$P_i = P_{Li} + P_{Si}, Q_i = Q_{Li} + Q_{Si}, \tag{13}$$

$$I_{Si} = (P_{Si} - jQ_{Si})/V_i^*, \tag{14}$$

$$V_{Si} = V_i - I_{Si}Z_{Si}. \tag{15}$$

According to the equivalent circuit of the STATCOM shown in Fig. 2, the power constrains of STATCOM are:

$$\begin{gathered} P_{Si} = |V_i|^2 \cdot g_{Si} - |V_i||V_{Si}| \cdot \\ \cdot [g_{Si} \cdot \cos(\delta_i - \delta_{Si}) + b_{Si} \cdot \sin(\delta_i - \delta_{Si})], \end{gathered} \tag{16}$$

$$\begin{gathered} Q_{Si} = -|V_i|^2 \cdot b_{Si} + |V_i||V_{Si}| \cdot \\ \cdot [-g_{Si} \cdot \sin(\delta_i - \delta_{Si}) + b_{Si} \cdot \cos(\delta_i - \delta_{Si})], \end{gathered} \tag{17}$$

that are the active and reactive power equations obtained from STATCOM at bus i, respectively. The voltage injection and capacity of STATCOM are bounded as follows:

$$\begin{gathered} V_{Si\min} \le V_{Si} \le V_{Si\max}, \\ Q_{Si\min} \le Q_{Si} \le Q_{Si\max}. \end{gathered} \tag{18}$$

The active and reactive power exchange via the DC-link are described by:

$$P_{STAT,i} - jQ_{STAT,i} = V_{Si}^* \cdot I_{Si}. \tag{19}$$

Therefore, we have:

$$\begin{gathered} P_{STAT,i} = |V_{Si}|^2 \cdot g_{Si} - |V_i||V_{Si}| \\ \cdot [g_{Si} \cdot \cos(\delta_i - \delta_{Si}) - b_{Si} \cdot \sin(\delta_i - \delta_{Si})], \end{gathered} \tag{20}$$

$$\begin{gathered} Q_{STAT,i} = -|V_{Si}|^2 \cdot b_{Si} + |V_i||V_{Si}| \\ \cdot [g_{Si} \cdot \sin(\delta_i - \delta_{Si}) + b_{Si} \cdot \cos(\delta_i - \delta_{Si})]. \end{gathered} \tag{21}$$

5. Indirect Algorithm

This algorithm was researched and published, for example in [6], [7], [8] and [9]. According to this algorithm, the system buses, where the STATCOM are installed, are made PV buses. In [6] and [7] the internal losses in the STATCOM (the losses in switching and transformer) were neglected and therefore, it is not accurate. In [8] and [9] the algorithm was improved, such losses are taken into consideration (they have been represented by resistance and reactance of STATCOM in the equivalent circuit). The procedure of this algorithm can be summarized as follows:

- Step 1. Input the system data and the information of buses (voltage magnitude and phase angle at a slack bus, voltage magnitude and active power at PV buses, active and reactive power at PQ buses). The system buses, where the STATCOMs are placed, are made PV buses. On the STATCOM buses, the limits of the voltage and reactive power of STATCOM should be included.
- Step 2. Set the initial value of reactive power exchange via the DC-link, for example $P_{STAT,i}^{(0)} = 0.01$.
- Step 3. Run normal load flow to calculate bus voltages, phase angle and injected reactive powers.
- Step 4. Calculate new value of $P_{STAT,i}$ using the following procedure:
  - Calculate Q_{Si} from the Eq. (13).
  - Calculate I_{Si} from the Eq. (14).
  - Calculate V_{Si} from the Eq. (15).
  - Calculate $P_{STAT,i}$ from the Eq. (20).
  - The errors of $P_{STAT,i}$ can be calculated as follows:

    $$\Delta P_{STAT,i} = 0 - P_{STAT,i}. \tag{22}$$
- Step 5. Update the active power of STATCOM and return to step 2.
- Step 6. The process is continued until the residuals $\Delta P_{STAT,i}$ are less than the specified accuracy.

