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Stabilization and Performance Analysis for Networked Flight Control System with Random Time Delays via a LMI Gridding Approach

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Abstract: The use of avionics networks for flight control system results in a reduced aircraft weight, system installation, integration and maintenance costs as well as an improved airborne system performance and flexibility. This paper studies the stability and control design problems with performance analysis for networked flight control systems (NFCS) with random time delays by modeling NFCS as a discrete-time switched linear system. Sufficient conditions are given for asymptotical stability and exponential stability of proposed NCS model and state feedback controller is designed via linear matrix inequality (LMI) approach. The bound of decay rate of the system is obtained by solving a LMI optimization problem. Gridding approach is introduced to guarantee the feasibility of proposed LMIs. Illustrative examples are presented to demonstrate the effectiveness of the proposed method and control performance with decay rate is analyzed based on the simulation results.

Keywords: Networked flight control systems (NFCS), Linear matrix inequality (LMI), Time delay, Switched system, Decay rate.

1. Introduction

A typical commercial/military aircraft consists of a number of safety-critical systems, such as aircraft engine control system, aircraft flight control systems, aircraft cabin environmental control system, structural and engine health monitoring systems. These systems demand a large number of real-time sensors and actuators for their optimal operation. Current systems are based on a centralized architecture in which all the sensors and actuators are individually connected to the engine controller. These point-to-point connections have a high obsolescence cost as well as a high maintenance cost [1, 2]. In recent decades, with the rapid development of computer, network and communication technology, aircraft manufacturers desire to consider replacing point-to-point nodes connections with faster and low-cost avionics networks to reduce aircraft weight, system installation, integration, reconfiguration, assembly and maintenance costs as well as improving airborne system performance, system flexibility and resources sharing [3-5].

Currently, widespread applications of avionics networks for the connection of onboard devices of the aircraft can be observed throughout flight control systems. Avionics networks provide numerous physical and logical configurations for avionics architecture, data packets, protocols, message traffic, etc. In avionics networks, three fundamental approaches to controlling bus access are used: contention resolution (CAN, Ethernet, AFDX), time

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slot allocation (ARINC 629, ARINC 659/SAFEbus, TTP), and token passing (FDDI, AS 4074, LTPB). Some protocols use mixed approaches. No matter what protocols are used, the adoption of avionics networks however, introduce potential issues, such as random time delays, stochastic packet dropouts and packet disorder, which may degrade a system's performance and even cause system instability [6].

The network-induced delay is the most serious challenge which contains sensor-to-controller delay, controller-to-actuator delay, and the controller computational delay. Sensor-to-controller delay and controller-to-actuator delay will depend on the network protocol and can be either constant, time varying or random in nature. As a new control system, only a few studies research on networked flight control system (NFCS) to solve network-induced issues [7-9]. However, Research on Networked control systems (NCS) has achieved a significant amount of results. How to solve the network induced issues of random time delay and packet dropout to guarantee the system stability and performance is the hot research point in literature. Different models are developed for NCS to study stability criteria or stabilizing controller design. A discrete-time modeling approach is employed for network delays in [10]. An augmented state vector method is used for NCS modeling over random time delays in [11]. NSCs with packet dropouts are modeled as discrete Markov jump system in [12]. Time delays are considered as perturbations of the whole feedback control system in [13]. Queuing strategy method is developed to deal with time delay in [14, 15]. Optimal gain is calculated by modeling NCS as a switched system in [16].

To adapt the results above to solve the network-induced delay issue of the NFCS, we first model the NFCS as a switched linear system with infinite switching rules, which is compose of a continuous-time plant, a discrete-time controller and the communication channels. Linear matrix inequality (LMI) method is adopted to design a stabilizing controller for NFCS and an LMI optimization problem is formulated to find the bound of the decay rate for the switched linear system which indicates the stability performance of NFCS controller. A "gridding" approach [17] is introduced to convert infinite switching rules into finite ones to guarantee the feasibility of proposed LMIs.

The remainder of this paper is organized as follows. Section 2 gives problem formulation. In section 3, Stability of the newly developed model is analyzed and an approximately stabilizing controller is designed. In section 4, Stability performance is analyzed and the bound of decay rate is found by solving an LMI optimization problem. Simulations and Results are present in section 5 and conclusions are given in section 6.

