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PROBABILISTIC HUMAN DEVELOPMENT INDICES

Annibal Parracho Sant'Anna[a]

[a] **Universidade Federal Fluminense** (UFF), Brazil

Abstract

Alternative indicators to the Human Development Index adopting the same components but using other forms of combination are discussed here. The basic idea is to transform the initial measurements into probabilities of achieving the worst performance. Joint probabilities are compared to the proximity to the frontier generated by the algorithm of Data Envelopment Analysis with constant returns to scale and constant inputs to scores applying the Choquet integral with respect to capacities that take into account the substitutability between the components of the index. A high correlation between the results of different forms of composition was found, demonstrating the robustness of the Human Development Index. On the other hand, advantages of different alternatives of composition could be noticed.

Keywords: Human Development Index, Composition of Probabilistic Preferences, Capacities, Choquet Integral, Data Envelopment Analysis

1. INTRODUCTION

The combination in a unique index of indicators of development in different directions has been employed to draw attention to the importance of achieving goals in dimensions that are difficult to quantify. The most significant result of efforts to combine these indices was reached with the creation and computation by the United Nations development Programme (UNDP) of the Human Development Index (HDI). Combining three dimensions, education, health and income, the HDI draws attention to the need to take into account social dimensions of development that are not covered by the economic growth indicators easier to build.

The HDI was designed to provide a counterpoint to development measures that do not take into account more important aspects of the quality of life in a country than the volume of goods sold in the market. While on one hand, to achieve this purpose it is important to have a single and simple index, on the other hand it is important to validate and refine this index. This suggests the study of the form of composition of the HDI and the creation of alternative indices.

The HDI has received many suggestions for improvement (Raworth *et* Stewart, 2002; Bourguignon *et* Chakravarty, 2003; Ranis *et al*, 2006; Alkire *et* Santos, 2010; Fuentes-Nieva *et* Pereira, 2010; Kovacevik, 2010; Mayer-Foulkes, 2010; Newmayer, 2010). Here other forms of composition of the components of the HDI using probabilistic composition are considered.

In 2010 the method of calculating the HDI was updated. The main change then performed in the computational algorithm consisted in substituting a geometric mean for a weighted average employed before. This approximates the computation to that proposed in Sant'Anna *et al.* (2011), which uses a joint probability calculated by the product of the probabilities of optimizing the three separate indicators of education, health and income.

The basic difficulty of combining social indicators is dealing with the differences between the forms in which indicators in different dimensions are measured. This difficulty grows as new dimensions are found necessary to consider. The probabilistic composition solves this problem by previously transforming the measurements in each particular dimension into probabilities of reaching a frontier of best or worst performance.

This transformation expands the distances at one end of the spectrum of evaluations observed while reducing the distances at the other. Thus it helps to focus attention on the extreme of most critical situations. This effect, which is already pursued since the beginning of the construction of the HDI by means of a logarithmic transformation of the

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income measurements, can be more smoothly attained by a probabilistic transformation in each dimension.

Another ever-present difficulty lies in the imprecision in the indicators records of the fact that they intend to evaluate. The composition of probabilistic preferences - CPP (Sant'Anna, 2002) has as its first step the modeling of the imprecision in the numerical measurements observed.

Different forms of composition are studied here, all avoiding the assignment of absolute weights to the index components. Alongside the simplest forms based on a calculation of joint probabilities, is used a composition by the DEA algorithm of constant input, which treats each component as an output of an input available by all in a same amount. It is also used a form of composition attaching equal importance to the three dimensions, which, to avoid overweighting the two educational performance measurements, employs, instead of marginal probabilities, a Choquet capacity in which these measurements appear as substitutable for one another.

Besides, a sorting approach is also described. It employs the trichotomic composition of probabilistic preferences - CPP-Tri (Sant'Anna *et al*, 2014), a methodology that combines the trichotomic principle of Electre-Tri-nC (Almeida Dias, 2010. 2012) to CPP. The result is rather than ranking the countries, allocating them into a small number of previously defined classes.

2. THE UNITED NATIONS HDI

Social values are complex and dense in subjectivity. People cannot measure them directly. Social indicators are objective measures of simple attributes that are supposed to be somehow correlated with the qualities which essentially interest. Therefore, there is a risk in the production of indicators. If used to set the goals to be attained, they can distort the search for the true values that they supposed to represent.

This is what happens with the gross national product or the per capita income. They reduce to the economic dimension the whole pursuit of development. The HDI was created to allow other indicators to share the importance attached to strictly economic indicators. But by including values more difficult to quantify, it suffers more sharply the two kinds of problems that, in the formulation of Carley (1981), plague all social indicators: political problems and methodological problems.

The political problems are associated with the subjective judgments that shape the construction of the indicator. The methodological problems, in turn, come from the difficulty of establishing a correlation between the measurable and the non-measurable.

Simple and objective indicators like the HDI have as its highest quality the ability to drive direct, objective and strong impact. The counterpart of this quality is the loss of relevant dimensions of the phenomenon. To highlight the idea that to assess the development economic terms are insufficient, the HDI combines measures in three basic dimensions of human life. But other important dimensions may still be left underrepresented. For instance, a dimension of political culture, involving democratic freedoms, legal stability and the protection of discriminated groups. One should also better consider the aspect of sustainability, contemplating the interests of future generations, particularly affected by environmental concerns and the need to preserve natural resources.

