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Adaptive Discrete Filters for Telephone Channels Based on the Wavelet Packet Transform

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Abstract

The wavelet transform provides good and in many times excellent results when used as a basic block transform in many systems such as electronic, communication, medical and even chemical systems. The paper uses the wavelet packet transform to adjust the tap gains of the adaptive filter used in channel equalization and estimation. The results using the wavelet technique achieve good improvements in convergence time over the ordinary LMS algorithm. The two systems were compared on full mathematical and simulation basis. Learning curves for adaptive channel equalization and adaptive channel estimation using wavelet packet transform with different mother functions, different level decompositions, different step sizes, different levels of signal to noise ratio, different telephone channels and different filter sizes were compared with conventional LMS adaptive channel equalization and channel estimation. The simulation results carried out using the MATLAB package version 6.1, demonstrate the efficiency of the proposed technique.

Keywords: Filters, Telephony.Waves

1. Introduction

The topic of adaptive filtering has evolved in the last forty years to become a solid framework in building many practical systems. The context of this study will be restricted to adaptive filters driven by the least mean square (LMS) adaptation algorithm, firstly proposed by Widrow and Hoff in 1960. This algorithm has been in use for many years in different engineering applications found in the fields of communications, control, biomedicine, radar, etc. The main advantages of such systems are their relative simplicity and their powerful ability to track the varying characteristics of the involved system. However the major defect of these systems is the unpredictable behavior in their convergence to the final solution, such as slow convergence or no convergence of the filter [1].

As a suggested cure for the problems mentioned above, the wavelet packet transform is used to modify the input data streams fed to the filter. The goal of most modern wavelet research is to create a set of basis functions and transforms that will give an informative, efficient, and useful description of a function or signal. If the signal is represented as a function of time, wavelets provide efficient localization in both time and frequency or scale [2]. This analysis is derived from the wavelet transform in its classical scheme.
(coefficient) calculation or adaptation. This study is performed by simulating the adaptation curves of such systems using various types of models of telephone channels. The results were evaluated and compared with those obtained from the conventional adaptive systems and with those obtained with different filter sizes.

2.The Discrete Linear System Model

The model of the employed system is shown in Fig.1. It is assumed to have a transmitter, a linear baseband transmission path, a sampler and a receiver. This model may represent a simple digital communication link. With simple variation, it can represent any other linear model that may be found in other fields such as control, digital signal processing (DSP), etc.

The transmitted information is expressed by a sequence of data symbols $\{s_i\}$, where $\{s_i\}$ may represent any alphabet. In this work, we have adopted the balanced binary model with

$$s_i \in \{-1,+1\} \quad (1)$$

The $\{s_i\}$ are statistically independent and equally likely to have any of their two possible values. The waveform $r(t)$ at the output of the linear baseband channel is

$$r(t) = \sum_i s_i . h(t - iT) + w(t) \quad (2)$$

Where w(*t*)is additive noise. Assuming full synchronization between the receiver and transmitter, the waveform $r(t)$ is sampled once per data symbol at the time instant t_i to yield the sample r_i which is fed to the adaptive filter.

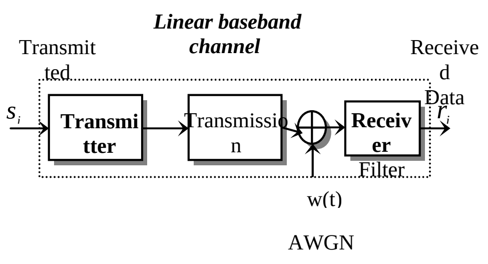


Figure (1) Model of the employed system

where $r_i = r(iT), h_j = h(jT)$ and $w_i = w(iT)$. Thus the sampled impulse response of the linear baseband channel is given by the $(g+1)$-component sequence H where

$$H = \begin{bmatrix} h_0 & h_1 & \cdots & h_g \end{bmatrix} \quad (4)$$

with z -transform of

$$H(z) = h_o + h_1 z^{-1} + \cdots + h_g z^{-g} \quad (5)$$

2.1 The LMS Adaptive Estimator

The adaptive channel estimator structure is shown in Fig. 2. This estimator uses the error in the estimated signal e_i to adjust the tap gains $\{c_i\}$ of the filter C. The error at time instant iT is defined as [2,3]:

$$e_i = x_i - r_i \quad (6)$$

where x_i is the estimated channel signal, which is givenb y:

$$x_i = \sum_{j=o}^{n} s_{i-j} . c_j \quad (7)$$

The Mean Square Error ε , which is given by:

$$\begin{aligned} \varepsilon &= \mathrm{E}\left[\, |x_i - r_i|^2 \right] \\ &= \mathrm{E}\left[(x_i - r_i)(x_i^T - r_i^T)\right] \\ &= \mathrm{E}\left[x_i . x_i^T - x_i . r_i^T - x_i^T . r_i + r_i . r_i^T\right] \end{aligned} \quad (8)$$

should be minimized by calculating the gradient of the expected value of the square error with respect to the coefficients of the filter, which is given by [4]:

$$\nabla_c(\varepsilon) = \left[\frac{\partial \varepsilon}{\partial c_0} \quad \frac{\partial \varepsilon}{\partial c_1} \cdots \frac{\partial \varepsilon}{\partial c_n} \right] \quad (9)$$

then the following quantity is computed:

$$\frac{\partial \varepsilon}{\partial c_j} = \mathrm{E}\left[\frac{\partial}{\partial c_j}\left(x_i.x_i^T - x_i.r_i^T - x_i^T.r_i + r_i.r_i^T\right)\right]$$

$$= \mathrm{E}\left[x_i.\frac{\partial x_i^T}{\partial c_j} + x_i.^T\frac{\partial x_i}{\partial c_j} - r_i^T.\frac{\partial x_i}{\partial c_j} - r_i.\frac{\partial x_i^T}{\partial c_j}\right]$$

$$= \mathrm{E}\left[2x_i.s_{i-j} - 2r_i.s_{i-j}\right]$$
$$= \mathrm{E}\left[2s_{i-j}\left(x_i - r_i\right)\right]$$
$$= \mathrm{E}\left[2s_{i-j}.e_i\right] \qquad (10)$$

Since ε is a quadratic function, it has a convex form in the multidimensional space representing the taps vector. Hence, a global minimum of this function at which the above derivatives are zero can be found. Using the gradient algorithm, the filter coefficients can be iterated to reach the optimum setting by using the equation [3]:

$$c_{j,new} = c_{j,old} - \alpha \mathrm{E}\left[2s_{i-j}.e_i\right]$$
$$= c_{j,old} - 2\alpha s_{i-j}.e_i \qquad (11)$$

The rate of convergence towards the desired adjustment of the channel estimator depends upon the step size. Hence, the updating equations in vector form are given by:

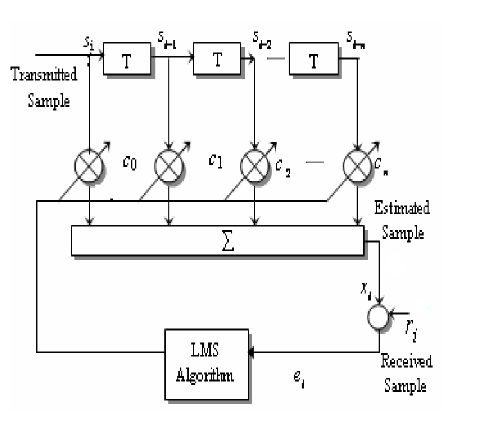
$$e_i = x_i - r_i \qquad (12)$$

$$C_{new} = C_{old} - \Delta e_i.S \qquad (13)$$

where $\Delta = 2\alpha$

$$C = \left[c_0 \quad c_1 \cdots c_n\right] \qquad (14)$$

$$S = \left[s_i \quad s_{i-1} \cdots s_{i-n}\right] \qquad (15)$$



Figure(2) Adaptive channel estimation

2.2 The LMS Adaptive Equalizer

The linear equalizer [5, 6, 7] can be implemented as a linear feed forward transversal filter. The tap coefficients of this equalizer are given by the *(n+1)*- component sequence.

$$C = \left[c_0 \quad c_1 \cdots c_n\right] \qquad (16)$$

with z-transform

$$C(z) = c_o + c_1.z^{-1} + \cdots + c_n.z^{-n} \qquad (17)$$

The equalizer acts on the received samples in such a manner that the signals r_i and x_i are those present at instant iT. By the reception of a sample, the stored symbolsare shifted one place and the output at the instant iT is the sum given by:

$$x_i = \sum_{j=o}^{n} r_{i-j}.c_j \qquad (18)$$

Since the sampled impulse response of the linear baseband channel is given by the *(g+1)*-component sequence H ,the z-transform of the sampled impulse response of the equalized channel is given by $D(z)$, where

$$D(z) = H(z).C(z) \qquad (19)$$

or alternatively, one can define the sequence D by:

$$D = conv(H,C) = \left[d_0 \quad d_1 \cdots d_{n+g}\right] \qquad (20)$$

where $conv$ is the convolution.

