

## Frequency Offset Correction for OFDM Systems

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**Abstract:** In this paper, we propose a robust method to estimate carrier frequency offset (CFO) for orthogonal frequency division multiplexing (OFDM) systems. A training symbol with two identical halves is employed to measure the fractional part of the CFO. While the integral part is estimated by using a novel noise subspace based metric. Simulation results demonstrate that the proposed method can achieve an estimation range equal to the whole bandwidth of the OFDM signal. *Copyright © 2013 IFSA.*

**Keywords:** Carrier frequency offset, Noise subspace, OFDM, Wireless communications.

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### 1. Introduction

Orthogonal frequency division multiplexing (OFDM) has received considerable interest in the last few years for its advantages in high-bit-rate transmissions over frequency-selective fading channels. In OFDM systems, the entire channel is divided into many narrow subchannels, which are transmitted in parallel, thereby increasing the symbol duration and inter-symbol interference (ISI) is reduced due to the cyclic prefix (CP) insertion.

However, OFDM is extremely sensitive to synchronization errors, especially the carrier frequency offset (CFO), which is induced by oscillator discrepancies between the transmitter and receiver and/or Doppler shifts [1, 2]. As a result, CFO estimation for OFDM became an active area of research and has received significant attention in recent years [3-9]. [3] and [4] focus on the analysis of Cramér-Rao bound (CRB) and identifiability problem in the estimation of CFO. In [5], Moose proposes a maximum likelihood estimator of the CFO offset by using two consecutive and identical training symbols. But, the estimation range of this method is limited

within half subcarrier spacing. In order to widen the estimation range, the structures of the training symbols are redesigned by Schmidl and Cox in [6]. The first training symbol has two identical halves and serves to measure the fractional part of the frequency offset. The left integral part is estimated by using the second training symbol which is redesigned to contain a pseudonoise sequence. Based on the Schmidl and Cox method, an improved frequency offset estimation method is proposed by Morelli et al. in [8], using only one training symbol with  $L$  identical components. By making  $L$  sufficiently large, this method can achieve the desired estimation range. However, shortening the duration of identical component will result in decreased estimation accuracy [10]. In order to reduce the interference introduced by the multipath channel, a zero-masking approach is proposed by Minn et al. in [11]. This zero-masking method can be applied if the number of samples in the basis part is much larger than the maximum channel delay spread.

In this paper, we propose a robust CFO estimation method which is developed based on the well known DFT-based channel estimation in [12]. A training

symbol with two identical halves is employed to measure the fractional part of the frequency offset. By using a novel noise subspace based metric to estimate the integral part of the CFO, the proposed method can achieve an estimation range equal to the whole bandwidth of the OFDM signal.

## 2. Brief System Description

We consider an OFDM system with  $N$  subcarriers, each having a frequency spacing of  $B_{sub}$ . Thus, the overall bandwidth is  $B=NB_{sub}$ . In an OFDM symbol, a vector  $\mathbf{X}=[X(0), X(1), \dots, X(N-1)]$  will be transmitted with  $X(k)$  being a symbol from a complex valued alphabet. The corresponding time domain vector  $\mathbf{x}=[x(0), x(1), \dots, x(N-1)]$  is obtained by applying the inverse discrete Fourier transform (IDFT) on  $\mathbf{X}$ , i.e.  $\mathbf{x}=\text{IDFT}\{\mathbf{X}\}$ . This vector corresponds to a series of time domain samples, spaced by sampling period  $T$  and,  $T=1/B$ . Before transmitting the signal, a CP, which is a copy of the last  $G$  samples in  $\mathbf{x}$ , is inserted. The CP length is chosen to be larger than the maximum delay spread to prevent any interference between adjacent symbols.

In this paper, we assume that the timing synchronization is perfect and the channel is constant during the transmission of one OFDM symbol. When these conditions are satisfied, the baseband equivalent discrete-time signal at receiver can be written as

$$y(k) = \sum_{l=0}^{L-1} h(l)x(k-l), \quad (1)$$

where  $L$  is the total number of paths.  $\{h(l)\}$  are the different path complex gains which are assumed to be zero-mean complex Gaussian random variables with  $E\{h(l)h^*(n)\}=0$  for  $l \neq n$  and  $E\{|h(l)|^2\}=\sigma_l^2$ .

