Chapter 3 once more

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Abstract. We revisit the famous chapter in Keynes' *General Theory* (*GT*) with the tools of modern microeconomics to 1. derive expressions for the elasticity of the supply curve, 2. define effective demand, 3. draw out Keynes' comparative statics. The reconstruction is inspired by the inhouse to and fro preceding and following the publication of the classic, collated in *The Collected Writings* (*CW*).
Keywords: effective demand; Keynes comparative statics.

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Resumen. Revisamos el famoso capítulo de la Teoría general (GT) de Keynes con las herramientas de la microeconomía moderna para 1. derivar expresiones para la elasticidad de la curva de oferta, 2. definir la demanda efectiva, 3. obtener la estadística comparativa de Keynes. La reconstrucción está inspirada en las idas y venidas previas a la publicación del texto clásico, recopilado en *The Collected Writings (CW)*.
Palabras clave: demanda efectiva; Estática comparativa de Keynes.

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1. Motivation from the master

The exchanges between Keynes and sympathetic and hostile members of his 'circus' were exemplary manifestations of the famous Cambridge oral tradition. We are concerned, in particular, with the debate surrounding demand and supply schedules. The complexity arose in subsuming both microeconomic and macroeconomic reasoning in describing the mechanics of the movement of both. These days we would begin with the primitives in setting up optimisation problems whose solution would give us the demand and supply functions we seek. There are elementary advantages in writing out equations and ensuring that their number is at least as large as the number of unknowns. Structural or general equilibrium practitioners are especially strong here and Roy Harrod, a technically astute colleague of Keynes, pointed out that Keynes could not deny the classicals the simultaneous solution of demand and supply equations. The two equations determined two unknowns, quantity and price (Harrod 1935, *CW* XIII, pp. 532-534). Keynes' response was a fine example of implicit theorising. He claimed that the two equations were not in fact independent (Keynes 1935, *CW* XIII, p. 538). Indeed, our equations 1 and 2 below may be regarded as an explicit rendition of his proposition. In our formulation, we will capture his insight that the demand for labour on the part of the firm is, at the same time, the supply of income to the worker. Another consequence of not having to write down the bare bones of a model everyone can agree upon is that the optimisation problems of different agents do not have to be specified. Keynes and his contemporaries worked with the entrepreneur in the forefront or her maximand as the primal problem, leaving the characteristics of the solution of the consumer or the worker

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in the background or as elements of the dual problem. R. G. Hawtrey refers to this practice with great subtlety when he observes that the concept of the marginal disutility of labour cannot be applied to a situation of unemployment. The equilibrium conditions of the worker were invoked in symmetry to the calculus of the employer in a purely static account of the problem in the classical theory (Hawtrey 1936, *CW* XIV, p. 18). In response, we spell out the entrepreneur and worker interlocking problems in the letter and, hopefully, the spirit of the Chapter referred to in order to write down an equilibrium with involuntary unemployment.

Our orientation is drawn from the opening lines of Chapter 3 in Keynes' *General Theory* (*GT* hereafter). We operate with "a given state of technique, resources and costs". The entrepreneur employs workers and the product of the wage rate, *w*, and the number of employed *N*, is "factor cost". Looked at by households, factor cost is wage income. In sum, we are concerned with a closed model of "total income" as the sum of wage income and the income of the entrepreneur. Entrepreneurs are believed to act so as to maximize their profits. We propose that households act likewise so as to maximize their utility.

The individual is both consumer and worker, demanding output *O* and supplying labor *N*. She takes prices, *P* and *w*, in both markets and maximizes her utility subject to her wage income. That is, her program is to maximize $u(O, N)$ subject to $PO \leq wN$. If the ratio of the marginal rate of substitution between output and labor service equals the ratio of their relative prices we must have marginal disutility of labor. In his own words, Keynes was concerned with the level of output for which the marginal product of a unit of labour equals the marginal disutility of that unit of labour (Keynes 1973b, *CW* XIII, p. 427). Distinguishing this condition from the classical, he wrote to R.G. Hawtrey that it is only under full employment that the marginal disutility of labour equaled the marginal utility of wages (Keynes 1936a, *CW* XIV, p. 26). We work with an indirect utility function, $v(P,w)$. In that case, using Roy's Identity our aggregate demand function is

$$O^* = -\frac{\frac{\partial v}{\partial P}}{\frac{\partial v}{\partial w}} N^* \qquad\qquad 1$$

We note that the starred values here and below denote optimal values of variables. Choice variables have been chosen to maximize a pair of interlocking programs. In our case, both capitalists and workers have maximized their profits and utility over the relevant domains. Furthermore, since the indirect utility function is homogenous of degree zero in the price level and the wage, $\frac{\partial v}{\partial P} P + \frac{\partial v}{\partial w} w = 0$

we can write equation 1 as

$$O^* = \frac{w}{P} N^* \qquad\qquad 2$$

Keynes' aggregate demand curve is the maximum level of (expected) profits associated with a given level of factor costs. Effective demand is the point of intersection of the aggregate supply curve (below) and the aggregate demand curve which determines the level of output O^*. Effective demand, in turn, is the sum of the consumption function and investment which is 'autonomous'. Our translation summarized in equations 1 or 2 is free of "the psychological characteristic of the community" in the first case and animal spirits in the latter. At the same time, the equations can be regarded as being expressed in terms of the wage-unit that Keynes made the case for in Chapter 4 of the *GT*.

