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Factors Affecting the Slenderness Limit for RC Columns-The Use of the (ACI-318M-05) Provisions

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Abstract:

The current ACI code and many other codes permit the moment magnification method for design of slender reinforced concrete (RC) columns. Manually, the design/analysis procedure is tedious, so that the codes state a pre-prediction if the column is slender or not. In other words, before starting design codes provide slenderness limits to be considered to decide whether a column is slender or not.

This work will concentrate on the slender RC column provisions of the latest ACI Code. Using this method some columns are found to be short. In the proposed method of this work, it is found that some of these "short" columns are found to be "long", thus needing moment magnification. Since column analysis/design represents dealing with one of the most critical parts of RC buildings, this proposed magnification is presented in this work.

Introduction:

Modern architectural and engineering requirements and space utilization have increasingly demanded the use of more slender columns for reinforced concrete (RC) structures. The use of high strength concrete (HSC) increases the probability of having slender columns in modern reinforced concrete structures.

Clause 10.10.0 of ACI 318M-08[2] states that for compression members not braced against sidesway, it shall be permitted to neglect the effects of slenderness when ($k\,\ell_u/r$) is less than 22.

The limit 22 is a function of many variables, Eq. (1). The main variables include area of gross section (A_g), compressive strength of concrete (f'_c) and the applied axial load on the column (P_u).

Mirza[5] stated that the expression of flexural rigidity in the ACI code [$EI = 0.4\,E_c\,I_g\,/\,(1+\beta_d)$] is in most cases less conservative than Eq. (10-14) ($EI = (0.2\,E_c\,I_g + E_s\,I_{se})\,/\,(1+\beta_d)$ ... (10-14)) of this Code. Therefore, based on the more accurate estimation of EI [Eq. (10-14)] neglecting the bar reinforcement between the extreme reinforcement layer about the bending direction (if present), the limit of slenderness for a column may be expressed as

$$\frac{k\,\ell_u}{r} \le \sqrt{\frac{E_s\,\phi_s\,(\zeta - C_m)\,\pi^2\,\rho\,\gamma^2}{0.36\,\zeta\,(1+\beta_d)} \cdot \frac{\left(\frac{0.36\,\alpha\,E_c}{\rho\,\gamma^2\,E_s}\right)}{P_u/A_g}} \quad (1)$$

where

k	=	Effective length factor,
ℓ_u	=	Unsupported column length,
r	=	Column radius of gyration,
E_c, Es	=	Modulii of elasticity for concrete and steel,
? s	=	stability resistance factor,
ζ	=	stability index,
C_m	=	factor relating actual moment diagram to an equivalent uniform moment diagram
ρ	=	Total reinforcing steel ratio of the column,
γ	=	Column cover index,
β_d	=	Creep factor,
α	=	Effective flexural rigidity factor,
P_u	=	Factored designed axial load, and
A_g	=	Column cross sectional area.

The derivation of this equation is based on the principle that slenderness is ignored if the magnification factor $\delta_s \le 1.05$ (i.e. $\zeta = 1.05$).

Effective Length Factor (k):

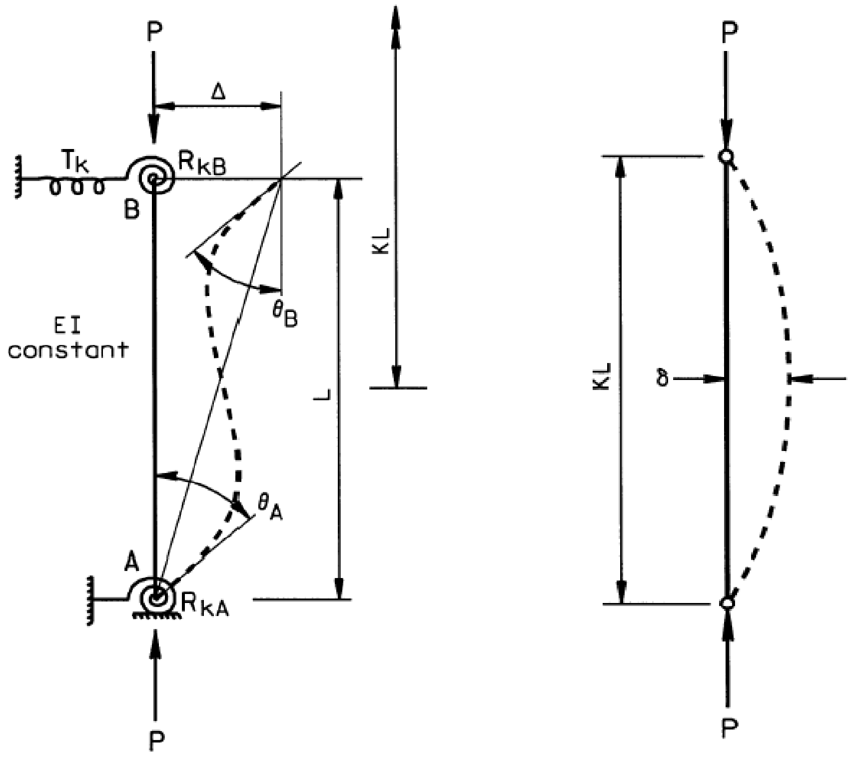
The concept of the effective length factors of columns has been well established and widely used by design engineers and plays an important role in compression member design. Most structural design codes and specifications have provisions concerning the effective length factor.

Mathematically, the effective length factor or the elastic k-factor is defined as:

$$k = \sqrt{\frac{P_e}{P_{cr}}} = \sqrt{\frac{\pi^2 EI}{\ell^2 P_{cr}}} \qquad (2)$$

In which EI is the flexural rigidity of the column, P_e is the Euler load, the elastic buckling load of a pin-ended column and P_{cr} is the elastic buckling load of an end-restrained framed column

Physically, the k-factor is a factor that when multiplied by the actual length of the end-restrained column (Fig. 1 a) gives the length of an equivalent pin-ended column (Fig. 1 b) whose buckling load is the same as that of the end-restrained column. It follows that effective length, $k\ell$, of an end-restrained column is the length between adjacent inflection points of its pure flexural buckling shape.



a- End restrained columns b- Pin ended columns

Fig. (1) Isolated columns[4]

