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A Multi Objective Programming Approach to Solve Integer Valued Neutrosophic Shortest Path Problems

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Abstract. Neutrosophic (NS) set hypothesis gives another way to deal with the vulnerabilities of the shortest path problems (SPP). Several researchers have worked on fuzzy shortest path problem (FSPP) in a fuzzy graph with vulnerability data and completely different applications in real world eventualities. However, the uncertainty related to the inconsistent information and indeterminate information isn't properly expressed by fuzzy set. The neutrosophic set deals these forms of uncertainty. This paper presents a model for shortest path problem with various arrangements of integer-valued trapezoidal neutrosophic (INVTpNS) and integer-valued triangular neutrosophic (INVTrNS). We characterized this issue as Neutrosophic Shortest way problem (NSSPP). The established linear programming (LP) model solves the classical SPP that consists of crisp parameters. To the simplest of our data, there's no multi objective applied mathematics approach in literature for finding the Neutrosophic shortest path problem (NSSPP). During this paper, we tend to introduce a multi objective applied mathematics approach to unravel the NSPP. The subsequent integer valued neutrosophic shortest path (IVNSSP) issue is changed over into a multi objective linear programming (MOLP) issue. At that point, a lexicographic methodology is utilized to acquire the productive arrangement of the subsequent MOLP issue. The optimization process affirms that the optimum integer valued neutrosophic shortest path weight conserves the arrangement of an integer valued neutrosophic number**.** Finally, some numerical investigations are given to demonstrate the adequacy and strength of the new model.

Keywords: Triangular neutrosophic fuzzy numbers; shortest path problem; network distribution; optimization technique;

1. Introduction and review of the literature:

The SPP, which uses on determining the shortest path (SP) between a specified source vertex (SV) and destination vertex (DV), is a well-known and fundamental combinatorial optimization problem. The SPP appears in many real life applications, e.g., routing [1], supply chain management problems[2], computer networks [3] etc. as a sub problem. Some effective algorithmic approaches were introduced by Dijkstra[4] and Floyd[5] in between 1950 and 1970. We refer these algorithms as classical algorithms. In classical algorithms for SPP, the costs of the arcs in a SPP are considered as real numbers, i.e., crisp number.

The arc costs of a SPP are used to represent the travelling cost, distance, time or other variable in the real world scenarios. In practical application of SPP, the edge weights in the path of a graph have some parameters which are very hard to find exactly, i.e., capacities, distance, costs, demands, traffic frequencies, etc. For example, the geographical distance between two cities may be recognized exactly, however, the travelling cost or travelling time may change due to weather, accident and traffic flow. So, the edge weights are nondeterministic in such situations and it is impossible to use the classical algorithm to find exact solution of the SPP in such uncertain environment. Many researchers thought that uncertainties due to nondeterministic environment adjust to randomness and they proposed the concept of probability SPP[6-7] and stochastic SPP[8-9]. Fuzziness is applied to randomness for dealing uncertainties of SPP. The SPP with fuzzy arc lengths (FAL), defined as FSPP, represents the type 1 fuzzy (T1-F) number as FAL. Dubois and Prade[10] first introduced FSPP based on classical Floyd and Ford-Moore-Bellman method. Till date, numerous researchers have worked on FSPP[11-22] T1-F variables are generally used in FSPP. However, if the arc lengths of a graph change under some specific condition such as travelling time or arc lengths are collected from more than one source which changes regularly. So, it becomes very hard to represent those lengths by using T1-F number. For example, we are generally unable to give the mathematical description of traffic frequency of a road in different time. These types of information are collected from a set of person using some questionnaires which consists of uncertain words. The classical fuzzy set is unable to handle these types of uncertainties as their membership values are completely crisp. These types of uncertainty can be modeled by type-2 fuzziness. In type 2 fuzzy(T2-F) sets have membership values that are also fuzzy. Few researchers have work on SPP with T2-F set as arc lengths. The membership degree in fuzzy set or T2-F set is unable to model the ambiguous situation of the SPP. To solve this problem, Atanassov[23] have introduced the idea of intuitionistic fuzzy (INF) set which is described by membership and non-membership degree and it can also deal the imprecision information. Recently,[24] has used INF set as arc length of SPP. However, T1-F set, INF set and T2-F set have no capability to understand all possibilities of indeterminate or inconsistent information of SPP.

We use multi objective linear programming (MOLP) optimization approach on this paper. The traditional MOLP is related to crisp parameters. However, in real world situation, the variables for multi criteria decision making (MCDM) troubles are obscure in nature which has been discussed above. Such vulnerabilities results to expanded troubles in the related streamlining endeavors. Basically evading these vulnerabilities is unfortunate as it might influence the choice making[25-26]. Henceforth, vulnerability ought to be taken care of by inaccurate LP techniques. Amid the most recent couple of decades, immense number of vague LP techniques has been set up to deal with different vulnerabilities. Those methods classified into three classes, i.e., single objective fuzzy linear programming, bi-objective fuzzy linear programming, and multi-objective fuzzy linear programming. Najafi&Edaltpanah [27], Hosseinzadeh &Edaltpanah[28], and so forth. There have been many researches in past regarding multi-objective linear programming technique under neutrosophic environment. Das and Roy [29] suggested an innovative application of neutrosophic optimization technique in riser design problem. Hezam et al. [30] came up with Taylor series approximation to solve these kinds of problems. Sarkar et al.[31] attained powerfully neutrosophic goal programming process using structural design optimization. Ahmed et al.[32] proposed an interesting technique for solving nonlinear problems using hesitant fuzzy computational algorithm. Fahmi et al.[33] proposed Triangular Cubic Hesitant Fuzzy Einstein Hybrid Weighted Averaging Operator, Islam & Ray[34] introduced Multi-objective portfolio selection model. To the excellent of our facts, there's no multi-goal LP version in literature for SPP with NS number.

In this paper, we represent the edge weight as neutrosophic number. In 1998, Smarandache[35-36] has introduced the idea of NS set which can deal with vague, indeterminate and inconsistent information of the problem that may exist in the real word scenarios. It is an extension of crisp set, T1-F set and INFu set. A NS set is described by three membership degrees of truth, indeterminate and a false. This three independent membership degrees are within the non-standard unit interval] 0, 1+[. However, the membership degree of fuzzy set lies between the interval [0,1]. Recently, the NS set is used for modeling many engineering applications because it can deal with incomplete information as well as the inconsistent and indeterminate information. Some of the recent works on NS related problem are available in references [37-46]. Moreover, there are many researchers who have introduced some significant operators for decision- making in engineering technicalities under neutrosophic environment. Ye [47] invented a new operator for trapezoidal neutrosophic set, Deli [48] proposed innovative operators on single valued trapezoidal neutrosophic numbers, Gulistan et al.[49] introduced Neutrosophic Cubic Mean Operators and Entropy , Khan et al.[50] attained Interval Neutrosophic Dombi Power Bonferroni Mean Operators, Fahmi et al.[33] proposed Triangular Cubic Hesitant Fuzzy Einstein Hybrid Weighted Averaging Operator, Khan et al.[51] suggested Neutrosophic Cubic Einstein Geometric Aggregation Operators and many others[52-58] These operators are used in handling different real life technicalities. We have tabulated all those important influences of different researchers who have introduced these real life applications in Table 1.

Table 1. Important influences of different researchers for real life applications of operators for decision-making under Neutrosophic environment.

Author and references	Year	Significance influences
Kour & Basu [59]	2015	Real-life transportation problem using extended fuzzy programming techniq-ue.
Shahzadi et al. [60]	2017	Single-Valued Neutrosophic Sets in Medical Diagnosis.
Mohamed et al. [61]	2017	Neutrosophic integer programming problem.
Kour & Basu [62]	2017	Sorting of transportation companies.
Abdel-Basset et al. [63]	2018	Linear programming problem applied in diary industries.
Mondal et al. [64]	2018	Single valued neutrosophic hyperbolic sine similarity mea-sure based on madm strategy.
Altinirmak et al. [65]	2018	Evaluation of mutual funds' performance via a case study carried out in Turkey.

Alava et al. [66]	2018	Analytic hierarchy process for project selection.
Dey et al. [67]	2018	Minimum spanning trees for undirected graphs.
Teruel et al. [68]	2018	Selection of cloud computing services based on consensus.
Broumi et al. [69]	2018	Spanning Tree Problem with Edge Weights.
Mohamed et al. [70]	2017	Critical path problem.

Table 2, charts some significant influences towards NSPP. Based on the previous discussions on SPP and currently available data as mentioned in below table, there are no existing methods which are available for MOLP under neutrosophic environment .Therefore, there is a need to establish a neutrosophic version multi objective linear programming for neutrosophic shortest path problems.