Besides the lack of important dimensions of human well-being, even in dimensions which it considers the HDI is to some extent incomplete. The way it evaluates and combines its three dimensions makes it susceptible to distortions. The quality of education and health may not be suitably measured by its quantitative indicators. Since the per capita GDP is the only indicator used to explain the income dimension of the HDI, improvements have been designed, for instance, to make it access the intensity of poverty that results from income inequality. Furthermore, the importance of the measured features varies from one region of the world to each other.

Thus there are two critical features to consider in the HDI: the imprecision in the measurement of each concept and the variety of dimensions combined. The probabilistic approach is appropriate to improve the index in both these aspects. The preliminary transformation of the measurements obtained into preference probabilities takes into account the possibility of errors in measurement. In addition, this transformation leads the measures of different dimensions to a same scale where originally heterogeneous values may be combined.

3. THE COMPUTATION OF THE HDI

By 2009, the partial indicators combined in the HDI were: life expectancy at birth, as the indicator of the country's development on the health dimension; literacy, weighing 2/3, combined with the gross rate of school enrollment (ratio between the total number of students attending school at the three levels of education and the number of people of school age), weighing 1/3, as education indicator; the Neperian logarithm of the gross domestic product per capita, measured using as a yardstick the purchase power

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parity - PPP (WORLD BANK, 2015), as an indicator of consumption pattern.

Each of these indicators were transformed to a scale from 0 to 1 by subtracting an absolute minimum and dividing by an absolute amplitude. The range for life expectancy in a country was fixed in 25 to 85 years. For GDP per capita, the limits were 100 and 40,000. Finally, the HDI was calculated as the arithmetic average of these three indicators.

Currently there are different partial indicators and a different composition rule, remaining the three dimensions. The health indicator is still longevity, measured as before, but with limits of 20 and 83.57 years. As income indicator, the GDP per capita was replaced by the Gross National Income per capita, with limits of 100 and 87 478 PPP. The biggest change occurred in the evaluation of education, whose indicator is now the geometric mean of two educational measures: the average number of years a person of twenty-five years today attended school and the expected value of the number of years a child of age six today will attend school. This second parameter is estimated from the distribution of school enrollments today extracted from censuses and national surveys and employing Barro *et* Lee (2010) estimation methodology.

Before entering the calculation of the geometric mean, each of these two indicators are standardized to values between zero and one, by dividing the first one by the observed maximum of 13.3 and the second by the upper bound of 18.

The use of the geometric mean of the three indicators, instead of the arithmetic mean, implies that the influence of the three factors is accumulated in a multiplicative base. Thinking about each of the partial indicators as a chance to meet a standard of well-being, what seems to be the real reason of building measures between zero and one, - one might think the HDI as the probability of full satisfaction in the three dimensions.

Another interesting feature of the HDI is the application of the logarithmic transformation to the income measures. With the logarithmic transformation the importance of changes in countries close to the upper bound of the indicator is reduced. A similar effect is obtained by transforming into probability of presenting the lowest value and constructing the index by a combination of the probabilities of minimization of the indicators for each of the dimensions. Thus the probabilistic approach eliminates the use of the logarithmic transformation.

4. CPP

CPP (Sant'Anna, 2002) is a methodology developed to take into account in the composition of multiple criteria the presence of imprecision in the preference assessments. It deals with rules to rank options evaluated by different criteria or different experts in terms of probabilities of choice. For example, the ranking can be made according to the probability of the option being the best evaluated by all the criteria or by at least one of them.

Its first step is to associate to each measurement a probability distribution, in a way similar to that employed in the Theory Fuzzy sets (Zadeh, 1965) to replace exact numbers by relevance measures for the points in intervals around them. The initial exact measurements are regarded as location parameters of probability distributions of the possible values that in other assessments under similar circumstances would be assigned to the same option.

This substitution of probability distributions for exact measurements constitutes the basis for the replacement, by a simple calculation, of each vector of assessments of preference for a set of options according to a given criterion by a vector of probabilities of each option in the set being the best according to such criterion.

In a last step, from these preference probabilities according to each criterion are derived global preference probabilities by suitable rules of composition.

The preference for an option according to a criterion is naturally measured by the probability of it being the best option among all available. The calculation of such probabilistic preference can be made starting from any initial assessment, be it verbal, ordinal or resulting from direct measurement of some attribute. To this initial assessment is associated a random error. The key idea of randomization is to treat each observed value as representing a position around which the measured attribute varies.

Thus the observed value signals only one point around which is the real value being evaluated. The model for the probability distribution of this random value is completed by adding to that observed value taken as a location parameter information about other parameters enough to precisely determine the distribution. General assumptions of independence and assumptions about the shape of the distribution simplify the identification of these parameters.

How to determine the shape of the probability distribution of the random error? In a first approximation, we can assume that it presents, as the distribution of disturbances in classical statistical models, a normal distribution, which depends only on the attribute and not on the particular option being evaluated. It may be more realistic, however, to allow for dispersions that depend, for instance, on the distance of the measure observed to extreme values.