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Specifications provide the resistance equations for pin-ended columns, while the resistance of framed columns can be estimated through the *k*-factor to the pin-ended column strength equation. Theoretical *k*-factor can be determined from an elastic eigenvalue analysis of the entire structural system, while practical methods for the *k*-factor are based on an elastic eigenvalue analysis of selected subassemblages. The effective length concept is the only tool currently available for the design of compression members, and it is an essential part of analysis procedures.[4]

In theory, the effective length factor *k* for any column in a framed structure can be determined using a stability analysis of the entire structural eigenvalue analysis. Methods available for stability analysis include the slope-deflection method, three-moment equation method and energy methods. In practice, however, such analysis is not practical, and simple models are often used to determine the effective length factors for framed columns.[6] One such practical procedure that provides an approximate value of the elastic *k*-factor is the alignment chart method. This procedure has been adopted by the AISC,[3] ACI 318-08[2] and AASHTO[1] specifications, among others. At present, most engineers use the alignment chart method in lieu of an actual stability analysis.[4]

Slenderness Limit for Sway Frames:

For RC slender columns in sway frames, C_m will be equal to 1 and the dominant definition of β_d is equal to zero. In addition, substituting of ? $_s$ = 0.75, $\alpha = 0.2$, $\rho = 0.025$, $E_s = 2 \times 10^5$ MPa and $E_c = 4700\ \sqrt{f'_c}$, and based on the safely assumption that $\gamma = 0.6$ in Eq. (1) yields the following limit of slenderness.

$$\frac{k\,\ell_u}{r} \le \sqrt{\frac{\pi^2\,\rho}{1.4\times10^{-4}}\cdot\frac{\left(\frac{0.0047}{\rho}\sqrt{f'_c}+1\right)}{P_u/A_g}} \qquad (3)$$

Eq. (3) indicates that the slenderness limit value will increase with f'_c and decreases with rising value of the stress P_u/A_g on the column. Table (1) illustrates the influence of f'_c and P_u/A_g one each other. It is clear that the influence of P_u/A_g is much greater on the slenderness limit than f'_c.

Table (1) Effects of axial stress and compressive strength of concrete on the slenderness limit of sway columns

f'_c (MPa)	P_u / A_g (MPa)									
	10	20	30	40	50	60	70	80	90	100
20	18.012	12.736	---	---	---	---	---	---	---	---
30	18.914	13.374	10.920	---	---	---	---	---	---	---
40	19.642	13.889	11.340	9.821	---	---	---	---	---	---
50	20.262	14.327	11.698	10.131	9.061	---	---	---	---	---
60	20.806	14.712	12.012	10.403	9.305	8.494	---	---	---	---
70	21.295	15.058	12.294	10.647	9.523	8.693	8.049	---	---	---
80	21.739	15.372	12.551	10.870	9.722	8.875	8.217	7.686	---	---
90	22.149	15.662	12.788	11.074	9.905	9.042	8.372	7.831	7.383	---
100	22.530	15.931	13.007	11.265	10.076	9.198	8.515	7.965	7.510	7.124
110	22.886	16.183	13.213	11.443	10.235	9.343	8.650	8.091	7.629	7.237

Note; for practical purpose (P_u/A_g) is limited such that $P_u/A_g \leq f'_c$ (Ignoring the steel contribution).

Figs. (2 and 3) show the slenderness limitation with respect to P_u/A_g and f'_c in MPa. It is shown that by increasing f'_c for the RC column, the slenderness limit will increase in its value. In contrast, for the same value of f'_c, increasing of

the applied stress on the RC column (P_u/A_g) will decrease the column slenderness limit.

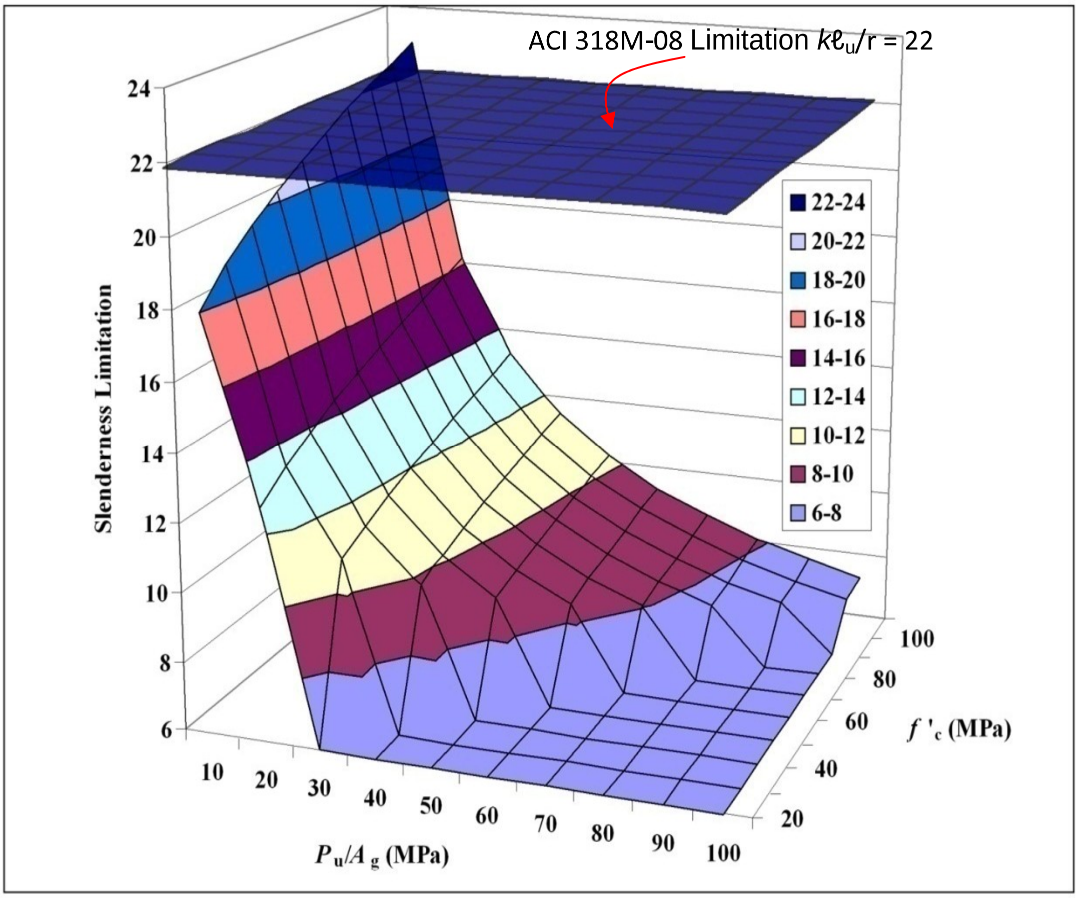

Fig. (2) Surface diagram showing the slenderness limitation for columns in sway frames with respect to the applied P_u/A_g and f'_c, MPa.

