Extended Second Order Sliding Mode Control for Mismatched Uncertain Systems with Only Output Measurable

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Abstract. *Most existing Second Order Sliding Mode Control (SOSMC) approaches are achieved under assumptions that 1) all of state variables must be accessible; 2) the second derivative of all state variables must exist, even though mathematical model of systems uses the first order equations. In this paper, a new adaptive SOSMC scheme is proposed for mismatched uncertain systems in which these above assumptions are required. In this proposed method, only output variables are used in the sliding surface and controller design. The advantage of no need of all state variables in controller design makes the method more useful and realistic since it can be applied to a wider class of systems. Finally, a vertical take-off and landing aircraft at the nominal airspeed of 135 knots is simulated to demonstrate the advantages and effectiveness of the proposed approach.*

Keywords

Adaptive control, linear matrix inequalities, output feedback controller, second order sliding mode control.

1. Introduction

Over the past three decades, there has been an increasing research interest in Sliding Mode Control (SMC) theory and application. The main advantages of SMC are fast global convergence, simplicity of implementation, order reduction, high robustness to external disturbances and insensitivity to model errors and system parameter variations [1]. Thanks to these advantages, the SMC theory has been successfully applied to a wide variety of practical engineering systems such as robot manipulators, aircrafts, underwater vehicles, spacecraft, flexible space structures, electrical motors, power systems, and automotive engines [1], [2] and [3].

Although the sliding mode controller guarantees robustness with respect to uncertainties and external disturbances, chattering is its main drawback. Chattering is the high frequency finite amplitude oscillations occurring because of the discontinuous control signal used in the SMC [4]. Such chattering has many negative effects in practical applications since it may damage the control actuator and excite the undesirable unmodeled dynamics, which probably leads to unforeseen instability [4]. Many authors have applied various techniques to reduce chattering problem across the sliding surface. Recently, some good results have been published in high quality journal such as [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18] and [21]. The authors of [5] have presented a direct way to reduce chattering problems by inserting a fixed or variable boundary layer near the sliding variable so that a smooth continuous control replaces the discontinuous one when the system is inside the boundary layer. This approach can produce a chattering-free system but a larger boundary layer width results in larger errors in control accuracy and a finite steady-state error may occur. Another approach to eliminate the chattering is carried out by using fuzzy control with sliding mode controller [6], using low-pass filtering [7] or nonlinear reaching law [8]. This method can give a chattering-free system but a finite steady-state error may remain.

One of the most effective methods to avoid chattering problems is to use the Second-Order Sliding Mode Control (SOSMC). The basic idea of the second order sliding mode controller is that the discontinuous sign function is made to act on the time derivative of the control inputs and the actual control signal obtained after integration is continuous and hence chattering is removed [9]. In addition, SOSMC allows driving to zero the sliding variable and its consecutive derivatives in the presence of the disturbances/uncertainties increasing the accuracy of the sliding variable stabilization [10]. Thanks to these advantages, the SOSMC with finite-time convergence has been successfully implemented for solution of real problems [11] and [12]. Another approach given in [13] was to present a modified second-order sliding mode control for single-input nonlinear systems. This study guarantees the finite time reaching of the sliding manifold and chattering reduction. The SOSMC proposed in [13] was extended by [14] for a class of uncertain multi input nonlinear systems but the disturbances were not considered in the above approach.

In [15], the second-order sliding mode control approach with additional capabilities of learning and control adaptation was developed to estimate and compensate for the uncertainty affecting the system's dynamics. This technique is capable of reducing the discontinuous control effort to an arbitrarily small quantity. However, the approach given in [15] could not be applied for systems with unknown upper bounds of uncertainties. The study of [16] proposed a robust adaptive SOSMC scheme for a class of uncertain nonlinear systems where the upper bounds of uncertainties are not required to be known in advance. As a result, a finite-time convergent second-order sliding mode is established and the chattering problem is eliminated. In [17], an adaptive second order sliding mode control law is proposed for the control of an electro pneumatic actuator. In order to reduce the overshoot and the settling time, the adaptive second order sliding mode controller with a nonlinear sliding surface is presented in [18]. The authors of [19] have proposed a chattering free adaptive sliding mode controller for stabilizing a class of multi-input multi-output systems. This approach can ensure asymptotical stability of the overall system and eliminate chattering in the control input. In [20], based on the linear quadratic regulator method, an optimal second order sliding mode controller was proposed for a class of matched uncertain systems. By designing a new sliding surface, the approach given in [21] can solve both the chattering and singularity problems in sliding mode control. A second-order sliding mode control method to handle sliding mode dynamics with mismatched term was presented in [22]. In [23], a robust chattering-free control scheme was proposed using second-order fast terminal sliding mode control technique for the tracking problem of a class of uncertain systems with matched and mismatched uncertainties.

However, it is worth pointing out that most of the previous results have been developed under the assumption that all the system states are available for the control law. It may be impossible or prohibitively expensive to measure all of the process variables in some practical systems [24]. For example, it is difficult to measure the variables describing the flexible motion, the modal position, and the velocity of flexible spacecraft [25]. Thus, it is very important to establish a new adaptive SOSMC method to control mismatched uncertain systems via output feedback. Herein, we intent to use the output information completely in the sliding surface and controller design but still remain the advantages of SOSMC such as the chattering-free, the maximum convergence time interval, and the dimension of neighbourhood of the origin to which the controlled trajectory converges [14].

In this paper, we extend the concept of second order sliding mode controller, introduced by [19], [22], [23] and [27], for the aim of stabilizing mismatched uncertain systems where only output variables are accessible. The main contributions of this paper are as follows:

- A new Lemma and a novel adaptive law are established for the aim of controller design using only output variables.
- New sufficient conditions in terms of Linear Matrix Inequalities (LMI) are derived such that the equivalent reduced-order system in the sliding mode is asymptotically stable.
- The two major assumptions by [19], [22], [23] and [27] (that all of state variables must be accessible, and that the second derivative of all state variables must exist) are both eliminated. Therefore, the proposed method can be applied to a wider class of mismatched uncertain systems.