A simple asymmetric approach is by triangular distributions with modes on the observed values. Assuming a triangular distribution, a smoother decline to the farthest end compensates a possible excessive deviation of the

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evaluator. The ends of the triangular distribution may be derived from the extremes permitted or from the extremes observed in the application of the criterion. For instance, for a nine points Likert (1932) scale, lower and upper ends eLj =0 and eUj=10 may be assumed. Or, alternatively, for ($e1j$, … , eoj) being the observed vector of evaluations of o options by the j-th criterion, $e0j$ = min{$e1j$, … , eoj}–1 and eLj = max{$e1j$, … , eoj}+1.

For this situation of bounded measurements, a beta distribution may be also assumed. This is specially suitable to the situation of each criterion being applied separately by several experts, so that the initial evaluation for option i according to criterion j is derived from a sample $(IJ_1, \dots, IJ_{nij})$ of assessments by n_{ij} experts. Here, although the set of experts may be fixed, some of them may not be able to manifest their preferences according to some criteria or about some option, so that assuming a more flexible number of evaluations may be useful.

Given the sample size n_{ij} and the lower and upper bounds A_j and B_j for the distributions, a beta distribution for the evaluation of the i-th option according to the j-th criterion can be determined by its mean. This mean can be estimated by the sample mean $M_{ij} = (IJ_1 + \dots + IJ_{nij})/n_{ij}$. The beta distribution has then a density $f_{ij}(x)=[(x-A_j)/(B_j-A_j)]^{\alpha ij-1}[(B_j-x)/(B_j-A_j)]^{\beta ij-1}/Beta(\alpha_{ij},\beta_{ij})$, with $Beta(\alpha_{ij},\beta_{ij})$ denoting the beta function evaluated at (α_{ij},β_{ij}), $\alpha_{ij} = n_{ij}(M_{ij}-A_j)/(B_j-A_j)$ and $\beta_{ij} = n_{ij}(B_j-M_{ij})/(B_j-A_j)$, for x varying between A_j and B_j.

This construction allows for an asymmetric modeling of the dispersion as a function of the estimated preference. In fact, the variance of a beta distribution with parameters α_{ij} and β_{ij} is given by $V_{ij} = [(B_j-A_j)^2\alpha_{ij}\beta_{ij}]/[(\alpha_{ij}+\beta_{ij})^2(\alpha_{ij}+\beta_{ij}+1)]$. For α_{ij} and β_{ij} derived from the sample in the form above, this is equal to $(M_{ij}-A_j)(B_j-M_{ij})/(n_{ij}+1)$. Thus, the variance decreases with the sample size and with the noncentrality of the mean.

This distribution is unimodal for $\alpha_{ij}>1$ and $\beta_{ij}>1$, with mode $A_j+(B_j-A_j)(\alpha_{ij}-1)/(\alpha_{ij}+\beta_{ij}-2)$ and mean $A_j+(B_j-A_j)\alpha_{ij}/(\alpha_{ij}+\beta_{ij}) = M_{ij}$.

To satisfy these conditions for unimodality, the extremes A_i and B_i must be modeled at a suitable distance from the values effectively employed in the preference assessments. If the experts are allowed to assign values between L_j and U_j and $\min_i n_{ij} = N_j$, then the conditions $\alpha_{ij}>1$ and $\beta_{ij}>1$ are satisfied for all i if $N_j(L_j-A_j)$ and $N_j(B_j-U_j)$ exceed the range B_j-A_j. This happens for $L_j-A_j = B_j-U_j = (B_j-A_j)/(N_j-1)$, what implies $U_j-L_j = ((1-2/(N_j-1))(B_j-A_j)$, what is equivalent to $(B_j-A_j)/(N_j-1) = (U_j-L_j)/(N_j-3)$.

Thus, it is recommended to count on samples of size greater than 3 and to fix the extremes A_i and B_i at $L_j-(U_j-L_j)/(N_j-3)$ and $U_j+(U_j-L_j)/(N_j-3)$.

The parameters α_{ij} and β_{ij} of the beta distribution may be also estimated from the mode of the sample $(IJ_1, \dots, IJ_{nij})$, but, for small samples, the fact that the estimation from the mean employs more completely the available information makes more robust the estimates above presented.

Once associated with each option a random distribution, the score of the i-th option is given by the probability that, in a sample of the random vector so determined, the highest value is found in the i-th coordinate. The probabilistic composition consists in deriving, from the vectors of scores so obtained for each criterion, a unique global vector of scores.

The easiest way to perform this composition is by treating the probabilities of being the best option according to each criterion as preferences conditional on the preference for such criterion. The probability of global preference may be then obtained combining these conditional probabilities by the Theorem of Total Probability. In this way, the global score is determined as a weighted average of the conditional probabilities. The weights for this average are the marginal probabilities of the conditioning events, which correspond to the preferences by the criteria.

However, the identification of these marginal probabilities is not easy. It is not reasonable, in general, suppose that the decision maker may depend on combining assessments according to multiple criteria to evaluate the options while counting on some simpler way to determine preferences among the criteria. By their more abstract character, it should be more difficult to compare the criteria objectively than to compare the options.

Besides, even if numerical preferences are obtained for the criteria, the application of these numerical preferences as weights in a weighted average may be distorted by interactions between criteria.

With the evaluations in respect to each criteria given in the same form, the form of probabilities, it is possible to combine them, rather than by a weighted average, by a Choquet integral (Choquet, 1953). Instead of the additive use of a probability distribution for the criteria, which would disregard the possibility of interactions, the Choquet integral uses a capacity to more realistically measure the preference by any set of criteria.