Table 2. Significance influences of different authors towards neutrosophic shortest path problem

Author and references	**Year**	**Significance influences**
Broumi et.al.[71]	2016b	Dijkstra principle for interval based data based problems.
Broumi et.al.[72]	2016c	Single valued trapezoidal NS numbers for dijkstra principle
Broumi et.al.[73]	2016d	Single valued NS Graphs in SPP
Broumi et.al.[74]	2017a	Neutrosophic setting along with trapezoidal fuzzy for proce-ssing SPP.
Broumi et.al.[75]	2017b	Bipolar neutrosophic environment.
Broumi et.al[76]	2017c	Interval-valued NS setting environment for process-ing SPP.
Broumi et.al[77]	2018	Invented decision-making problem for maximization of deviation method with partial weight under the neutrosophic environment
Kumar et al.[78]	2018	A new algorithm based on score function for finding the neutrosophic shortest path problems.
Broumi et.al.[79]	2019	Interval valued trapezoidal and triangular neutrosophic envir-onment using improved score and center of gravity function for finding SPP.

To the nice of our facts, there are no multi-objective linear programming (MOLP) models in literature for SPP under NS environment. Additionally, the currently available methods for solving SPP have a significant number of limitations and drawbacks which have been explained in different sections of this paper. This complete scena-rio has motivated us to come up with a new method for solving SPP with neutrosophic range which are formu-lated and solved with the use of multi-goal linear programming model for the first time.

NS set theory is documented technique to manage uncertainty in optimization problem. SPP with NS variety etc. area unit represented by few researchers. The most contributions of this paper as follows.

- This approach helps to resolve a new set of problem with NS number.
- We define the SP problem below integer valued neutrosophic surroundings and recommend an efficient answer technique to locate the ultimate integer valued neutrosophic path weight and the corresponding integer valued neutrosophic course.
- First time within the literature of neutrosophic set, we tend to introduce a lexicographical approach in conjunction with multi objective linear programming method.

Whatever remains of the paper is systemized as pursues: In Segment 2, some fundamental ideas of whole number esteemed neutrosophic numbers are exhibited. In Segment 3, the scientific detailing of the SP issue under whole number esteemed neutrosophic condition is given.and also, another technique is proposed for taking care of a similar issue. In Segment 4, a numerical precedent is given to represent the proposed arrangement system. In segment 5, result and disscussion. At last, we conclude the paper in Segment 6.

2. Preliminaries

Definition 2.1: [80] : Let X be a space point or objects, with a genetic element in X denoted by *x*. A single-valu-ed NS, V in X is characterised by three independent parts, namely truth-MF T_V, indeterminacy-MF I_V and falsity -MF F_V, such that $T_V : X \to [0,1], I_V : X \to [0,1], \ and\ F_V : X \to [0,1]$.

Now, V is denoted as $V = \{ < x, (T_V(x), I_V(x), F_V(x)) > \ | x \in X \}$, satisfying $0 \le T_V(x) + I_V(x) + F_V(x) \le 3$.

Definition 2.2: [81]: Let $\hat{r}^N = \left\langle \left(\overline{r_{ij,l}}, \overline{r_{ij,m}}, \overline{r_{ij,k}} \right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k} \right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}} \right) \right\rangle$ is a special NS on the real number set R, whose truth-MF $\Delta_{\tilde{r}}(x)$, indeterminacy-MF $\nabla_{\tilde{r}}(x)$, and falsity-MF $\mho_{\tilde{r}}(x)$ are given as follows:

$$\Delta_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(x-\overline{r_{ij,l}}\right)}{\left(\overline{r_{ij,m}}-\overline{r_{ij,l}}\right)} & \overline{r_{ij,l}}\le x<\overline{r_{ij,m}},\\ 1 & x=\overline{r_{ij,m}},\\ \dfrac{\left(\overline{r_{ij,k}}-x\right)}{\left(\overline{r_{ij,k}}-\overline{r_{ij,m}}\right)} & \overline{r_{ij,m}}<x\le\overline{r_{ij,k}},\\ 0 & otherwise.\end{cases} \quad (1)$$

$$\nabla_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(r_{ij,m}-x\right)}{\left(r_{ij,m}-r_{ij,l}\right)} & r_{ij,l}\le x<r_{ij,m},\\ 0 & x=r_{ij,m},\\ \dfrac{\left(x-r_{ij,m}\right)}{\left(r_{ij,k}-r_{ij,m}\right)} & r_{ij,m}<x\le r_{ij,k},\\ 1 & otherwise.\end{cases} \quad (2)$$

$$\mho_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(\underline{r_{ij,m}}-x\right)}{\left(\underline{r_{ij,m}}-\underline{r_{ij,l}}\right)} & \underline{r_{ij,l}}\le x<\underline{r_{ij,m}},\\ 0 & x=\underline{r_{ij,m}},\\ \dfrac{\left(x-\underline{r_{ij,m}}\right)}{\left(\underline{r_{ij,k}}-\underline{r_{ij,m}}\right)} & \underline{r_{ij,m}}<x\le\underline{r_{ij,k}},\\ 1 & otherwise.\end{cases} \quad (3)$$

Where $0\le\Delta_{\tilde{r}}(x)+\nabla_{\tilde{r}}(x)+\mho_{\tilde{r}}(x)\le 3,\quad x\in\hat{r}^N$

The parametric form is defined as follows

$\left(\hat{r}^N\right)_{\delta,\nu,\eta}=\left[\Delta_{\hat{r}^{N1}}(\delta),\Delta_{\hat{r}^{N2}}(\delta);\nabla_{\hat{r}^{N1}}(\nu),\nabla_{\hat{r}^{N2}}(\nu);\mho_{\hat{r}^{N1}}(\eta),\mho_{\hat{r}^{N2}}(\eta)\right]$, Where $\delta,\nu,\eta\in(0,1)$

$\Delta_{\hat{r}^{N1}}(\delta)=\overline{r_{ij,l}}+\delta\left(\overline{r_{ij,m}}-\overline{r_{ij,l}},\right)$

$\Delta_{\hat{r}^{N2}}(\delta)=\overline{r_{ij,k}}-\delta\left(,\overline{r_{ij,k}}-\overline{r_{ij,m}}\right)$

$\nabla_{\hat{r}^{N1}}(\nu)=r_{ij,m}-\nu\left(r_{ij,m}-r_{ij,l}\right)$

$\nabla_{\hat{r}^{N2}}(\nu)=r_{ij,m}+\nu\left(r_{ij,k}-r_{ij,m}\right)$

$\mho_{\hat{r}^{N1}}(\eta)=\underline{r_{ij,m}}-\eta\left(\underline{r_{ij,m}}-\underline{r_{ij,l}}\right)$

$\mho_{\hat{r}^{N2}}(\eta)=\underline{r_{ij,m}}+\eta\left(\underline{r_{ij,m}}-\underline{r_{ij,k}}\right)$

Example 2.1: Now consider the a integer value neutrosophic number i.e. <(0.3,0.4,0.6);(0.1,0.4,0.5);(0.3, 0.5, 0.7)> then we get the membership function as shown in figure 1 where red graph denotes truth membership , green denotes falsity and blue denotes indeterminacy function.

Definition 2.3:[61-63]: Arithmetic operation).Let $\hat{r}^N=\left\langle\left(\overline{r_{ij,l}},\overline{r_{ij,m}},\overline{r_{ij,k}}\right),\left(r_{ij,l},r_{ij,m},r_{ij,k}\right),\left(\underline{r_{ij,l}},\underline{r_{ij,m}},\underline{r_{ij,k}}\right)\right\rangle$ and $\hat{s}^N=\left\langle\left(\overline{s_{ij,l}},\overline{s_{ij,m}},\overline{s_{ij,k}}\right),\left(s_{ij,l},s_{ij,m},s_{ij,k}\right),\left(\underline{s_{ij,l}},\underline{s_{ij,m}},\underline{s_{ij,k}}\right)\right\rangle$ be two arbitrary SVTNNs, and $\theta\ge 0$; then:

$$\hat{r}^N \oplus \hat{s}^N = \left\langle \left(\overline{r_{ij,l}} + \overline{s_{ij,l}}, \overline{r_{ij,m}} + \overline{s_{ij,m}}, \overline{r_{ij,k}} + \overline{s_{ij,k}}\right), \left(r_{ij,l} + s_{ij,l}, r_{ij,m} + s_{ij,m}, r_{ij,k} + s_{ij,k}\right), \left(\underline{r_{ij,l}} + \underline{s_{ij,l}}, \underline{r_{ij,m}} + \underline{s_{ij,m}}, \underline{r_{ij,k}} + \underline{s_{ij,k}}\right) \right\rangle$$

$$\hat{r}^N \otimes \hat{s}^N = \left\langle \left(\overline{r_{ij,l}} \cdot \overline{s_{ij,l}}, \overline{r_{ij,m}} \cdot \overline{s_{ij,m}}, \overline{r_{ij,k}} \cdot \overline{s_{ij,k}}\right), \left(r_{ij,l} \cdot s_{ij,l}, r_{ij,m} \cdot s_{ij,m}, r_{ij,k} \cdot s_{ij,k}\right), \left(\underline{r_{ij,l}} \cdot \underline{s_{ij,l}}, \underline{r_{ij,m}} \cdot \underline{s_{ij,m}}, \underline{r_{ij,k}} \cdot \underline{s_{ij,k}}\right) \right\rangle$$

$$\theta \hat{r}^N = \left\langle \left(\theta \overline{r_{ij,l}}, \theta \overline{r_{ij,m}}, \theta \overline{r_{ij,k}}\right), \left(\theta r_{ij,l}, \theta r_{ij,m}, \theta r_{ij,k}\right), \left(\theta \underline{r_{ij,l}}, \theta \underline{r_{ij,m}}, \theta \underline{r_{ij,k}}\right) \right\rangle \ if \ (\theta > 0)$$

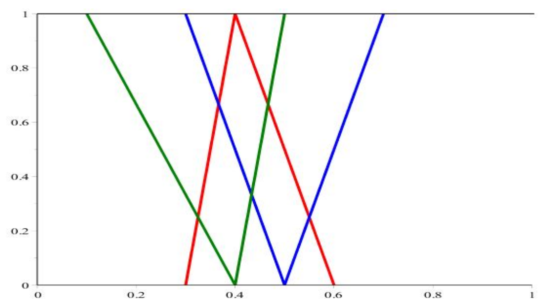

Figure 1: membership of function of example 2.1.