2. System Description and Preliminary Results

Consider the following mismatched uncertain systems:

$$\begin{aligned} \dot{x} &= [A + \Delta A(x,t)]\, x + B\, [u + \xi(x,t)]\,, \\ y &= Cx. \end{aligned} \tag{1}$$

Here $x \in R^n$, $u \in R^m$ and $y \in R^p$ denote the state variables, inputs and outputs, respectively. $\mathbf{A} \in R^{n\times n}$ is state matrix, $\mathbf{B} \in R^{n\times m}$ is the input matrix and $\mathbf{C} \in R^{p\times n}$ is the output matrix. The terms $\Delta\mathbf{A}$ and $\xi(x,t)$ represent the system matrix and the input matrix uncertainties, respectively. We assume that:

Assumption 1. The matrix pair $(\mathbf{A}, \mathbf{B})$ is completely controllable.

Assumption 2. The mismatched uncertainty $\Delta\mathbf{A}(x,t)$ is a norm-bounded time varying uncertainty as follows:

$$\Delta\mathbf{A}(x,t) = \mathbf{DF}(x,t)\mathbf{E}, \quad \| \mathbf{F}(x,t) \leq 1 \|,$$

where $\mathbf{D}$ and $\mathbf{E}$ are known constant real matrices with appropriate dimensions that characterize the structure of the uncertainty, and $\mathbf{F}(x,t)$ is a norm-bounded unknown matrix.

Assumption 3. $rank(\mathbf{CB}) = m$.

From [28], Assumption 3 implies that there exists a non-singular linear coordinate transformation $z = \tilde{T}x$ such that the triple $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ with respect to the new coordinates has the structure

$$\tilde{\mathbf{A}} = \begin{bmatrix} \tilde{A}_1 & \tilde{A}_2 \\ \tilde{A}_3 & \tilde{A}_4 \end{bmatrix}, \tilde{\mathbf{B}} = \begin{bmatrix} 0 \\ \tilde{B}_2 \end{bmatrix}, \tilde{\mathbf{C}} = \begin{bmatrix} 0 & \tilde{C}_2 \end{bmatrix}, \tag{2}$$

where $\tilde{A}_1 \in R^{(n-m)\times(n-m)}$, $\tilde{B}_2 \in R^{m\times m}$ are non-singular and $\tilde{C}_2 \in R^{p\times p}$ is orthogonal.

Assumption 4: The triple $(\tilde{A}_1, \tilde{A}_2, \Xi)$ is output feedback stabilisable, where $\Xi = \begin{bmatrix} 0_{(p-m)\times(n-p)} & I_{(p-m)} \end{bmatrix}$.

Assumption 4 implies that there exist matrix $\tilde{\mathbf{K}}$ such that the matrix $A_1 = \tilde{A}_1 - \tilde{A}_2\tilde{K}\Xi$ is stable. From [28], the coordinate transformation $z = \bar{T}\tilde{x}$ where

$$\bar{\mathbf{T}} = \begin{bmatrix} I & 0 \\ -\tilde{K}\Xi & I \end{bmatrix}, \tag{3}$$

will transform the triple $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ to the following form in the new coordinate system z

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \begin{bmatrix} 0 & C_2 \end{bmatrix}, \tag{4}$$

where $A_1 = \tilde{A}_1 - \tilde{A}_2\tilde{K}\Xi \in R^{(n-m)\times(n-m)}$ is stable and both these matrices $B_2 \in R^{m\times m}$, $C_2 \in R^{p\times p}$ are non-singular.

Remark 1: In [19], [22], [23] and [27], all of state variables $x \in R^n$ must be accessible and the second derivative of all state variables $\ddot{x}$ exist, even though mathematical model of systems is of the first order. The proposed method needs only a subset of state variables $y \in R^p$ to be accessible and the second derivative of output variables $\ddot{y}$ exists. Therefore, the proposed approach can be applied to a wider class of mismatched uncertain systems.

Remark 2: The output feedback SOSMC scheme is proposed in [26]. However, there are three major conditions set by [26]:

- The system under consideration is assumed to be matched.
- The exogenous disturbances are bounded by a known constant value. That is $\| f \| \leq \pi$ where π is known. This condition is quite restrictive.
- The sliding matrix $\mathbf{F}$ satisfies that the matrix $\mathbf{FCAB}$ is invertible to guarantee sliding condition $S(t) = Fy(t) + w(t) = 0$. This limitation is really strong.

Remark 3: The SOSMCs using output variables were subject of many recently researches [29] and [30]. However, all these methods require more hardware and increase system dimension. In this paper, an adaptive output feedback SOSMC scheme is proposed for mismatched uncertain systems where above limitations are eliminated.

3. Adaptive Output Feedback Second Order Sliding Mode Control Design

In this section, we introduce a systematic design procedure of an adaptive output feedback Second Order Sliding Mode Control (SOSMC) scheme. There are three steps involved in the design of an adaptive output feedback SOSMC for system, see Eq. (1). In the first step, a sliding surface is designed to depend on only output variables. In the second step, we derive appropriate Linear Matrix Inequalities (LMI) stability conditions by the Lyapunov method to guarantee that the system in the sliding mode is asymptotically stable. In the third step, we design an adaptive output feedback second order sliding mode controller in a way such that the system states reach the sliding manifold in finite time and remain it thereafter.