For the aggregate supply curve, we need the profit function $\pi(P,w)$. The supply function is given by the first part of Hôtelling's lemma below.

$$O^* \quad \frac{\partial \pi}{\partial P} \text{ and } N^* = -\frac{\partial \pi}{\partial w} \qquad\qquad 3$$

Here as well, our supply function can be regarded as the 'dual' of Keynes' 'primal' aggregate supply function which is the aggregate supply price of *O* from employing *N* workers. The price element enters on the right-hand side of the first equation. The second equation is the complement of the first like Keynes' employment function defined in Chapter 20 is the inverse of the supply function of the *GT*. Both the employment functions are defined in terms of the wage-unit.

Coming to the present, since we are within the confines of the *GT* we are advised to cleave to its neoclassical microeconomics (Palley 2017). A new microeconomics for the task at hand is yet to be written. We deploy

the contemporary version of old microeconomics. However, while the real wage equals the marginal product of labour in equation 2, the latter is determined by the level of output which is a combination of equations 1 or 2 and 3. The equations do not determine the full-employment level of income. R.B. Bryce offered a neat summary of the master's thesis which Keynes approved of (Bryce 1935, *CW* XXIX, pp. 132-133). Keynes retained the postulate of the equality of the marginal product of labour with the wage rate or marginal cost in his "monetary theory of production" but rejected the assumption of the equality of the marginal disutility of labour with the wage rate. Secondly, mainstream macroeconomics today even under the rallying cry of agent heterogeneity is not able to model different preferences because there are no classes therein (Stiglitz 2017). The propensities to consume and save of capitalists and workers are identical. We observe as a footnote that no complex debt dynamics Keynes-driven or otherwise would be necessary in elaborations of the model. Even in a first-best world with no debt, if taxes and expenditures are socially calibrated, the balanced budget multiplier can be large. In a recent paper Jochen Hartwig (2017) treads common ground

with us and we follow in his tracks as a reference point to develop our theme.

2. 'Elasticity pessismism' and the money wage share

The *Collected Writings* are replete with references to the elasticities of demand and supply essential to making out the signs of comparative statics. Keynes' conclusions in this regard were hesitant and delicate. We provide a few illustrations that also reveal Keynes' skepticism about deriving propositions concerning the price level both as a dependent as well as an explanatory variable. Thus, when there is a fall in variable costs and incomes per unit of output, we cannot arrive at a sharp conclusion about the impact on the volume of output. The elasticities of demand of entrepreneurs and workers might cancel each other out (Keynes 1973a, *CW* XIII, pp. 392-393). Then, in a letter to A.C. Pigou, he writes that the amount of unemployment arising out of a fall in prices depends on the extent to which producers curtail output, that is, on the elas-

ticity of supply (Keynes 1930b, *CW* XX, p. 422). Finally, it might be that a fall in wages is passed on *pari passu* by a producer in a fall in prices. However, Keynes regarded it as equally likely that the producer did not pass through the fall in costs to prices (Keynes 1931, *CW* XIII, p. 469).

Therefore, before we proceed to equalize equations 1 and 3 and perform comparative statics we enter the discussion surrounding an opaque footnote by Keynes where he offered two allegedly inconsistent statements, that the elasticity of the supply curve is unity as well as equals the reciprocal of the money wage. Professor Hartwig, following others, proves that if the money wage is replaced by the money wage share, the latter proposition is true. Our approach prompts the following treatment. The profit function is homogenous of degree one in both arguments. Thus,

$$P\frac{\partial \pi}{\partial P} + w\frac{\partial \pi}{\partial w} = \pi(P, w)$$

Following Hartwig's lead, we take the derivative of the above expression with our attention focused on the term on the extreme left-hand side. Since we are dealing with the 'dual' of 'primal' functions as well as seeking a conventional derivation of supply elasticity, the derivative is with respect to price.

$$\frac{P}{w}\frac{\frac{\partial^2 \pi}{\partial P^2}}{\frac{\partial N^*}{\partial P}} = 1$$