Notation: The notation used throughout the paper is fairly standard. A^T represents the transpose of matrix A, the notation $P>0\,(P<0)$ means that P is positive definite (negative definite), I and 0 represent identity matrices and zero matrices with appropriate dimensions, respectively. * denotes the entries of matrices implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate dimensions.

2. Problem Formulation

To describe a flight dynamics system controlled via a network, a typical abstract mathematical model combined with the characteristics of the networked control system is considered in this paper. A single-input single-output dynamic plant is described by

$$\dot{x}(t) = Ax(t) + Bu(t)\,, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the input vector. A and B are constant matrices of appropriate dimensions.

The framework of networked flight control system is shown in Fig. 1, which is a feedback control system closed via a shared band limited digital communication network, which connect sensor nodes, actuator nodes and controller nodes together. Actuator accepts the digital signal from sensor via some feedback calculating by controller.

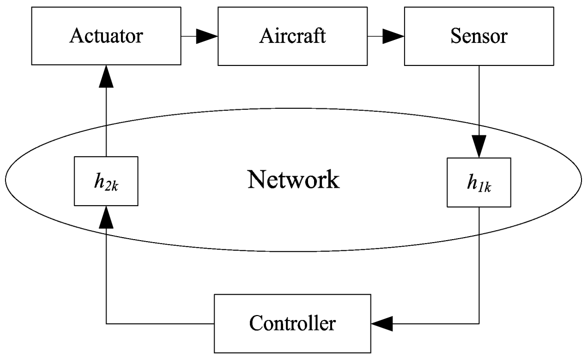

Fig. 1. The framework of networked flight control system.

Data collisions and communication failure will inevitably happen because of limited capacity of transmission channel. Based on the characteristics of the flight control bus, the following assumptions about the data transmission process and the network are made:

Assumption 1: Sensor is time-driven, the sampling period is T . Actuator and controller are event-driven.

Assumption 2: The total transmission delay induced by the communication network from sensor to actuator is less than one sampling period but time-varying.

Assumption 3: All control and measurement information in the network is sent in a single packet.

Assumption 4: Between two implemented control signals, actuator operates in a zero order hold (ZOH) fashion.

Assumption 5: No packet losses occur in the communication network.

Packet transmission conditions via network are shown in Fig. 2. Define t_k as the arrival time at the actuator of the kth sampled packet from sensor, and assume that total transmission delay from sensor to actuator of the kth sampled packet is τ_k. Then during the time interval $[t_k, t_{k+1})$.

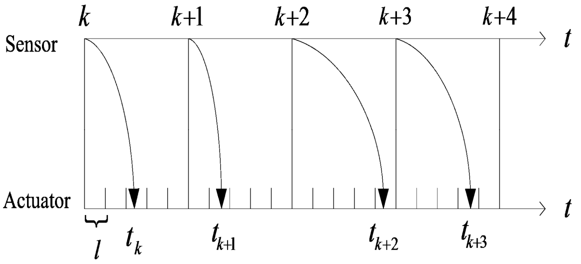

Fig. 2. Packet transmission conditions via network.

The feedback controller can be described as

$$u(t) = u(k) = K(\tau_k)x(k), \quad t_k \le t < t_{k+1} \tag{2}$$

with t_{k+1} being the next packet arrival instant of the ZOH after t_k.

Note that the total network induced time delay of each sampled packet is time-varying. Thus, the state-feedback control gains in (3) are not constant but varying with every updating interval $[t_k, t_{k-1})$ to ensure the stability of the time varying sampled system.

Sampling with the constant sampling period T gives the discrete-time NFCS with the total time delay from sensor to actuator as follows:

$$x(k+1) = \Phi x(k) + \Gamma_0(\tau_k)u(k) + \Gamma_1(\tau_k)u(k-1), \tag{3}$$

where $\Phi = e^{AT}$,

$\Gamma_0(\tau_k) = \int_0^{T-\tau_k} e^{As} dsB, \ \Gamma_1(\tau_k) = \int_{T-\tau_k}^{T} e^{As} dsB$.