3.1. Sliding Surface Design

From Eq. (11), Eq. (12) and Eq. (13) of paper [28], it follows that under Assumptions 3 and 4, there exists a non-singular matrix $\mathbf{T}$ such that in the new coordinates $z = Tx$ the system Eq. (1) can be described as:

$$\begin{aligned} \dot{z} = &\left(\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} F \begin{bmatrix} E_1 & E_2 \end{bmatrix} \right) z + \\ &+ \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \left[u + \xi(T^{-1}z, t) \right], \end{aligned} \tag{5}$$

and

$$\mathbf{y} = \begin{bmatrix} 0 & C_2 \end{bmatrix} z, \tag{6}$$

where $z = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T$, $z_1 \in R^{n-m}$, $z_2 \in R^m$, $\mathbf{TAT}^{-1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$, $\mathbf{TDFET}^{-1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} F \begin{bmatrix} E_1 & E_2 \end{bmatrix}$, $\mathbf{TB} =$

$\begin{bmatrix} 0 \\ B_2 \end{bmatrix}$ and $\mathbf{CT}^{-1} = \begin{bmatrix} 0 & C_2 \end{bmatrix}$. The matrices $B_2 \in R^{m\times m}$, $C_2 \in R^{p\times p}$ are non-singular and $A_1 = \tilde{A}_1 - \tilde{A}_2\tilde{K}\Xi \in R^{(n-m)\times(n-m)}$ is stable.

It follows from Eq. (5) that

$$\dot{z}_1 = (A_1 + D_1FE_1)\, z_1 + (A_2 + D_1FE_2)\, z_2, \tag{7}$$

and

$$\begin{aligned} \dot{z}_2 = (A_3 + D_2FE_1)\, z_1 + (A_4 + D_2FE_2)\, z_2 + \\ + B_2\,[u + \xi]\,. \end{aligned} \tag{8}$$

For the systems Eq. (5) and Eq. (6), consider a sliding surface

$$\sigma(y(t)) = \mathbf{K}C_2^{-1}y = 0, \tag{9}$$

where $\mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & \mathbf{K}_2 \end{bmatrix} = \begin{bmatrix} 0_{m\times(p-m)} & \mathbf{K}_2 \end{bmatrix}$. The matrix $\mathbf{K}_2 \in R^{m\times m}$ is the form of

$$K_2 = \Psi\mathbf{P}\Psi^T, \tag{10}$$

in which $\mathbf{P} \in R^{(n-m)\times(n-m)}$ is defined later and the matrix $\Psi \in R^{m\times(n-m)}$ is selected such that the matrix $\mathbf{K}_2 \in R^{m\times m}$ is non-singular. According to Eq. (6), the sliding surface Eq. (9) can be rewritten as:

$$\begin{aligned} \sigma(y(t)) = \mathbf{K}C_2^{-1}y = \\ = \mathbf{K}\begin{bmatrix} N & 0_{(p-m)\times m} \\ 0_{m\times(n-m)} & I_{m\times m} \end{bmatrix} z = \mathbf{K}_2 z_2 = 0, \end{aligned} \tag{11}$$

where $\mathbf{N} = \begin{bmatrix} O_{(p-m)\times(n-p)} & I_{(p-m)\times(p-m)} \end{bmatrix}$. From Eq. (11) and since $\mathbf{K}_2 \in R^{m\times m}$ is non-singular, in the coordinate z, the sliding surface Eq. (9) can be described by:

$$\{col(z_1, z_2) \mid z_2 = 0\}\,. \tag{12}$$

Using Eq. (12), the dynamic equation in sliding mode is:

$$\dot{z}_1 = (A_1 + D_1FE_1)\, z_1. \tag{13}$$

3.2. Stability Analysis of Sliding Motion

In last section, we have designed an output sliding surface. There are still two important tasks that should be done. The first task is to derive appropriate LMI stability conditions by the Lyapunov method to guarantee that the sliding mode dynamics Eq. (13) is asymptotically stable. The second task is to design an adaptive output feedback SOSMC in a way such that the system states reach the sliding manifold in finite time and stay on it thereafter. Now, we are going to do the former task by considering the following LMI:

$$\begin{bmatrix} A_1^TG + G^TA_1 + \Theta & A_1^TG - G^T + P + \Theta & E_1^T \\ G^TA_1 - G + P + \Theta & -G - G^T + \Theta & 0 \\ E_1 & 0 & -\varphi I \end{bmatrix} < 0, \tag{14}$$

where $\mathbf{G} \in R^{(n-m)\times(n-m)}$ is general and non-zero matrix, $\mathbf{P} \in R^{(n-m)\times(n-m)}$ is any positive matrix, $\Theta = \varphi G^TD_1D_1^TG$ and the scalar $\varphi > 0$. Then, we can establish the following theorem.

Theorem 1. Suppose that Assumptions 1-3 hold. Then, the sliding mode dynamics Eq. (13) is asymptotically stable if the matrices $\mathbf{P} > 0$ and non-zero matrix $\mathbf{G}$ satisfy Eq. (14).

Proof: For the sliding mode dynamics Eq. (13), consider a candidate Lyapunov function function

$$V = z_1^TPz_1, \tag{15}$$

where $\mathbf{P} > 0$ satisfies Eq. (14). The time derivative of V along the trajectories of Eq. (13) is given by

$$\dot{V} = z_1^T\left[(A_1 + D_1FE_1)^TP + P(A_1 + D_1FE_1)\right] z_1. \tag{16}$$

From Eq. (16), if $(A_1 + D_1FE_1)^TP + P(A_1 + D_1FE_1) < 0$ then $\dot{V} < 0$ and the sliding mode dynamics Eq. (13) is asymptotically stable.

Before proving $\dot{V} < 0$, we recall the following Lemmas.

Lemma 1. [31]: Let $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{F}$ be matrices of compatible dimension then $XFY + Y^TF^TX^T < \varphi^{-1}XX^T + \varphi Y^TY$ for any $\mathbf{F}$ satisfying $\|\mathbf{F}\| \leq 1$ and a scalar $\varphi > 0$.

Lemma 2. [31]: Given a symmetric matrix $\mathbf{W}$ and two matrices $\mathbf{\Gamma}$ and, $\mathbf{\Sigma}$ and consider the problem of finding some matrix $\mathbf{G}$ such that $W + \Sigma G\Gamma + (\Sigma G\Gamma)^T < 0$.

Denote $\mathbf{\Sigma}^{\perp}$ and $\mathbf{\Gamma}^{\perp}$ any matrices whose columns form the bases of the null spaces of $\mathbf{\Sigma}$ and $\mathbf{\Gamma}$, respectively. Then the above inequality is solvable for $\mathbf{G}$ if and only if $\Sigma^{\perp}W\Sigma^{\perp T} < 0$, $\Gamma^{T\perp}W\Gamma^{T\perp T} < 0$.