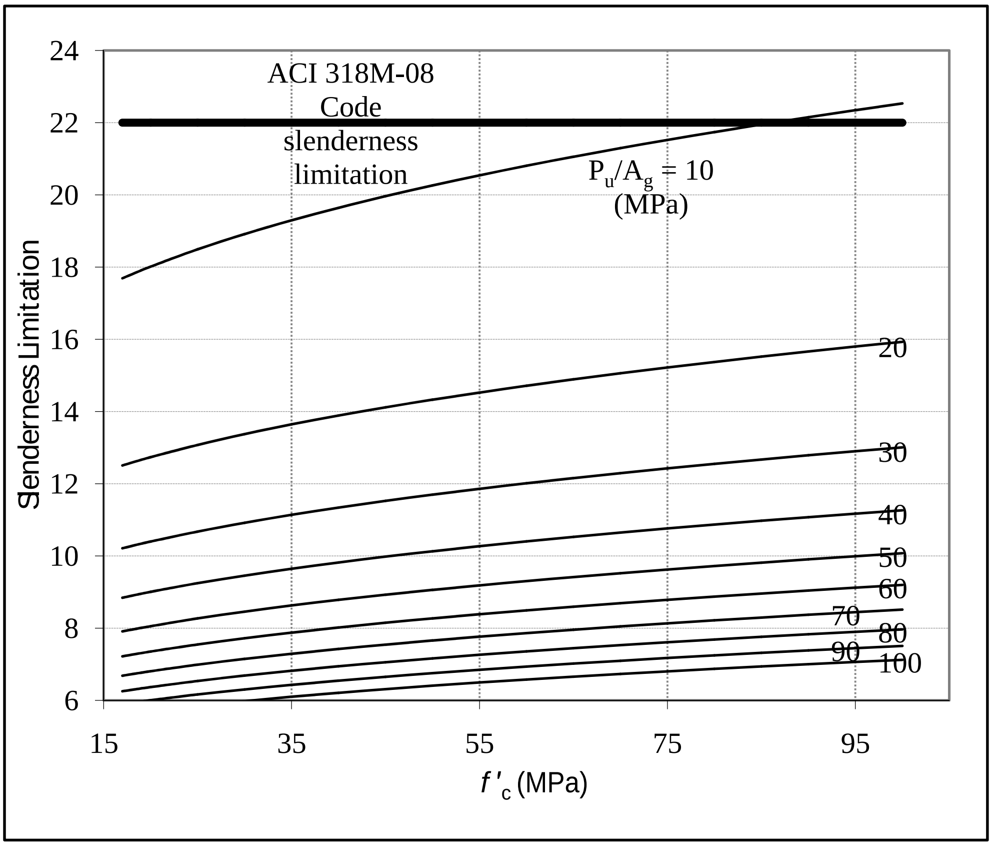

Fig. (3) Line diagram showing the slenderness limitation for columns in sway frames with respect to the applied P_u/A_g and f'_c, MPa.

If a proposed value of ρ = 0.025 is selected, Eq. (3) will be simplified to Eq. (4) for sway frames.

$$\frac{k\,\ell_u}{r} \le 42 \,.\, \sqrt{\frac{0.188\sqrt{f'_c}+1}{P_u/A_g}} \qquad (4)$$

when the left hand side of Eq. (4) is less than the right one, effects of slenderness can be neglected.

Slenderness Limit for Nonsway Frames:

Based on the same assumptions of the previous section, the slenderness ratio of columns in nonsway frames can be obtained as:

$$\frac{k\,\ell_u}{r} \le 188\sqrt{\frac{1.05-C_m}{1+\beta_{dns}} \,.\, \frac{\left(0.188\sqrt{f'_c}+1\right)}{P_u/A_g}} \qquad (5)$$

when $k\ell_u/r$ is less than this limit, effects of slenderness can be neglected.

Figs. (4 a to 5 f) show the slenderness limitation of Eq. (5) with respect to (β_{dns} = 0 and 1) and (P_u/A_g = 10 to 100 MPa) with respect to M_1/M_2. The subscript ns in (β_{dns}) refers to nonsway creep factor.

NUMERICAL EXAMPLES:

1. **EXAMPLE [1]:**

A 325 by 300 mm rectangular RC tied column is reinforced with (8-12 mm), four in each side about the minor axis (h = 300 mm), providing total area of steel = 904.7787 mm^2. Cover distance is 60 mm to the steel center in each direction. The column is a part of ***sway*** frame with effective length of 1.970 m.

Material strengths are f'_c = 44 MPa and f_y = 400 MPa. A factored load P_u of 1130 kN is to be applied with eccentricity e about the minor axis of 90 mm. By assuming that all the columns in the story have the same factored axial load and the same critical load capacity, check the adequacy of the column by:

a- using ACI 318M-08 approach, without any modification.
b- using ACI 318M-08 approach modified by Eq. (4).

SOLUTION:

a- According to the ACI 318M-08 Code, in sway columns, slenderness effects can be neglected if $k\ell_u/r \leq 22$. With r = 0.3 × 300 = 90 mm, $k\ell_u/r$ = 1970 / 90 = 21.889 < 22, so that the slenderness effects may not be included in the analysis calculations.

By trials and interpolation process, selected c = 186.172 mm, (neutral axis depth). From this value of c, when the concrete reaches its ultimate strain limit of 0.003, the strains at the locations of the reinforcing bars can be found from strain compatibility, after which, the stresses are found by multiplying strains times E_s (= 2×10^5 MPa) and applying the limit value f_y:

ε_s' = 0.002033155 f_s' = 406.631 MPa compressive, (> f_y = 400 MPa).

ε_s' = 0.000867382 f_s = 173.476 MPa tensile.

for f'_c = 44 MPa, β_1 = 0.736.

The concrete compressive resultant is $C_c = \alpha_1 f'_c \beta_1$ c b

C=0.85×44×0.736×186.172×325/1000
=1664.867 kN.