To determine such a capacity is also not easy, in general. A form of identification of a capacity to be used in the joint evaluation by multiple criteria of options whose evaluation according to each criterion is put in terms of probabilities of being the best is proposed in Sant'Anna (2014a). The basic principle of this form of identification is the maximization of the power to discriminate between the options evaluated. In the Choquet integral, the increase in the capacity of a set applies to the increase observed in the integrand, so maximization occurs in the composition of probabilities of preference by multiple criteria if the importance of the set is raised proportionally to the largest increase in the

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probability of preference according to whatever criterion in it.

A simpler form of composition is based on joint probabilities. Rules to derive scores by joint probabilities vary along two main aspects: the view adopted by the decision maker to build the joint probability and the joint modeling of the disturbances that affect the evaluations according to the multiple criteria.

Four main forms of composition can be obtained according to the position in which lies the decision maker on two axes, of conservatism and of pessimism. It has proven much easier the identification of extremes on these two axes to guide the decision than the choice of a set of weights or capacities for the criteria.

In the first axis, the decision maker in the progressive extreme considers the probabilities of maximizing the preference according to the criteria, while that in the conservative extreme prefers to consider the probabilities of not minimizing it. The progressive decision maker pays attention to distances to the extremes of excellence, while the conservative decision maker pays attention to distances to the extreme of worst performance. Conservative, in this terminology, is related to care about avoiding losses, while progressive is related to center attention on attaining gains.

On the other hand, in the optimistic-pessimistic axis, the optimistic extreme is to consider satisfactory being the best or not being the worst in at least one criterion. All the criteria are taken into account, but the composition uses the connective "or." The global sore is determined by the probability of maximizing (or not minimizing, if the view along the other axis is conservative) preference according to at least one among the multiple criteria. Alternatively, in the pessimistic extreme, preference is measured by the probability of maximizing or not minimizing the preference according to all the criteria.

A way to deal with the possibility of interactions is to compare joint probabilities computed under the hypotheses of extreme dependence and of independence and to compare the results. While the hypothesis of independence leads to calculate the probability of the intersection by the product of the probabilities of events that are intersected, the hypothesis of maximum dependence leads to calculate this probability by the minimum between these probabilities in a form of composition equivalent to the application of the principles the necessity and sufficiency of Fuzzy logic (Zadeh, 1978).

The probabilistic approach can be applied in the context of assessing the efficiency of production units employing compositions of inputs to generate sets of outputs. In this kind of problem, the nonparametric methodology most employed since Charnes *et al.* (1978) is Data Envelopment Analysis (DEA). In the probabilistic approach to this problem, both, the output/input ratios and the volume of each input and of each output separately, can be considered as criteria.

If this last approach is taken, the probabilities of preference according to each criterion are, respectively, those of maximizing revenue by selling each output and of minimizing cost in acquiring each input. Taking the ratios as the criteria and combining optimistically by joint probabilities will be similar to the DEA approach.

Conversely, the composition of any kind of criteria may be done by taking the probabilities of preference according to each criterion as volumes of outputs generated by a constant volume of input and evaluating the options by their DEA efficiency scores.

The criteria may be also divided not in a group of inputs and another of outputs, but in a group of highest priority and another of secondary priority. The composition of the criteria of high priority would be done from a pessimistic point of view, while secondary criteria would be combined from an optimistic point of view. Outputs may be treated from a progressive perspective, while inputs would attract a conservative one.

5. CPP-TRI

Trichotomic composition of probabilistic preferences (CPP-Tri) was developed in Sant'Anna *et al.* (2015) to deal with the multicriteria problem of allocation in predetermined classes. Let $G = \{g_1, \ldots, g_m\}$ be a set of m criteria; $A = (a_1, \ldots, a_m)$, a vector containing the evaluations of an option A according to these m criteria; $C = \{C_1, \ldots, C_r\}$, a set of r classes, in which the option will be classified in such a manner that the better the option according the set of criteria the higher the index of the class in which it is allocated.

For each i, to identify class C_i, is previously determined a number p of profiles, $C_{i1} = (C_{i11}, \ldots, C_{im1}), \ldots, C_{ip} = (C_{i1p}, \ldots C_{imp})$. For any j, the coordinates c_{ijk} are typical values of possible evaluations of options by criterion j such that, for i_1 and i_2 from 1 to r with $i_1<i_2$, for all k_1 and k_2 from 1 to p, $C_{i1jk1} \leq C_{i2jk2}$, and, for some k_1 and k_2, $C_{i1jk1}<C_{i2jk2}$.

In practice, the allocation is not of an only option, but a larger number of options are allocated along a small number of classes and the representative profiles of the classes can be determined from the observation of the values assigned to the options by the multiple criteria. A typical profile for the *i-th* of r classes can be the vector whose j-th coordinate is the quantile of order $(2i-1)/(2r)$ of a sample of evaluations of options by the j-th criterion. For example, in the case of five classes, the j-th coordinates would be the first, third, fifth, seventh and ninth deciles of the sample of evaluations by the j-th criterion.

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Each coordinate a_j is treated as a location parameter of the statistical distribution of a random variable X_j. Denoting, respectively, by A_{ij}^+ and A_{ij}^- the probability of option A presenting, in p independent assessments by the j-th criterion, evaluations above and below the j-th coordinates of the p profiles representing the i-th class, $A_{ij}^+ = \Pi_k P[X_j > C_{ijk}]$ and $A_{ij}^- = \Pi j P[X_j < C_{ijk}]$.