Now, we are going to prove $\dot{V} < 0$. Let us first define $\mathbf{\Gamma}^{\perp} = \begin{bmatrix} I \\ I \end{bmatrix}$, $\mathbf{W} = \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix}$, $\mathbf{\Sigma} = \begin{bmatrix} (A_1 + D_1FE_1)^{\mathbf{T}} \\ -I \end{bmatrix}$, $\mathbf{\Sigma}^{\perp} = \begin{bmatrix} I & (A_1 + D_1FE_1)^T \end{bmatrix}$, $\mathbf{\Gamma}^{T\perp} = \begin{bmatrix} I & -I \end{bmatrix}$ and $\mathbf{G}$ is defined in LMI Eq. (14). Then, we have

$$\begin{aligned} \mathbf{\Lambda} = W + \Sigma G\Gamma + (\Sigma G\Gamma)^T \\ = \begin{bmatrix} A_1^TG + G^TA_1 & A_1^TG - G^T + P \\ G^TA_1 - G + P & -G - G^T \end{bmatrix} \\ + \begin{bmatrix} G^T & D_1 \\ G^T & D_1 \end{bmatrix} F\,\begin{bmatrix} E_1 & 0 \end{bmatrix} + \begin{bmatrix} E_1^T \\ 0 \end{bmatrix} F^T\begin{bmatrix} D_1^TG & D_1^TG \end{bmatrix}. \end{aligned} \tag{17}$$

Applying Lem. 1 to Eq. (17), we achieve

$$\begin{aligned} \mathbf{\Lambda} \leq \begin{bmatrix} A_1^TG + G^TA_1 + \Theta & A_1^TG - G^T + P + \Theta \\ G^TA_1 - G + P + \Theta & -G - G^T + \Theta \end{bmatrix} \\ + \varphi^{-1}\begin{bmatrix} E_1^T \\ 0 \end{bmatrix}\begin{bmatrix} E_1 & 0 \end{bmatrix}, \end{aligned} \tag{18}$$

where $\Theta = \varphi G^T D_1 D_1^T G$ and the scalar $\varphi > 0$.

Using Schur complement formula, LMI Eq. (14) can be rewritten as

$$\begin{bmatrix} A_1^T G + G^T A_1 + \Theta & A_1^T G - G^T + P + \Theta \\ G^T A_1 - G + P + \Theta & -G - G^T + \Theta \end{bmatrix} + \varphi^{-1} \begin{bmatrix} E_1^T \\ 0 \end{bmatrix} \begin{bmatrix} E_1 & 0 \end{bmatrix} < 0. \tag{19}$$

From Eq. (18) and Eq. (19), it can be observed that

$$\mathbf{\Lambda} \leq \begin{bmatrix} A_1^T G + G^T A_1 + \Theta & A_1^T G - G^T + P + \Theta \\ G^T A_1 - G + P + \Theta & -G - G^T + \Theta \end{bmatrix} + \varphi^{-1} \begin{bmatrix} E_1^T \\ 0 \end{bmatrix} \begin{bmatrix} E_1 & 0 \end{bmatrix} < 0. \tag{20}$$

It follows from Eq. (20) and Lem. 2 that

$$\Sigma^{\perp} W \Sigma^{\perp T} < 0. \tag{21}$$

Since the fact $\mathbf{W} = \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix}$, and $\mathbf{\Sigma}^{\perp} = \begin{bmatrix} I & (A_1 + D_1 F E_1)^T \end{bmatrix}$, we obtain:

$$\Sigma^{\perp} W \Sigma^{\perp T} = (A_1 + D_1 F E_1)^T P + P(A_1 + D_1 F E_1) < 0. \tag{22}$$

According to Eq. (16) and Eq. (22), it is obvious that

$$\dot{V} < 0. \tag{23}$$

Note that Eq. (23) verifies that Eq. (14) holds, which further implies that sliding motion is asymptotically stable. The following new Lemma is derived for controller design using only output variables.

Lemma 3. Consider the reduced-order system Eq. (7). If the matrix $\mathbf{A}_1$ is stable then $\| z_1(t) \|$ is bounded by $\eta(t)$ for all time, where $\eta(t)$ is the solution of

$$\dot{\eta}(t) = (k \| D_1 \| \| E_1 \| + \lambda)\, \eta(t) + k\,(k \| A_2 \| + \| D_1 \| \| E_2 \|) \| K_2^{-1} K C_2^{-1} \| \| y \|, \tag{24}$$

where $k \| D_1 \| \| E_1 \| + \lambda < 0$, the scalar $k > 0$ and λ is the maximum eigenvalue of the matrix $\mathbf{A}_1$.

Proof: Solving Eq. (7) gives

$$\begin{aligned} \|z_1(t)\| \leq \int_0^t &\|\exp\{A_1(t-\tau)\}\| \times [\|D_1 F E_1\| \|z_1\| \\ &+ (\| A_2 \| + \| D_1 F E_2 \|) \| z_2 \|]\, \mathrm{d}\tau \\ &+ \| \exp(A_1 t) \| \| z_1(0) \| . \end{aligned} \tag{25}$$

The stable matrix $\mathbf{A}_1$ satisfy the constraint

$$\| \exp(A_1 t) \| \leq k \exp(\lambda t), \tag{26}$$

where $k > 0$ and $\lambda < 0$ are defined in Eq. (3). According to Eq. (25) and Eq. (26) we have

$$\begin{aligned} \|z_1(t)\| \leq \int_0^t & k\, \|\exp\{\lambda(t-\tau)\}\| \times [\|D_1\| \|E_1\| \|z_1\| \\ &+ (\| A_2 \| + \| D_1 \| \| E_2 \|) \| z_2 \|]\, \mathrm{d}\tau \\ &+ k \| \exp(\lambda t) \| \| z_1(0) \| . \end{aligned} \tag{27}$$