When exact equalization takes place, the sampled impulse response of the linear baseband channel and equalizer becomes:

$$D_y = [0 \quad 0 \cdots 0 \quad 1 \quad 0 \quad 0 \cdots 0] \qquad (21)$$

where y is a positive integer in the range 0 to $n+g$, which represents a delay of y sampling instants in the equalized sampled impulse response. eq. (21) above cannot be satisfied exactly without having an infinite number of taps in the equalizer.

The Mean Square Error is defined as the expected value of the squared difference between the output from the adaptive filter and input signal. Considering the delay in transmission y, the Mean Square Error is given by:

$$\varepsilon = \mathrm{E}\left[\,|e_i|^2\right] = \mathrm{E}\left[|x_i - s_{i-y}|^2\right] \qquad (22)$$

where $\mathrm{E}[.]$ is the expected value.

The adaptive linear equalizer structure is shown in Fig. 3. This equalizer uses the error in the equalized e_i to adjust the tap gains $\{c_i\}$ of the filter C. The error at time instant iT is defined as [1,3]:

$$e_i = x_i - s_{i-y} \qquad (23)$$

where x_i is the equalized signal that is given by:

$$x_i = \sum_{j=o}^{n} r_{i-j} \,.\, c_j \qquad (24)$$

The same procedure for the adaptive channel estimator is applied here, yielding

$$C_{new} = C_{old} - \Delta e_i . R \qquad (25)$$

where

$$C = [c_0 \quad c_1 \cdots c_n] \qquad (26)$$

and

$$R = [r_i \quad r_{i-1} \cdots r_{i-n}] \qquad (27)$$

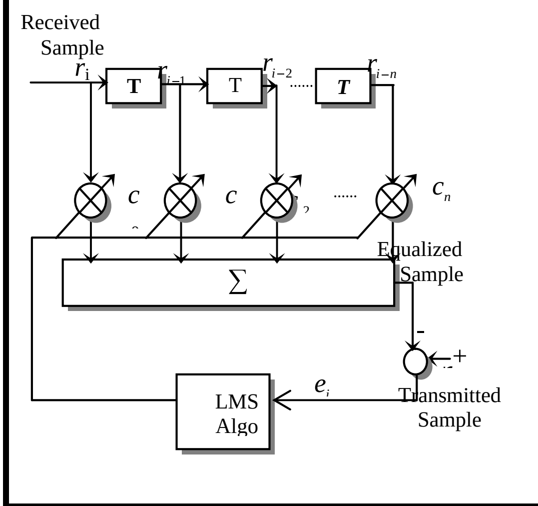


Figure (3) Adaptive channel equalization

3. The Wavelet Packet Analysis

The wavelet packet method is a generalization of wavelet decomposition that offers a richer range of possibilities for signal analysis. In wavelet analysis, a signal is split into an approximation and a detail. The approximation is then itself split into a second level approximation and detail, and the process is repeated. For a *j*-level decomposition, there are $j+1$ possible ways to decompose or encode the signal as shown in Fig. 4.

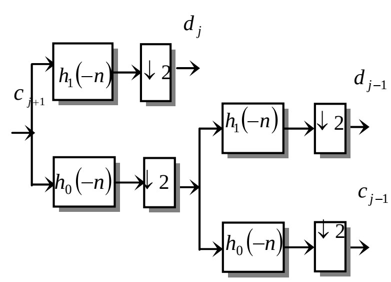


Figure (4) Two-stage wavelet decomposition and the DWT analysis tree

In wavelet packet analysis, the details as well as the approximations can be split. This yields 2^j different ways to encode the signal. Fig. 5 represents this decomposition.

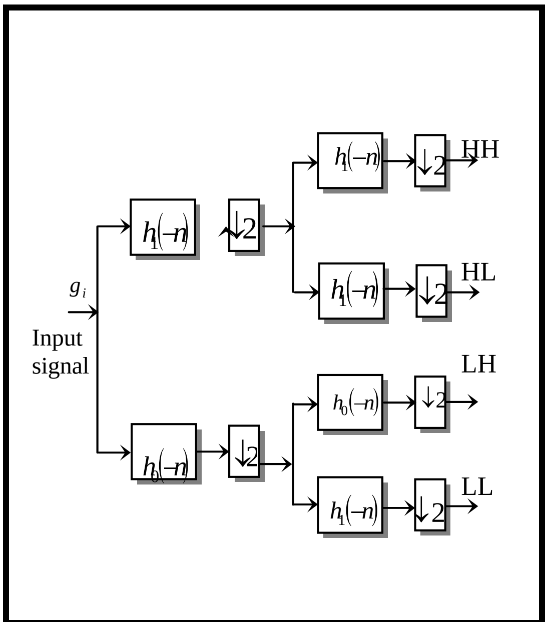


Figure (5) Analysis tree for the two-stage wavelet packet decomposition (DWPT)

Wavelet packet functions are designed by generalizing the filter bank tree that relates wavelets and conjugate mirror filters [1, 8]. The frequency axis division of wavelet packets is implemented with an appropriate sequence of iterated convolutions with conjugate mirror filters [8, 9, 10, 11]. Fast numerical wavelet packet decompositions are thus implemented with discrete filter banks [1, 9, 12].