Definition 2.4: [47]: Let $\hat{r}^N = \left\langle \left(\overline{r_{ij,l}}, \overline{r_{ij,m}}, \overline{r_{ij,k}}\right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k}\right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}}\right) \right\rangle$ then the score function and the accuracy function are defined as follows:

$$s(\tilde{r}) = \frac{1}{12}\left[8 + \left(\overline{r_{ij,l}} + 2 \cdot \overline{r_{ij,m}} + \overline{r_{ij,k}}\right) - \left(r_{ij,l} + 2 \cdot r_{ij,m} + r_{ij,k}\right) - \left(\underline{r_{ij,l}} + 2 \cdot \underline{r_{ij,m}} + \underline{r_{ij,k}}\right)\right]$$

$$H(\tilde{r}) = \frac{1}{4}\left[\left(\overline{r_{ij,l}} + 2 \cdot \overline{r_{ij,m}} + \overline{r_{ij,k}}\right) - \left(\underline{r_{ij,l}} + 2 \cdot \underline{r_{ij,m}} + \underline{r_{ij,k}}\right)\right]$$

Definition 2.5. [47]: $\hat{r}^N = \left\langle \left(\overline{r_{ij,l}}, \overline{r_{ij,m}}, \overline{r_{ij,k}}\right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k}\right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}}\right) \right\rangle$ and $\hat{s}^N = \left\langle \left(\overline{s_{ij,l}}, \overline{s_{ij,m}}, \overline{s_{ij,k}}\right), \left(s_{ij,l}, s_{ij,m}, s_{ij,k}\right), \left(\underline{s_{ij,l}}, \underline{s_{ij,m}}, \underline{s_{ij,k}}\right) \right\rangle$ be two arbitrary SVTNNs, the ranking of $\tilde{r}$ and $\tilde{s}$ by score function is described as follows:

1. $if \ s(\tilde{r}) \prec s(\tilde{s}) \ then \ \tilde{r} \prec \tilde{s}$
2. $if \ s(\tilde{r}) \approx s(\tilde{s}) \ and \ if$
   a. $H(\tilde{r}) \prec H(\tilde{s}) \ then \ \tilde{r} \prec \tilde{s}$
   b. $H(\tilde{r}) \succ H(\tilde{s}) \ then \ \tilde{r} \succ \tilde{s}$
   c. $H(\tilde{r}) \approx H(\tilde{s}) \ then \ \tilde{r} \approx \tilde{s}$

3. The Proposed model

Before we start the main algorithm, we introduce a sub-section i.e., shortcoming and limitation of some of the existing models:

3.1 Discussion on shortcoming of some of the existing methods

At first, we talked about the inadequacy and constraint of the current techniques under various kinds of NS condition.

Broumi et al.[73,75-77] proposed various techniques to locate the most brief way under various sorts of NS condition. In any case, we watch few shortcoming and confinement of the current techniques which is talked about beneath.

1. Author utilized some invalid numerical presumption to tackle the issue. This has been talked about in detail in Example 3.1.
2. Author utilized some numerical definition for anticipating the best course among the two continuous ways. In any case, we see that this presumption can't anticipate the best course if both the way has a similar score work. This has been talked about in detail in Example 3.2.
3. Author used some invalid mathematical assumption to resolve the matter. This has been mentioned well in Example 3.3. Here, we have a tendency to observe that the authors used score and accuracy functions with constant price. However, by this formulation, it's troublesome to settle on the shortest path in between the 3 consecutive nodes. Therefore, we can't decide the higher route to achieve the destination and therefore the journey is terminated

Therefore, we tend to conclude that the present technique isn't valid because of higher than mentioned shortcomings and limitations. This has driven us to propose few new strategies that overcome such limitations and also the projected strategies are careful below in section 4.

Example 3.**1:** Broumi et al:[73] Here authors have considered two arbitrary i.e., $\tilde{e},\tilde{f}$ be the following IVTrNS numbers:

$$\tilde{e}=\langle(0.36,0.58,0.75),(0.06,0.25,0.36),(0.04,0.1,0.18)\rangle,$$

$$\tilde{f}=\langle(0.4,0.6,0.8),(0.2,0.4,0.5),(0.1,0.3,0.4)\rangle.$$

We observe that the authors used the identical invalid mathematical assumption to solve the hassle i.e.,

$$S(\tilde{e}+\tilde{f})=S(\tilde{e})+S(\tilde{f})$$

Our goal is to show that the above-taken into consideration assumption isn't legitimate for all instances such as

$$S(\tilde{e}+\tilde{f})\neq S(\tilde{e})+S(\tilde{f}).$$

Solution : In this example, the authors Broumi et al. (2016), kind of like strategies of Broumi et al.[73,75-77] contemplate the incorrect assumption. in keeping with the strategy of Broumi et al. [73] [see; iteration five, page no 173, ref. Broumi et al. (2016)] , we have::

$$\tilde{e}+\tilde{f}=\left\langle\begin{matrix}(0.36,0.58,0.75),(0.06,0.25,0.36),\\(0.04,0.1,0.18)\end{matrix}\right\rangle\oplus\left\langle\begin{matrix}(0.4,0.6,0.8),(0.2,0.4,0.5),\\(0.1,0.3,0.4)\end{matrix}\right\rangle$$

$$\tilde{e}+\tilde{f}=\langle(0.616,0.832,0.95),(0.012,0.1,0.18),(0.004,0.03,0.072)\rangle.$$

Therefore, we get, $S(\tilde{e}+\tilde{f})=0.8225$. but $S(\tilde{e})+S(\tilde{f})$ = 1.1394.

Hence, It is clear that $S(\tilde{e}+\tilde{f})\neq S(\tilde{e})+S(\tilde{f})$.

Subsequently, In this manner, we can say that the technique for Broumi et al.[73] isn't substantial.

Example 3.**2:** Let us keep in mind a network proven in figure 1. The supply node i.e. 1 is connected to a few special nodes i.e. node 2, node three, node four with the subsequent values:

$$Arc(1,2)=\langle(1,3,15),(1,2,3),(2,4,12)\rangle,$$

$$Arc(1,3)=\langle(3,5,14),(1,1,5),(4,6,11)\rangle,$$

$$Arc(1,4)=\langle(7,10,15),(0,2,4),(8,11,12)\rangle,$$

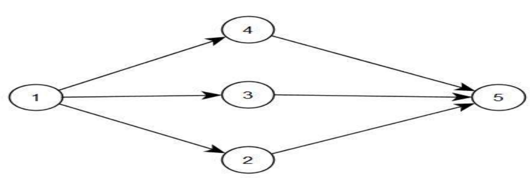

Figure 2.The network with 6 edges and 5 nodes.

Here author aim is to predict the simplest path best route among the three consecutive ways.

we observe that we won't predict the simplest route if a number of the trail have a similar score perform.

Solution: To succeed in the destination node from the supply node, there square measure 3 doable ways that, i.e., via node 2, node 3 or node 4. However, in keeping with the Definition 2.4-2.5, the projected technique gives:

$$s(Arc(1,2))=s(Arc(1,3))=s(Arc(1,4))=0\ ,\ H(Arc(1,2))=H(Arc(1,3))=H(Arc(1,4))=0$$

Here, we have a tendency to observe that the authors used score and accuracy functions with constant price. However, by this formulation, it's troublesome to settle on the shortest path in between the 3 consecutive nodes. Therefore, we have a tendency to can't decide the higher route to achieve the destination and therefore the journey is terminated.

Example 3.**3:** We consider two arbitrary, i.e., $\tilde{e}, \tilde{f}$ be the following two different Type IVTpNS numbers:

$\tilde{e} = \langle (1,3,15),(1,2,3),(2,4,12) \rangle$

$\tilde{f} = \langle (3,5,14),(1,1,5),(4,6,11) \rangle$.

