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Time Delay Estimation by Bispectrum Interpolation

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Abstract: Time delay is an important parameter for the characterization of signal propagation of sensors. In order to improve the precision of time delay estimation, this article proposes an algorithm based on bispectrum interpolation. The algorithm first calculates the characteristic spectrum function on the basis of the bispectrum of the two signals. The frequency domain interpolation is done on the characteristic spectrum function. By conducting the inverse Fourier transform and the maxima seeking method, the calculation formula for the time delay of the two signals is deduced. The simulation test results show that compared with the bispectrum-based time delay estimation method, the bispectrum interpolation-based time delay estimation method has improved greatly in terms of the estimation precision and temporal resolution. The algorithm has the advantages of high precision, small error and strong ability to suppress the correlative Gaussian noise. *Copyright © 2013 IFSA.*

Keywords: Time delay, Estimation, Cross-correlation, Bispectrum, Interpolation.

1. Introduction

Time delay estimation is a very important research topic in signal processing. It is widely applied in radar, sensor, biological sciences, sonar, oil exploration, communications, geophysics, underwater acoustics, speech signal enhancement and other scientific fields [1-3]. The core part of time delay estimation is the received target signals. On the basis of the known reference signal, the time delay of the received signal in relation to the reference signal is calculated fast and accurate.

Correlation analysis is a basic time delay estimation method. Time delay estimation methods based on correlation analysis include basic correlation method and generalized correlation method. In the basic delay estimation algorithms, the most typical algorithm is cross-correlation algorithm, and it is also the base of the second-order delay estimation algorithm. The basic correlation method is advantaged by its easy realization, but is susceptible to noise and required that signal and noise should be independent from each other. In order to eliminate or reduce the influence of noise on the correlation-based time delay estimation, Knappy et al proposed a generalized correlation method for time delay estimation [4]. This method first preprocesses the signal contaminated with noise, to improve the SNR. This method could better yield the time delay estimation in the case of low SNR. But when the instantaneous correlation energy value of the noise exceeds that of the target signal, the algorithm would consider the noise as valid signal, resulting in error. In general, the relevant delay estimation methods assume that the noises of the signal model are all Gaussian white noise and independent from each other. However, in practical applications, noise may be correlated and non-Gaussian. In this case, the correlation-based methods could not accurately estimate the time delay. To solve this problem, researchers have tried to use bispectrum-based method for delay estimation [5-14]. This method uses shiftinvariance property of bispectrum. The performance of conventional bispectrum-based signal processing method is also worth improving. First, if the input SNR is large enough, the bispectrum method will introduce specific distortions in the reconstructed signal. What's more, there are many ways to improve the performance of bispectrum-based signal processing by combining it with robust estimation and nonlinear filtering. Nikias and Pan proposed the time delay estimation method based on third-order statistics [5]. This method could estimate the time delay of non-Gaussian signals in an unknown Gaussian noise environment. However, this method involves the matrix inversion. When the inverse matrix is absent, the time delay could not be estimated. Hinich employed the Fourier transform over the time delay method based on third-order statistics to derive the bispectrum-based time delay method [6]. Robust estimation was also applied at the stage of bispectrum estimate. Both non-adaptive and adaptive delay estimation were used instead of conventional averaging. The most prominent advantage of bispectrum-based method is that it completely eliminates Gaussian noise. However, the performance of this method is affected by the sample. When the sampling frequency of the signal is low, the calculated resolution of the correlation function is also low.

In order to improve the time domain resolution, this article proposes a bispectrum interpolation-based delay estimation algorithm using the third-order cumulant. It improves the precision of time delay estimation by implementing the frequency domain interpolation. This algorithm combines the suppression of the correlated noises and refinement of frequency domain in bispectrum-based algorithm that improve temporal resolution. The experimental results show that relative to bispectrum-based technique, the precision of time delay estimation using bispectrum interpolation-based technique is higher, and the error is smaller.