Therefore,

$$P\frac{\partial^2 \pi}{\partial P^2} + \frac{\partial \pi}{\partial P} - w\frac{\partial N^*}{\partial P} = \frac{\partial \pi}{\partial P}$$

Note that unity is the value of an approximation to the elasticity of the supply curve. Once again, the expression on the left-hand side is expressed in terms of the wage-unit. To derive an exact expression for the elasticity of the supply curve we need to multiply both sides of equation 4 by . We get

$$\frac{w\frac{\partial N^*}{\partial P}}{O^*}$$

$$\frac{P}{O^*}\frac{\partial^2\pi}{\partial P^2} = \frac{w\frac{\partial N^*}{\partial P}}{O^*}$$

The expression on the right-hand side is only an approximation to the wage share but the left-hand side is the elasticity of the supply curve. It looks like Keynes was approximately consistent!

3. Effective demand and comparative statics

The supply curve of equation 3 must be equated to the demand curve of equation 1 to define the point of effective demand. Thereafter, we can conduct comparative static exercises. Thus,

$$\frac{\partial O^*}{\partial P} = -\frac{\frac{\partial^2 v}{\partial P^2}}{\frac{\partial v}{\partial w}}N^* = \frac{\partial^2\pi}{\partial P^2}$$

The ambiguity here is familiar. Both the indirect utility and the profit functions are convex in output price. The indirect utility function is increasing in the wage (income). Thus, the extreme right-hand term is nonnegative while the term in the center is nonpositive. In keeping with the state of exegesis, the influence of the price level on the level of activity is uncertain. Keynes' hesitation at arriving at clear conclusions concerning the influence of the price level, suggested above, is borne out. For instance, in his discussions on the slump with Sir Josiah Stamp in 1930, he expresses mystification that prices can fall sharply while wages can remain constant (Keynes 1930a, *CW* XX, pp. 318-319). Again, this time in the context of an open economy, he opines that the fall in prices is a double-edged sword. The positive aspect is reflected in a reduced cost of living so that the money wage of workers can fetch a larger basket of basic goods. The negative aspect is reflected in the fall in the value of a country's exports. The net effect might go either way: an increase in real wages arising out of cheaper imports and unemployment in the export goods industries (Keynes 1930c, *CW* XX, p 424).

Coming to the wage rate,

$$\frac{\partial O^*}{\partial w} = \frac{\frac{\partial v}{\partial P}}{\frac{\partial^2 v}{\partial w^2}}\frac{\partial\pi}{\partial w} + \frac{\frac{\partial v}{\partial p}}{\frac{\partial v}{\partial w}}\frac{\partial^2\pi}{\partial w^2}$$

Consider the right-hand side. The profit function is convex in the wage rate. If we can show that the term in the denominator of the first term is positive, we get a clear 'Keynes' sign for this comparative static exercise. We know that the indirect utility function is convex (actually quasi-convex) in prices. That is, for two points (P', w) and (P'', w),

$$v(P'w) \geq v(P'', w) + \frac{\partial v(P'',w)}{\partial P''}(P' - P'')$$

We need to show that the function is convex in the wage rate or for two points

(P, w') and (P, w''),   5

$$v(P, w') \geq v(P, w'') + \frac{\partial v(P,w'')}{\partial w''}(w' - w'')$$

Consider our original two points (P',w) and (P'',w). Multiply each of the vectors by scalars greater than unity, say β and γ, transforming them to (P,w') and (P,w'') respectively. Let Since the indirect utility function is homogenous of degree zero in both arguments,

$$v(P, w') \geq v(P, w'') + \frac{\partial v(P,w'')}{\partial w''}(w' - w'')$$

Now consider equation $w' \geq w''$ 5. By construction, and the partial derivative cannot be negative (indirect utility is increasing in income). Our claim is vindicated.

We summarise this section by juxtaposing Keynes' combination of the two comparative statics exercises with our own formulation. In depressed economic conditions, how do we induce entrepreneurs to hire labour so that the marginal product of their employment is brought to equality with the wages they will receive, he asks. We mention two of the three scenarios he entertains. Scenario A consists in increasing money wages without increasing prices (our second comparative static exercise). Scenario B consists in increasing prices relative to the increase in money wages. Policies would consist of some combination of the two (Keynes 1930d, *CW* XX, pp. 429-430). Our answer is provided in equation 1 constituting our definition of effective demand. Money wages and the price level must move proportionately.

4. Conclusion

We offer a simple and analytical treatment of Keynes' definitions in the *General Theory*. Our contribution is to complete the specification of the class configuration implicit in the book by introducing the optimization problem of a member of the working class. Our rendition is purely structural without any behavioral elements. In the process we attempt to clear up cobwebs in Keynes' treatment of the elasticity of the supply curve. Above all, we endorse the powerful Keynes comparative statics: an increase in the money-wage rate raises the equilibrium level of output and employment.

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