Multiplying both sides of Eq. (27) by $\exp(-\lambda t)$, gives

$$\begin{aligned} \|z_1(t)\| \exp(-\lambda t) \leq & \int_0^t k \exp(-\lambda\tau) \|D_1\| \|E_1\| \|z_1\| \mathrm{d}\tau \\ &+ \int_0^t k \exp(-\lambda\tau) \times (\|A_2\| + \|D_1\| \|E_2\|)\, \|z_2\|\, \mathrm{d}\tau \\ &+ k \| z_1(0) \| . \end{aligned} \tag{28}$$

Letting $h(t)$ is the right term of Eq. (28) and taking the time derivative of $h(t)$, yields

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} h(t) = & k \exp(-\lambda t) \| D_1 \| \| E_1 \| \| z_1 \| \\ &+ k \exp(-\lambda t) \times (\| A_2 \| + \| D_1 \| \| E_2 \|) \| z_2 \| . \end{aligned} \tag{29}$$

According to Eq. (28) and Eq. (29) we achieve

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} h(t) \leq & k \| D_1 \| \| E_1 \| h(t) \\ &+ k \exp(-\lambda t)\, (\| A_2 \| + \| D_1 \| \| E_2 \|) \| z_2 \| . \end{aligned} \tag{30}$$

Multiplying both sides of Eq. (30) by $\exp(-k \| D_1 \| \| E_1 \| t)$, gives

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \{h(t) \exp(-k \| D_1 \| \| E_1 \| t)\} \leq & k \exp(-\lambda t) \cdot \\ (\|A_2\| + \|D_1\| \|E_2\|)\, \|z_2\| & \exp(-k\, \|D_1\| \|E_1\|\, t). \end{aligned} \tag{31}$$

Integrating both sides of Eq. (31), we obtain

$$\begin{aligned} h(t) \leq k \| z_1(0) \| \exp(\bar{k} t) + \exp(\bar{k} t) \times \int_0^t k \exp(-\lambda\tau) \\ (\| A_2 \| + \| D_1 \| \| E_2 \|) \| z_2 \| \exp(-\bar{k}\tau) d\tau, \end{aligned} \tag{32}$$

where $\bar{k} = k \| D_1 \| \| E_1 \|$. Since $\| z_1(t) \| \exp(-\lambda t) \leq h(t)$ and $K C_2^{-1} y = K_2 z_2$, therefore it can be obtained that

$$\begin{aligned} \| z_1(t) \| \leq \eta(0) \exp\left[(\bar{k} + \lambda)t\right] + \int_0^t k(\| A_2 \| + \| D_1 \| \\ \|E_2\|) \| K_2^{-1} K C_2^{-1} \| \| y \| \exp\left[(\bar{k} + \lambda)(t-\tau)\right] \mathrm{d}\tau \\ = \eta(t), \quad \text{if} \quad \eta(0) \geq k \| z_1(0) \|, \end{aligned} \tag{33}$$

where $\eta(t)$ is defined in Eq. (3). Therefore, it is easy to conclude that $\eta(t) \geq \| z_1(t) \|$ for all time, if $\eta(0)$ sufficiently large.

3.3. Adaptive Output Feedback Second Order Sliding Mode Controller Design

In the last section, we dealt with the first and second elements of the design process. In this section, we design an adaptive output feedback second order sliding mode controller such that the system states reach the sliding manifold in finite time and stay on it thereafter. Let us begin with defining the sliding manifold $s(t)$ such that

$$s(t) = \dot{\sigma} + X\sigma \tag{34}$$

and

$$\dot{s}(t) = \ddot{\sigma} + X\dot{\sigma}, \tag{35}$$

where $\mathbf{X} \in diag(\chi_1, \chi_2, ...\chi_m)$ is any diagonal matrix. The main idea behind the second-order sliding mode is to act on the second-order derivative of the sliding variable $\ddot{\sigma}$ rather than the first derivative as in conventional sliding mode. The second-order sliding mode is determined from the basic equality condition $\dot{\sigma} = \ddot{\sigma} = 0$ reaches in finite time, whereas the proposed controller reaches the condition asymptotically.

According to Eq. (9), we obtain

$$s(t) = KC_2^{-1}\dot{y} + X\sigma \tag{36}$$

and

$$\dot{s}(t) = KC_2^{-1}\ddot{y} + X\dot{\sigma}. \tag{37}$$

Since $KC_2^{-1}y = K_2 z_2$ and Eq. (8), Eq. (37) can be rewritten as:

$$\dot{s}(t) = K_2\left[A_3\dot{z}_1 + A_4\dot{z}_2\right] + B_2\dot{u} + \dot{\psi} + X\dot{\sigma}, \tag{38}$$

where $\psi = K_2D_2FE_1z_1 + K_2D_2FE_2z_2 + B_2\xi$.

Assumption 5. The disturbance $\xi(t)$ of Eq. (38) in the domain of interest satisfies

$$\dot{\psi}(t) \leq \sum_{i=0}^{r} a_i \parallel x \parallel^i, \tag{39}$$

where a_i are unknown positive constants, r is a designed positive integer.

The proposed adaptive output feedback second order sliding mode controller for tackling the system uncertainty is designed as follows:

$$\begin{aligned} u(t) = u(0) - \int_0^t (K_2B_2)^{-1}\left[\kappa(t)\right. \\ \left. + \rho\eta(t) + \bar{\rho} \parallel y \parallel + \hat{\rho} \parallel \dot{y} \parallel + \alpha\right] \frac{s(t)}{\parallel s(t) \parallel} \mathrm{d}t, \end{aligned} \tag{40}$$