The respective steel forces are:

$C_s' = A_{st} / 2 (f_y - \alpha_1 f'_c)$ = 164.036 kN

$T_s = A_{st} / 2 (f_s)$ = -78.479 kN

The thrust and moment, which would be considered as design capacities, are:

? P_n = 0.65 (1664.867 + 164.036 - 78.478) = 1137.776 kN.

? $M_n = 0.65 [C_c \times (h/2 - \beta_1 c/2) + (h-2d')/2 \times (C_s' - T_s)]$ = 102.4 kN.m.

Then ? $P_n > P_u$ (= 1130 kN) therefore, the column is <u>safe and adequate</u> per ACI 318M-08 Code. (1137.776 / 1130 = 100.688 %).

b- According to Eq. (4), slenderness effects can be neglected if $k\ell_u/r \leq$ the right hand of this equation. With r = 0.3 × 300 = 90 mm, $k\ell_u/r$ = 1970/90=21.889.

$$42\sqrt{\frac{0.188\sqrt{44}+1}{1130\times10^3/(325\times300)}} = 18.5$$

The $k\ell_u/r$ (21.889) > 18.5, so that the slenderness effects must be included in the analysis calculations.

According to the ACI 318M-08 Code, $EI = 0.2 \times 4700\sqrt{f'_c} \times bh^3/12 + E_s \times A_{st} (d_s)^2$.

$EI = 0.2 \times 4700\sqrt{44} \times 325\ (300)^3/12 + 2 \times 10^5 \times 904.78\ (90)^2 = 6.025\times10^{12}$ N.mm^2.

$P_c = \pi^2 EI/(k\ \ell_u)^2$ = 15323.004 kN.

Selected c = 172.109 mm, and by the procedure of example (1-a),

? P_n = 1034.745 kN < P_u = 1130 kN, (<u>Not Safe</u>). (1034.745 / 1130 =91.57 %)

? M_n = 102.3 kN.m.

Where

$$\delta = \frac{1}{1-\frac{P_u}{0.75\,P_c}} = 1.1.$$

The ratio between solution a and b is = (1034.745/1137.776) = 0.909.

2. EXAMPLE [2]:

A 225 by 400 mm rectangular concrete column section is reinforced symmetrically with A_{st} = 2400 mm^2 of steel, half at each of the two critical faces. The centroid of each group of bars is 60 mm from the near edge. The column is a part of ***sway*** frame that has an effective length of 2.63 m. The concrete has cylinder strength of 60 MPa. The steel has yield strength of 400 MPa. The load acts eccentrically with respect to the major axis of the column section are P_u = 2580 kN, M_u = 129 kN.m.

By assuming that all the columns in the story have the same factored axial load and the same critical load capacity, check the adequacy of the column by:
a- using ACI 318M-08 approach and
b- using ACI 318M-08 approach with Eq. (4).

SOLUTION:

By the same procedure followed in example (1) before;

a- Slenderness effect may not be included in the analysis calculations;
selecting of c = 471.341 mm obtain

? P_n = 2648 kN > P_u = 2580 kN.

Therefore the column is <u>safe and adequate</u> per ACI 318M-08. (2648 / 2580 = 102.636 %).

b- $k\ \ell_u / r$ = 2630 / (0.3 × 400) = 21.92.

$$42\sqrt{\frac{0.188\sqrt{60}+1}{2580\times10^3/(225\times400)}} = 12.29$$

therefore, slenderness effects must be included in the analysis calculations.

Selected c = 449.545 mm.

? P_n = 2525.96 kN < P_u = 2580 kN, (<u>Not Safe</u>). (2525.96 / 2580 = 97.905 %).

? M_n = 145.1834 kN.m.

Where $\delta = \frac{1}{1-\frac{P_u}{0.75\,P_c}} = 1.15.$

The ratio between solution a and b is
= (2525.96/2648) = 0.95.

3. EXAMPLE [3]:

In high rise buildings where $k\ \ell_u$ / r should be determined carefully, 300 by 350 mm rectangular concrete column section is reinforced symmetrically by (8-20 mm; A_{st} = 2400 mm^2) of steel, four bars at each of the two critical faces, (h = 350 mm). The centroid of each group of bars is 62 mm from the near edges. The column is a part of ***nonsway*** frame has an effective length of 2.65 m. The concrete has cylinder strength of 120 MPa. The steel has yield strength of 520 MPa. The load acts eccentrically with respect to one major axis of the column section are P_u = 3200 kN, M_{u1} = 200 kN.m and M_{u2} = 320 kN.m causing single column curvature.

By assuming that the creep factor β_d = 0.35, check the adequacy of the column by:

a- using ACI 318M-08 approach and

b- using ACI 318M-08 approach with Eq. (5).

SOLUTION:

a- According to the ACI 318M-08 Code, in nonsway columns, slenderness effects can be neglected if ($k\ell_u$/r ≤ 34 -12 M_1 /M_2). With r = 0.3 × 350 = 105 mm.

$k\ell_u$/r of the column = 2650 / 105 = 25.238.

The limit of the slenderness provision used by the ACI 318M-08 Code is 34 – 12 (200 / 320) = 26.5.

$k\ell_u$/r of the column = 25.238 < 26.5 limitation of the ACI 318M-08 Code.

Hence, per the ACI 318M-08 Code, the slenderness effects may not be included in the analysis calculations.

By trials and interpolation process, selected c = 249.610 mm, (neutral axis depth). From this value of c, when the concrete reaches its ultimate strain limit of 0.003, the strains at the locations of the reinforcing bars can be found from strain compatibility, after which, the stresses are found by multiplying strains times E_s (= 2×10^5 MPa) and applying the limit value f_y:

f_s' = 450.968 MPa compressive, (< f_y = 520 MPa).

f_s = 92.279 MPa tensile.

for f'_c = 120 MPa, β_1 = 0.65.

The concrete compressive resultant is $C_c = \alpha_1 f'_c \beta_1$ c b

C_c = 0.85×120×0.65×249.610×300/1000

= 4964.747 kN.

The respective steel forces are:

$C_s' = A_{st} / 2\ (f_y - \alpha_1 f'_c)$ = 438.526 kN

$T_s = A_{st} / 2\ (f_s)$ = -115.962 kN

The thrust and moment, which would be considered as design capacities, are:

? P_n = 0.65 (4964.747 + 438.526 - 115.962) = 3436.757 kN.

? M_n = 0.65 [C_c × (h/2 - β_1 c/2) + (h-2 d′)/2 × (C_s' - T_s)] = 343.675 kN.m.