The idea behind this decomposition can be approached by noting that the highpass filter $h_1(n)$ gives the detail part of the signal dyadic decomposition, while the lowpass filter $h_0(n)$ represents the approximate part. This would give us great flexibility in decomposing the signals, as used in wavelet packet analysis.

The computation scheme for wavelet packet generation, is easy when using an orthogonal wavelet. It starts with the two filters of length N, denoted by $h_0(n)$ and $h_1(n)$ corresponding to the reversed versions of the lowpass and highpass decomposition filters respectively divided by $\sqrt{2}$. Now by induction, let us define the following sequence of functions $W_n(t)$, n= 0,1,2,… by [13, 14]:

$$W_{2n}(t) = \sqrt{2}\sum_{k=0}^{N-1} h_0(k).W_n(2t-k) \qquad (28)$$

$$W_{2n+1}(t) = \sqrt{2}\sum_{k=0}^{N-1} h_1(k).W_n(2t-k) \qquad (29)$$

where $W_0(t)=\varphi(t)$ is the scaling function and $W_1(t)=\psi(t)$ is the wavelet function. Starting from the functions $(W_n(t), n \in N)$ and following the same line leading to orthogonal wavelets, we consider the three-indexed family of analyzing functions (the waveforms) :

$$W_{j,n,k,}(t) = 2^{-j/2} W_n\left(2^{-j}t-k\right) \qquad (30)$$

where $n \in N$ and $(j,k) \in Z$.

The set of functions: $W_{j,n} = \left(W_{j,n,k}(t),\ k \in Z\right)$ is the (j,n) wavelet packet. For each scale j, the possible values of parameter n are: 0, 1,…, 2j-1, and

$$W_{0,0} = \varphi(t-k) \qquad k \in Z \qquad (31)$$

4. Model Structure for the Adaptive Systems Using the Wavelet Packet Transform

The general setup of the proposed adaptive estimation filter has the form given in Fig. 6, while the adaptive equalizer filter has the arrangement given in Fig. 7. In the former figure the transmitted signal generated as a random sequence of {1, -1} is split into two equal numbers of samples by the action of the scaling filter $h_0(-n)$ and the wavelet filter $h_1(-n)$. When the system consists of two or more decomposition levels, the two outputs from the scaling and wavelet filters are further processed by another set of these filters as shown in Fig.5

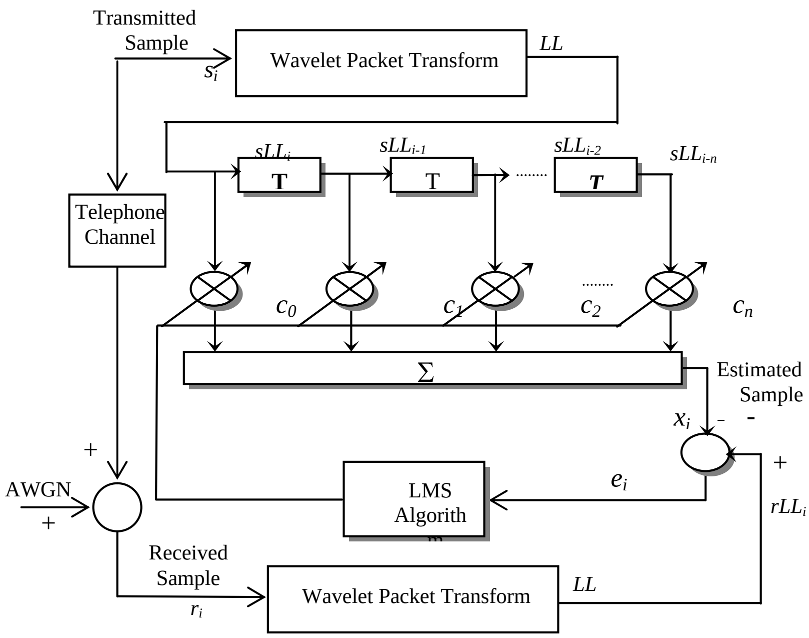


Figure (6) General arrangement of adaptive channel estimator using the wavelet packet transform

The symbols H.and *L* denote highpass and lowpass filters respectively. This means that the output of the scaling filter is in fact a highpass response of the input signal and the wavelet filter is the lowpass side of the processed input signal. Between any two stages of the wavelet packet transform blocks there is a down sampling process, which reduces the number of samples by half. The term *HL* means that the samples are processed through highpass filter followed by lowpass one, while *LH* is the processing of the sequence by lowpass filter followed by highpass one. Generally, the feeding input of the adaptive filter can be taken from any output stage of the wavelet packet transform block taking into consideration the convergence time of the adaptation.