6. Direct Algorithm

6.1. Newton-Raphson Load Flow Formulation with STATCOM

The direct algorithm was presented, for example, in [10], [14], [15], [16] and [17]. In [10], a STATCOM was presented as a new bus and new branch. In other publications [14], [15], [16] and [17], a STATCOM is represented as independent variables. In this method, the Newton-Raphson load flow algorithm for electrical power system incorporating STATCOM requires some modifications:

- The Jacobian matrix needs to be extend. The new sub-blocks related to the STATCOM device should be included.
- The mismatch vector also needs to be extended. The residuals of power contributed by STATCOM at the connected buses should be included.

With these modifications, the Jacobian matrix is increased according to the number of STATCOMs. Acha et al. in [16] proposed the phase angles and voltage magnitudes of STATCOM are the independent variables with the mismatch vectors ΔP_S and ΔQ_S (the difference between specified and calculate values of active and reactive power of STATCOM, respectively). However, the disadvantage of this method is the unspecified reactive power of STATCOM. We introduce a new approach to solve this problem. Below, the system of linearized load flow equations of the power systems with STATCOM is shown:

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta PE \\ \Delta F \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P}{\partial \delta} & \dfrac{\partial P}{\partial |V|} & \dfrac{\partial P}{\partial \delta_S} & \dfrac{\partial P}{\partial |V_S|} \\ \dfrac{\partial Q}{\partial \delta} & \dfrac{\partial Q}{\partial |V|} & \dfrac{\partial Q}{\partial \delta_S} & \dfrac{\partial Q}{\partial |V_S|} \\ \dfrac{\partial PE}{\partial \delta} & \dfrac{\partial PE}{\partial |V|} & \dfrac{\partial PE}{\partial \delta_S} & \dfrac{\partial PE}{\partial |V_S|} \\ \dfrac{\partial F}{\partial \delta} & \dfrac{\partial F}{\partial |V|} & \dfrac{\partial F}{\partial \delta_S} & \dfrac{\partial F}{\partial |V_S|} \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta \\ \Delta |V| \\ \Delta \delta_S \\ \Delta |V_S| \end{bmatrix}. \tag{23}$$

According to Eq. (23):

- PE is the active power exchange via the DC-link of STATCOM, it is given in Eq. (20).
- F is the voltage magnitude of the bus where the STATCOM is installed.
- ΔP and ΔQ are given in Eq. (8) and Eq. (9).
- For a trial set of variables $\Delta \delta_S$, $\Delta |V_S|$, the mismatch vectors are added, represented by:

$$\Delta PE_i^{(k)} = 0 - P_{STAT,i}^{(k)}, \tag{24}$$

$$\Delta F_i^{(k)} = |V_i^{Con}| - |V_i^{(k)}|. \tag{25}$$

Assume that the power system has n buses including: 1 slack bus, m PV buses, $(n-1-m)$ PQ buses and p STATCOMs. Accordingly, the Jacobian matrix of this algorithm is in the order $(2n-2-m+2p)\times(2n-2-m+2p)$ while it is in the order $(2n-2-m)\times(2n-2-m)$ in indirect algorithm. The elements of the Jacobian matrix in Eq. (23) are given in App. B.

6.2. The Newton-Raphson Load Flow Algorithm with STATCOM

The procedure for the load flow solution with the power system incorporating STATCOM of this algorithm is summarized as follows:

Tab. 1: Comparison of the complex bus voltage of the IEEE 5-bus system with STATCOM between two algorithms.