2. Methods

Suppose the transmitted signal and the received signal to be $x_1(t)$ and $x_2(t)$ respectively.

$$\begin{aligned} x_1(t) &= s(t) + n_1(t) \\ x_2(t) &= s(t-D) + n_2(t) \end{aligned} , \tag{1}$$

where $s(t)$ is the original signal without noise. $n_1(t)$ and $n_2(t)$ are the noises superimposed on the two signals, and they may be correlated. D is the time delay between the two signals. t is the time.

The classical time delay estimation is based on analyzing the cross correlation between $x_1(t)$ and $x_2(t)$ as the two signals of interest and the lag that corresponds to the maximum value of cross correlation is the time delay. The cross correlation function for $x_1(t)$ and $x_2(t)$ is given as

$$R_{x_1x_2}(\tau) = \int_{-\infty}^{+\infty} x_1(t)x_2(t-\tau)dt \,. \tag{2}$$

In practical applications, noise may be correlated. In this case, the correlation-based methods could not accurately estimate the time delay.

Because the noises are correlated, the bispectrum algorithm is used for the calculation of time delay. It is known that the bispectrum can eliminate the correlative noise.

The third-order self-correlation function of $x_1(t)$ is

$$\begin{aligned} R_{111}(\tau_1,\tau_2) = E[x_1(t)x_1(t+\tau_1) \\ x_1(t+\tau_2)] = R_{sss}(\tau_1,\tau_2) \end{aligned} , \tag{3}$$

where τ_1 and τ_2 are time delay variables.

The third-order correlation functions of $x_1(t)$ and $x_2(t)$ is

$$\begin{aligned} R_{121}(\tau_1,\tau_2) = E[x_1(t)x_2(t+\tau_1) \\ x_1(t+\tau_2)] = R_{sss}(\tau_1 - D,\tau_2) \end{aligned} . \tag{4}$$

The third-order self-correlation function and cross-correlation functions after FFT transform have the following form:

$$\begin{aligned} B_{111}(w_1,w_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} R_{111}(\tau_1,\tau_2) * \\ e^{-2j\pi(w_1\tau_1 + w_2\tau_2)} d_{\tau_1} d_{\tau_2} \end{aligned} , \tag{5}$$

$$\begin{aligned} B_{121}(w_1,w_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} R_{121}(\tau_1,\tau_2) * \\ e^{-2j\pi(w_1\tau_1 + w_2\tau_2)} d_{\tau_1} d_{\tau_2} \end{aligned} ,. \tag{6}$$

Their discrete forms are:

$$B_{111}(k_1,k_2)=\sum_{m_1=0}^{2N-1}\sum_{m_2=0}^{2N-1}R_{111}(m_1,m_2)* e^{\frac{-2j\pi(k_1m_1+k_2m_2)}{2N}}, \quad (7)$$

$$B_{121}(k_1,k_2)=\sum_{m_1=0}^{2N-1}\sum_{m_2=0}^{2N-1}R_{121}(m_1,m_2)* e^{\frac{-2j\pi(k_1m_1+k_2m_2)}{2N}}, \quad (8)$$

As could be seen from the above equation, the only difference is a linear phase shift between $B_{111}(w_1,w_2)$ and $B_{121}(w_1,w_2)$ in the frequency domain.

Characteristic function:

$$g(\tau)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{B_{111}(w_1,w_2)}{B_{121}(w_1,w_2)}* e^{2j\pi w_2\tau}d_{w_1}d_{w_2} = 4\pi^2\delta(\tau-D), \quad (9)$$

As could be seen from the above equation, the maximum value point of $g(\tau)$ is the time delay D between $x_1(t)$ and $x_2(t)$.

Given the discrete system, it can be known from the sampling theorem that the characteristic function $g(t)$ with continuous time domain and limited frequency spectrum is sampled with the period of Ts. The frequency spectrum $G(k)$ corresponding to the sequence $g(n)$ is the periodic repetition of the frequency spectrum $G(f)$ of the sequence $g(n)$. The repetition interval is equal to the sampling frequency f_s. As long as the sampling frequency is greater than or twice as the highest frequency of the signal, $g(n)$ could be recovered based on $G(k)$ with no distortion.