The random time delay determines the time-varying input matrices $\Gamma_0(\tau_k)$ and $\Gamma_1(\tau_k)$, so the system (3) is a time-varying discrete dynamic system. To deal with the time-varying part of system (3), let us introduce a new matrix variable $F(\tau_k)$ to transform system (3) into a discrete linear system with parameter uncertainty as follows:

$$\begin{aligned}\Gamma_0(\tau_k) &= \int_0^{T-\tau_k} e^{As} ds \cdot B = \int_0^{T-T/2-\Delta\tau_k} e^{As} ds \cdot B \\ &= \int_0^{T/2} e^{As} ds \cdot B + e^{A(T/2)} \int_0^{-\Delta\tau_k} e^{As} ds \cdot B\end{aligned} \tag{4}$$

$$\begin{aligned}\Gamma_1(\tau_k) &= \int_{T-\tau_k}^{T} e^{As} ds \cdot B = \int_{T-T/2-\Delta\tau_k}^{T} e^{As} ds \cdot B \\ &= \int_{T/2}^{T} e^{As} ds \cdot B + e^{A(T/2)} \int_0^{-\Delta\tau_k} e^{As} ds \cdot (-B)\end{aligned} \tag{5}$$

$$\tau_k = \frac{T}{2} + \Delta\tau_k, \ -\frac{T}{2} \le \Delta\tau_k \le \frac{T}{2} \tag{6}$$

define $\Gamma_0 = \int_0^{T/2} e^{As} Bds, \ \Gamma_1 = \int_{T/2}^{T} e^{As} Bds, \ D = e^{AT/2}$, $F(\tau_K) = \int_0^{-\tau_k} e^{As} Bds$, $E_0 = B$, $E_1 = B$, then, the time-varying input matrices $\Gamma_0(\tau_k)$ and $\Gamma_1(\tau_k)$ in (3) can be written as

$$\begin{aligned}\Gamma_0(\tau_k) &= \Gamma_0 + DF(\tau_k)E_0 \\ \Gamma_1(\tau_k) &= \Gamma_1 + DF(\tau_k)E_1\end{aligned} \tag{7}$$

where $F^T(\tau_k)F(\tau_k) \le I$.

Let us introduce a new augmented state $z(k) = \left[x(i_k) \quad u(i_{k-1})\right]^T$. Therefore, from (2), (3) and (7), we can get the following augmented closed-loop system

$$z(k+1) = \Psi_k z(k), \tag{8}$$

where

$$\Psi_k = \begin{bmatrix} \Phi + \Gamma_0 + DF(\tau_k)E_0K(\tau_k) & \Gamma_1 + DF(\tau_k)E_1 \\ K(\tau_k) & 0 \end{bmatrix} \tag{9}$$

It can be seen that the newly generated augmented model is a discrete switched linear system with infinite switching rules. Therefore, the problem of stabilizing control for NCS is changed into the stabilization problem of discrete switched linear system.

Based on the analysis above, we will discuss questions below in the following sections:

Q1) Sufficient conditions for asymptotic stability: Is it possible to specify conditions such that the autonomous switched system is asymptotically stable?

Q2) Asymptotical controller design: If the answer to Q1 is yes, how to obtain the control law for the autonomous switched system?

Q3) Sufficient conditions for exponential stability with some decay rate: Is it possible to specify conditions such that the autonomous switched system is exponentially stable with some decay rate?

Q4) Decay rate: Can a bound of the decay rate of the autonomous switched system be found? And what is the bound?

3. Asymptotical Stability Analysis and Controller Design

As in the discrete switched linear system (8), continuously varying time delay τ_k leads to the infinite switching rules. To obtain the control law for the autonomous switched system, the formulation of an LMI optimization problem must be set for a finite set of switching rules. In order to solve the problem, every sampling period of the time axis is partitioned into equidistant small time intervals. Suppose the split number is N , then the length of the split interval $T_0 = T/N$ and the value of time delay τ_k can be redefined as follows:

$$\tau_k = \begin{cases} 2/T_0, \quad kT \le \tau_k < kT + T_0 \\ T_0 + 2/T_0, \quad kT + T_0 \le \tau_k < kT + 2T_0 \\ \vdots \\ (N-1)T_0 + 2/T_0, \\ \qquad kT + (N-1)T_0 \le \tau_k < kT + NT_0 \end{cases} \tag{10}$$