where the scalar $\alpha > 0$,
$\rho = \parallel K_2 \parallel\parallel A_3 \parallel (\parallel A_1 \parallel + \parallel D_1 \parallel\parallel E_1 \parallel)$,
$\kappa(t) = \sum_{i=0}^{r} \hat{a}_i(t)(\parallel H_1 \parallel \eta(t) + \parallel H_2 \parallel\parallel K_2^{-1}KC_2^{-1} \parallel\parallel y \parallel)^i$,
$\hat{\rho} = \parallel K_2 \parallel\parallel A_4 \parallel\parallel K_2^{-1}KC_2^{-1} \parallel + \parallel X \parallel\parallel KC_2^{-1} \parallel$,
$\bar{\rho} = \parallel K_2 \parallel\parallel A_3 \parallel (\parallel A_2 \parallel + \parallel D_1 \parallel\parallel E_2 \parallel) \parallel K_2^{-1}KC_2^{-1} \parallel$, and $\begin{bmatrix} H_1 & H_2 \end{bmatrix} = T^{-1}$. The adaptive gains $\hat{a}_i(t)$ are given by

$$\begin{aligned} \dot{\hat{a}}_i(t) = q_i \left[-\breve{q}_i\hat{a}_i + (\parallel H_1 \parallel \eta \right. \\ \left. + \parallel H_2 \parallel\parallel K_2^{-1}KC_2^{-1} \parallel\parallel y \parallel)^i\right]. \end{aligned} \tag{41}$$

The time function $\eta(t)$ is the solution of Eq. (3) and q_i, $\breve{q}_i$ are positive constants. The major drawback of sliding mode control is so-called chattering phenomenon. Such a phenomenon consists of the oscillation of the control signal, tied to the discontinuous nature of the control strategy, at a frequency and with an amplitude capable of disrupting, damaging or, at least, wearing the controlled physical system. It should be pointed out that the controller Eq. (40) is a continuous control input and uses only output variables. Hence the undesired high frequency chattering of the control signal is eliminated. This is a new contribution of the proposed method.

Now let us discuss the reaching conditions in the following theorem.

Theorem 2. Consider the uncertain dynamic system defined by Eq. (1) with the assumptions 1–4. If the sliding manifold and the adaptive sliding mode controller are designed as Eq. (34) and Eq. (40), respectively. Then, the system states reach the sliding manifold $s(t)$ in finite time and stay on it thereafter.

Proof: Define a Lyapunov function candidate as follows:

$$V(t) = \parallel s \parallel + \frac{1}{2}\sum_{i=0}^{r}\frac{\tilde{a}_i^2}{q_i}, \tag{42}$$

where $\tilde{a}_i(t) = a_i - \tilde{a}_i(t)$, $i = 0, 1, ..., r$ are the estimation errors of the adaptive gains. Now taking the derivative of $V(t)$ yields

$$\dot{V}(t) = \frac{s^T}{\parallel s \parallel}\dot{s} - \sum_{i=0}^{r}\frac{1}{q_i}\dot{\hat{a}}_i\tilde{a}_i. \tag{43}$$

Using Eq. (7), Eq. (38) and Eq. (43) and property $\parallel AB \parallel \leq \parallel A \parallel\parallel B \parallel$, it generates

$$\begin{aligned} \dot{V}(t) \leq \parallel K_2 \parallel\parallel A_3 \parallel [(\parallel A_1 \parallel + \parallel D_1 \parallel\parallel E_1 \parallel) \parallel z_1 \parallel \\ + (\parallel A_2 \parallel + \parallel D_1 \parallel\parallel E_2 \parallel) \parallel z_2 \parallel] + \parallel K_2 \parallel\parallel A_4 \parallel\parallel \dot{z}_2 \parallel \\ + \frac{s^T}{\parallel s \parallel}K_2B_2\dot{u} + \parallel \dot{\psi} \parallel + \parallel X \parallel\parallel \dot{\sigma} \parallel - \sum_{i=0}^{r}\frac{1}{q_i}\dot{\hat{a}}_i\tilde{a}_i. \end{aligned} \tag{44}$$

From Assumption 4 and $\parallel x \parallel \leq \parallel H_1 \parallel\parallel z_1 \parallel + \parallel H_2 \parallel\parallel z_2 \parallel$ where $\begin{bmatrix} H_1 & H_2 \end{bmatrix} = T^{-1}$. We can obtain that

$$\begin{aligned} \dot{V}(t) \leq \tfrac{s^T}{\|s\|}K_2B_2\dot{u} - \textstyle\sum_{i=0}^{r}\tfrac{1}{q_i}\dot{\hat{a}}_i\tilde{a}_i + \parallel K_2 \parallel\parallel A_3 \parallel \\ [(\parallel A_1 \parallel + \parallel D_1 \parallel\parallel E_1 \parallel) \parallel z_1 \parallel + (\parallel A_2 \parallel + \parallel D_1 \parallel \\ \parallel E_2 \parallel) \parallel z_2 \parallel] + \parallel X \parallel\parallel \dot{\sigma} \parallel + \parallel K_2 \parallel\parallel A_4 \parallel\parallel \dot{z}_2 \parallel \\ + \textstyle\sum_{i=0}^{r} a_i(\parallel H_1 \parallel\parallel z_1 \parallel + \parallel H_2 \parallel\parallel z_2 \parallel)^i. \end{aligned} \tag{45}$$

Equation (11) implies that

$$z_2 = K_2^{-1} K C_2^{-1} y. \tag{46}$$

According to Lem. 3, we have

$$\| z_1(t) \| \leq \eta(t). \tag{47}$$

From Eq. (41), Eq. (45), Eq. (46) and Eq. (47), it can be observed that

$$\begin{aligned} \dot{V}(t) &\leq \rho\eta + \bar{\rho} \| y \| + \hat{\rho} \| \dot{y} \| + \sum_{i=0}^{r} \breve{q}_i \hat{a}_i (a_i - \hat{a}_i) \\ &+ \sum_{i=0}^{r} \hat{a}_i \left(\| H_1 \| \eta + \| H_2 \| \| K_2^{-1} K C_2^{-1} \| \| y \| \right)^i \\ &+ \frac{s^T}{\| s \|} K_2 B_2 \dot{u}. \end{aligned} \tag{48}$$

Substituting the controller Eq. (40) into Eq. (48), we achieve

$$\dot{V}(t) \leq -\alpha - \sum_{i=0}^{r} \breve{q}_i (\hat{a}_i - \frac{a_i}{2})^2 + \sum_{i=0}^{r} \breve{q}_i \frac{a_i^2}{4}. \tag{49}$$

Then, from Eq. (49), it is easy to see that the uniform ultimate boundedness can be guaranteed.