When the sampling frequency is low, the calculated correlation function resolution is low. In order to improve the resolution of correlation peak, there are generally two ways: increasing the sampling frequency and the correlation peak interpolation.

The higher the sampling frequency, the smaller the estimation error. But the higher the sampling frequency of the chip performance requirements will be higher. For example, if requires the estimation error of 1ns, clock frequency will need to increase to 1 GHz. The present digital signal processing chip used is difficult to achieve such a high processing rate. At the same time, the problem of electromagnetic interference will also give circuit board layout, material selection, processing brings higher requirements.

Interpolation calculation of the correlation peak, such as the use of the least square method or three spline interpolation, may not only increase the amount of calculation, but also will introduce new errors.

We present an algorithm to improve the estimation method of bispectrum interpolation delay accuracy by interpolation in frequency domain when the sampling rate is low.

If $G(k)$ is expanded in the frequency domain, it is equivalent to expanding the repetition interval of the frequency spectrum. The inverse transform of $g(n)$ waveform does not change. It would not bring new error, but the sampling rate of $g(n)$ could improve at the same time.

Based on the core idea of algorithm for spectrum interpolation, i.e. frequency domain zero padding could improve the resolution of the time-domain waveform in the characteristic function, $G(k)$ sequence with sampling length of $2N$ points is zero padded at the spectrum interval of the characteristic function. $N_1 \geq 2N$ is assumed to construct $G_1(k)$ sequence.

$$\begin{aligned} &G_1(k)=G(k) \quad k=0,1,2,...N-1 \\ &G_1(k)=0 \quad k=N,N+1,...N_1-N-1. \\ &G_1(k)=G(2N-N+k) \\ &\quad k=N_1-N,N_1-N+1,...N_1-1 \end{aligned} \quad (10)$$

Then we do the inverse Fourier transform on $G_1(k)$ to get $g_1(n)$. It is equivalent to improving the sampling rate of the correlation function g (n) to $\frac{N_1}{2N}\bullet f_s$. In order to give time delay estimation with high precision, the sampling rate of $g(k)$ has to be maximized. N_1 should be large enough. But this directly increases the number of points in the inverse Fourier transform, thereby increasing the overall computation work. Therefore, the precision and computation workload of time delay estimation should be balanced depending on the actual conditions.

3. Experimental Results and Discussion

To verify the effect of the bispectrum interpolation-based time delay estimation algorithm, the numerical simulation test is performed. The algorithm is compared with the cross-correlation and bispectrum algorithms. When the signal is a band-limited signal, and SNR is a constant, the time delay is set as 0 s. The number of bispectrum interpolation points is N1 = 4N. The experimental results of the cross-correlation-based estimation, bispectrum interpolation-based method and bispectrum-based estimation are shown in Fig. 1. From the figure, it could be seen that in the presence of noise, the time delay positioning of bispectrum interpolation-based algorithm is more accurate, with higher precision than the bispectrum-based estimation and cross-correlation-based estimation.

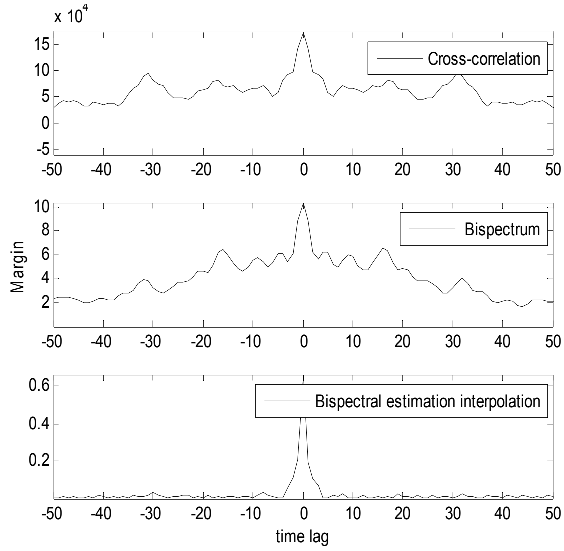

Fig. 1. Comparison of three time delay estimation algorithms.