Then continuously varying time delay τ_k is quantized into a finite set. Let us define a compact set I where switching rules of (8) lie in, then we can get $I = \{1,2,3...,N\}$. Denote $\hat{A}_i$ as the switching rule determined by τ_k, then, augmented system (8) can be written into a discrete switched system

$$z(k+1) = \hat{A}_i z(k), \quad \forall i \in I \,, \tag{11}$$

where

$$A_i = \begin{bmatrix} \Phi + \Gamma_0 + DF_iE_0K_i & \Gamma_1 + DF_iE_1 \\ K_i & 0 \end{bmatrix} \tag{12}$$

Considering the switching nature of our system (11), LMI-based quadratic Lyapunov function for asymptotic stability is introduced to check system stability and design stabilizing controller.

Theorem 1: If there exists N symmetric positive definite matrices $P_1, P_2, \dots, P_N$ satisfying

$$\begin{bmatrix} P_i & \hat{A}_i^T P_j \\ P_j \hat{A}_i & P_j \end{bmatrix} > 0, \ \forall (i,j) \in I \tag{13}$$

then the system (11) is asymptotically stable.

Proof: Choose the following form of the Lyapunov function:

$$V(k) = z^T(k) P_i z(k) \tag{14}$$

the difference of Lyapunov function is given by

$$\begin{aligned} \Delta V &= V(k+1) - V(k) \\ &= z^T(k+1) P_j z(k+1) - z^T(k) P_i z(k) \\ &= z^T(k) \hat{A}_i^T P_j \hat{A}_i z(k) - z^T(k) P_i z(k) \\ &= z^T(k)(\hat{A}_i^T P_j \hat{A}_i - P_i) z(k) \end{aligned} \tag{15}$$

according to the Schur complement formula, (10) is equivalent to

$$P_i - \hat{A}_i^T P_j \hat{A}_i > 0 \tag{16}$$

thus, if condition (10) holds, then $\Delta V < 0$, which implies that system (11) is asymptotically stable.

The following part will study the design of stabilizing controller for NCS. Let us denote the following matrixes:

$$\begin{aligned} \tilde{A}_i &= \begin{bmatrix} \Phi_i & \Gamma_1 + DF_iE_1 \\ 0 & 0 \end{bmatrix}, \\ \tilde{B}_i &= \begin{bmatrix} \Gamma_0 + DF_iE_0K_i \\ I \end{bmatrix}, \\ \tilde{K}_i &= \begin{bmatrix} K_i & 0 \end{bmatrix} \end{aligned} \tag{17}$$

Then, system (11) can be rewritten into the following form:

$$z(k+1) = (\tilde{A}_i + \tilde{B}_i \tilde{K}_i) z(k) \tag{18}$$

Theorem 2: If there exists symmetric positive definite matrices G_i and V_i , matrices R_i ($\forall i \in I$) satisfying

$$\begin{bmatrix} G_i & 0 & G_i^T \Phi_i^T + R_i^T \Gamma_{i,0}^T & R_i^T \\ * & V_i & V_i^T \Gamma_{i,1}^T & 0 \\ * & * & G_j & 0 \\ * & * & * & V_j \end{bmatrix} > 0, \ \forall i, j \in I \tag{19}$$

Then the state feedback control given by (2) with

$$K_i = R_i G_i^{-1}, \quad i \in I \tag{20}$$

stabilizes asymptotically the system (11).

Proof: Assume that there exist symmetric positive definite matrices G_i and V_i , matrices R_i ($\forall i \in I$) such that (19) is satisfied. From (20) we get

$$R_i = K_i G_i, \quad i \in I \tag{21}$$

replacing R_i in (19) by $K_i G_i$, we can get

$$\begin{bmatrix} G_i & 0 & G_i^T (\Phi_i + \Gamma_{i,0} K_i)^T & G_i^T K_i^T \\ * & V_i & V_i^T \Gamma_{i,1}^T & 0 \\ * & * & G_j & 0 \\ * & * & * & V_j \end{bmatrix} > 0, \tag{22}$$
$$\forall i, j \in I$$