The attenuated signal from the output of telephone channel is added to an additive white Gaussian noise denoted by AWGN. To this point, a received signal r_i is ready to estimate the mean square error of the LMS block which leads to the estimation of the total convergence time.

Finally, the error signal e_i is generated by subtracting the equalized sample output of the adaptive filter x_i from an output similar to the wavelet packet transform block but with an input signal r_i.

The adaptive estimator shown in the block diagram of Fig. 6 is somehow similar to the adaptive equalizer shown in the block diagram of Fig. 7 with some differences represented by the input signal and location of the telephone channel and AWGN addition.

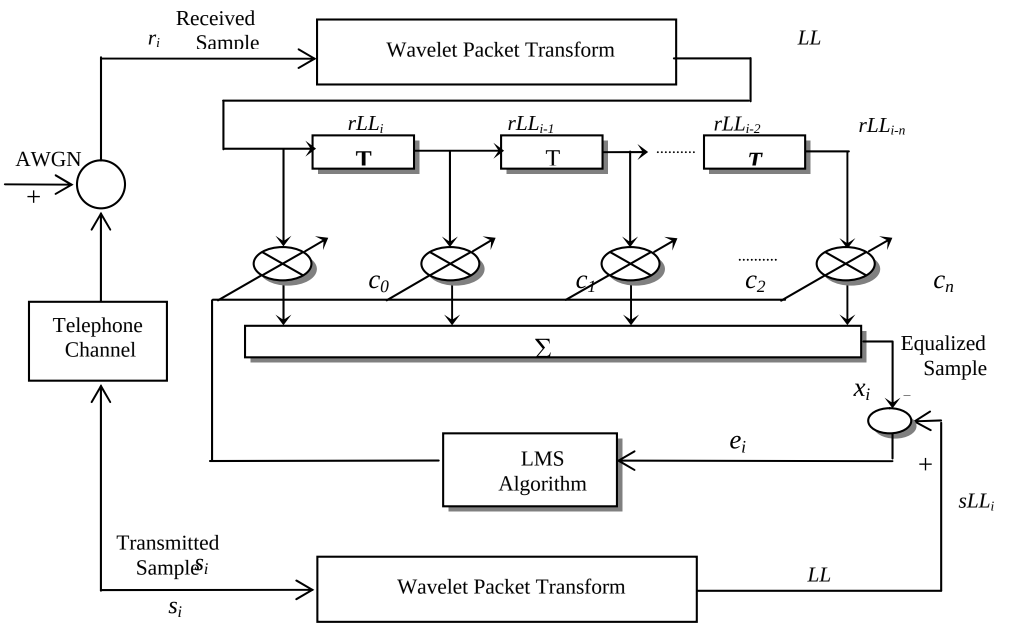


Figure (7)
General arrangement of adaptive channel equalizer using the wavelet packet transform

The parameters of the channels used in the adaptive equalizer and adaptive estimator are shown in Tables 1 and 2 respectively, where n represents the number of adaptive filter taps, g is the number of telephone channel taps, y is the delay between the two sequences for the equalizer case and Δ assigns the step size of adaptation.

Table (1) Parameters of the estimation LMS filter

Channel	Ch1	Ch2
n	10	20
g	14	18
Δ	0.0075 0.005 0.001	0.0075 0.005 0.001

Table (2) Parameters of the equalization LMS filter

Channel	Ch1	Ch2
n	20	35
g	14	18
y	5	10
Δ	0.0075 0.005 0.001	0.0075 0.005 0.001

5. Simulation and Test Results

In the simulation, different families of wavelets were tested with different decomposition levels. Haar, Daubechies, Symlet, Morlet, and Coiflet wavelet families were tested, while decomposition levels of up to four have been used.

The given results show the learning curve of the adaptive filter in both its direct form (estimator) and inverse form (equalizer). Two distortion schemes, as related to two typical channels, have been used to generate the test data stream. Further details about such channels can be found elsewhere [6]. The transmitted stream is kept fixed as a random sequence of $\{1,-1\}$ with long repetition period. The block diagram of the ordinary LMS adaptive system as shown in Figs. 2 and 3 is used for performance comparison with the proposed wavelet packet adaptive system. The software implementation is carried out with Matlab 6.1.