Global probabilities A_i^+ and A_i^- of A being above and below the profiles of the i-th class may be obtained applying composition rules like weighted average, Choquet integral or joint probabilities as listed in the previous section.

For instance, assuming independence between evaluations by different criteria, the joint probabilities of being above and below all the profiles of the i-th class by all the criteria are given by $A_i^+ = \prod_k A_{ik}^+ \prod_k A_{ik}^+$ and $A_i^- = \prod_k A_{ik}^- \prod_k A_{ik}^-$. Assuming maximal dependency, minima will substitute for these products.

The final classification rule is simple: option A is allocated in the class i for which the difference $Ai^+ - Ai^-$ is nearest zero. There may be a tie between contiguous classes, but with continuous distributions, it is unlikely that the same numerical value is found for this difference. Even so, this possibility of indecision in the allocation must be accepted as a natural consequence of imprecision in the subjective process of determination of preferences, so that, in principle, the class to which the alternative belongs may be given by a point or by an interval.

6. BUILDING PROBABILISTIC INDICES

CPP was applied in Sant'Anna *et al*. (2011) to combine the components of the HDI. Results of other applications, to data of 2012 (Sant'Anna, 2014b) are presented in Tables 1 and 2. Table 1 presents the probabilities of being the worst for 30 selected countries, 17 selected by a high value in some component and 13 by a low value.

The probabilities in Table 1 were obtained by calculating the probabilities of being the worst in the entire population of One hundred eighty-seven countries considered, adopting triangular distributions with equal amplitudes for all the countries and extremes extended by 10% for all the criteria. With the use of the probability of being the worst, all the countries in the first group of seventeen have the same evaluation, when approximated to the third decimal place, by the longevity criterion. For the other criteria, it can also be seen that the evaluations of the first countries on the list are much closer together than those of the other thirteen.

Table 2 presents the results of the application of four different forms of composition to the probabilities of not being worse than the elements of a fixed sample of nine fictitious countries formed with the values of the observed deciles.

In the first two forms, a joint probability score is obtained by assuming maximal dependence between the criteria, what leads to the use as the lowest of the four probabilities for the probability of the intersection. The first adopts the pessimistic approach, i.e., evaluates the country by the probability of not being worse than the sample by every criterion. In the second, on the contrary, the approach is optimistic: the country is evaluated by the probability of not being worse than the sample by at least one of the criteria.

The third form of composition is also optimistic, but uses the DEA algorithm for constant input. The score is given by the proximity of the DEA frontier in the weighting most benevolent for the country.

Finally, the fourth score is given by the integral of Choquet, modeling the preference among the criteria by a capacity with preferences equal to 1/3 for each criterion, assuming the education measurements as nonadditive, but rather fully substitutive for each other.

Table 2 shows the close concordance between the ranks derived from the different approaches with each other and with the vector of HDI ranks. In fact, in the set one hundred eighty-seven countries, the lowest correlation with the HDI, for the optimistic ranking by joint probability, is 94%. The others correlation with the HDI are of 95% for the composition by DEA, 96% for the pessimistic composition and 99% for the composition by the capacity.

For one hundred sixty-five of the one hundred sixty-five countries analyzed, the difference between the ranks by the current UN IDH model and by the Choquet integral for full substitutability between the educational components is less than ten. There is also a Spearman correlation of 99% between the HDI and the result of application of the HDI geometric mean algorithm to the probabilities of reaching the worst performance extremes instead of to the initial values of the components.

This high concordance demonstrates the robustness of the approach, features of the calculation little influencing the final result. Nevertheless, it is possible to perceive differences in the rankings. Given the goal of evidencing the importance of all the components considered, is especially interesting to highlight the peculiarities in the results of the pessimistic composition, which maximizes this feature.

In Table 2, stands out, as examples of countries whose pessimistic score is improved because they have good performance in all components: Switzerland, Israel, France and Sweden. On the other hand, Singapore, Kuwait, Papua New Guinea and Djibouti have a lower classification from the pessimistic point of view for no distancing the worst performance in all the criteria. The first two present good performances in terms of income, which put them in a good position by some forms of composition. This is not

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accompanied by good performance in the educational components. The other two, instead, with poor performance in the income measurement, present a much better performance in the other components, especially in the health component.