Then ? P_n > P_u (= 3200 kN) therefore, the column is <u>safe and adequate</u> per ACI 318M-08 Code. (3436.757 / 3200 = 107.4 %).

b- According to Eq. (5), slenderness effects can be neglected if $k\ell_u$/r ≤ the right hand of this equation. With r = 0.3 × 350 = 105 mm, $k\ell_u$/r ≤ = 2650 / 105 = 25.238.

$C_m = 0.6 + 0.4\ M_1/M_2$ = 0.6 + 0.4 (200/320) = 0.85 > 0.4, Then C_m = 0.85.

Slenderness limit is equals

$$188\sqrt{\frac{(1.05-0.85)\times\left(0.188\sqrt{120}+1\right)}{(1+0.35)\times 3200\times 10^3/(300\times 350)}}$$
$$= 22.927$$

The column slenderness $k\ell_u$/r (25.238) > 22.927, so that the slenderness effects must be included in the analysis calculations.

According to the ACI 318M-08 Code, $EI = 0.2 \times 4700\sqrt{f'_c} \times bh^3/12 + E_s \times A_{st}\ (d_s)^2$.

$EI = 0.2 \times 4700\sqrt{120} \times 300\ (350)^3/12 + 2 \times 10^5 \times 2513.274\ (113)^2 = 1.746\times 10^{13}$ N.mm^2.

$P_c = \pi^2 EI/(k\ \ell_u)^2$ = 24532.686 kN.

Selected c = 210.488 mm, and by the procedure of example (1-a),

? P_n = 2803.239 kN < P_u = 3200 kN, (Not Safe). (2803.239 / 3200 = 87.601 %).

? M_n = 102.3 kN.m.

where

$$\delta = \frac{1}{1-\frac{P_u}{0.75\,P_c}} = 1.306.$$

The ratio between solution a and b is = (2803.239/3436.757) = 0.816.

From examples (1), (2) and (3), it can be concluded that the solution gives different capacities for the same column.

This indicates that with the application of Eq. (4) as well as Eq. (5), safer RC slender columns design can be obtained.

Summary and Conclusions:

This study is conducted to investigate the conservative conditions for predicting the slender column magnification. Nonsway and sway RC columns are studied using the current ACI 318 Code magnification method. Three examples are presented using new formulas for slenderness RC column prediction. These proposed formulas giving more conservative (especially when the axial load is high) and accurate estimation of slenderness limit values than the ACI 318 Code

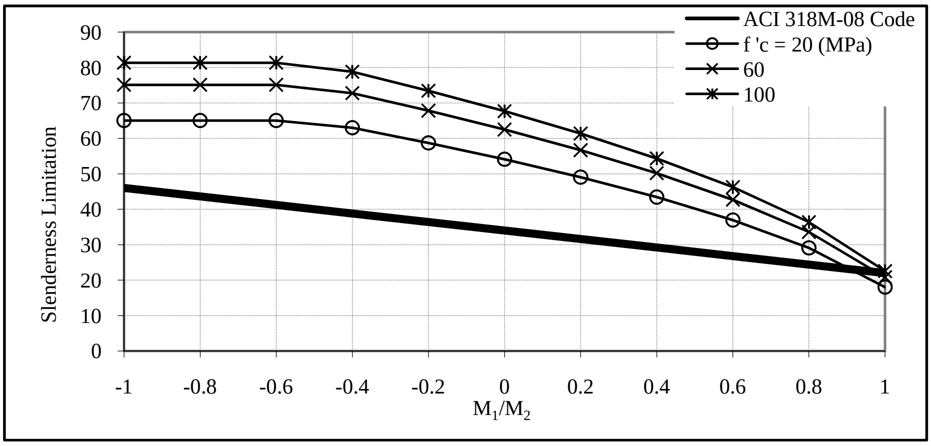

Fig. (4 a) The slenderness limitation for columns in nonsway frames, P_u/A_g = 10 MPa and β_d = 0.

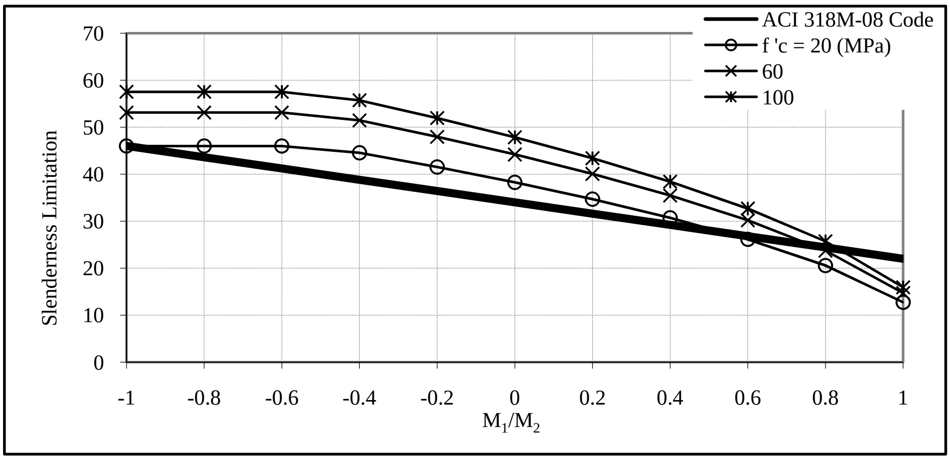

Fig. (4 b) The slenderness limitation for columns in nonsway frames, P_u/A_g = 20 MPa and β_d = 0.