We observe that the authors used the identical invalid mathematical assumption to solve the hassle, i.e.,

$$if\ s(\tilde{e}) = s(\tilde{f})\ and\ \ H(\tilde{e}) = H(\tilde{f})\ then\ \tilde{e} = \tilde{f}$$

Our goal is to show that the above-taken into consideration assumption isn't legitimate for all instances..

This above circumstance is valid simplest when $\tilde{e} = \tilde{f}\ then\ only\ H(\tilde{e}) = H(\tilde{f})\ and\ s(\tilde{e}) = s(\tilde{f})$

but not necessarily it is valid $if\ s(\tilde{e}) = s(\tilde{f})\ and\ \ H(\tilde{e}) = H(\tilde{f})\ then\ \tilde{e} = \tilde{f}$

Solution: Broumi et al. [63] suggested mathematical formulas show that

$$s(\tilde{e}) = s(\tilde{f}) = 0\ ,\ H(\tilde{e}) = H(\tilde{f}) = 0$$

Here, we observe that $if\ s(\tilde{e}) = s(\tilde{f})\ and\ \ H(\tilde{e}) = H(\tilde{f})\ then\ \tilde{e} = \tilde{f}$

but in this case, $\tilde{e} \neq \tilde{f}$. Hence, we conclude that the method suggested by Broumi et al.[73] IVTrNS environment is scientifically incorrect.

3.2. Existing crisp model in SPP

In this section, we have a tendency to study the notation and existing linear model in crisp and proposed neutrosophic SPPs.

Notations

Ω **:** Starting node

$\mho$ **:** Final destination node

$\sum_{k=1}^{s} x_{mk}$: The total flow out of node *s*.

$\sum_{k=1}^{s} x_{mk}$: The total flow into node *s*.

RK_{mk} *:* The shortest distance from associate degree mth node to kth node.

The crisp SPP problem within the applied math model is as follows [78,82]

$$Min = \sum_{m=1}^{s}\sum_{k=1}^{s} RK_{mk} \cdot x_{mk}$$

Subject to: (4)

$$\sum_{m=1}^{s} x_{mk} - \sum_{k=1}^{s} x_{km} = \underset{\sim}{\kappa}_m$$

for all $x_{mk} \in \Re$ and non-negative where $m,k = 1,2,.....,s$ and:

$$\underset{\sim}{\kappa}_m = \begin{cases} 1\ if & m = \Omega, \\ 0\ if & m = \Omega+1, \Omega+2, ..., \mho - 1 \\ -1\ if & m = \mho. \end{cases} \quad (5)$$

3.3. Transformation of crisp SPP model into nutrosophic SPP

If we tend to replaced the parameter RK_{mk} into neutrosophic parameters, i.e.. RK_{mk}^{N} , then the applied math model of the neutrosophic surroundings is as follows: (Kumar et al. [78])

$$Min = \sum_{m=1}^{s}\sum_{k=1}^{s} RK_{mk}^{N} .\ x_{mk}$$

Subject to: (6)

$$\sum_{m=1}^{s} x_{mk} - \sum_{k=1}^{s} x_{km} = \underset{\sim}{\kappa}_m \quad m,k = 1,2,.....,s.$$

$x_{mk} \in \Re$ And are non-negative.

3.4. Algorithm: A novel approach for finding the SPP under IVTrNS environment

We consider a directed acyclic graph whose arc lengths are represented by neutrosophic number. Our proposed algorithm finds the shortest path from the source node *s* to the destination *d* of the graph. The steps of the algorithm are as follows:

In this section, we tend to provide a completely unique methodology for locating absolutely the NSP in conjunction with the NSPL. we tend to think about IVTrNS numbers for the parameters.

Step 1: Consider the neutrosophic model that is as follows.

$$Min\ NSC = \left\langle \left(\overline{NS_{ij,l}}, \overline{NS_{ij,m}}, \overline{NS_{ij,k}}\right), \left(NS_{ij,l}, NS_{ij,m}, NS_{ij,k}\right), \left(\underline{NS_{ij,l}}, \underline{NS_{ij,m}}, \underline{NS_{ij,k}}\right) \right\rangle x_{ab} \tag{7}$$

Subject to constraints

Subject to:

$$\sum_{m=1}^{s} \overline{\pi}_{mk} - \sum_{k=1}^{s} \overline{\pi}_{mk} = \tilde{\kappa}_m$$

for all $\overline{\pi}_{mk} \in \Re$ and non-negative where $m, k = 1, 2, ..., s$ and

$$\Gamma\ \tilde{\kappa}_m = \begin{cases} 1\ if & m = \Gamma, \\ 0\ if & m = \Gamma+1, \Gamma+2, ..., \Lambda-1 \\ -1\ if & m = \Lambda. \end{cases} \tag{8}$$

Step 2: Use the arithmatic operation from the definition 2.3, then we get

$$NSC = \left\langle \begin{array}{l} \left(\begin{array}{l} \sum_{a=1}^{k}\sum_{b=1}^{k} \overline{NC_{ij,l}} \cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k} \overline{NC_{ij,m}} \cdot x_{ab}, \\ \sum_{a=1}^{k}\sum_{b=1}^{k} \overline{NC_{ij,k}} \cdot x_{ab}, \end{array} \right); \left(\begin{array}{l} \sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ij,l} \cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ij,m} \cdot x_{ab}, \\ \sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ij,k} \cdot x_{ab}, \end{array} \right); \\ \left(\begin{array}{l} \sum_{a=1}^{k}\sum_{b=1}^{k} \underline{NC_{ij,l}} \cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k} \underline{NC_{ij,m}} \cdot x_{ab}, \\ \sum_{a=1}^{k}\sum_{b=1}^{k} \underline{NC_{ij,k}} \cdot x_{ab}, \end{array} \right) \end{array} \right\rangle \tag{9}$$

With subject to constraints (8)

Step 3: Solve the following crisp SP problem using standard algorithm such as

$$\overline{NC_l}^{*} = Min\ \overline{NC_l} = \sum_{a=1}^{k}\sum_{b=1}^{k} \overline{NC_{ab,l}} \cdot x_{ab} \tag{10}$$

With subject to constraints (8)

The optimum value of model 10, is $\overline{NC_l}^{*}$

Step 4: Once more solve the subsequent crisp LPP mistreatment customary algorithmic program.

$$\overline{NC_m}^{*} = Min\ \overline{NC_m} = \sum_{a=1}^{k}\sum_{b=1}^{k} \overline{NC_{ab,m}} \cdot x_{ab} \tag{11}$$

With subject to constraints

$$\sum_{a=1}^{m}\sum_{b=1}^{m} \overline{NC_{ab,l}} \cdot x_{ab} = \overline{NC_l}^{*}$$

Constraints of model (10)

The optimal value of model 11 is $\overline{NC_m}^{*}$

Step 5: Again solve the subsequent crisp LPP mistreatment customary algorithmic program.

$$\overline{NC_k}^{*} = Min\ \overline{NC_k} = \sum_{a=1}^{k}\sum_{b=1}^{k} \overline{NC_{ab,k}} \cdot x_{ab} \tag{12}$$

With subject to constraints

$$\sum_{a=1}^{m}\sum_{b=1}^{m} \overline{NC_{ab,m}} \cdot x_{ab} = \overline{NC_m}^{*}$$

Constraints of model 11

The optimal value of model 12 is $\overline{NC_k}^{*}$

Step 6: Similarly, solve the subsequent crisp LPP mistreatment customary algorithmic program.

$$NC_l^* = Min\ NC_l = \sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ab,l} \cdot x_{ab} \tag{13}$$

With subject to constraints

$$\sum_{a=1}^{m}\sum_{b=1}^{m} \overline{NC_{ab,k}} \cdot x_{ab} = \overline{NC_k}^*$$

Constraints of model 12

The optimal value of model 13 is

Step 7: Again solve the subsequent crisp LPP mistreatment customary algorithmic program.

$$NC_m^* = Min\ NC_m = \sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ab,m} \cdot x_{ab} \tag{14}$$

With subject to constraints

$$\sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ab,l} \cdot x_{ab} = NC_l^*$$

Constraints of model 13

The optimal value of model 14 is NC_m^*

Step 8: Again solve the subsequent crisp LPP mistreatment customary algorithmic program.

$$NC_k^* = Min\ NC_k = \sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ab,k} \cdot x_{ab} \tag{15}$$

With subject to constraints

$$\sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ab,m} \cdot x_{ab} = NC_m^*$$

Constraints of model 14

The optimal value of model 15 is NC_k^*

Step 9: Again solve the subsequent crisp LPP mistreatment customary algorithmic program.

$$\underline{NC_l}^* = Min\ \underline{NC_l} = \sum_{a=1}^{k}\sum_{b=1}^{k} \underline{NC_{ab,l}} \cdot x_{ab} \tag{16}$$

With subject to constraints

$$\sum_{a=1}^{k}\sum_{b=1}^{k} NC_{ab,k} \cdot x_{ab} = NC_k^*$$

Constraints of model 15

The optimal value of model 16 is $\underline{NC_l}^*$

Step 10: Again solve the subsequent crisp LPP mistreatment customary algorithmic program.

$$\underline{NC_m}^* = Min\ \underline{NC_m} = \sum_{a=1}^{k}\sum_{b=1}^{k} \underline{NC_{ab,m}} \cdot x_{ab} \tag{17}$$

With subject to constraints

$$\sum_{a=1}^{k}\sum_{b=1}^{k} \underline{NC_{ab,l}} \cdot x_{ab} = \underline{NC_l}^*$$

Constraints of model 16

The optimal value of model 17 is $\underline{NC_m}^*$

Step 11: Again solve the subsequent crisp LPP mistreatment customary algorithmic program.

$$\underline{NC_k}^* = Min\ \underline{NC_k} = \sum_{a=1}^{k}\sum_{b=1}^{k} \underline{NC_{ab,k}} \cdot x_{ab} \tag{18}$$

With subject to constraints

$$\sum_{a=1}^{k}\sum_{b=1}^{k} \underline{NC_{ab,m}} \cdot x_{ab} = \underline{NC_m}^*$$

Constraints of model 17

The optimal value of model 18 is $\underline{NC_k}^*$

Theorem 1: The optimal value of model (18) provides the optimum value of IVNSSP problem (7).