Table 1 – Probabilities of minimization of single criteria

Country	Longevity	Mean schooling	Expected Schooling	**Income**
Switzerland	0.0026	0.0029	0.0034	0.0017
Australia	0.0026	0.0026	0.0026	0.0019
Israel	0.0026	0.0027	0.0034	0.0024
France	0.0026	0.0030	0.0033	0.0021
Sweden	0.0026	0.0027	0.0033	0.0019
Norway	0.0026	0.0025	0.0030	0.0014
Japan	0.0025	0.0027	0.0035	0.0020
Iceland	0.0026	0.0030	0.0028	0.0022
Netherlands	0.0027	0.0027	0.0031	0.0018
New Zealand	0.0027	0.0025	0.0026	0.0025
Germany	0.0027	0.0026	0.0032	0.0019
Hong Kong	0.0025	0.0032	0.0034	0.0015
United States	0.0028	0.0024	0.0031	0.0016
Singapore	0.0027	0.0029	0.0035	0.0018
Qatar	0.0029	0.0043	0.0047	0.0009
Liechtenstein	0.0028	0.0031	0.0048	0.0009
Kuwait	0.0032	0.0052	0.0038	0.0013
Mali	0.0134	0.0159	0.0096	0.0090
EquatorialGuinea	0.0144	0.0059	0.0088	0.0027
Chad	0.0185	0.0213	0.0098	0.0086
Burkina Faso	0.0086	0.0246	0.0111	0.0087
Mozambique	0.0160	0.0266	0.0069	0.0089
CentralAfric.Rep.	0.0218	0.0091	0.0114	0.0091
Congo	0.0239	0.0091	0.0078	0.0096
Sierra Leone	0.0279	0.0096	0.0100	0.0090
PapuaNewGuinea	0.0053	0.0081	0.0154	0.0077
Djibouti	0.0071	0.0083	0.0160	0.0077
Niger	0.0093	0.0228	0.0228	0.0091
Eritrea	0.0056	0.0093	0.0271	0.0093
Sudan	0.0057	0.0102	0.0288	0.0081

Table 2 - Indices by Different Composition Rules

	Pessimistic		Optimistic		DEA		Tridimensional		UN HDI
	score	rank	score	Rank	score	rank	score	Rank	Rank
Switzerland	0.9539	1	0.9767	11	0.9999	17	0.9974	10.5	9
Australia	0.9534	2	0.9752	17	1	5	0.9976	2	2
Israel	0.9533	3	0.9685	29.5	0.9999	17	0.9972	26.5	16
France	0.9531	4	0.9725	24	0.9999	17	0.9973	19.5	20
Sweden	0.953	5	0.9757	14	0.9999	17	0.9974	10.5	7.5
Norway	0.9527	6	0.9783	5	1	5	0.9977	1	1
Japan	0.9525	7	0.9743	21.5	1	5	0.9973	19.5	10
Iceland	0.9524	9	0.9715	25	1	5	0.9974	10.5	13.5
Netherlands	0.9522	11.5	0.9760	12	0.9999	17	0.9975	4	4
New Zealand	0.9522	11.5	0.9663	32	1	5	0.9974	10.5	6
Germany	0.952	15	0.9755	15.5	0.9999	17	0.9974	10.5	5

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Hong Kong	0.9498	22	0.9777	8	1	5	0.9975	4	13.5
United States	0.9487	25	0.9773	9	1	5	0.9975	4	3
Singapore	0.9461	29	0.9789	3.5	0.9999	17	0.9974	10.5	18
Qatar	0.9196	72	0.9821	1	1	5	0.9972	26.5	36
Liechtenstein	0.9139	77.5	0.9819	2	1	5	0.9972	26.5	24
Kuwait	0.8966	94	0.9789	3.5	0.9999	17	0.9968	36.5	54
Mali	0.7171	165.5	0.8194	184	0.9930	185.5	0.9872	174.5	182
EquatorialGuinea	0.7162	167	0.9626	40	0.9982	138	0.9914	157	136.5
Chad	0.7048	171	0.8258	177	0.9930	185.5	0.9839	186	184
Burkina Faso	0.7029	172	0.8248	178	0.9939	182	0.9860	179.5	183
Mozambique	0.7011	174	0.8201	182	0.9957	174.5	0.9828	187	185
CentralAfric.Rep.	0.6995	175.5	0.8176	185	0.9933	183	0.9859	181	180
Congo	0.697	179.5	0.8125	187	0.9948	180	0.9858	182.5	186.5
Sierra Leone	0.6936	182	0.8197	183	0.9929	187	0.9844	184	177
PapuaNewGuinea	0.6877	183	0.8852	159	0.9972	156	0.9905	167	156
Djibouti	0.6863	184	0.8453	172	0.9954	177	0.9897	170	164
Niger	0.6768	185	0.8173	186	0.9932	184	0.9863	178	186.5
Eritrea	0.6737	186	0.8750	163	0.9969	162	0.9860	179.5	181
Sudan	0.6728	187	0.8729	165	0.9968	165	0.9858	182.5	171

Information may be also sought for a strategy that encourages the countries to seek escaping from underdevelopment in some particular aspect, hoping that the momentum generated in some dimension has the effect of causing advances in others. In this case, the joint probability optimistic approach and the DEA algorithm should be preferred.

There are some differences between the results of application of these two approaches, the use of DEA rewarding steeper advances in any indicator. In the optimistic probabilistic approach, less significant progresses, combined in two different dimensions will have a positive effect on the overall assessment that does not appear in the composition by the DEA algorithm.

For example, countries like Liechtenstein and Qatar, which stand out for their income but do not have such a good evaluation in the educational components, have a better assessment when used as global score the proximity to the DEA efficiency frontier and have a worse rank by the optimistic probabilistic composition, though not as bad as by the pessimistic composition.

It is also interesting to notice the cases of Japan and Iceland, hitting the DEA frontier and rated better by the pessimistic approach that rewards the homogeneity of the assessments by all criteria, than by the optimistic composition, which ranks better countries with greater proximity to the position of excellence on two of the components.

At the other extreme, call attention in Table 2 the positions of Eritrea and Sudan, with the last positions by the pessimistic composition, but escaping such positions in the optimistic probabilistic composition and in the composition by the DEA algorithm, due to their performances in longevity. On the other hand, Sierra Leone, Congo and Central African Republic, despite the worst performance on this component, have a better position in the pessimistic assessment due to a better performance in other components.