The proposed adaptive SOSMC, using the output information completely in the sliding surface and controller design, offers following advantages. Firstly, conservatism is reduced and the robustness is enhanced. Secondly, an improved transient performance can be obtained without the knowledge about the upper bound of the system uncertainties. Finally, the chattering in the control input is removed.

Design procedure: The proposed adaptive output feedback SOSMC scheme can be simultaneously designed by the following steps.

- Step 1: Find a feasible solution of LMI Eq. (14) and calculate the scaling of sliding surface parameter K_2 using Eq. (10).
- Step 2: Design the sliding surface $\sigma(t)$ according to Eq. (9).
- Step 3: Design the sliding manifold $s(t)$ using Eq. (36).
- Step 4: Design the adaptive output feedback SOSMC $u(t)$ according to Eq. (40).

4. Numerical Example

In order to demonstrate the validity and effectiveness of the proposed method, in this section, we are going to apply the adaptive output feedback SOSMC given in previous sections for a Vertical Take-Off and Landing (VTOL) aircraft at the nominal airspeed of 135 knots, which is modified from [19].

$$\begin{aligned} \dot{x} &= \left(\begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.0455 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.42 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \Delta A \right) x + \\ &+ \begin{bmatrix} -0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix} (u + \xi), \end{aligned} \tag{50}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x, \tag{51}$$

where $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$, $u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T$ with x_1 is the horizontal velocity (knots), x_2 is the vertical velocity (knots), x_3 is the pitch rate (degrees per second) and x_4 is the pitch angle (degrees), u_1 is the collective pitch control, u_2 is the longitudinal cyclic pitch control. It is assumed that x_1, x_2, and x_4 are the output signals. The mismatched uncertainty is given as $\Delta A = DFE$ with $\mathbf{D} = \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}^T$, $\mathbf{E} = \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}$ and

$$F = \sin\left(x_2 \times t + 4 x_1 \times \pi \times t + x_1 x_3 x_4 \times t \right). \tag{52}$$

The disturbance is assumed to satisfy the following condition $\| \dot{\psi}(t) \| \leq 2 + 0.1 \| x \|$. For this work, the following parameters are selected as follows: $\alpha = 300.1$, $\varphi = 10.0109$, and $k = 1.001$.

Assumptions 2 and 3 can be shown to hold. The coordinate transformation $z = Tx$ is given by:

$$\mathbf{T} = \begin{bmatrix} -1 & 0.038 & 0.10514 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.841 & 0.540 & 0.758 \\ 0 & -0.540 & 0.841 & 0.539 \end{bmatrix}.$$

Solving LMI Eq. (14), we have the solution $\mathbf{P} = \begin{bmatrix} 0.320 & -0.045 \\ -0.045 & 2.254 \end{bmatrix}$ and $\mathbf{G} = \begin{bmatrix} 0.269 & 0 \\ 0 & 0.942 \end{bmatrix}$. The matrix $\mathbf{K}_2 = \begin{bmatrix} 0.3204 & 0.00025 \\ 0.00025 & 0.27545 \end{bmatrix}$ is non-singular. The sliding surface is given as $\mathbf{\Sigma}(x) = \begin{bmatrix} 1.64858 & 0.20519 & 0.24317 \\ 2.20577 & -0.23462 & 0.27545 \end{bmatrix} y = 0.$

The controller for the system Eq. (50) and Eq. (51) is the solution of the following equation:

$$\begin{aligned} u(t) = u(0) - \int_0^t \Bigg\{ & \begin{bmatrix} -0.94575 & -0.5525 \\ -0.7877 & 0.00072 \end{bmatrix} \Big[\kappa(t) + \\ & +1.346\eta(t) + 13.376\|y\| + 15.338\|\dot{y}\| + 300 \Big] \frac{s}{\| s \|} \Bigg\} \mathrm{d}t, \end{aligned} \tag{53}$$

where $\kappa(t) = \sum_{i=0}^{1} 2.999 \hat{a}_i(t) (1.375\eta(t) + 10.109 \| y \|)^i$, $\dot{\hat{a}}_i(t) = -180 \hat{a}_i(t) + 2.999 (1.375\eta(t) + 10.109 \| y \|)^i$,

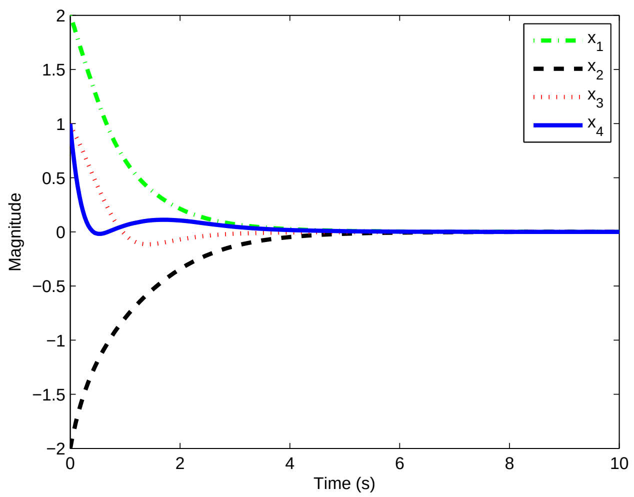

Fig. 1: Time responses of states x_1 (dash-dot), x_2 (dashed), x_3 (dotted), x_4 (solid).