Bus	**Indirect algorithm**		**Direct algorithm**		**Differences**	
No.	$\lvert V\rvert(p.u)$	$\delta(degree)$	$\lvert V\rvert(p.u)$	$\delta(degree)$	$\lvert V\rvert(p.u)$	$\delta(degree)$
1	1.0600	0.00	1.0600	0.00	0.0000	0.00
2	1.0000	-2.05	1.0000	-2.05	0.0000	0.00
3	1.0000	-4.84	1.0000	-4.84	0.0000	0.00
4	0.9942	-5.11	0.9944	-5.11	0.0002	0.00
5	0.9751	-5.80	0.9752	-5.80	0.0001	0.00

- Step 1. Input the system data and the information of buses (voltage magnitude and phase angle at a slack bus, voltage magnitude and active power at PV buses, active and reactive power at PQ buses) and STATCOM (location, impedance, the limits of voltage and capacity).
- Step 2. Calculate ΔP_i, ΔQ_i, ΔPE_i, ΔF_i from Eq. (8), Eq. (9), Eq. (24) and Eq. (25).
- Step 3. The elements of the Jacobian matrix are calculated from Eq. (26) to Eq. (58) (see App. B).
- Step 4. Solve Eq. (23) for corrections of voltage magnitudes and phase angles at buses and STATCOMs. The limits of voltage and capacity of STATCOM should be included.
- Step 5. Update the voltage magnitude and phase angle by adding to the previous values and return to step 2.
- Step 6. The process is continued until the residuals ΔP_i, ΔQ_i, ΔPE_i, ΔF_i are less than the specified accuracy.

7. Case Studies and Results

MATLAB programs based on these algorithms were developed for the load flow analysis of electrical power systems incorporating STATCOM. In order to investigate the performance of the indirect and direct algorithm, these flow programs were tested on IEEE 5-bus, modified IEEE 14-bus and IEEE 30-bus systems (see App. A).

7.1. Case I: IEEE 5-bus

From the results obtained from load flow for the 5-bus system without STATCOM (see Tab. 2), one can observe that the voltage magnitudes at bus 3, 4 and 5 are lower than 1.0 pu. Therefore, one STATCOM was installed at bus 3 to regulate the voltage magnitude at 1.0 pu. The STATCOM data is given in Tab. 3. Table 1 and Tab. 4 provide the details of simulation results of two algorithms including a comparison of voltage magnitudes and phase angles at all buses, and the injected power of STATCOM.

Tab. 2: The complex bus voltage of the IEEE 5-bus system without STATCOM.

Bus No.	**1 (Slack)**	**2 (PV)**	**3 (PQ)**	**4 (PQ)**	**5 (PQ)**
$\lvert V\rvert(pu)$	1.0600	1.000	0.9872	0.9841	0.9717
$\delta(degree)$	0.00	-2.06	-4.63	-4.96	-5.77

Tab. 3: STATCOM parameters.

$\boldsymbol{R_S(pu)}$	$\boldsymbol{X_S(pu)}$	$\boldsymbol{Q_{Smin}(pu)}$	$\boldsymbol{Q_{Smax}(pu)}$
0.01	0.10	-0.5	0.5

Tab. 4: Comparison of the voltage and reactive power of the STATCOM between two algorithms in the IEEE 5-bus system.

	$\boldsymbol{\lvert V_S\rvert(pu)}$	$\boldsymbol{\delta_S(degree)}$	$\boldsymbol{Q_S(pu)}$
Indirect algorithm	1.0198	-4.96	-0.2047
Direct algorithm	1.0205	-4.96	-0.2049
Differences	0.0007	0.00	0.0002

7.2. Case II: IEEE 14-bus

The second case study in this paper used the IEEE-14 bus system with a small modification, PV buses at bus 6 and 8 are 1.07 pu and 1.09 pu, respectively, which are changed to 1.05 pu. From the results obtained by a program for this system without STATCOM (see Tab. 5), we can observe that all the voltage magnitudes are greater than 1.0 pu. Assume that a STATCOM is installed at bus 11 to ensure the voltage at bus 11 is 1.0 pu. The simulation results of two algorithms are illustrated in Tab. 6 and Tab. 7, including the comparison of voltage magnitudes, phase angles at all buses, and the injected power of STATCOM. In this case, the STATCOM absorbed the reactive power, therefore, its voltage magnitude is smaller than at the bus which the STATCOM was installed.

7.3. Case III: IEEE 30-bus

Similar results was also obtained from the modified IEEE 30-bus system and hence these are not repeated here. Two STATCOMs were installed at bus 26 and 30 in order to regulate the voltage to 1.0 pu. The voltage magnitudes of all buses with and without STAT-

Tab. 5: The complex bus voltage of the IEEE 14-bus system without STATCOM.