Proof:let x^*_{ab} be the ideal arrangement of model (18) and $\tilde{x}_{ab}$ be an arbitrary neutrosophic viable solution of IVNSSP (7). the solution system of the proposed technique confirms that the most excellent solution of problem (18) is the greatest answer of the problems (7)-(17). Owing the optimality of x^*_{ab} for problem (10) and feasibility of $\tilde{x}_{ab}$ for problem (10), we conclude that $\overline{NC_l}^* = \sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ab,l}}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ab,l}}\cdot \tilde{x}_{ab}$ moreover, owing the optimality of x^*_{ab} for hassle (11) and we conclude that $\overline{NC_m}^* = \sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ab,m}}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ab,m}}\cdot \tilde{x}_{ab}$ similar discussions make sure that

$$\overline{NC_k}^* = \sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ab,k}}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ab,k}}\cdot \tilde{x}_{ab},\ NC_l^* = \sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ab,l}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ab,l}\cdot \tilde{x}_{ab}$$

$$NC_m^* = \sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ab,m}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ab,m}\cdot \tilde{x}_{ab},\ NC_k^* = \sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ab,k}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ab,k}\cdot \tilde{x}_{ab}$$

$$\underline{NC_l}^* = \sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ab,l}}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ab,l}}\cdot \tilde{x}_{ab},\ \underline{NC_m}^* = \sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ab,m}}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ab,m}}\cdot \tilde{x}_{ab},$$

$$\underline{NC_k}^* = \sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ab,k}}\cdot x^*_{ab} \le \sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ab,k}}\cdot \tilde{x}_{ab},$$

Therefore

$$NSC = \left\langle \begin{array}{l} \left(\overline{NC_{ab,l}}^*, \overline{NC_{ab,m}}^*, \overline{NC_{ab,k}}^*\right), \\ \left(NC_{ab,l}^*, NC_{ab,m}^*, NC_{ab,k}^*\right), \\ \left(\underline{NC_{ab,l}}^*, \underline{NC_{ab,m}}^*, \underline{NC_{ab,k}}^*\right) \end{array} \right\rangle \preceq \left\langle \begin{array}{l} \left(\sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ij,l}}\cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ij,m}}\cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k}\overline{NC_{ij,k}}\cdot x_{ab},\right); \\ \left(\sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ij,l}\cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ij,m}\cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k}NC_{ij,k}\cdot x_{ab},\right); \\ \left(\sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ij,l}}\cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ij,m}}\cdot x_{ab}, \sum_{a=1}^{k}\sum_{b=1}^{k}\underline{NC_{ij,k}}\cdot x_{ab},\right) \end{array} \right\rangle$$

4. Example of network application:

To justify our proposed algorithms, we consider a network shown in Figure 3 [Broumi et al.[73,75-77], Kumar et al. [78]] and figure 4 [Kumar et al. [11]]

Example 4.1: Consider a network (Figure 3), with six nodes and eight edges, where node 1 is the SV and node 6 is the DV. The IVTrNS cost is given in Table 3[73].

Table 3. The conditions of Example 4.1 [73]

T	H	IVTrNS Cost	T	H	IVTrNS Cost
1	2	<(0.1,0.2,0.3);(0.2,0.3,0.5);(0.4,0.5,0.6)>	3	4	<(0.2,0.3,0.5);(0.2,0.5,0.6);(0.4,0.5,0.6)>
1	3	<(0.2,0.4,0.5);(0.3,0.5,0.6);(0.1,0.2,0.3)>	3	5	<(0.3,0.6,0.7);(0.1,0.2,0.3);(0.1,0.4,0.5)>
2	3	<(0.3,0.4,0.6);(0.1,0.2,0.3);(0.3,0.5,0.7)>	4	6	<(0.4,0.6,0.8);(0.2,0.4,0.5);(0.1,0.3,0.4)>
2	5	<(0.1,0.3,0.4);(0.3,0.4,0.5);(0.2,0.3,0.6)>	5	6	<(0.2,0.3,0.4);(0.3,0.4,0.5);(0.1,0.3,0.5)>

*IVTrNS: Integer valued triangular neutrosophic

Solution: Applying steps 1-11 in proposed Algorithm, we get the NSSP as $1 \to 2 \to 5 \to 6$ with the NSPL is <(0.4, 0.8, 1.1), (0.8, 1.1, 1.5), (0.7, 1.1, 2.1)>.

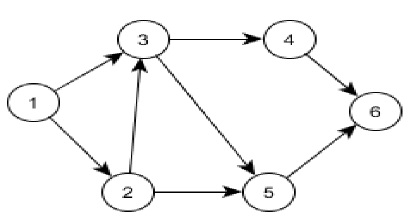

Figure 3.A network where node 1 is the SV and node 6 is the DV[Broumi et al.[73,75-77], Kumar et al. [78]

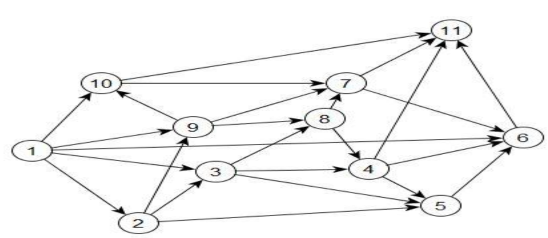

Figure 4. A network with eleven vertices and twenty-five edges Kumar et al. [11]

Solution approach is shown in below steps:

Step 1: The GVNSSP model is detailed as shown in equation 7:

$$Min\ GVC = <(0.1,0.2,0.3);(0.2,0.3,0.5);(0.4,0.5,0.6)> \cdot x_{12} + <(0.2,0.4,0.5);(0.3,0.5,0.6);(0.1,0.2,0.3)> \cdot x_{13}$$
$$+ <(0.3,0.4,0.6);(0.1,0.2,0.3);(0.3,0.5,0.7)> \cdot x_{23} + <(0.1,0.3,0.4);(0.3,0.4,0.5);(0.2,0.3,0.6)> \cdot x_{25}$$
$$+ <(0.2,0.3,0.5);(0.2,0.5,0.6);(0.4,0.5,0.6)> \cdot x_{34} + <(0.3,0.6,0.7);(0.1,0.2,0.3);(0.1,0.4,0.5)> \cdot x_{35}$$
$$<(0.4,0.6,0.8);(0.2,0.4,0.5);(0.1,0.3,0.4)> \cdot x_{46} + <(0.2,0.3,0.4);(0.3,0.4,0.5);(0.1,0.3,0.5)> \cdot x_{56}$$

Subject to constraints in line with equation (8)

$x_{12} + x_{13} = 1;$ $x_{23} + x_{25} = x_{12};$ $x_{34} + x_{35} = x_{13} + x_{23};$

$x_{46} = x_{34};$ $x_{56} = x_{25} + x_{35};$ $x_{56} + x_{46} = 1;$

Step 2: Execute the arithmatic operation on equation (7) then proceed to step 3

Step 3: Now we get the linear standard equation (10)

$$\overline{NGV_r}^* = Min\ \overline{NGV_r} = 0.1 \bullet x_{12} + 0.4 \bullet x_{46} + 0.2 \bullet x_{13} + 0.2 \bullet x_{34} + 0.1 \bullet x_{56} + 0.2 \bullet x_{25} + 0.3 \bullet x_{23} + 0.3 \bullet x_{35}$$

Subject to constraints in line with equation (8)

After excecuting the LPP then the optimal solution is 0.4.