The investigation of the effects on the learning curve covered the following parameters: The choice of the mother function (Haar, Daubechies, etc.); the number of the decomposition levels in the wavelet tree; the step size of the LMS algorithm; the signal to noise ratio (SNR = 30 dB and 40 dB); the type of the telephone channel (ch1, and ch2) and length of adaptation filter size.

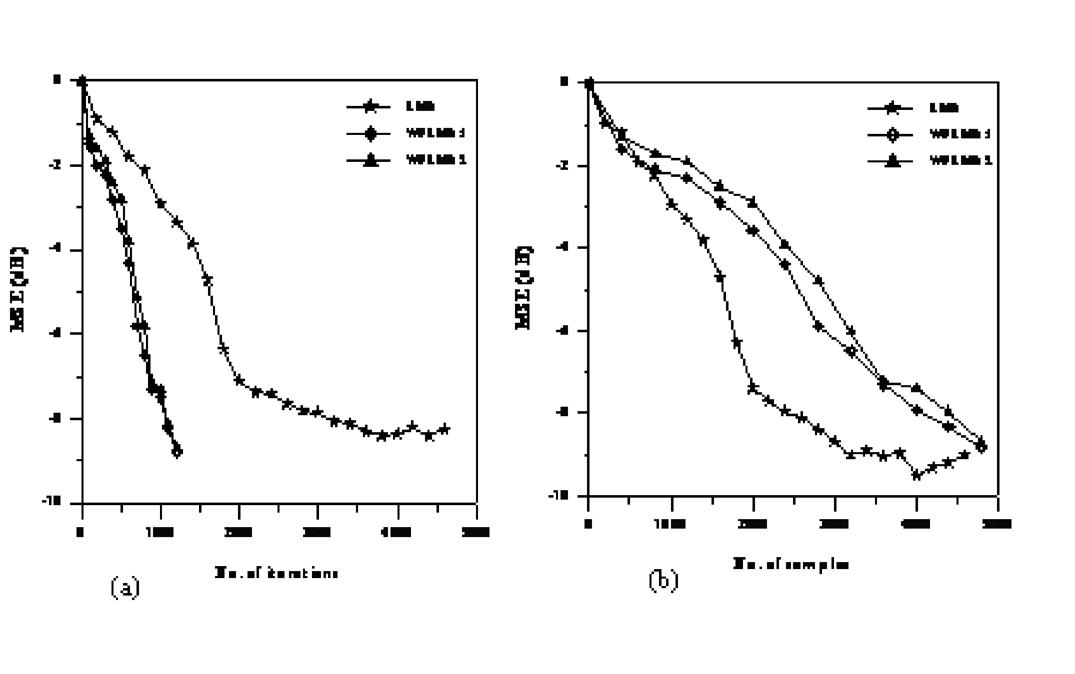
In the case of different decomposition levels the results for channel 1 are shown in Fig. 8 for adaptive estimation and in Fig.9 for adaptive equalization. Figs. 8(a) and 9(a) show the learning curve with the error as a function of the number of iterations, while Figs. 8(b) and 9(b) show the same curves with the number of the received samples as the argument of the variation. The difference between the two presentations is crucial to the explanation of our conclusions. As shown, the difference illustrates itself for the case of the wavelet packet curve. This is so because any sample of the data in their wavelet packet representation comes out of 2^j samples of the received data, where j is the level of the wavelet packet decomposition. Here, WPLMS1 and WPLMS2 are for 2 and 4 level decomposition respectively.

The results using this technique achieved good improvements in convergence time over the ordinary LMS algorithm. The change of mother function had little effect on the performance. It was found that the use of more than two decomposition levels leads to divergence, while the increase in step size speeds up the convergence. The improvement due to the WPLMS becomes more noticeable for higher levels of SNR.

The improvement in convergence time in the case of adaptive channel estimation using the WPLMS was nearly 42%, and in the case of adaptive channel equalization using the WPLMS it was nearly 33%. This demonstrates the validity and effectiveness of the proposed technique.

In the case of different mother functions in wavelet packet transform the results are shown in Fig. 10 and Fig. 11. On the other hand, Figs.12 and 13 show the convergence for different signal to noise ratios (SNR).

It is clear from the results presented in Figs.8 – 13 that the proposed algorithm performs better in channel 1. It has been found that the WPLMS with channel 2 requires larger filter sizes and more iterations to give the same convergence level as channel 1. This is because the level of distortion over channel 1 is lower than that over channel 2.