Denoting by  $\nu$  the CFO normalized to  $B_{sub}$ , the received OFDM symbol has the following form

$$r(k) = e^{j2\pi\nu k/N} y(k) + n(k), \quad (2)$$

where  $n(k)$  is the white complex Gaussian noise with zero mean and variance  $\sigma_n^2 = E\{|n(k)|^2\}$ . The signal-to-noise ratio (SNR) is defined as  $SNR \equiv \sigma_s^2 / \sigma_n^2$  with  $\sigma_s^2 \equiv E\{|x(k)|^2\}$ .

The training symbol consists of two identical halves which are generated by transmitting a complex pseudonoise sequence on the even subcarriers, while zeros are used on the odd subcarriers. Since the training symbol has the same structure as that in [6], the fractional part of the frequency offset can be estimated using the method proposed by Schmidl. In this letter, we focus on the estimation of integral part

of the frequency offset and assume that  $\nu$  is an integer in the following analysis.

## 3. Estimation Method

When  $\nu=0$ , the received signal on the even subcarriers can be written in a matrix form as

$$\bar{\mathbf{R}} = \frac{1}{\sqrt{N}} \mathbf{F}_1 \bar{\mathbf{r}} = \mathbf{X}_p \mathbf{H} + \mathbf{V}, \quad (3)$$

where  $\mathbf{X}_p$  is the diagonal matrix of the transmitted pseudonoise sequence, i.e.,  $\mathbf{X}_p = \text{diag}[\mathbf{X}_p(0), \mathbf{X}_p(1), \dots, \mathbf{X}_p(N/2-1)]$  and  $\mathbf{V}$  is the white Gaussian noise in the frequency domain.  $\bar{\mathbf{r}} = [r(G), r(G+1), \dots, r(N+G)]^T$  is the received signal in the time domain with CP removed.  $\mathbf{F}_1$  is a discrete Fourier transform (DFT) matrix retaining only those rows corresponding to the even subcarriers. In other words, the elements of  $\mathbf{F}_1$  are given by

$$[\mathbf{F}_1]_{p,q} = e^{-\frac{j2\pi pq}{N}}, \quad \text{for } p=0,2,\dots,N-2 \quad (4)$$

and  $q=0,1,\dots,N-1$ .

$\mathbf{H} = [H(0), H(2), \dots, H(N-2)]^T$  is a  $(N/2) \times 1$  vector containing the channel frequency response (CFR) at the even subcarriers. According to [13, 14], the CFR can be expressed by

$$H(k) = \sum_{l=0}^{L-1} h(l) e^{-j\frac{2\pi kl}{N}}. \quad (5)$$

Therefore, the channel impulse response (CIR) can be estimated by [15, 16]

$$\hat{\mathbf{h}} = \left[ \hat{h}(0), \hat{h}(1), \dots, \hat{h}\left(\frac{N}{2}-1\right) \right]^T, \quad (6)$$

$$= \mathbf{F}_2^{-1} \mathbf{X}_p^{-1} \bar{\mathbf{R}}$$

where  $\mathbf{F}_2$  is the DFT matrix retaining the first  $N/2$  columns and the even rows, i.e.,

$$[\mathbf{F}_2]_{p,q} = e^{-\frac{j2\pi pq}{N}}, \quad \text{for } p=0,2,\dots,N-2 \quad (7)$$

and  $q=0,1,\dots,N/2-1$ .

According to [17],  $\hat{\mathbf{h}}$  is the minimum mean-square error (MMSE) estimate of CIR. It should be noted that since the CIR has at most  $G$  taps, all the other samples in  $\hat{\mathbf{h}}$  compose the noise subspace.

When  $\nu > 0$ , the phases of the samples in  $\bar{\mathbf{r}}$  are rotated by a diagonal matrix  $\mathbf{D}(\nu)$  given by

$$\mathbf{D}(\nu) = \begin{bmatrix} e^{j\frac{2\pi\nu G}{N}} & & & 0 \\ & e^{j\frac{2\pi\nu(G+1)}{N}} & & \\ & & \ddots & \\ 0 & & & e^{j\frac{2\pi\nu(G+N-1)}{N}} \end{bmatrix} \quad (8)$$

However, if the phase rotations can be compensated,  $\hat{\mathbf{h}}$  is still the MMSE estimate of CIR. Otherwise,  $\hat{\mathbf{h}}$  is only a noise vector.

