https://doi.org/10.1051/epjconf/201920401021

Analytical description of hadron production in hadron-hadron and nuclear-nuclear collisions in the mid-rapidity region

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Abstract. It is shown that the inclusive spectra of the produced hadrons in hadron-hadron and nuclear-nuclear collisions can be presented as the universal function dependent of the self-similarity parameter in the analytical form. The article gives a description of the self-similarity parameter depending on the rapidity in the mid-rapidity region. The experimental data are in good agreement with the results of our calculations in a wide energy range from a few GeV to a few TeV in the central rapidity region.

1 Introduction

Almost all theoretical approaches operate the relativistic invariant Mandelstam variables s, t, u to analyze the hadron inclusive spectra in the mid-rapidity region. However, there is another approach to analyze multiple hadron production in hadron-hadron and nuclear-nuclear collisions at high energies, which operates four velocities of the initial and final particles. It is the so called "self-similarity" approach, which demonstrates the similarity of inclusive spectra of the hadrons produced in the hadron-hadron and nuclear-nuclear collisions, as a function of the similarity parameter [1].

In this work we have used the approach based on the law of similarity. For example, when planning large expensive hydraulic structures it is necessary to carry out physical modeling. Geometrically, the body of the model is made similarly to the nature-body [2].

As the main parameters of the problem we take the following: l the characteristic size of the body model, l^0 is the size of the nature body, l^0/l is the coefficient of geometric similarity, and U is the velocity of the impinging flow, μ is the viscosity of the fluid, ρ is the fluid density.

These parameters define the system of units: L - length, M - mass, T - time, and have the following dimensions:

$$[l] = L, [U] = L \cdot T^{-1}, [\mu] = M \cdot L^{-1}T, [\rho] = M \cdot L^{-3}.$$

From the defining parameters we can construct only one dynamic similarity parameter (a dimensionless combination, independent of the choice of measuring units):

$$\Pi = \rho Ul/\mu = Re.$$

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This invariant is called the Reynolds number. To provide the similarity, it is required to have equality of this parameter for the model and nature.

Let us briefly present here the main idea of this study. Consider, for example, the production of hadrons 1, 2, etc. in the collision of nucleus A with nucleus B:

$$A + B \to 1 + 2 + \ldots \tag{1}$$

According to this assumption more than one nucleon in the nucleus A can participate in the interaction. The value of N_A is the efficient number of nucleons inside the nucleus A, participating in the interaction which is called the cumulative number.

Its values lie in the region of $0 \leq N_A \leq A_A$ (A_A - atomic number of nucleus A). The cumulative area complies with $N_A > 1$.

Of course, the same situation will be for the nucleus B, and one can enter the cumulative number of N_B.

For the reaction with the production of the inclusive particle 1

$$A + B \to 1 + \ldots \tag{2}$$

we can write the conservation law of four-momentum in the following form:

$$(N_A P_A + N_B P_B - p_1)^2 = (N_A m_0 + N_B m_0 + M)^2, \tag{3}$$

where N_A and N_B – the number of the nucleons involved in the interaction or the fraction of four momenta transmitted by the nucleus A and nucleus B; P_A, P_B, p_1 are four momenta of the nuclei A and B and particle 1, respectively; m_0 is the mass of the nucleon; M is the mass of the particle providing the conservation of the baryon number, strangeness, and other quantum numbers.

For π mesons $m_1 = m_\pi$ and $M = 0$.

For antinuclei and K^- mesons $M = m_1$.

For nuclear fragments $M = -m_1$.

For K^+ mesons $m_1 = m_K$ and $M = m_\Lambda - m_K$, m_Λ is the mass of the Λ baryon.

In [2] the parameter of self-similarity is introduced, which allows one to describe the differential cross section of the yield of a large class of particles in relativistic nuclear collisions:

$$\Pi = \min 1/2 \cdot (u_A N_A + u_B N_B)^{1/2}, \tag{4}$$

where u_A and u_B are four velocities of the nuclei A and B.

Then the inclusive spectrum of the produced particle 1 in AA collision can be presented as the universal function dependent of the self-similarity parameter:

$$E \cdot d^3\sigma/dp^3 = C_1 A_A^{\alpha(N_A)} \cdot A_B^{\alpha(N_B)} \cdot \exp(-\Pi/C_2), \tag{5}$$

where $\alpha(N_A) = 1/3 + N_A/3$, $\alpha(N_B) = 1/3 + N_B/3$, $C_1 = 1.9 \cdot 10^4$ mb $\cdot$ GeV$^{-2} \cdot c^3 \cdot$ st^{-1} and $C_2 = 0.125 \pm 0.002$.