Bus No.	**1 (slack)**	**2 (PV)**	**3 (PV)**	**4 (PQ)**	**5 (PQ)**	**6 (PV)**	**7 (PQ)**
$\lvert V\rvert(pu)$	1.0600	1.0450	1.0100	1.0067	1.0104	1.0500	1.0270
$\delta(degree)$	0.00	-5.01	-12.81	-10.18	-8.69	-14.49	-13.26
Bus No.	**8 (PV)**	**9 (PQ)**	**10 (PQ)**	**11 (PQ)**	**12 (PQ)**	**13 (PQ)**	**14 (PQ)**
$\lvert V\rvert(pu)$	1.0500	1.0127	1.0116	1.0269	1.0331	1.0266	1.0001
$\delta(degree)$	-13.26	-14.91	-15.13	-14.93	-15.37	-15.41	-16.20

Tab. 6: Comparison of the complex bus voltage of the IEEE 14-bus system with STATCOM between two algorithms.

Bus	**Indirect algorithm**		**Direct algorithm**		**Differences**	
No.	$\lvert V\rvert(pu)$	$\delta(degree)$	$\lvert V\rvert(pu)$	$\delta(degree)$	$\lvert V\rvert(pu)$	$\delta(degree)$
1	1.0600	0.00	1.0600	0.00	0.0000	0.00
2	1.0450	-5.01	1.0450	-5.02	0.0000	0.01
3	1.0100	-12.83	1.0100	-12.84	0.0000	0.01
4	1.0052	-10.17	1.0051	-10.17	0.0001	0.00
5	1.0093	-8.70	1.0092	-8.70	0.0001	0.00
6	1.0500	-14.63	1.0500	-14.62	0.0000	-0.01
7	1.0224	-13.24	1.0222	-13.24	0.0002	0.00
8	1.0500	-13.24	1.0500	-13.24	0.0000	0.00
9	1.0035	-14.89	1.0033	-14.90	0.0002	0.01
10	0.9969	-14.98	0.9967	-14.99	0.0002	0.01
11	1.0000	-14.44	1.0000	-14.45	0.0000	0.01
12	1.0324	-15.51	1.0315	-15.50	0.0009	-0.01
13	1.0252	-15.53	1.0243	-15.53	0.0009	0.00
14	0.9942	-16.25	0.9937	-16.25	0.0005	0.00

Tab. 7: Comparison of the complex voltage and reactive power of the STATCOM between two algorithms in the IEEE 14-bus system.

	$\boldsymbol{\lvert V_S\rvert(pu)}$	$\boldsymbol{\delta_S(degree)}$	$\boldsymbol{Q_S(pu)}$
Indirect algorithm	0.9806	-14.33	0.1920
Direct algorithm	0.9808	-14.34	0.1916
Differences	-0.0002	0.01	0.0004

COM corresponding to two algorithms are shown in Fig. 3. Table 8 shows the simulation results of two algorithms, including the comparison of voltage magnitudes, phase angles and the injected powers of STATCOMs. The maximum difference of voltage magnitudes and phase angles between two algorithms are 0.0011 pu and 0.02 degrees, respectively. In this case, the STATCOMs compensate reactive power, therefore their voltage magnitudes are larger than at the buses which the STATCOMs were installed at.

7.4. Discussion

In three case studies, it is easy to see that the accuracy of the two algorithms is nearly the same. To determine the effectiveness of the different algorithms, the number of iterations, the total time took off the execution to complete the solution on three systems have been studied and compared. If the desired accuracy decreases, the accuracy of the solution and the number of iterations increase. In this paper, the desired accuracy is 0.0001. Table 9 provides detailed information of the comparison between two algorithms for IEEE 5-bus, 14-bus and 30-bus systems. From the information, we can see that:

- The results of two algorithms are slightly different. This demonstrates that the direct algorithm is quite good in comparison with the well-known indirect algorithm.
- The convergence rate for Newton-Raphson method is fast and the number of iterations are less dependent on the number of buses in the system [12], [13], [14], [15], [16], [17], [18], [19] and [20]. In three case studies, the number of iterations of indirect algorithm is less than or equal to the direct one.
- However, the indirect algorithm is slower than the direct algorithm. Indeed, the loop of indirect algorithm includes the computing time of solving the n-bus system that does not appear in the direct algorithm. As a result, it is profitable when applying the latter to large scale systems.