applying the Schur complement formula, we get

$$\begin{aligned} &\begin{bmatrix} G_i & \\ & V_i \end{bmatrix} - \begin{bmatrix} G_i & \\ & V_i \end{bmatrix} \begin{bmatrix} \Phi_i + \Gamma_{i,0} K_i & \Gamma_{i,1} \\ K_i & 0 \end{bmatrix}^T \\ &\begin{bmatrix} G_j & \\ & V_j \end{bmatrix}^{-1} \left[\begin{bmatrix} \Phi_i + \Gamma_{i,0} K_i & \Gamma_{i,1} \\ K_i & 0 \end{bmatrix} \right] \\ &\begin{bmatrix} G_i & \\ & V_i \end{bmatrix} > 0, \forall (i,j) \in I \end{aligned} \tag{23}$$

let us denote

$$P_i = \begin{bmatrix} G_i & \\ & V_i \end{bmatrix}^{-1} \tag{24}$$

and notice that

$$\begin{aligned} &\begin{bmatrix} \Phi_i + \Gamma_{i,0} K_i & \Gamma_{i,1} \\ K_i & 0 \end{bmatrix} \\ &= \begin{bmatrix} \Phi_i & \Gamma_{i,1} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \Gamma_{i,0} \\ I \end{bmatrix} \begin{bmatrix} K_i & 0 \end{bmatrix} \\ &= \tilde{A}_i + \tilde{B}_i \tilde{K}_i \end{aligned} \tag{25}$$

then replacing related item in (23) by(24) (25), pre-multiply and post-multiply P_i^{-1}, respectively, then we can get

$$\begin{aligned} &P_i - (P_j \tilde{A}_i + P_j \tilde{B}_i \tilde{K}_i)^T P_j^{-1} (P_j \tilde{A}_k + P_j \tilde{B}_k \tilde{K}_k) > 0, \\ &\forall (i,j) \in I \end{aligned} \tag{26}$$

by applying the Schur complement formula, we get

$$\begin{aligned} &\begin{bmatrix} P_i & (\tilde{A}_i + \tilde{B}_i \tilde{K}_i)^T P_j \\ P_j (\tilde{A}_i + \tilde{B}_i \tilde{K}_i) & P_j \end{bmatrix} > 0, \\ &\forall (i,j) \in I \end{aligned} \tag{27}$$

thus, Theorem 1 indicates the sufficient condition for asymptotic stability of closed-loop NCS (11).

4. Exponential Stability and the Bound of Decay Rate

Theorem 3: For given scalar $\alpha \, (\alpha > 1)$, if there exists N symmetric positive definite matrices $P_1, P_2, \ldots, P_N$ satisfying

$$\begin{bmatrix} P_i & \alpha \hat{A}_i^T P_j \\ \alpha P_j \hat{A}_i & P_j \end{bmatrix} > 0, \ \forall (i,j) \in I \tag{28}$$

Then the closed-loop NCS (11) is exponentially stable with a decay rate α.

Proof: Choose the following form of the Lyapunov function:

$$V(k) = z^T(k) P_i z(k) \tag{29}$$

define $\zeta(k) = \alpha^k z(k)$ and choose another Lyapunov function:

$$W(k) = \zeta^T(k) P_i \zeta(k) \tag{30}$$

then the difference for $W(k)$ is given by

$$\begin{aligned} \Delta W &= W(k+1) - W(k) \\ &= \zeta^T(k+1) P_j(k+1) \zeta(k+1) - \zeta^T(k) P_i \zeta(k) \\ &= \alpha^{2(k+1)} z^T(k+1) P_j(k+1) z(k+1) - \alpha^{2k} z^T(k) P_i z(k) \\ &= \alpha^{2(k+1)} z^T(k) \hat{A}_i^T P_j \hat{A}_i z(k) - \alpha^{2k} z^T(k) P_i z(k) \\ &= \alpha^{2k} z^T(k) (\alpha^2 \hat{A}_i^T P_j \hat{A}_i - P_i) z(k) \end{aligned} \tag{31}$$

according to the Schur complement formula, (23) is equivalent to

$$P_i - \alpha^2 \hat{A}_i^T P_j \hat{A}_i > 0 \tag{32}$$

then we can obtain that $\Delta W < 0$, which implies $W(k) < W(0)$. Thus we have

$$V(k) = \alpha^{-2k} W(k) < \alpha^{-2k} W(0) = \alpha^{-2k} V(0) \tag{33}$$

by the result of theorem 1in [33], we can come to the conclusion that the closed-loop NCS (11) is exponentially stable with a decay rate α. This completes the proof.