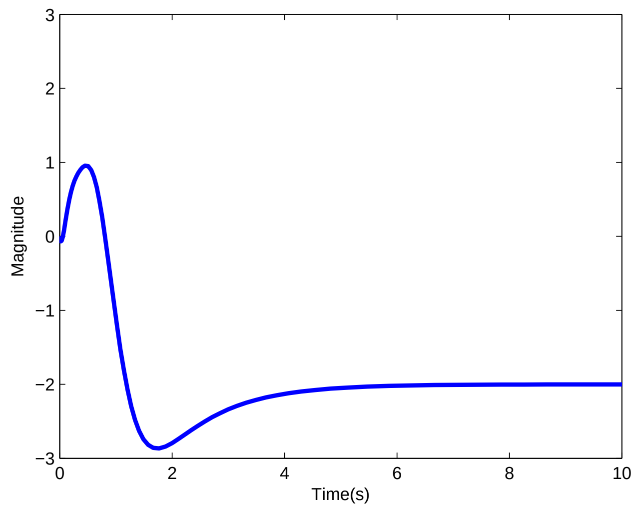

Fig. 2: Control input u_1.

$i = 0, 1$ and $\dot{\eta}(t) = -0.048996\eta(t) + 10.109 \parallel y \parallel$, $\eta(0) = 0.1$.

The initial conditions for the above system are selected to be $\mathbf{x}(0) = \begin{bmatrix} 2 & -2 & 1 & 1 \end{bmatrix}^T$ and $\mathbf{u}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. The system states and the control input of the VTOL aircraft system using the proposed adaptive output feedback SOSMC are shown in Fig. 1, Fig. 2 and Fig. 3. It is evident from Fig. 1 that the proposed adaptive output feedback SOSMC produces faster convergence of the system states to equilibrium as compared to the method proposed by [19]. It is observed from Fig. 2 and Fig. 3 that the actual control input obtained by the proposed method is smooth and chattering free. Convergence of the sliding surface is shown in Fig. 4 and Fig. 5. Figure 4 and Fig. 5, clearly show that the proposed sliding surface is smooth and approach to equilibrium point quickly.

Remark 4: The method proposed by [19] cannot be applied for the system Eq. (50) and Eq. (51) if the state variable x_3 is unmeasurable. This limitation has been removed by the proposed adaptive output feedback SOSMC Eq. (53) because the proposed controller Eq. (53) only uses three output variables (x_1, x_2, and x_4).

Remark 5: The mismatched parameter uncertainties in the state matrix of the system Eq. (50) and Eq. (51) are non-linear and time-varying. Thus, the approaches given in [26] could not be applied for the system defined by Eq. (50) and Eq. (51).

5. Conclusion

This paper has presented a new adaptive Second Order Sliding Mode Control (SOSMC) for mismatched uncertain systems where only output information is available. The proposed SOSMC is guaranteed that the system in sliding mode is asymptotically stable and the state trajectories reach the sliding manifold in finite time and stay on it thereafter. Furthermore, system performance using the proposed control is good without and no chattering phenomenon exists. The most significant advantage of the proposed SOSMC scheme is that the measurement of all the system state variables which are required in most existing SOSMCs is removed. This is valuable for cases in which the system state variables are unavailable.

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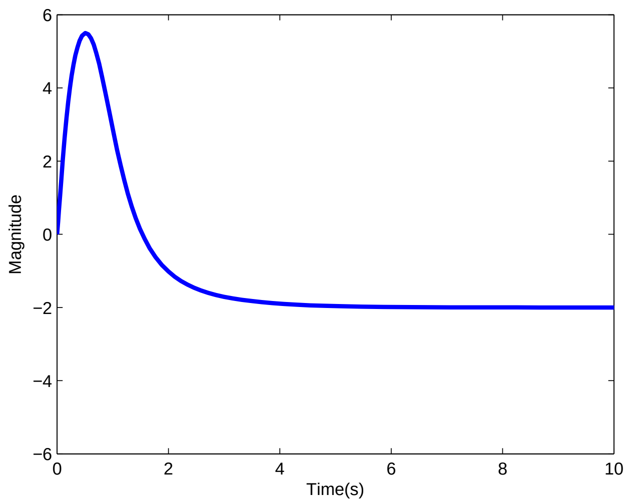

Fig. 3: Control input u_2.

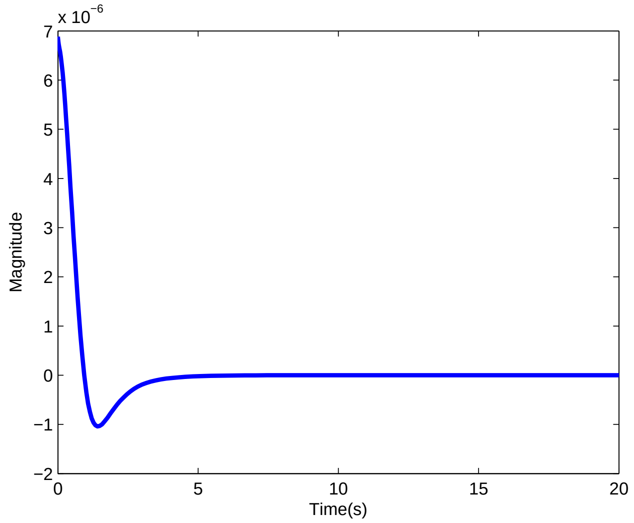

Fig. 4: Sliding surface σ_1.

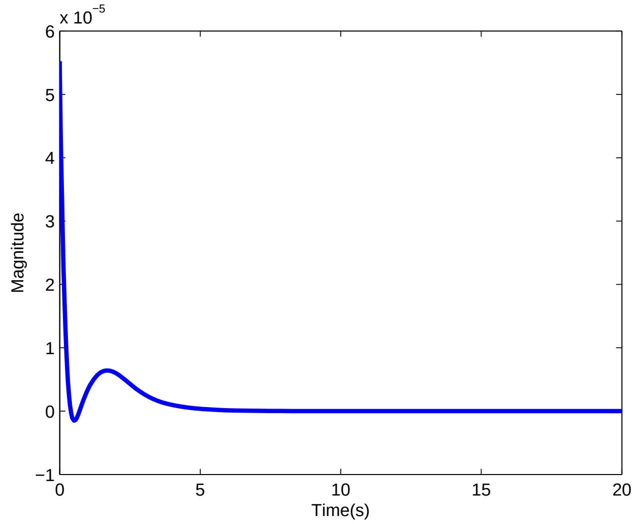

Fig. 5: Sliding surface σ_2.

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