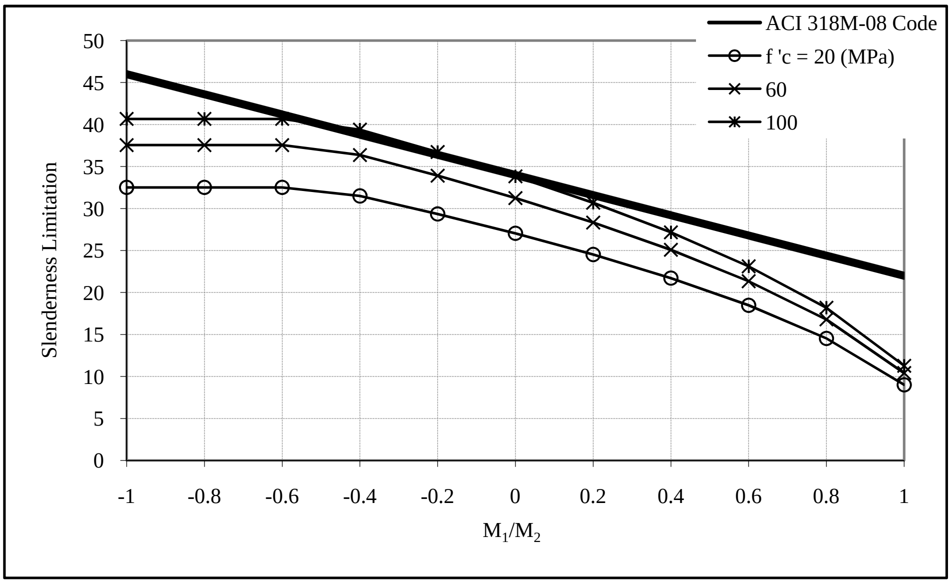

Fig. (4 c) The slenderness limitation for columns in nonsway frames, P_u/A_g = 40 MPa and β_d = 0.

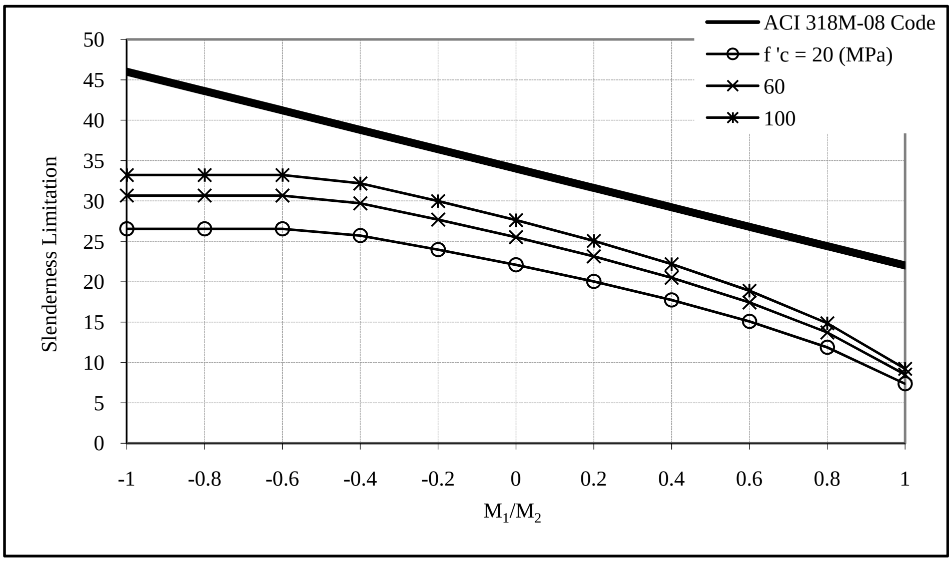

Fig. (4 d) The slenderness limitation for columns in nonsway frames, P_u/A_g = 60 MPa and β_d = 0.

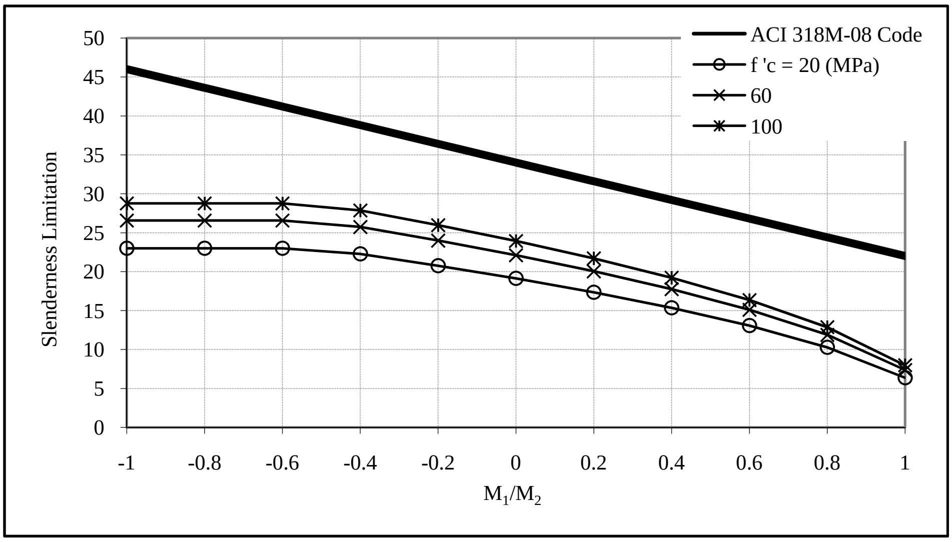

Fig. (4 e) The slenderness limitation for columns in nonsway frames, P_u/A_g = 80 MPa and β_d = 0.

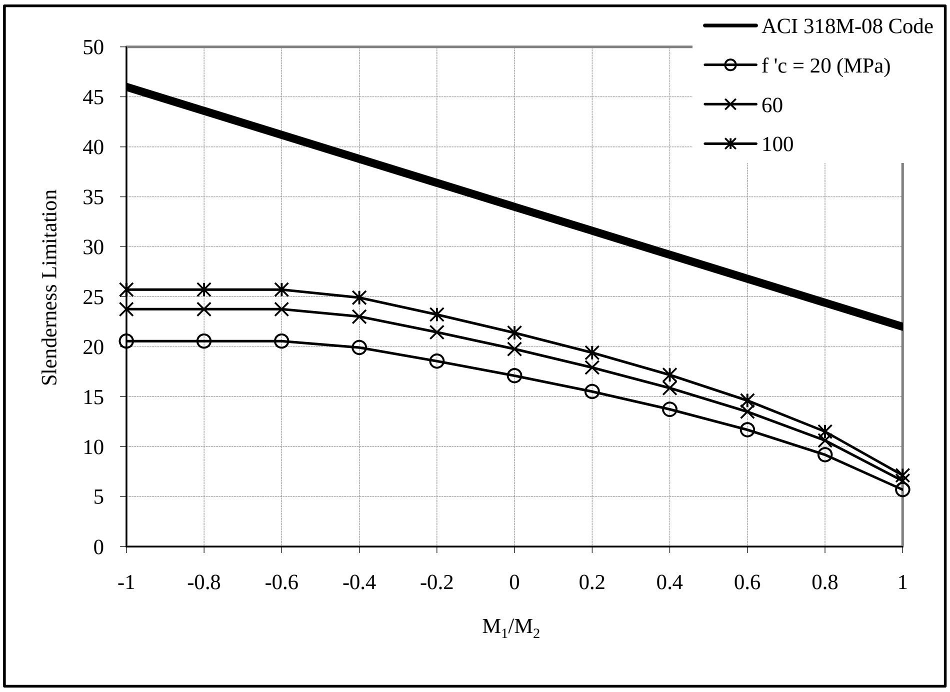

Fig. (4 f) The slenderness limitation for columns in nonsway frames, P_u/A_g = 100 MPa and β_d = 0.