N1 = 4N and time delay value set 0.1 and 0.5 s. The sampling rate is twice the original rate, and SNR is set to be a constant. The calculated time delay is shown in Fig. 2 and Fig. 3. As could be seen from those figures, bispectrum interpolation-based algorithm has high estimation precision for time delay positioning.

Given the initial values of several time delays, the time delay estimation results of several methods are shown in Table 1 below. It could be seen from Table 1 that the time delay estimation obtained by IFFT operation after bispectrum interpolation is more accurate than directly using bispectrum to calculate time delay. The time delay errors and relative error calculated according to Table 1 under certain SNR conditions are shown in Tables 2 and 3. It could be known from Tables that when the time delay value increases, the measurement precision increases as well. The precision of the bispectrum interpolation-based algorithm of this article is significantly higher than that of the other two algorithms.

The number of interpolations has certain influence on the precision of time delay estimation. When the time delay value is 0.1, 0.5 and 0.7 s, estimation precisions of interpolating different numbers of values are shown below in Fig. 4 to Fig. 6. As could be seen from those figures, with the interpolation points increasing, time delay estimation precision increases as well. But when it reaches a certain precision, time delay estimation precision would improve slowly. Therefore, in the actual situation, the number of interpolation points has to be reasonably chosen.

4. Conclusions

In the absence of noise or when the noises are not correlated, the classic cross correlation-based algorithm could have a good positioning precision for time delay. But in actual conditions, the noises are often correlated. This article uses the bispectrum-based technique to suppress the influence of correlated Gaussian noise and its autocorrelation on the entire time delay. The frequency domain interpolation of the bispectrum is carried out to refine the frequency domain and increase temporal resolution, thereby increasing the precision of time delay estimation.

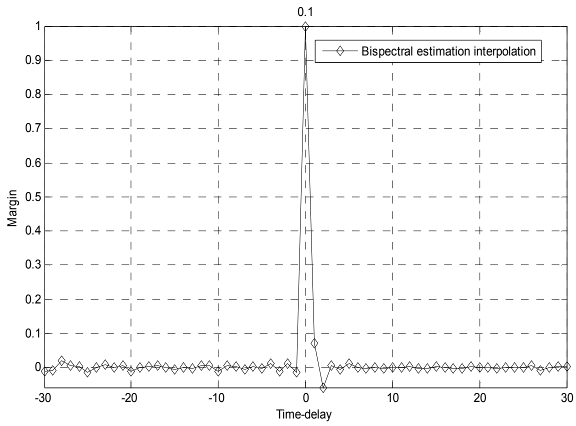

Fig. 2. Position precision of bispectrum interpolation-based time delay estimation at the set time 0.1 s.

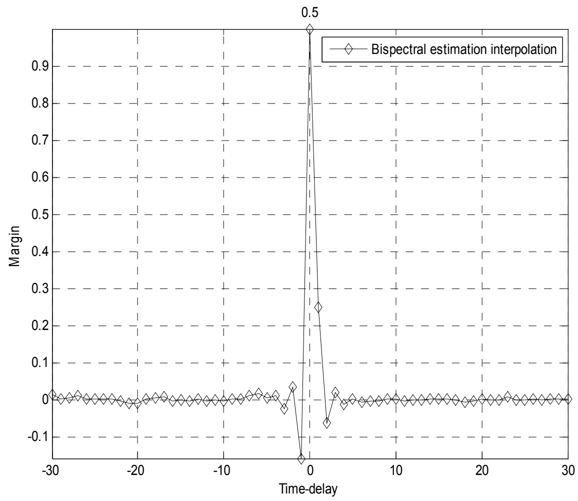

Fig. 3. Position precision of bispectrum interpolation-based time delay estimation at the set time 0.5 s.

Table 1. Time delay comparison at different set time.