**Figure(8) Learning curves of adaptive channel estimation over channel 1 with a step size = 0.005 at SNR = 30 dB ; Variation of MSE with :
(a) number of iterations (b) number of samples)**

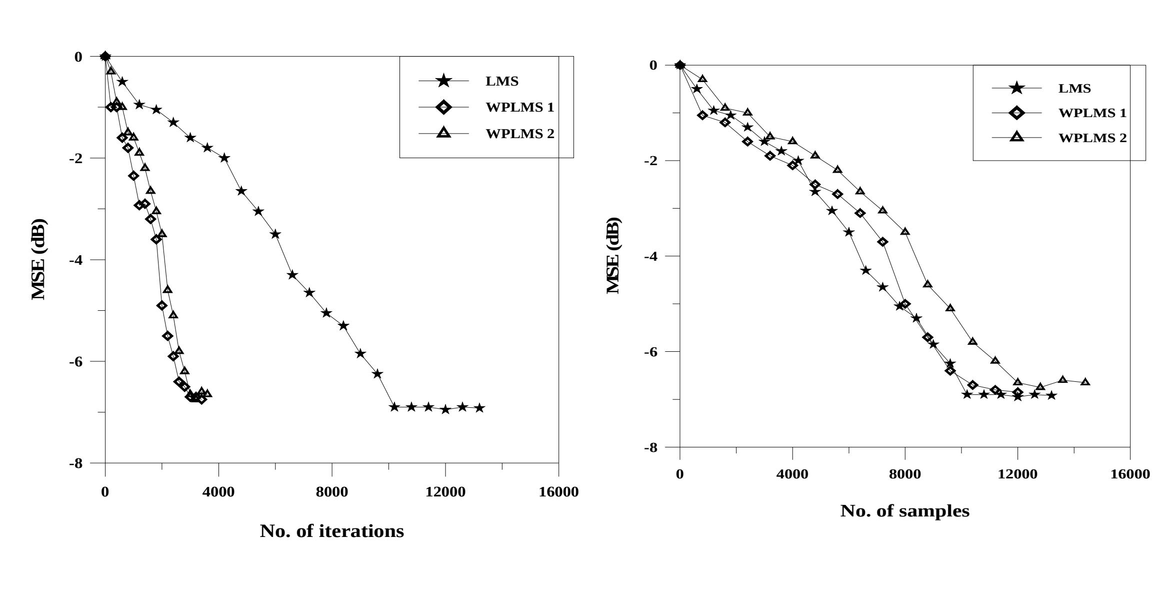


**Figure (9) Learning curves of adaptive channel equalization over Channel 1 with a step size = 0.005 at SNR = 30 dB ;Variation of MSE with :
(a)number of iterations,.(b) number of samples**

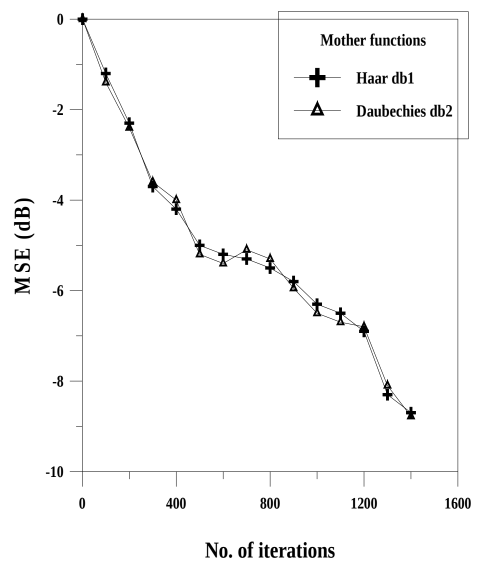


Figure (10) Adaptive channel estimation over channel 2 using wavelet packet transform with SNR=40 dB and step size = 0.0075

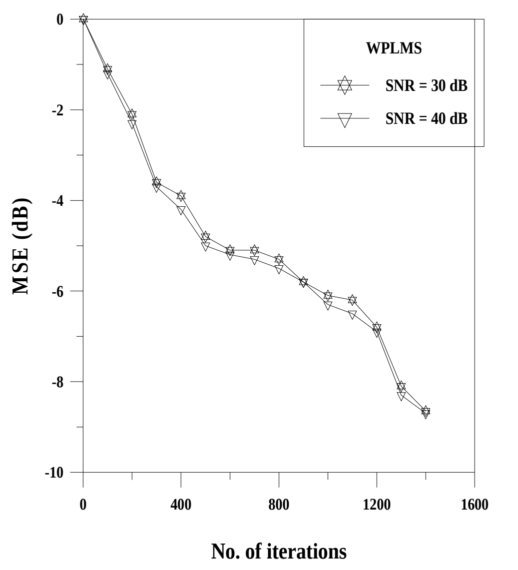


Figure (12) Learning curves for the WPLMS algorithm used in adaptive channel estimation over channel 2 with step size = 0.0075