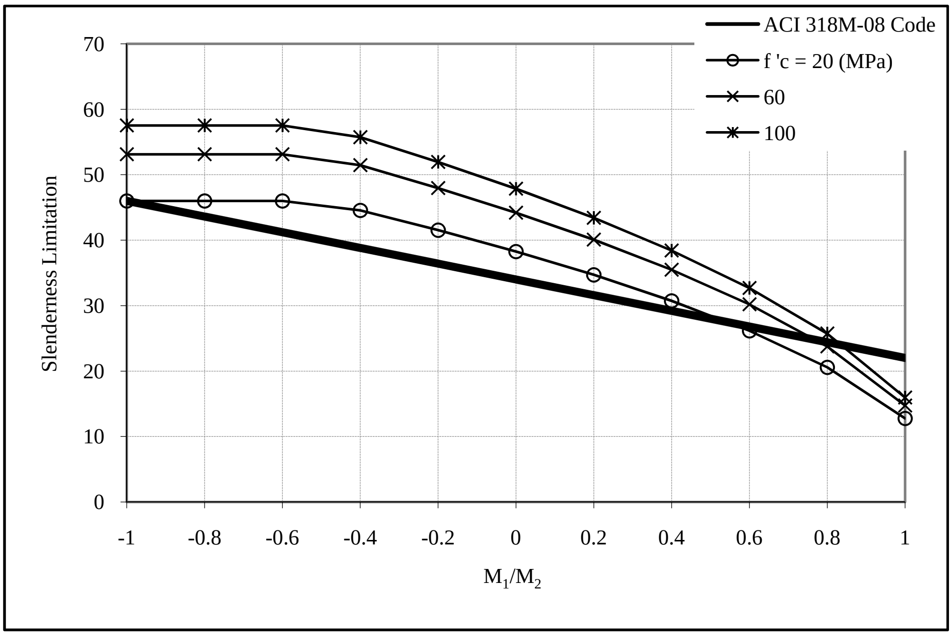

Fig. (5 a) The slenderness limitation for columns in nonsway frames, P_u/A_g = 10 MPa and β_d = 1.

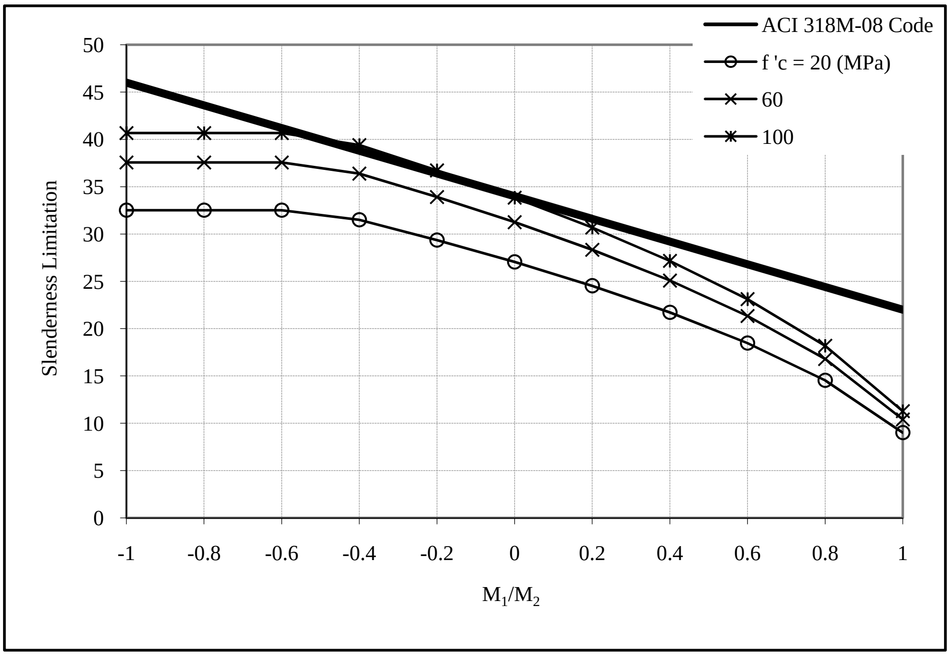

Fig. (5 b) The slenderness limitation for columns in nonsway frames, P_u/A_g = 20 MPa and β_d = 1.

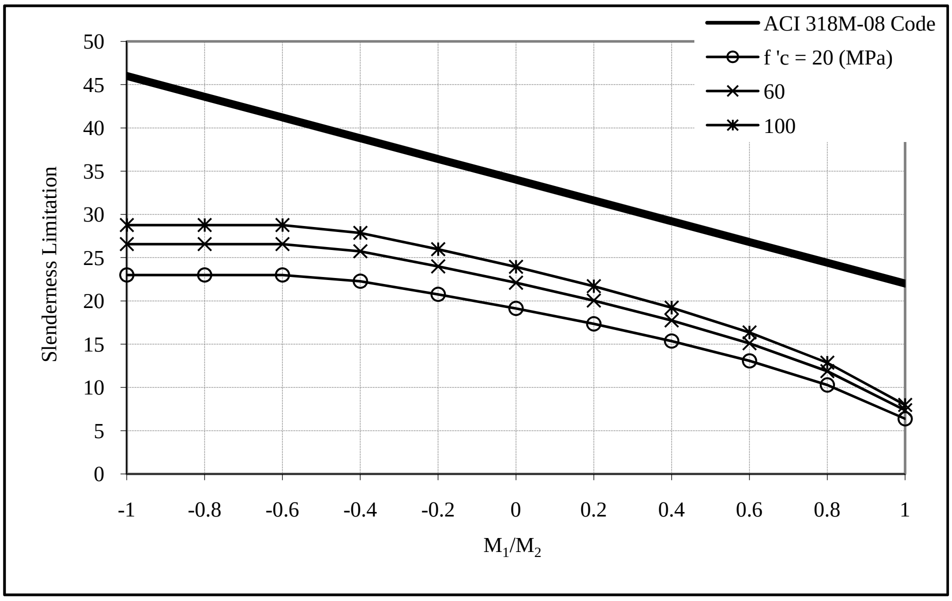

Fig. (5 c) The slenderness limitation for columns in nonsway frames, P_u/A_g = 40 MPa and β_d = 1.