Now we will discuss how to get the controller gain for a given decay rate.

Theorem 4: For given scalar $\alpha \, (\alpha > 1)$, If there exists symmetric positive definite matrices G_i and V_i, matrices R_i ($\forall i \in I$) satisfying

$$\begin{aligned} &\begin{bmatrix} G_i & 0 & \alpha G_i^T \Phi_i^T + \alpha R_i^T \Gamma_{i,0}^T & \alpha R_i^T \\ * & V_i & \alpha V_i^T \Gamma_{i,1}^T & 0 \\ * & * & G_j & 0 \\ * & * & * & V_j \end{bmatrix} > 0, \\ &\forall i, j \in I \end{aligned} \tag{34}$$

Then the state feedback control given by (3) with

$$K_i = R_i G_i^{-1}, \quad i \in I \tag{35}$$

exponentially stabilizing the system (5) with decay rate α.

Proof: Replacing $\hat{A}_i$ in Theorem 3 by $\tilde{A}_i + \tilde{B}_i \tilde{K}_i$ and substitute (14) respectively for $\tilde{A}_i, \tilde{B}_i, \tilde{K}_i$, we can easily obtain the conclusion of Theorem 4.

Theorem 4 indicates that if we find feasible solution to (25), we can obtain the control law for a given decay rate. But what is the maximum value of decay rate? To obtain the bound of the decay rate, we should consider the following optimization LMI problem:

Maximize α

Subject to

$$\begin{aligned} &\begin{bmatrix} G_i & 0 & \alpha G_i^T \Phi_i^T + \alpha R_i^T \Gamma_{i,0}^T & \alpha R_i^T \\ * & V_i & \alpha V_i^T \Gamma_{i,1}^T & 0 \\ * & * & G_j & 0 \\ * & * & * & V_j \end{bmatrix} > 0 \\ &G_i > 0, \quad V_i > 0, \quad \alpha > 1, \forall i, j \in I \end{aligned} \tag{36}$$

In order to use Matlab LMI control toolbox setting up LMI problem (33), a remedy should be given to convert (33) into a generalized LMI optimizing problem like the following form:

Minimize λ

Subject to

$$C(x) < D(x), B(x) > 0, A(x) < \lambda B(x) \tag{37}$$

Pre-multiply and post-multiply $diag\{I, I, 1/\alpha * I, 1/\alpha * I\}$ by (33), we can get

$$\begin{bmatrix} G_i & 0 & G_i^T \Phi_i^T + R_i^T \Gamma_{i,0}^T & R_i^T \\ * & V_i & V_i^T \Gamma_{i,1}^T & 0 \\ * & * & \frac{1}{\alpha^2} G_j & 0 \\ * & * & * & \frac{1}{\alpha^2} V_j \end{bmatrix} > 0 \tag{38}$$

Replacing $(1/\alpha)G_j$ by Y_i, $(1/\alpha)V_j$ by Z_i and then replacing $(1/\alpha)Y_i$ by M_i, $(1/\alpha)Z_i$ by N_i, and defining $\mu = 1/\alpha$, optimization LMI problem (33) can be converted into:

Minimize μ

Subject to

$$\begin{bmatrix} G_i & 0 & G_i^T \Phi_i^T + R_i^T \Gamma_{i,0}^T & R_i^T \\ * & V_i & V_i^T \Gamma_{i,1}^T & 0 \\ * & * & M_i & 0 \\ * & * & * & N_i \end{bmatrix} > 0 \tag{39}$$

$$G_i, V_i, M_i, N_i > 0$$

$$Y_i < \mu G_i, M_i < \mu Y_i$$

$$Z_i < \mu V_i, N_i < \mu Z_i$$

By solving the optimization LMI problem (36), we can obtain the bound of decay rate.