8. Conclusion

In this paper, two algorithms of Newton-Raphson method for load flow analysis in power system incorporating STATCOM were presented. MATLAB programs based on these algorithms were developed and implemented for the IEEE 5-bus, IEEE 14-bus and IEEE 30-bus systems. The simulation results show that the STATCOM is able to inject or absorb reactive

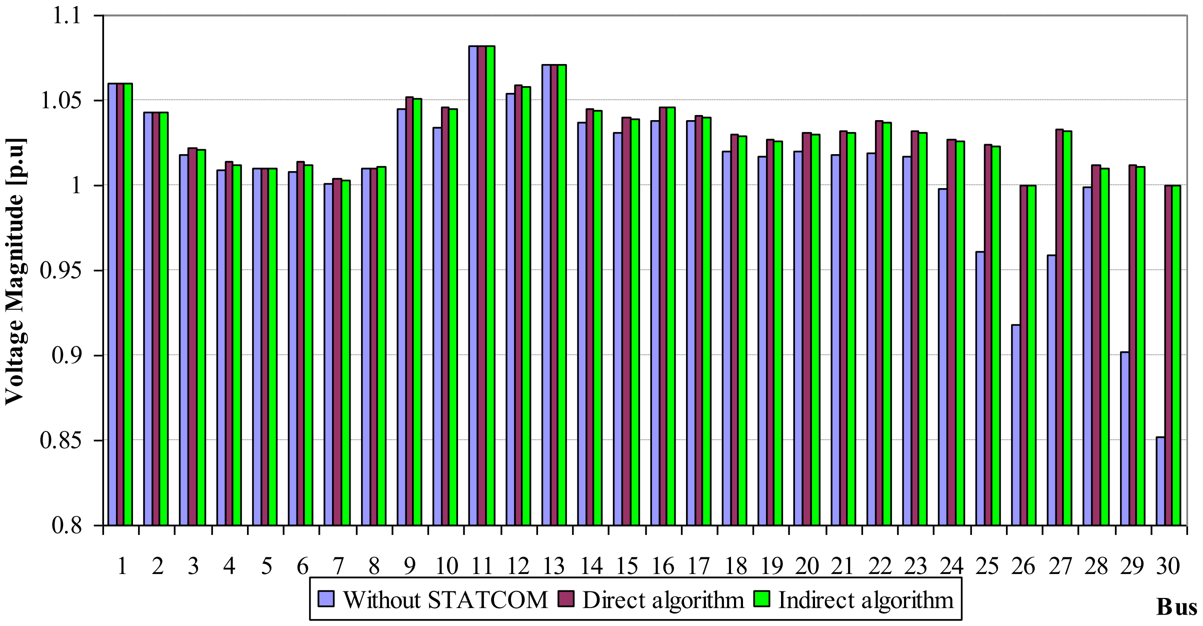

Fig. 3: The voltage of all buses of the IEEE 30-bus system without and with STATCOMs calculated by two algorithms.

Tab. 8: Comparison of the complex voltage and reactive power of the STATCOM between two algorithms in the IEEE 30-bus system.

Bus	**Indirect algorithm**			**Direct algorithm**			**Differences**		
No.	$\|V_S\|(pu)$	$\delta_S(degree)$	$Q_S(pu)$	$\|V_S\|(p.u)$	$\delta_S(degree)$	$Q_S(pu)$	$\|V_S\|(p.u)$	$\delta_S(degree)$	$Q_S(p.u)$
26	1.0041	-20.29	-0.0436	1.0043	-20.32	-0.0432	-0.0002	0.03	-0.0004
30	1.0170	-23.69	-0.1735	1.0173	-23.71	-0.1730	-0.0003	0.02	-0.0005

Tab. 9: The computing time of two algorithms.