Set Time(s)	Cross-correlation(s)	Bispectrum (s)	Bispectral interpolation (s)
0.1	0.0978	0.1019	0.1011
0.2	0.1956	0.2037	0.2023
0.3	0.3057	0.2944	0.3031
0.4	0.4071	0.3940	0.4039
0.5	0.4924	0.4930	0.4952
0.6	0.6083	0.5927	0.6055
0.7	0.7094	0.6927	0.7037

Table 2. Estimation error comparison at different set time.

Set Time(s)	Cross-correlation(s)	Bispectrum(s)	Bispectral interpolation(s)
0.1	0.0022	0.0019	0.0011
0.2	0.0044	0.0037	0.0023
0.3	0.0057	0.0056	0.0031
0.4	0.0071	0.0060	0.0039
0.5	0.0076	0.0070	0.0048
0.6	0.0083	0.0073	0.0055
0.7	0.0094	0.0073	0.0037

Table 3. Estimation relative error at different set time.

Set Time (s)	Cross-correlation (%)	Bispectrum (%)	Bispectral interpolation (%)
0.1	2.20	1.90	1.10
0.2	2.20	1.85	1.15
0.3	1.90	1.87	1.03
0.4	1.78	1.50	0.97
0.5	1.52	1.40	0.96
0.6	1.38	1.22	0.92
0.7	1.34	1.04	0.53

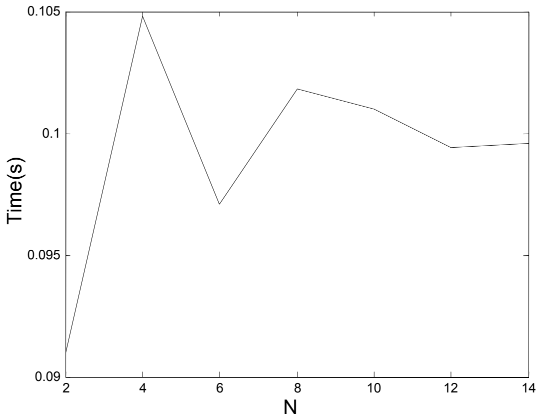

Fig. 4. Time delay estimation of bispectrum interpolation-based algorithm at set time 0.1 s.

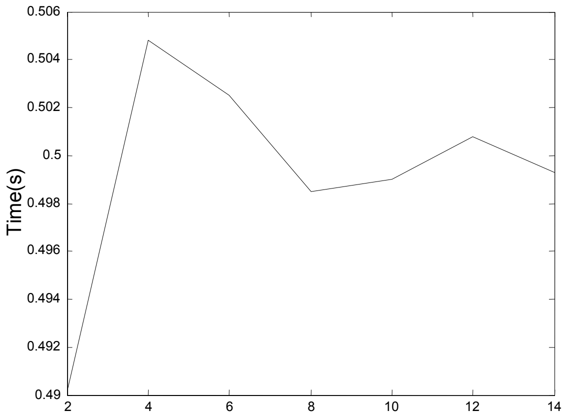

Fig. 5. Time delay estimation of bispectrum interpolation-based algorithm at set time 0.5 s.

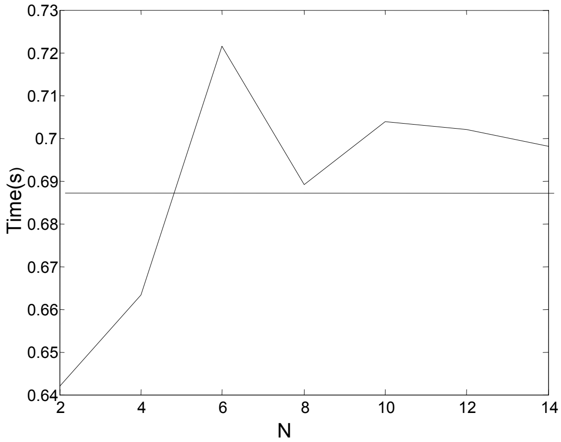

Fig. 6. Time delay estimation of bispectrum interpolation-based algorithm at set time 0.7 s.

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