The biggest discrepancy between assessments is offered by Equatorial Guinea, a country to which per capita income guarantees the fortieth position in the optimistic probabilistic classification, while ranked in position one hundred, sixty-seven in the pessimistic composition, which takes into account more strongly the proximity of the worst performance in the other components.

7. CLASSES OF COUNTRIES

In this section, CPP-Tri is employed, initially to classify countries by HDI criteria into five classes.

Each class is identified by three representative profiles. In the first profile, the coordinate vectors are given by approximate values of the first, third, fifth, seventh and ninth observed deciles for each component, corresponding to positions 19, 56, 93, 131 and 168 in the set of 187 countries in 2012. Other profiles are formed by the values observed for the four components in the five countries in such ranks by the HDI (Austria, Romania, Iraq, Algeria and Ivory Coast). A third type of profile is formed with equally spaced values close to the values observed in the other two profiles. This set of profiles is shown in Table 3.

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The use of CPP-Tri to allocate countries based on the probabilities of being above and below these profiles was discussed in Sant'Anna (2014c). The application of different forms of composition led to similar results consistent with the HDI ranks. For instance, assuming maximum dependence, all countries with higher HDI than New Zealand end up in class 5 for all compositions and all countries with an HDI lower than Sierra Leone in Class 1.

Table 3. Profiles of five classes

Profiles	Longevity	Mean schooling	**Expected schooling**	**Income**
9th decile	81	12	16	34000
Austria	81	11	15	36000
Class 5	76	12	16	18000
7th decile	76	10	14	14000
Romania	74	10	15	11000
Class 4	74	10	14	14000
5th decile	73	8	13	8000
Algeria	74	8	14	7000
Class 3	72	8	12	10000
3th decile	68	6	11	3000
Iraq	70	6	10	4000
Class 2	70	6	10	6000
1st decile	55	3	9	1000
Côte d´Ivoire	56	4	7	2000
Class1	68	4	8	2000

Another classification is presented here based on the means of the years from 2005 to 2013. From these means and the number nine of years accumulated were derived beta distributions. The extremes of these beta distributions are constant along the countries, with the values of -20 and 140 for income, 0 and 100 for longevity, 0 and 15 for observed mean number of years of schooling and 0 and 20 for expected numbers of years of schooling. To avoid means excessively close to the extremes, the values of these two last components are truncated at a distance of 1/8 of the range.

The classification is in nine classes represented by one profile each. The profiles were derived from the nine observed deciles. The composition of the probabilities of being above and below each coordinate of a profile employs the capacity determined by the maximization principle. The capacity of each set of criteria is shown in Table 4.

Table 4. Maximization capacity

Unitary sets		Binary sets		Ternary sets	
Income (I)	0.069	I and II	0.310	I, II and III	0.778
Health (II)	0.257	I and III	0.759	I, II and IV	0.503
Mean Schooling (III)	0.713	I and IV	0.393	I, III and IV	0.965
Expected Schooling (IV)	0.337	II and III	0.732	II, III and IV	0.954
		II and IV	0.456		
		III and IV	0.919		

It can be seen in Table 4 that the highest importance is assigned to the isolated effect of the mean years of schooling observed. This is the criterion with highest power to discriminate between the countries with the lowest assessments.

Small importance is given to the interactions between the different components. In fact, this capacity corresponds to Shapley values (Shapley, 1953) of 0.05 for Income, 0.12 for Longevity, 0.58 for Expected Schooling and 0.25 for Observed Schooling, which keep considerable proportionality to the

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individual contribution of each component, with a larger loss of importance in the interactions only to the longevity component.

The one hundred, sixty-seven countries are spread along the nine classes. The classes with fewer countries are classes 2, 3 and 7 with 16 countries each. The class with the largest number of countries is Class 9 with thirty-three countries followed by Class 1 with twenty-nine countries. This light concentration in the extremes may be explained by the truncation procedures adopted.

In spite of the conceptual differences, this classification presented also results consistent with the results of the other analysis. From the twenty countries with highest HDI, only two, Singapore and Hong Kong, were not allocated to Class 9, but to Class8. On the other hand, from the 20 with lowest HDI, only Malawi was not allocated to Class1 but to Class 2.

8. CONCLUSION

A new approach to combine the components of the HDI was discussed here. The same health, education and income components are used, but probabilistic forms of composition are used to combine them. A main advantage of this new approach is its ability to easily deal with more dimensions, more components in each direction and more kinds of measurement of the components.

The strategy that allowed for the new approach was based on exploring the transformation of the initial measures into probabilities of reaching the extreme worst performances. With this transformation, attention is shifted to the efforts of the countries presently at lower levels of development.

Combination by joint probabilities is studied. In one form of combination, the pessimistic classification, the final scores are given by the probabilities of avoiding the worst performance jointly across all of the criteria. In the other form, optimistic, they are derived from the joint probability of moving away from the worst performance frontier in at least one criterion.

Other scores are also built. For instance, in one of them the distance to the frontier is calculated by the DEA algorithm for constant returns to scale and identical inputs. Another, considering that the two educational components present values close to each other, employs a capacity allowing for interaction between them, treated as substitute.

There is high correlation between the results of different approaches. Applying this latter approach, the scores derived by a Choquet integral obtain a correlation with the current HDI ranks of 99%.