5. Design Example

Consider the following longitudinal motion equation of aircraft:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{40}$$

$$A = \begin{bmatrix} -1.5 & -8.65 & 0 \\ -1.5 & -0.38 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 \\ 5.6 \\ 0 \end{bmatrix}, \tag{41}$$

where $x = [q \quad a \quad \theta]$, q is the pitch angular rate, a is the attack angle, θ is the pitch angle, $u = \xi_e$, ξ_e is the elevating rudder deflection angle. Suppose the sampling period is 8 ms and the length of gridded equidistant small interval T_0 is $2\,ms$, the values of possible time delays are $\tau_1 = 1\,ms$, $\tau_2 = 3\,ms$, $\tau_3 = 5\,ms$, $\tau_4 = 7\,ms$. To obtain feedback gains, we can use the Matlab LMI Control Toolbox to solve the LMI feasible problem presented in (19). Table 1 shows the feedback controller gains based on the switching rules.

Table. 1. Switching mode related feedback controller gains.

$\tau_1 = 1ms$	$K_1 = [\,-17.9144\;\; -17.5080\;\; -18.0312\,]$
$\tau_2 = 3ms$	$K_2 = [-20.5357\;\; -20.0698\;\; -20.6695]$
$\tau_3 = 5ms$	$K_3 = [-17.0806\;\; -16.6931\;\; -17.1919]$
$\tau_4 = 7ms$	$K_4 = [27.2913\;\; -26.6722\;\; -27.4692]$

Fig. 3 shows the state trajectory of aircraft longitudinal motion with the feedback control law proposed in this paper. We can see that the mode dependent control law effectively solved the time delay problem of networked flight control system, which guarantee the asymptotical stability of the system. Fig. 4 shows how the decay rate impacts the stabilizing performance of networked flight control system. We can see that decay rate determines the setting time of the state trajectory towards constant value which is the system stable state. The larger the decay rate is, the shorter time the system needs to reach the stable state with better stabilizing performance.

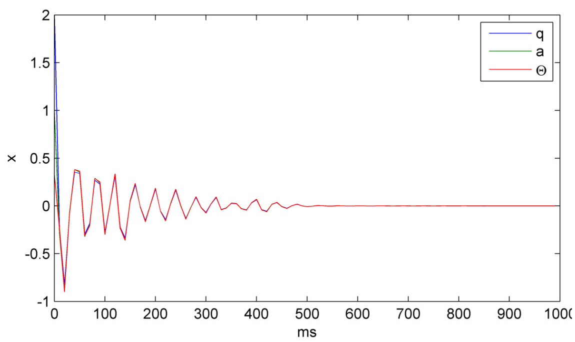

Fig. 3. The state trajectory of aircraft longitudinal motion.

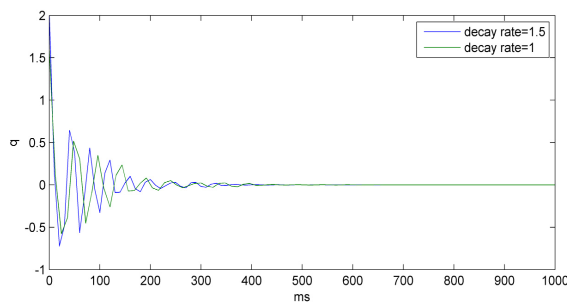

Fig. 4. Stabilizing performance of the system with decay rate.

6. Conclusions

The multitude of embedded sensors, controllers and actuators found on aircraft need to communicate with each other to collect information, transmit control signal and monitor system condition. Rapid developments of computer, network and communication technology result in progressive forward of a number of new and existing data buses used in flight control systems. In this paper, we study the stability problem of networked flight control system with random time delays and analysis control performance based on the decay rate. An augmented state vector is introduced to successfully convert NFCS into a discrete-time switched linear system. Sufficient conditions for asymptotical stability and exponential stability of proposed NFCS model are given. To solve the LMIs for obtaining feedback gains and the bound of decay rate, the "gridding approach" is adopted to guarantee LMI set up for a discrete switched system with finite switching rules. Numerical examples illustrate the effectiveness of the proposed strategy for the asymptotical stabilizing and exponential stabilizing controller over NFCS, finally the control performance with decay rate is analyzed based on the simulation results.

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