Based on the observation that the correct CFO corresponds to the MMSE estimate of CIR, we propose the following noise subspace based metric to estimate  $\nu$

$$\hat{\nu} = \min_{\varepsilon \in (-N/2, N/2]} \rho^\varepsilon = \min_{\varepsilon \in (-N/2, N/2]} \frac{\sum_{l=G}^{N/2-1} |\hat{h}^\varepsilon(l)|^2}{\sum_{l=0}^{G-1} |\hat{h}^\varepsilon(l)|^2} \quad (9)$$

where

$$\begin{aligned} \hat{\mathbf{h}}^\varepsilon &= \left[ \hat{h}^\varepsilon(0), \hat{h}^\varepsilon(1), \dots, \hat{h}^\varepsilon\left(\frac{N}{2}-1\right) \right]^T, \\ &= \frac{1}{\sqrt{N}} \mathbf{F}_2^{-1} \mathbf{X}_p^{-1} \mathbf{F}_1 \mathbf{D}(\varepsilon) \bar{\mathbf{r}}. \end{aligned} \quad (10)$$

$\mathbf{D}(\varepsilon)$  is the diagonal matrix written by

$$\mathbf{D}(\varepsilon) = \begin{bmatrix} e^{-j\frac{2\pi\varepsilon G}{N}} & & & 0 \\ & e^{-j\frac{2\pi\varepsilon(G+1)}{N}} & & \\ & & \ddots & \\ 0 & & & e^{-j\frac{2\pi\varepsilon(G+N-1)}{N}} \end{bmatrix} \quad (11)$$

Fig. 1 shows an example of the proposed metric in a run of simulation for SNR=30 dB. The simulation parameters are shown in section IV. In the simulation, we assume that the channel remains unchanged during one OFDM symbol period and the total power of the channel is normalized to unity. In Fig. 1, the correct CFO is indexed by 10 which corresponds to the  $\varepsilon$  with the minimal value of  $\rho^\varepsilon$ .

In fact, when  $\varepsilon = \nu$ , each of the first  $G$  elements in  $\hat{\mathbf{h}}^\varepsilon$  consists of not only the CIR, but also the noise samples. So, the expectation of the denominator in Eq. (9) can be derived as

$$\mu_1 = \mathbb{E} \left\{ \sum_{l=0}^{G-1} |\hat{h}^\varepsilon(l)|^2 \right\} = \sum_{l=0}^{L-1} \sigma_l^2 + \frac{G}{N} \frac{1}{\text{SNR}}. \quad (12)$$

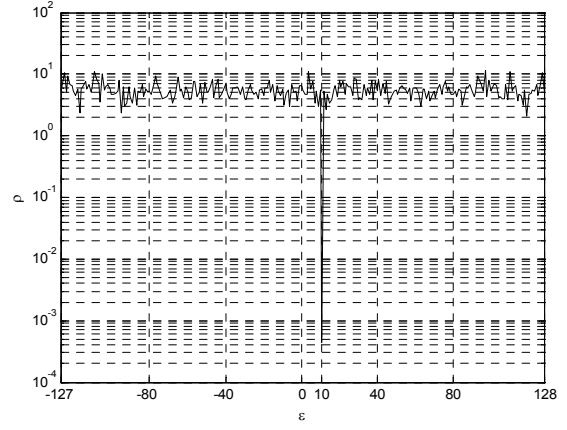


Fig. 1. An example of metric for SNR=30 dB.

Since the last samples correspond to the noise, the expectation of the numerator can be written by

$$\mu_2 = \mathbb{E} \left\{ \sum_{l=G}^{N/2-1} |\hat{h}^\varepsilon(l)|^2 \right\} = \frac{1}{N} \frac{1}{\text{SNR}} \left( \frac{N}{2} - G \right). \quad (13)$$