Step 4: Now we get the linear standard equation (11)

$$\overline{NGV_a}^* = Min\ \overline{NGV_a} = 0.2 \bullet x_{12} + 0.6 \bullet x_{46} + 0.4 \bullet x_{13} + 0.3 \bullet x_{34} + 0.3 \bullet x_{56} + 0.3 \bullet x_{25} + 0.3 \bullet x_{23} + 0.3 \bullet x_{35}$$

Subject to constraints in line with equation (10) and

$$0.1 \bullet x_{12} + 0.4 \bullet x_{46} + 0.2 \bullet x_{13} + 0.2 \bullet x_{34} + 0.1 \bullet x_{56} + 0.2 \bullet x_{25} + 0.3 \bullet x_{23} + 0.3 \bullet x_{35} = 0.4;$$

After excecute the LPP then the optimal solution is 0.8

Similarly proceed from step 5 to step 11, we get the final optimum solution the NSSP is $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$ and the GVNSSPL is <(0.4, 0.8, 1.1), (0.8, 1.1, 1.5), (0.7, 1.1, 2.1)>.

Finally the shortest route is shown in figure 5:

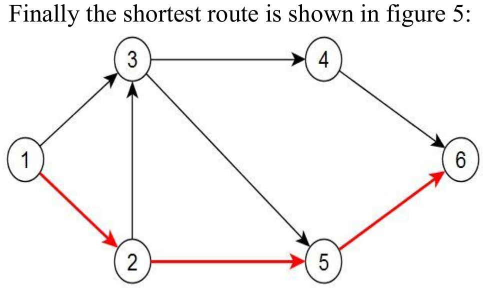

Figure 5: shown the suggested shortest route

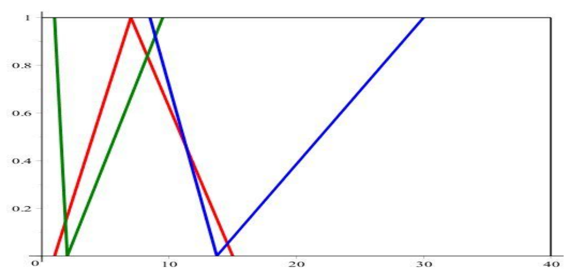

Figure 6. Membership graph for solution obtained in example 4.2.

Example 4.2. Consider figure 4 with 11 nodes and 25 edges, where SV is 1 and DV is 11. The IVTrNS time is given in Table 4[81].

Table 4. The conditions of Example 4.2 [81]

T	H	TrNS time	T	H	TrNS time
1	2	<(1,3,15);(1,2,3);(2,4,12)>	9	7	<(5,10,15); (2.5,5,7.5);(10,17.5; 19)>
1	6	<(3,5,14);(1,1,5);(4,6,11)>	10	7	<(4,6,8); (3,6,9);(1,1.75,2.5)>
1	9	<(7,10,15);(0,2,4);(8,11,12)>	4	6	<(9,16,23);(5.5,11,16.5);(11,19.25,25)>
1	10	<(1,3,4); (1,1,5);(1,2,6)>	10	11	<(0,4,11);(0,1,4.5);(7.5,11.75,24)>
2	3	<(10,15,20);(14,16,22); (12,15,19)>	1	3	<(0,1,3);(0,1,6);(1,1,2)>
2	5	<(20,60,120);(7.5,30,67.5);(10,30.63,62.5)>	4	11	<(1,2,3);(0.5,1.5,2.5);(1.2,2.7,3.5)>
3	4	<(12,18,24);(9,18,27);(3,5.25,7.5)>	2	9	<(0.5,1.5,2.5);(0.3,1.3,2.2);(0.7,1.7,2.2)>
3	5	<(0.3,1.2,2.8);(0.5,1.5,2.5);(0.8,1.7,2.7)>	3	8	<(1,3,5);(0.5,1.5,2.5);(1.2,2.7,4.5)>
4	5	<(1,5,8);(1.5,3,6.5);(4,7,9)>	6	11	<(1,4,7);(1,3,5);(3.5,6,7.5)>
5	6	<(2,4,6);(1.5,2.5,3.5);(3,5,7)>	7	11	<(1,5,9);(1.5,4.5,6.5);(4,7,10)>
7	6	<(1,5,8);(1.5,3.5,6.5);(4,6,8.5)>	9	8	<(10,15,20);(14,16,22); (12,15,19)>
8	4	<(12,18,24);(9,18,27);(3,5.25,7.5)>	9	10	<(4,6,8); (3,6,9);(1,1.75,2.5)>
8	7	<(9,16,23);(5.5,11,16.5);(11,19.25,25)>			

Solution: Applying steps 1-11 in proposed Algorithm, the NSSP is $1 \rightarrow 10 \rightarrow 11$ and the NSSPL is <(1, 7, 15);(1, 2, 9.5);(8.5, 13.75, 30)>. The result is shown in Table 4. And the final NSSPL shown in figure 6 where red graph is the truth value , green value is falsity value and blue value indeterminancy value.

5. Result and Discussion

At first, we examined the Example 4.1, which is considered by Broumi et al.[73], We found that the proposed calculation gives indistinguishable SPP recommended by Broumi et al. [73] however in Model 4.1, their proposed least neutrosophic time taken is <(0.352, 0.608, 0.748), (0.018, 0.048,0.125), (0.002, 0.018, 0.09)> which isn't like our proposed neutrosophic time<(0.4,0.8, 1.1),(0.8, 1.1, 1.5), (0.7, 1.1, 2.1)>. So as to assess which time taken are progressively exact we execute def. 2.4-2.5, we found that Broumi et al. [73] recommended time gives 0.8291 scoring value, if there should arise an occurrence of Example 4.1 while our proposed technique gives 0.1333 scoring value in the event of Example 4.1. So obviously our recommended time is increasingly precise. So plainly our recommended neutrosophic times are increasingly exact. Also, our proposed techniques foresee the better neutrosophic ongoing values as contrasted and the referenced existing strategies. This can be logically shown as Broumi et al. Method [73] $\succ$ our proposed method. The best part about our proposed calculations is that it gives the fresh ideal cost esteems as contrasted and the present existing technique. This is appeared in Table 5 (Numerical examination with existing techniques) individually.

Presently, we consider Example 4.2, any of the current strategies cannot take care of these two issues because of the impediment of the current techniques which is examined before in Section 3. As a result of these restrictions, the current technique isn't substantial. Presently, subsequent to executing proposed Calculation; in Example 4.2 , the NSSP and the NSRT is $1 \rightarrow 10 \rightarrow 11$ and is <(1, 7, 15);(1, 2, 9.5);(8.5, 13.75, 30)>Obviously the current techniques are not material wherever our proposed strategies execute the ideal arrangement.

Table 5. Numerical Comparison of our proposed method with the existing methods.

Ex	The method's name	Proposed path	SVNSPP
4.1	Process 1 [75]	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	NA
	Process 2 [73]		NSOC: <(0.352, 0.608, 0.748), (0.018, 0.048,0.125), (0.002, 0.018, 0.09)>
	Process 3 [76]	-	NA
	Process 4 [77]	-	NA
	Suggested Algorithm	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	Suggested NSOC: <(0.4,0.8, 1.1),(0.8, 1.1, 1.5), (0.7, 1.1, 2.1)>.
4.2	Process 1 [75]		NA
	Process 2 [73]		NA
	Process 3 [76]		NA
	Process 4 [77]		NA
	Suggested Algorithm	$1 \rightarrow 10 \rightarrow 11$	Suggested NSOT: is <(1, 7, 15);(1, 2, 9.5);(8.5, 13.75, 30)>

*NSOC: Neutrosophic optimum cost. *NSOT: Neutrosophic optimum time.

Because of these capabilities, we can say that our proposed algorithms are superior to the existing methods.

6. Conclusion

Conventional SPP expect exact qualities for the curve loads which isn't generally the situation in genuine circumstances. In this paper, a SPP having number esteemed neutrosophic circular segment loads has been explored. We intially figured the issue in the number esteemed neutrosophic condition. At that point, we proposed another arrangement appraoch for understanding whole number esteemed neutrosophic SPP. We changes over the IVNSSPP issue under thought into multi-objective LP issues which can be illuminated utilizing the standard LP calculations. According to the proposed optimization manner, the integer-valued neutrosophic source weight has preserved the shape of an integer-valued neutrosophic quantitiy. Furthermore, the shortcoming of the prevailing algorithms are pointed out and to show the benefits of the proposed algorithms. For this purpose, we have considered NSSPP and evaluate with existing methods. The numerical results show that the new algorithms outperform the present day stratgies. In future, we will extend the method to more complicated community issues involving integer-valued neutrosophic costs.

References:

[1] R. Montagné, M. Gamache, and M. Gendreau, "A shortest path-based algorithm for the inventory routing problem of waste vegetable oil collection," *Journal of the Operational Research Society*, vol. 0, pp. 1-12, 2018.

[2] S. K. Srivastava, "Green supply-chain management: A state-of-the-art literature review," *International Journal of Management Reviews*, vol. 9, pp. 53-80, 2007.

[3] W. Huang, C. Yan, J. Wang, and W. Wang, "A time-delay neural network for solving time-dependent shortest path problem," *Neural Networks*, vol. 90, pp. 21-28, 2017.

[4] E. W. Dijkstra, "A note on two problems in connexion with graphs," *Numerische Mathematik*, vol. 1, pp. 26 9-271, 1959.