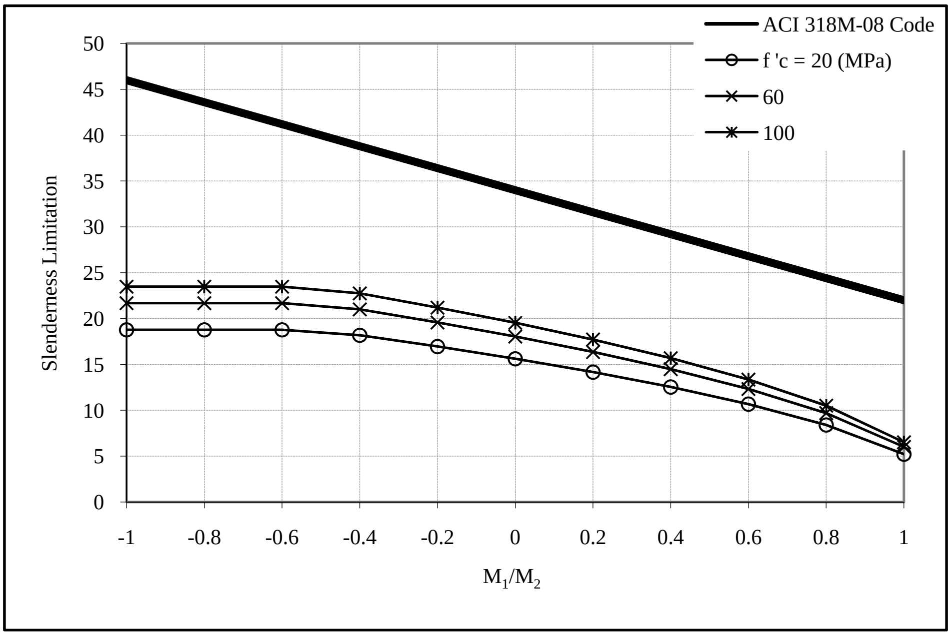

Fig. (5 d) The slenderness limitation for columns in nonsway frames, P_u/A_g = 60 Mpa and β_d = 1.

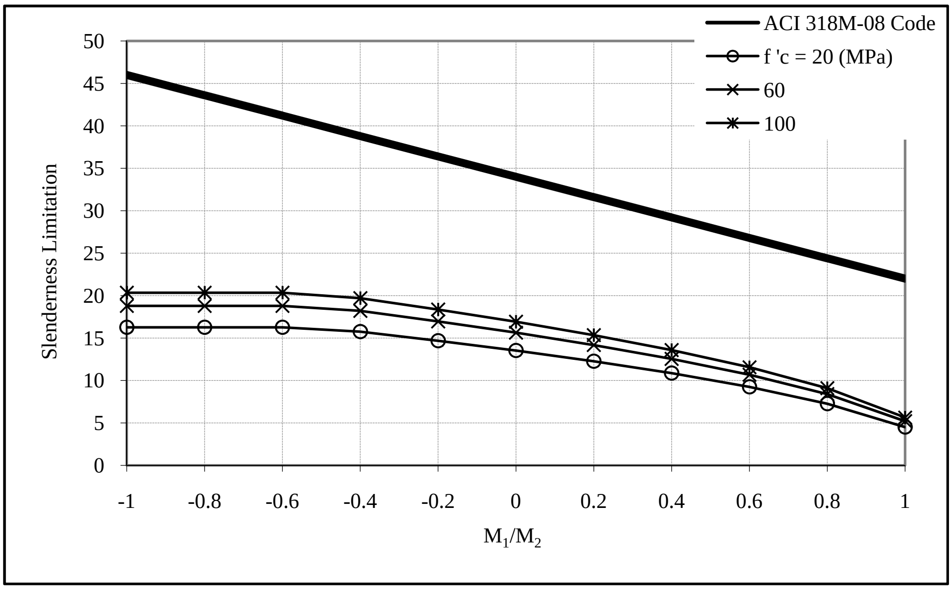

Fig. (5 e) The slenderness limitation for columns in nonsway frames, P_u/A_g = 80 MPa and β_d = 1.

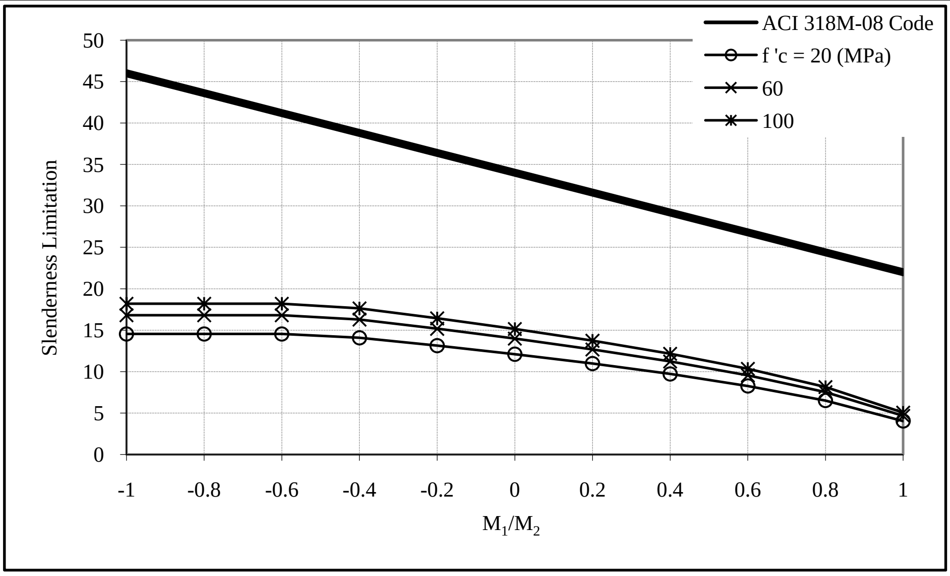

Fig. (5 f) The slenderness limitation for columns in nonsway frames, P_u/A_g = 100 MPa and β_d = 1.

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