When  $\varepsilon \neq \nu$ , two cases need to be considered. If  $|\varepsilon - \nu|$  is an odd number, the residual CFO will shift the training sequence from even subcarriers to odd subcarriers. In this case, the elements in  $\hat{\mathbf{h}}^\varepsilon$  are only the noise samples. So, the expectations of the denominator and the numerator can be derived respectively as

$$\mu_1 = \frac{G}{N} \frac{1}{\text{SNR}}. \quad (14)$$

and

$$\mu_2 = \frac{1}{N} \frac{1}{\text{SNR}} \left( \frac{N}{2} - G \right). \quad (15)$$

If  $|\varepsilon - \nu|$  is an even number, the shifted training sequence is still located on the even subcarriers. In this case, the expectations of the denominator and the numerator will change to

$$\mu_1 = \frac{G}{N} \left( 2 \sum_{l=0}^{L-1} \sigma_l^2 + \frac{1}{\text{SNR}} \right) \quad (16)$$

and

$$\mu_2 = \frac{1}{N} \left( 2 \sum_{l=0}^{L-1} \sigma_l^2 + \frac{1}{\text{SNR}} \right) \left( \frac{N}{2} - G \right). \quad (17)$$

Fig. 2 to Fig. 4 show the values of  $\mu_1$  and  $\mu_2$  for  $\varepsilon=10$ ,  $\varepsilon=59$  and  $\varepsilon=110$ , respectively. In the simulation, we assume the correct CFO is indexed by 10 and the power of transmitted signal  $\sigma_s^2$  is normalized to unity. The other simulation parameters are given in section IV.

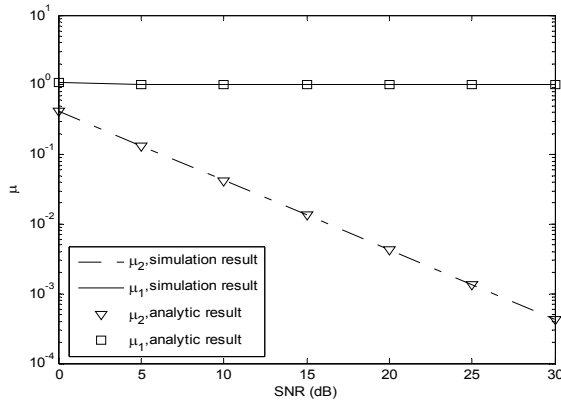


Fig. 2. Expectations of numerator and denominator for  $\varepsilon = 10$ .

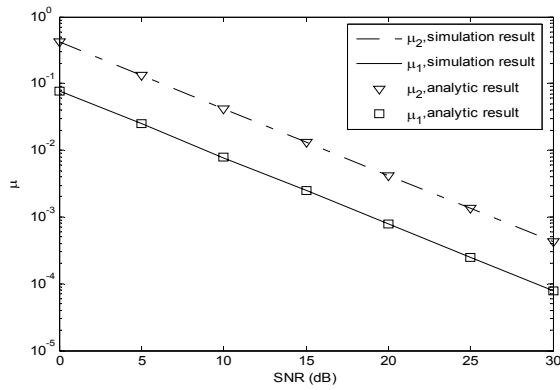


Fig. 3. Expectations of numerator and denominator for  $\varepsilon = 59$ .

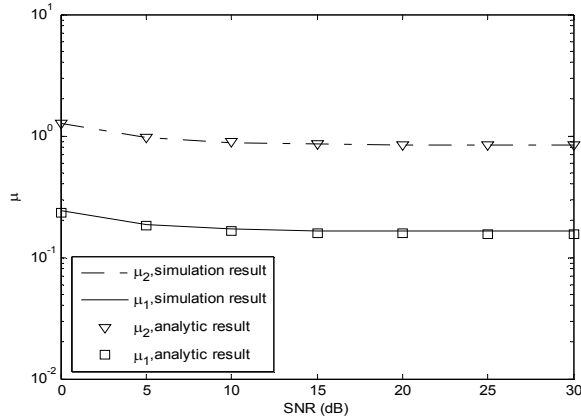


Fig. 4. Expectations of numerator and denominator for  $\varepsilon = 110$ .

In Fig. 2 to Fig. 4, the analytical results are calculated from (12) to (17) which are found to coincide well with the simulation results. Actually, as long as  $\mu_2/\mu_1$  at  $\varepsilon=v$  is smaller than its value at  $\varepsilon \neq v$ , the metric will have the minimal value at the correct CFO in the mean sense. It should be noted that when the CFO is suppose to have fractional part, we need to estimate the

fractional part firstly and then substitute  $\mathbf{D}(\varepsilon)$  by  $\mathbf{D}(\varepsilon + \hat{v}_f)$  in the metric, where  $\hat{v}_f$  is the estimate of the fractional part.