[5] R. W. Floyd, "Algorithm 97: Shortest Path," *Commun. ACM*, vol. 5, pp. 345, June 1962.

[6] Y. Nie and X. Wu, "Shortest path problem considering on-time arrival probability," *Transportation Research Part B: Methodological*, vol. 43, pp. 597-613, 2009.

[7] E. Conde, "A minimum expected regret model for the shortest path problem with solution-dependent probability distributions," *Computers & Operations Research*, vol. 77, pp. 11-19, 2017.

[8] D. P. Bertsekas and J. N. Tsitsiklis, "An analysis of stochastic shortest path problems," *Mathematics of Operations Research*, vol. 16, pp. 580-595, 1991.

[9] G. H. Polychronopoulos and J. N. Tsitsiklis, "Stochastic shortest path problems with recourse," *Networks*, vol. 27, pp. 133-143, 1996.

[10] D. Dubois and H. Prade, "Ranking fuzzy numbers in the setting of possibility theory," *Information Sciences*, vol. 30, pp. 183-224, 1983.

[11] R. Kumar, S. Jha, and R. Singh, "A different appraoch for solving the shortest path problem under mixed fuzzy environment," *International journal of fuzzy system applications*, vol. 9 (2), p. article 6, forthcoming.

[12] A. Dey, A. Pal, and T. Pal, "Interval type 2 fuzzy set in fuzzy shortest path problem," *Mathematics*, vol. 4, pp. 1-19, 2016.

[13] A. Tajdin, I. Mahdavi, N. Mahdavi-Amiri, and B. Sadeghpour-Gildeh, "Computing a fuzzy shortest path in a network with mixed fuzzy arc lengths using α-cuts," *Computers & Mathematics with Applications*, vol. 60 , pp. 989-1002, 2010.

[14] A. Dey, R. Pradhan, A. Pal, and T. Pal, "A genetic algorithm for solving fuzzy shortest path problems with interval type-2 fuzzy arc lengths," *Malaysian Journal of Computer Science*, vol. 31, pp. 255-270, 2018.

[15] A. Dey, R. Pradhan, A. Pal, and T. Pal, "The fuzzy robust graph coloring problem," in *Proceedings of the 3rd International Conference on Frontiers of Intelligent Computing: Theory and Applications (FICTA) 2014*, Cham, 2015, pp. 805-813.

[16] R. Hassanzadeh, I. Mahdavi, N. Mahdavi-Amiri, and A. Tajdin, "A genetic algorithm for solving fuzzy shortest path problems with mixed fuzzy arc lengths," *Mathematical and Computer Modelling*, vol. 57, pp. 84-99, 2013, Mathematical and Computer Modelling in Power Control and Optimization.

[17] A. Dey, L. H. Son, P. K. Kishore Kumar, G. Selvachandran, and S. G. Quek, "New concepts on vertex and edge coloring of simple vague graphs," *Symmetry*, vol. 10, 2018.

[18] A Dey and A Pal, "Fuzzy graph coloring technique to classify the accidental zone of a traffic control," *Annals of Pure and Applied Mathematics* , vol. 3, no. 2, pp. 169-178, 2013.

[19] X. Zhang, and Q. Wang, and A. Adamatzky, and F. T. S. Chan, and S. Mahadevan, and Y. Deng, "A biologically inspired optimization algorithm for solving fuzzy shortest path problems with mixed fuzzy arc lengths," *Journal of Optimization Theory and Applications*, vol. 163, pp. 1049-1056, Dec. 2014.

[20] S. Broumi, F. Smarandache, and M. Dhar, "Rough neutrosophic sets.," *Neutrosophic Sets and Systems*, vol. 3, pp. 60-66, 2014.

[21] N. Deo and C-Y. Pang, "Shortest-path algorithms: taxonomy and annotation," *Networks*, vol. 14, pp. 275-323, 1984.

[22] R. K. Saini, A. Sangal, and O. Prakash, "Fuzzy transportation problem with generalized triangular-trapezoidal fuzzy number," in *Soft Computing: Theories and Applications*, Singapore, 2018, pp. 723-734.

[23] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, p. 87, 1986.

[24] G. Kumar, R. K. Bajaj, and N. Gandotra, "Algorithm for shortest path problem in a network with interval-valued intuitionistic trapezoidal fuzzy number," *Procedia Computer Science*, vol. 70, pp. 123-129, 2015, Proceedings of the 4th International Conference on Eco-friendly Computing and Communication Systems.

[25] A. Ruszczyński, "Decomposition methods in stochastic programming," *Mathematical Programming*, vol. 79, pp. 333-353, Oct. 1997.

[26] M. Common, "Uncertainty and the environment: implications for decision-making and environmental policy. Richard A. Young. Edward Elgar, Cheltenham, uk; northampton, MA, 2001. ISBN 1 84064 626 8 (hardcover, 59.95 pounds sterling, 85.00)," *Journal of Economic Psychology*, vol. 24, pp. 422-424, 2003.

[27] H. S. Najafi and S. A. Edalatpanah, "A note on "A new method for solving fully fuzzy linear programming problems"," *Applied Mathematical Modelling*, vol. 37, pp. 7865-7867, 2013.

[28] A. Hosseinzadeh and S. A. Edalatpanah, "A new approach for solving fully fuzzy linear programming by using the lexicography method," *Advances in Fuzzy Systems*, 2016.

[29] P. Das and T. K. Roy, "Multi-objective non-linear programming problem based on neutrosophic optimization technique and its application in riser design problem," *Neutrosophic Sets and Systems*, vol. 9, pp. 88-95, 2015.

[30] I. M. Hezam, M. Abdel-Baset, and F. Smarandache, "Taylor series approximation to solve neutrosophic multiobjective programming problem," *Neutrosophic Sets and Systems*, vol. 10, pp. 39-45, 2015.

[31] M. Sarkar, S. Dey, and T. Kumar Roy, "Multi-objective structural design optimization using neutrosophic goal programming technique," *Neutrosophic Sets and Systems*, vol. 15, pp. 8-17, 2017.

[32] F. Ahmad, A. Y. Adhami, and F. Smarandache, "Single valued neutrosophic hesitant fuzzy computational algorithm for multiobjective nonlinear optimization problem," *Neutrosophic Sets and Systems*, vol. 22, pp. 76-88, 2018.

[33] A. Fahmi, F. Amin, F. Smarandache, M. Khan, and N. Hassan, "Triangular cubic hesitant fuzzy einstein hybrid weighted averaging operator and its application to decision making," *Symmetry*, vol. 10, 2018.

[34] S. Islam and P. Ray, "Multi-objective portfolio selection model with diversification by neutrosophic optimization technique," *Neutrosophic Sets and Systems*, vol. 21, pp. 74-83, 2018.

[35] F. Smarandache, , Minh Perez, Ed.: American Research Press, 1999.

[36] F. Smarandache, "Neutrosophic set – a generalization of the intuitionistic fuzzy set," *International Journal of Pure and Applied Mathematics*, vol. 24, pp. 287-297, 2005.

[37] M. Abdel-Basset, G. Manogaran, M. Mohamed, and N. Chilamkurti, "Three-way decisions based on neutrosophic sets and AHP-QFD framework for supplier selection problem," *Future Generation Comp. Syst.*, vol .89 , pp. 19-30, 2018.

[38] M. Abdel-Basset, M. Mohamed, and A. K. Sangaiah, "Neutrosophic ahp-delphi group decision making model based on trapezoidal neutrosophic numbers," *J. Ambient Intelligence and Humanized Computing*, vol. 9, pp. 1427-1443, 2018.

[39] M. Abdel-Basset, M. Mohamed, and F. Smarandache, "An extension of neutrosophic AHP-SWOT analysis for strategic planning and decision-making," *Symmetry*, vol. 10, p. 116, 2018.

[40] M. Abdel-Basset, M. Mohamed, A.-N. Hussien, and A. K. Sangaiah, "A novel group decision-making model based on triangular neutrosophic numbers," *Soft Computing*, vol. 22, pp. 6629-6643, 2018.

[41] M. Abdel-Basset, M. Mohamed, A. K. Sangaiah, and V. Jain, "An integrated neutrosophic ahp and swot method for strategic planning methodology selection," *Benchmarking: An International Journal*, vol. 25, pp. 2546-2564, 2018.

[42] M. Abdel-Basset, M. Mohamed, Y. Zhou, and I. M. Hezam, "Multi-criteria group decision making based on neutrosophic analytic hierarchy process," *Journal of Intelligent and Fuzzy Systems*, vol. 33, pp. 4055-4066, 2017.

[43] M. Abdel-Basset, Y. Zhou, M. Mohamed, and V. Chang, "A group decision making framework based on neutrosophic VIKOR approach for e-government website evaluation," *Journal of Intelligent and Fuzzy Systems*, vol. 34, pp. 4213-4224, 2018.

[44] M. Abdel-Basset, M. Mohamed, and A. K. Sangaiah, "Neutrosophic ahp-delphi group decision making model based on trapezoidal neutrosophic numbers," *Journal of Ambient Intelligence and Humanized Computing*, vol. 9, pp. 1427-1443, 2018.