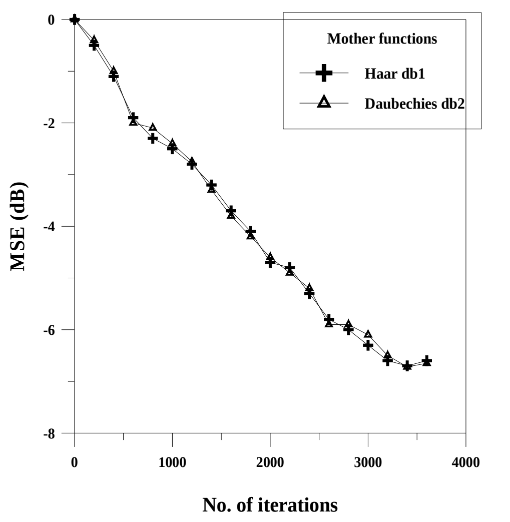


Figure (11) Adaptive channel equalization over channel 2 using wavelet packet transform with SNR = 40 dB and step size = 0.0075

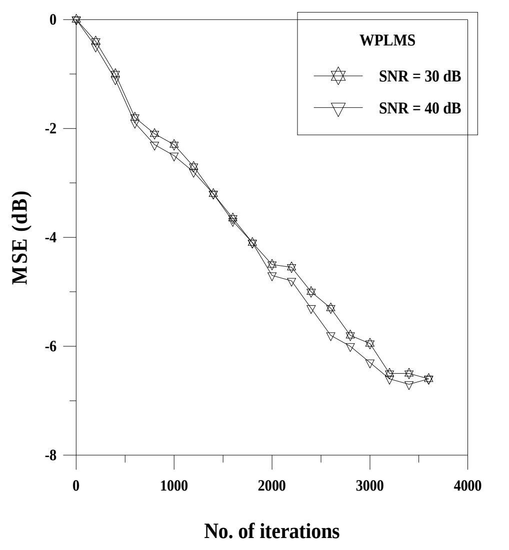


Figure (13) Learning curves for the WPLMS algorithm used in adaptive equalization over channel 2 with step size = 0.0075

6. Conclusion

The main conclusion obtained from the current research is the suitability of the compact representation of the signal (represented by its wavelet packet transform) to obtain a stable adaptive filter.

Related to the above note, the suggested system necessitates the use of a separate wavelet transformer before the adaptive filter adapts itself. This arrangement is supposed to deliver one sample to the filter out of 2^j input samples. So, each iteration in the adaptation process has 2^j times the sampling period to perform the required calculations. This is an advantage for real time applications.

The obtained results indicate that the proposed system works well with both direct and inverse adaptive modeling on the condition that the noise levels should not be high, and the distortion produced by the channel is not severe. Also, results have shown that deeper decomposition may not be efficient in the adaptation process

6. References

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المرشحات المتمايزة المتكيفة للقنوات الهاتفية و المؤسسة على تحويل الرزمة المويجية

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و الإتصالات	و تكنولوجيا المعلومات	و الإتصالات
كلية الهندسة / جامعة النهرين	الجامعة التكنولوجية	كلية الهندسة / جامعة النهرين

الخلاصة

ان تحويل المويجة يوفر نتائج جيدة و في بعض الأحيان ممتازة عند استخدامه ككتلة تحويل في العديد من الأنظمة مثل أنظمة الالكترونيات والاتصالات والطبية و حتى الكيمياوية. أن العمل المقدم في هذا البحث يربط بين تحويل المويجة مع أنظمة الترشيح المتكيفة لتكوين أنظمة ترشيح مويجة متكيفة.

إن نتائج استخدام تقنيات المويجة حسنت من وقت الحصول على حل مقارنة مع خوارزمية LMS الطبيعية. تم مقارنة النظامين على اساس رياضي و محاكاة كما تم الحصول على منحنيات التعلُم لمعدِل القناة المتكيف و مخمن القناة المتكيف باستخدام تحويل المويجة الحزمي بدوال اصل مختلفة و مستويات تفكيك مختلفة و حجم عتبة مختلف و نسب قدرة اشارة الى ضوضاء مختلفة و مع قنوات هاتفية مختلفة و حجم مرشحات مختلفة للحصول على مقارنات واضحة في الأداء مقارنة مع معدل و مخمن LMS المتكيف. ان نتائج المحاكاة التي تم الحصول عليها باستخدام MATLAB 6.1 أوضحت كفاءة التقنية المقترحة.