System	**Indirect algorithm**			**Direct algorithm**		
	Number of Iterations	Time per Iteration (ms)	Computing time (ms)	Number of Iterations	Time per Iteration (ms)	Computing time (ms)
5-bus	3	1.909	5.727	4	0.748	2.993
14-bus	4	13.277	53.109	5	3.243	16.217
30-bus	4	48.413	193.652	5	12.528	62.643

power to regulate the voltage magnitude of the buses where it is connected. From the obtained results, the accuracy of two algorithms was confirmed. However, the direct algorithm is more effective and reliable in terms of computing speed.

Acknowledgment

This research was partially supported by the SGS grant from VSB–Technical University of Ostrava (No. SP2019/20) and by the project TUCENET (No. LO1404).

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Appendix A
The data of modified IEEE system

Tab. 10: Modified IEEE-14 bus system.

Bus	Initial IEEE-14 bus	Modified
No.	$\lvert V\rvert (pu)$	$\lvert V\rvert (pu)$
6 (PV)	1.07	1.05
8 (PV)	1.09	1.05

Tab. 11: Modified IEEE-30 bus system.

Bus	Initial IEEE-30 bus		Modified	
No.	P (MW)	W (Mvar)	P (MW)	W (Mvar)
26	3.5	2.3	7.0	5.6
30	10.6	1.9	21.2	13.1

Appendix B
Jacobian matrix

- The sub-matrix $\mathbf{J}_{11}$ is of order $(n-1)\times(n-1)$ and the elements of $\mathbf{J}_{11}$ are:

$$\begin{aligned}\frac{\partial P_i}{\partial \delta_i} = \sum_{j\neq i}^{n} |V_i||V_j|[-G_{ij}\sin(\delta_i-\delta_j)\\ +B_{ij}\cos(\delta_i-\delta_j)],\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{\partial P_i}{\partial \delta_j} = |V_i||V_j|[G_{ij}\sin(\delta_i-\delta_j)\\ -B_{ij}\cos(\delta_i-\delta_j)],\\ j\neq i.\end{aligned} \tag{27}$$

If bus i connect with STATCOM,

$$\begin{aligned}\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{n} |V_i||V_j|[-G_{ij}\sin(\delta_i-\delta_j)\\ +B_{ij}\cos(\delta_i-\delta_j)] + |V_i||V_{Si}|[g_{Si}\sin(\delta_i-\delta_{Si})\\ -b_{Si}\cos(\delta_i-\delta_{Si})].\end{aligned} \tag{28}$$

- The sub-matrix $\mathbf{J}_{12}$ is of order $(n-1)\times(n-1-m)$ and the elements of $\mathbf{J}_{12}$ are:

$$\begin{aligned}\frac{\partial P_i}{\partial |V_i|} = 2|V_i|G_{ii} + \sum_{j\neq i}|V_j|[G_{ij}\cos(\delta_i-\delta_j)\\ +B_{ij}\sin(\delta_i-\delta_j)],\end{aligned} \tag{29}$$

$$\begin{aligned}\frac{\partial P_i}{\partial |V_j|} = |V_i||[G_{ij}\cos(\delta_i-\delta_j) + B_{ij}\sin(\delta_i-\delta_j)],\\ j\neq i.\end{aligned} \tag{30}$$

If bus i connect with STATCOM,

$$\begin{aligned}\frac{\partial P_i}{\partial |V_i|} = 2|V_i|G_{ii} + 2|V_i|g_{Si} +\\ \sum_{j\neq i}|V_j|[G_{ij}\cos(\delta_i-\delta_j) + B_{ij}\sin(\delta_i-\delta_j)]\\ -|V_{Si}|[g_{Si}\cos(\delta_i-\delta_{Si}) + b_{Si}\sin(\delta_i-\delta_{Si})].\end{aligned} \tag{31}$$

- The sub-matrix $\mathbf{J}_{13}$ is of order $(n-1)\times p$ and the elements of $\mathbf{J}_{13}$ are:

  If bus i connect with STATCOM,

$$\begin{aligned}\frac{\partial P_i}{\partial \delta_{Si}} =\\ -|V_i||V_{Si}|[g_{Si}\sin(\delta_i-\delta_{Si}) - b_{Si}\cos(\delta_i-\delta_{Si})].\end{aligned} \tag{32}$$

Otherwise,

$$\frac{\partial P_i}{\partial \delta_{Sj}} = 0, j\neq i. \tag{33}$$

- The sub-matrix $\mathbf{J}_{14}$ is of order $(n-1)\times p$ and the elements of $\mathbf{J}_{14}$ are:

  If bus i connect with STATCOM,

$$\frac{\partial P_i}{\partial |V_{Si}|} = -|V_i|[g_{Si}\cos(\delta_i-\delta_{Si}) + b_{Si}\sin(\delta_i-\delta_{Si})]. \tag{34}$$

  Otherwise,

$$\frac{\partial P_i}{\partial |V_{Sj}|} = 0, j \neq i. \tag{35}$$

- The sub-matrix $\mathbf{J}_{21}$ is of order $(n-1-m)\times(n-1)$ and the elements of $\mathbf{J}_{21}$ are:

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j\neq i}^{n} |V_i||V_j|[G_{ij}\cos(\delta_i-\delta_j)+B_{ij}\sin(\delta_i-\delta_j)], \tag{36}$$

$$\begin{aligned} \frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j|[G_{ij}\cos(\delta_i-\delta_j) \\ +B_{ij}\sin(\delta_i-\delta_j)], \\ j \neq i. \end{aligned} \tag{37}$$

  If bus i connect with STATCOM,

$$\begin{aligned} \frac{\partial Q_i}{\partial \delta_i} = \sum |V_i||V_j|[G_{ij}\cos(\delta_i-\delta_j) \\ +B_{ij}\sin(\delta_i-\delta_j)] - |V_i||V_{Si}|[g_{Si}\cos(\delta_i-\delta_{Si}) \\ +b_{Si}\sin(\delta_i-\delta_{Si})], \\ j \neq i. \end{aligned} \tag{38}$$

- The sub-matrix $\mathbf{J}_{22}$ is of order $(n-1-m)\times(n-1-m)$ and the elements of $\mathbf{J}_{22}$ are:

$$\begin{aligned} \frac{\partial Q_i}{\partial |V_i|} = -2|V_i|B_{ii}+ \\ +\sum_{j\neq i}|V_j|[G_{ij}\sin(\delta_i-\delta_j) - B_{ij}\cos(\delta_i-\delta_j)], \end{aligned} \tag{39}$$

$$\begin{aligned} \frac{\partial Q_i}{\partial |V_j|} = |V_i|[G_{ij}\sin(\delta_i-\delta_j) - B_{ij}\cos(\delta_i-\delta_j)], \\ j \neq i. \end{aligned} \tag{40}$$

  If bus i connect with STATCOM,

$$\begin{aligned} \frac{\partial Q_i}{\partial |V_i|} = -2|V_i|B_{ii} - 2|V_i|b_{Si} \\ -\sum_{j\neq i}|V_j|[G_{ij}\sin(\delta_i-\delta_j) - B_{ij}\cos(\delta_i-\delta_j)] \\ -|V_{Si}|[g_{Si}\sin(\delta_i-\delta_{Si}) - b_{Si}\cos(\delta_i-\delta_{Si})]. \end{aligned} \tag{41}$$

- The sub-matrix $\mathbf{J}_{23}$ is of order $(n-1-m)\times p$ and the elements of $\mathbf{J}_{23}$ are:

  If bus i connect with STATCOM,

$$\frac{\partial Q_i}{\partial \delta_{Si}} = |V_i||V_{Si}|[g_{Si}\cos(\delta_i-\delta_{Si})+b_{Si}\sin(\delta_i-\delta_{Si})]. \tag{42}$$

  Otherwise,

$$\begin{aligned} \frac{\partial Q_i}{\partial \delta_{Sj}} = 0, \\ j \neq i. \end{aligned} \tag{43}$$