[45] M. Abdel-Basset, G. Manogaran, A. Gamal, and F. Smarandache, "A hybrid approach of neutrosophic sets and dematel method for developing supplier selection criteria," *Design Automation for Embedded Systems*, pp. 1-22, 2018.

[46] M. Abdel-Basset, G. Manogaran, A. Gamal, and F. Smarandache, "A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection," *Journal of Medical Systems*, vol. 43, pp. 38:1-38:13, 2019.

[47] J. Ye, "Trapezoidal neutrosophic set and its application to multiple attribute decision-making," *Neural Computing and Applications*, vol. 26, pp. 1157-1166, July 2015.

[48] I. Deli, "Operators on single valued trapezoidal neutrosophic numbers and svtn-group decision making," *N-eutrosophic Sets and Systems*, vol. 22, pp. 131-150, 2018.

[49] M. Gulistan, H. A. Wahab, F. Smarandache, Salma Khan, and Sayed Inayat Ali Shah, "Some linguistic neu-trosophic cubic mean operators and entropy with applications in a corporation to choose an area supervisor," *Symmetry*, vol. 10, 2018.

[50] Q. Khan, P. Liu, T. Mahmood, F. Smarandache, and K. Ullah, "Some interval neutrosophic dombi power bonferroni mean operators and their application in multi–attribute decision–making," *Symmetry*, vol. 10, 2018.

[51] M. Khan, M. Gulistan, N. Yaqoob, M. Khan, and F. Smarandache, "Neutrosophic cubic einstein geometric aggregation operators with application to multi-criteria decision making method," *Symmetry*, vol. 11, 2019.

[52] J. Ye, "A multicriteria decision-making method using aggregation operators for simplified neutrosophic set-s," *Journal of Intelligent & Fuzzy Systems*, vol. 26, no. 5, pp. 2459-2466, 2014.

[53] J-j. Peng, J-q. Wang, X-h. Wu, J. Wang, and X-h. Chen, "Multi-valued neutrosophic sets and power aggreg-ation operators with their applications in multi-criteria group decision-making problems," *International Jou-rnal of Computational Intelligence Systems*, vol. 8, no. 2, pp. 345-363, 2015.

[54] Peide Liu and Lanlan Shi, "The generalized hybrid weighted average operator based on interval neutrosop-hic hesitant set and its application to multiple attribute decision making," *Neural Computing and Applicati-ons*, vol. 26, no. 2, pp. 457-471, 2015.

[55] P. Li, H. Liu, "Multiple attribute decision-making method based on some normal neutrosophic Bonferroni mean operators," *Neural Computing and Applications*, vol. 28, no. 1, pp. 179-194, 2017.

[56] P. Liu and Y. Wang, "Interval neutrosophic prioritized OWA operator and its application to multiple attrib-ute decision making," *Journal of Systems Science and Complexity*, vol. 29, no. 3, pp. 681-697, 2016.

[57] P. Liu and L. Shi, "Some neutrosophic uncertain linguistic number Heronian mean operators and their appli-cation to multi-attribute group decision making," *Neural Computing and Applications*, vol. 28, no. 5, pp. 10 79-1093, 2017.

[58] J-q. Wang, Y. Yang, and L. Li, "Multi-criteria decision-making method based on single-valued neutrosoph-ic linguistic Maclaurin symmetric mean operators," *Neural Computing and Applications*, vol. 30, no. 5, pp. 1529-1547, 2018.

[59] D. Kour and K. Basu, "Application of extended fuzzy programming technique to a real-life transportation problem in neutrosophic environment," *Neutrosophic Sets and Systems*, vol. 10, pp. 74-86, 2015.

[60] G. Shahzadi, M. Akram, and A. B. Saeid, "An application of single-valued neutrosophic sets in medical dia-gnosis," *Neutrosophic Sets and Systems*, vol. 18, pp. 80-88, 2017.

[61] M. Mohamed, M. Abdel-Basset, A. N. Zaied, and F. Smarandache, "Neutrosophic integer programming pr-oblem," *Neutrosophic Sets and Systems*, vol. 15, pp. 3-7, 2017.

[62] D. Kour and K. Basu, "Selection of transportation companies and their mode of transportation for interval valued data," *Neutrosophic Sets and Systems*, vol. 18, pp. 67-79, 2017.

[63] M. Abdel-Basset, M. Gunasekaran, M. Mohamed, and F. Smarandache, "A novel method for solving the fully neutrosophic linear programming problems," *Neural Computing and Applications*, 2018

[64] K. Mondal, S. Pramanik, and Bibhas C. Giri, "Single valued neutrosophic hyperbolic sine similarity meas-ure based madm strategy," *Neutrosophic Sets and Systems*, vol. 20, pp. 3-11, 2018.

[65] S. Altinirmak, Y. Gul, B. O. Okoth, and C. Karamasa, "Performance evaluation of mutual funds via single valued neutrosophic set (svns) perspective: a case study in turkey," *Neutrosophic Sets and Systems*, vol. 23, pp. 110-125, 2018.

[66] M. V. Alava, S. P. D. Figueroa, H. M. B. Alcivar, and M. L. Vázquez, "Single valued neutrosophic numbers and analytic hierarchy process for project selection," *Neutrosophic Sets and Systems*, vol. 21, pp. 122-130, 2018.

[67] A. Dey, B. Said; L.H. Son, A. Bakali, M. Talea,F. Smarandache, Florentin, "A new algorithm for finding minimum spanning trees with undirected neutrosophic graphs," *Granular Computing*, vol. 4, no. 1, pp. 63-69, 2019.

[68] K. P. Teruel, J. C. Cedeno, H. L. Gavilanez, C. B. Diaz, and M. L. Vazquez, "A framework for selecting cl-oud computing services based on consensus under single valued neutrosophic numbers," *Neutrosophic Sets and Systems*, vol. 22, pp. 39-49, 2018.

[69] S. Broumi, A. Bakali, M. Talea,F. Smarandache, A Dey, L.H.Son;"Spanning tree problem with neutrosoph-ic edge weights," in *proceedings of the first international conference on intelligent computing in data scien-ces, icds2017*, vol. 127, 2018, pp. 190-199.

[70] M. Abdel-Baset, M. Mohamed, Y. Zhou, and F. Smarandache, "A critical path problem using triangular neutrosophic number," in *2017 IEEE International Conference on INnovations in Intelligent SysTems and Applications (INISTA),* , Gdynia Maritime University, Gdynia, Poland, 3-5 July 2017, pp. 403-407.

[71] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu, "Computation of shortest path problem in a network with sv-triangular neutrosophic numbers," in *IEEE International Conference on Innovations in Intelligent Systems and Applications (INISTA)*, Gdynia, Poland, 2017, pp. 426-431.

[72] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu, "Computation of shortest path problem in a network with SV-trapezoidal neutrosophic numbers," in *2016 International Conference on Advanced Mechatronic Systems (ICAMechS)*, 2016, pp. 417-422.

[73] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu, "Shortest path problem under triangular fuzzy neutrosophic information," in *2016 10th International Conference on Software, Knowledge, Information Management & Applications (SKIMA)*, 2016, pp. 169-174.

[74] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "Shortest path problem under trapezoidal neutrosophic information," in *Infinite Study*, London, UK, 2017.

[75] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and M. ALi, "Shortest path problem under bipolar neutrosphic setting.," *Applied Mechanics & Materials*, vol. 859, pp. 59-66, 2017.

[76] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "Application of Dijkstra algorithm for solving interval valued neutrosophic shortest path problem," in *2016 IEEE Symposium Series on Computational Intelligence (SSCI)*, 2016, pp. 1-6.

[77] S. Broumi,A. Bakali; M. Talea; F. Smarandache; KP Krishnan Kishore; R. Şahin; "Shortest path problem under interval valued neutrosophic setting," *Journal of Fundamental and Applied Sciences*, vol. 10, pp. 168-174, 2018.

[78] R. Kumar, S. A. Edaltpanah, S. Jha, S. Broumi, and A. Dey, "Neutrosophic shortest path problem," *Neutrosophic Sets and Systems*, vol. 23, pp. 5-15, 2018.

[79] S. Broumi, and D. Nagarajan, and A. Bakali, and M. Talea, and F. Smarandache, and M. Lathamaheswari,"The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment," *Complex & Intelligent Systems*, Feb. 2019

[80] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Technical sciences and applied mathematics*, pp. 10-14, 2004.

[81] A. Chakraborty, and S. P. Mondal, and A. Ahmadian, and N. Senu, and S. Alam, and S. Salahshour, "Different forms of triangular neutrosophic numbers, de-neutrosophication techniques, and their applications," *Symmetry*, vol. 10, no. 8, pp. 1-28, 2018.

[82] M. S. Bazaraa, J. J. Jarvis, and H. D. Sherali, *Linear programming and network flows. fourth Edition.*: A John Wiley & Sons, Inc., 2010.

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