#### 4. Simulation and Results

Several computer simulations are carried out in this section to evaluate the performance of the proposed CFO estimation method. The main simulation parameters for an OFDM system are chosen as follows: the sampling frequency is 5 MHz, the carrier frequency is 2 GHz, the number of subcarriers  $N$  is 256, the mobile speed is 100 km/h, and the CP takes the value of 20. The multipath fading channel is modeled by a  $T$ -spaced tapped-delay line filter with tap gains generated by the Jakes method [18]. The channel is assumed to have 16 paths, with path delays of 0, 1, ..., 15 samples and an exponential power delay profile  $E\{h(l)^2\} = \exp(-(l-1)/8)$ ,  $l=1,2,\dots,16$ .

In the first simulation test, we assume that the CFO is an integer and generated randomly from the range  $(-128,128)$ . The proposed method is evaluated under the SNR of 0, 4, 8, 12, 16 and 20 dB, respectively. Simulation results show that the probabilities of the correct estimation are all one which confirmed that the proposed method has an estimation range equal to the whole bandwidth.

Since the proposed method is basically a ‘‘Hypothesis test’’ style method, the computational complexity is proportional to the desired estimation range. If the desired estimation range is reduced from  $(-N/2,N/2]$  to  $(-N/64,N/64]$ , the search space is also reduced from  $N$  to  $N/32$  and the metric in Eq. (9) becomes

$$\hat{v} = \min_{\varepsilon \in \left(-\frac{N}{64}, \frac{N}{64}\right]} \rho^\varepsilon = \min_{\varepsilon \in \left(-\frac{N}{64}, \frac{N}{64}\right]} \frac{\sum_{l=0}^{N/2-1} |\hat{h}^\varepsilon(l)|^2}{\sum_{l=0}^{G-1} |\hat{h}^\varepsilon(l)|^2}, \quad (18)$$

On the other hand, the DFT and IDFT matrices in Eq. (10), i.e.  $\mathbf{F}_2^{-1}\mathbf{X}_p^{-1}\mathbf{F}_1$ , remain unchanged during the whole estimation procedure. So, if we calculate  $\mathbf{F}_2^{-1}\mathbf{X}_p^{-1}\mathbf{F}_1$  in advance and save their result at receiver, the computational complexity can be further reduced.

Fig. 5 illustrates the mean square estimation error (MSEE) as a function of SNR. For the purpose of comparison, the zero-masking method in [11] with  $L=8$  and  $H=4$  is also simulated. In this simulation, the CFO is not restricted to an integer and generated randomly from the range  $(-4,4]$  subcarrier spacing for both of the methods. According to the desired estimation range, the search space of the proposed method is reduced from 256 to 8. As can be seen from Fig. 5, the proposed method can achieve a MSEE gain about 3dB than the method in [11].

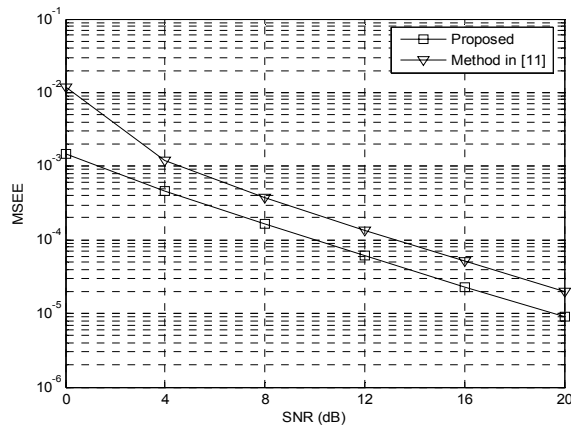


Fig. 5. MSEE comparison.

## 5. Conclusions

In this paper, we have proposed a novel noise subspace based metric to estimate the integral part of CFO in OFDM systems. By using the proposed method, a training symbol with two identical halves can achieve an estimation range equal to the whole bandwidth. Compared with the conventional method, the proposed method provides better estimation accuracy in the simulation.

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