Each form of composition, although not leading to distant results, has its own characteristics that can be exploited. A different contribution may be expected of each of them to different strategies to encourage the promotion of human development.

The probabilistic approach is simpler even with respect to the algorithm adopted today in the HDI. It permits for instance to give up the logarithmic transformation of income.

The strategy of building representative profiles of classes from the deciles of the distributions observed in the set of options to be analyzed was successfully tested. The use of weighted averages and of geometric means was compared with the Choquet integral with respect to different capacities. The results were very similar, demonstrating the robustness of the IDH classification.

9. REFERENCES

Alkire, S., *et* Santos M. (2010) Acute Multidimensional Poverty: A New Index for Developing Countries. Human Development Research Paper 11, New York: UNDP–HDRO.

Almeida-Dias, J., Figueira, J. *et* Roy, B. (2010) Electre Tri-C: A multiple criteria sorting method based on characteristic reference actions. European Journal of Operational Research, Vol. 204, No.3, pp. 565-580.

Almeida-Dias, J., Figueira, J. *et* Roy, B. (2012) A multiple criteria sorting method where each category is characterized by several reference actions: The Electre Tri-nC method. European Journal of Operational Research, Vol. 217, No. 3, pp. 567-579.

Barro, R. J. *et* Lee, J. W. (2010) A New Data Set of Educational Attainment in The World, 1950–2010, National Bureau of Economic Research Working Paper 15902. Avail. at http://www.nber.org/papers/w15902

Bourguignon, F. and Chakravarty, S. (2003) The Measurement of Multidimensional Poverty. Journal of Economic Inequality, Vol. 1, No. 1, pp. 25–49.

Carley, M. (1981) Social Measurement and Social Indicators: Issues of Policy and Theory. London: G. Allen and Unwin.

Charnes, A., Cooper, W. W. *et* Rhodes, E. (1978) Measuring the efficiency of decisionmaking units. European Journal of Operational Research, Vol. 2, pp. 429-444.

Choquet, G. (1953) Theory of capacities. Annales de l'Institut Fourier, Vol. 5, pp. 131–295.

Fuentes-Nieva, R. *et* Pereira, I. (2010) The Disconnect Between Indicators of Sustainability and Human Development. Human Development Research Paper 34. New York: UNDP–HDRO.

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Volume 12, Número 1, 2015, pp. 136-146
DOI: 10.14488/BJOPM.2015.v12.n1.a13

Kovacevic, M. (2010) Review of Critiques to HDI and Potential Improvements. Human Development Research Paper 33. New York: UNDP–HDRO.

Likert, R. (1932) A technique for the measurement of attitudes. Archives of Psychology, Vol. 140, pp. 1–55.

Mayer-Foulkes, D. (2010) Divergences and Convergences in Human Development. Human Development Research Paper 20. New York: UNDP–HDRO.

Neumayer, E. (2010) Human Development and Sustainability. Human Development Research Paper 5. New York: UNDP–HDRO.

Raworth, K. *et* Stewart, D. (2002) Critiques of the Human Development Index: A Review. In Fukuda-Parr, S. and Shiva-Kumar, A. K. (eds.) Readings in Human Development, Concepts, Measures and Policies for a Development Paradigm. New York: Oxford University Press.

Ranis, G., Stewart, F. *et* Samman, E. (2006) Human Development: Beyond the Human Development Index. Journal of Human Development, Vol. 7, No.3, pp. 323–358.

Sant'Anna, A. P. (2002) Data Envelopment Analysis of Randomized Ranks. Pesquisa Operacional, Vol. 22, No. 2, pp. 203-215.

Sant'Anna, A. P. (2014a) Probabilistic Composition of Preferences, Theory and Applications. Heidelberg: Springer.

Sant'Anna, A. P. (2014b). Alternativas Probabilísticas para a Composição do Indicador de Desenvolvimento Humano. In: Anais do X Congresso Nacional de Excelência em Gestão, p. 1-16.

Sant'Anna, A. P. (2014c) Aplicação do CPP-Tri à Classificação dos Países pelos Critérios do IDH. Anais do Encontro Nacional de Engenharia de Produção, 2014. pp. 1-15.

Sant'Anna, A. P., Dutt-Ross, S. and Ribeiro, R. (2011) Employing the Components of the Human Development Index to Drive Resources to Educational Policies, Social Indicators Research, Vol. 104, No. 3, pp. 523-532.

Sant'Anna, A. P., Costa, H. G. and Pereira, V. (2015) CPP-TRI: a sorting method based on the probabilistic composition of preferences. International Journal of Information and Decision Sciences, in press.

Shapley, L. S. (1953) A Value for n-person Games. In Contributions to the Theory of Games, volume II, by H.W. Kuhn and A.W. Tucker (Eds) Annals of Mathematical Studies, Vol. 28, pp. 307–317 Princeton: Princeton University Press.

WORLD BANK (2015). \PPP conversion factor (GDP) to market exchange rate ratio. Available at http://data.worldbank.org/indicator/PA.NUS.PPPC.RF

Zadeh, L. A. (1965) Fuzzy sets. Information and Control, Vol. 8, pp. 338–353, 1965.

Zadeh, L. (1978) A. Fuzzy sets as the basis for a Theory of Possibility. Fuzzy Sets and Systems, Vol. 1, pp. 3-28.