- The sub-matrix $\mathbf{J}_{24}$ is of order $(n-1-m)\times p$ and the elements of $\mathbf{J}_{24}$ are:

  If bus i connect with STATCOM,

$$\frac{\partial Q_i}{\partial |V_{Si}|} = -|V_i|[g_{Si}\sin(\delta_i-\delta_{Si}) - b_{Si}\cos(\delta_i-\delta_{Si})]. \tag{44}$$

  Otherwise,

$$\begin{aligned} \frac{\partial Q_i}{\partial |V_{Sj}|} = 0, \\ j \neq i. \end{aligned} \tag{45}$$

- The sub-matrix $\mathbf{J}_{31}$ is of order $p\times(n-1)$ and the elements of $\mathbf{J}_{31}$ are:

$$\frac{\partial PE_i}{\partial \delta_i} = |V_i||V_{Si}|[g_{Si}\sin(\delta_i-\delta_{Si})+b_{Si}\cos(\delta_i-\delta_{Si})], \tag{46}$$

$$\begin{aligned} \frac{\partial PE_i}{\partial \delta_j} = 0, \\ j \neq i. \end{aligned} \tag{47}$$

- The sub-matrix $\mathbf{J}_{32}$ is of order $p\times(n-1-m)$ and the elements of $\mathbf{J}_{32}$ are:

$$\begin{aligned} \frac{\partial PE_i}{\partial |V_i|} = \\ -|V_{Si}|[g_{Si}\cos(\delta_i-\delta_{Si}) - b_{Si}\sin(\delta_i-\delta_{Si})], \end{aligned} \tag{48}$$

$$\begin{aligned} \frac{\partial PE_i}{\partial |V_j|} = 0, \\ j \neq i. \end{aligned} \tag{49}$$

- The sub-matrix $\mathbf{J}_{33}$ is of order $p\times p$ and the elements of $\mathbf{J}_{33}$ are:

  If bus i connect with STATCOM,

$$\begin{aligned} \frac{\partial PE_i}{\partial \delta_{Si}} = \\ -|V_i||V_{Si}|[g_{Si}\sin(\delta_i-\delta_{Si}) + b_{Si}\cos(\delta_i-\delta_{Si})]. \end{aligned} \tag{50}$$

  Otherwise,

$$\begin{aligned} \frac{\partial PE_i}{\partial \delta_{Sj}} = 0, \\ j \neq i. \end{aligned} \tag{51}$$

- The sub-matrix $\mathbf{J}_{34}$ is of order pxp and the elements of $\mathbf{J}_{34}$ are:

  If bus i connect with STATCOM,

  $$\frac{\partial PE_i}{\partial |V_{Si}|} = 2|V_i|g_{Si} \\ -|V_i|[g_{Si}\cos(\delta_i - \delta_{Si}) - b_{Si}\sin(\delta_i - \delta_{Si})]. \tag{52}$$

  Otherwise,

  $$\frac{\partial PE_i}{\partial |V_{Sj}|} = 0, \\ j \neq i. \tag{53}$$

- The sub-matrix $\mathbf{J}_{41}$ is of order $p \times (n-1)$ and the elements of $\mathbf{J}_{41}$ are:

  $$\frac{\partial F}{\partial \delta} = 0. \tag{54}$$

- The sub-matrix $\mathbf{J}_{42}$ is of order $p \times (n-1-m)$ and the elements of $\mathbf{J}_{42}$ are:

  $$\frac{\partial F_i}{\partial |V_i|} = 1. \tag{55}$$

  Otherwise,

  $$\frac{\partial F_i}{\partial |V_j|} = 0, \\ j \neq i. \tag{56}$$

- The sub-matrix $\mathbf{J}_{43}$ is of order $p \times p$ and the elements of $\mathbf{J}_{43}$ are:

  $$\frac{\partial F}{\partial \delta_S} = 0. \tag{57}$$

- The sub-matrix $\mathbf{J}_{44}$ is of order $p \times p$ and the elements of $\mathbf{J}_{44}$ are:

  $$\frac{\partial F}{\partial |V_{Si}|} = 0. \tag{58}$$