2 Analytical solution for self-similarity parameter

An analytical solution for the self-similarity parameter Π was found in [3]. Here we give a more detailed obtaining of the parameter and consider its behavior at small values of particle 1 rapidity. Equation (3) can be written as follows:

$$N_A \cdot N_B - \Phi_A \cdot N_A - \Phi_B \cdot N_B = \Phi_M, \tag{6}$$

where relativistic invariant dimensionless values have been introduced:

$$\Phi_A = [(m_1/m_0) \cdot (u_A u_1) + M/m_0]/[(u_A u_B) - 1],$$

$$\Phi_B = [(m_1/m_0) \cdot (u_B u_1) + M/m_0]/[(u_A u_B) - 1],$$

$$\Phi_M = (M^2 - m_1^2)/[2m_0^2((u_A u_B) - 1)].$$

Equation (6) can be written as follows:

$$[(N_A/\Phi_B) - 1] \cdot [(N_B/\Phi_A) - 1] = 1 + [\Phi_M/(\Phi_A \cdot \Phi_B)]. \tag{7}$$

Minimum Π is found from the following:

$$d\Pi/dN_A = 0, \qquad d\Pi/dN_B = 0. \tag{8}$$

Let us introduce the intermediate variables:

$$F_A = [(N_A/\Phi_B) - 1], \qquad F_B = [(N_B/\Phi_A) - 1].$$

From the above we obtain: $F_A \cdot F_B = 1 + \Phi_M/(\Phi_A \cdot \Phi_B)$.
Then (8) is also equal to 0 as

$$d\Pi/dF_A = 0, \quad d\Pi/dF_B = 0.$$

From (4) we can obtain:

$$4\Pi^2 = N_A^2 + N_B^2 + 2N_A \cdot N_B \cdot (u_A u_B),$$

$$4\Pi^2 = (F_A + 1)^2\Phi_B^2 + (F_B + 1)^2\Phi_A^2 + 2\Phi_A \cdot \Phi_B(F_A + 1) \cdot (F_B + 1) \cdot (u_A u_B),$$

$$F_B = \alpha/F_A.$$

The condition of the minimum $d(4\Pi^2)/dF_A = 0$ gives the equation for F_A:

$$F_A^4 + F_A^3 - (\Phi_A/\Phi_B)^2 \cdot (\alpha^2 + \alpha F_A) + (u_A u_B) \cdot (\Phi_A/\Phi_B) \cdot (F_A^3 - \alpha F_A) = 0,$$

or

$$F_A^4 + F_A^3[1 + (u_A u_B)/z] - (\alpha/z) \cdot F_A \cdot [(u_A u_B) + (1/z)] - \alpha^2 z^2 = 0,$$

where $z = \Phi_B/\Phi_A$.
When changing A to B: $z \to (1/z)$, $F_1 \to (\alpha/F_B)$,

$$(\alpha/F_B)^4 + (\alpha/F_B)^3[1 + (u_A u_B) \cdot z] - \alpha z(\alpha/F_B)[(u_A u_B) + z] - \alpha^2 z^2 = 0,$$

or

$$F_B^4 + F_B^3[1 + (u_A u_B) \cdot z] - z\alpha \cdot F_B \cdot [z + (u_A u_B)] - \alpha^2 z^2 = 0.$$

Thus, at $z = 1 \to F_A = F_B$, $\quad \Phi_A = \Phi_B = \Phi$.
But since $F_A = F_B$, then $(N_A/\Phi - 1) = (N_B/\Phi - 1)$ and $N_A = N_B$,

$$F^2 = \alpha, \qquad F_A = F_B = \alpha^{1/2} = [1 + (\Phi_M/\Phi^2)]^{1/2},$$

$$N_A = N_B = N = (1 + F) \cdot \Phi = 1 + [1 + (\Phi_M/\Phi^2)]^{1/2} \cdot \Phi,$$

$$\Pi = 1/2[2N^2 + 2N^2(u_A u_B)]^{1/2} = (N/\sqrt{2})[1 + (u_A u_B)]^{1/2} = N \cdot chY.$$

Let us express the scalar product of four-dimensional velocities using the rapidity:

$$(u_A u_B) = ch2Y,$$
$$(u_A u_1) = (m_{1t}/m_1) \cdot ch(-Y - y) = (m_{1t}/m_1) \cdot ch(Y + y),$$
$$(u_B u_1) = (m_{1t}/m_1) \cdot ch(Y - y).$$

Here m_{1t} is the transverse mass of the particle 1, $m_{1t} = (m_1^2 + p_{1t}^2)^{1/2}$, Y - rapidity of interacting nuclei, y - rapidity particle 1.

If $y \ll 1$ it is possible to decompose hyperbolic functions into a series and write an approximate expression for the parameter Π,

$$(u_A u_1) = (m_{1t}/m_1) \cdot ch(-Y - y) = (m_{1t}/m_1) \cdot ch(Y + y),$$
$$(u_B u_1) = (m_{1t}/m_1) \cdot ch(Y - y),$$
$$ch(Y + y) = chY \cdot chy + shY \cdot shy,$$
$$e^y \approx 1 + y + y^2/2,$$
$$e^{-y} \approx 1 - y + y^2/2,$$
$$shy = 1/2(e^y - e^{-y}) \approx 1/2(1 + y + y^2/2 - 1 + y - y^2/2) = y,$$
$$chy = 1/2(e^y + e^{-y}) \approx 1/2(1 + y + y^2/2 + 1 - y + y^2/2) = 1 + y^2,$$

$$(u_A u_1) = (m_{1t}/m_1) \cdot ch(Y + y) = (m_{1t}/m_1) \cdot (chY \cdot chy + shY \cdot shy) \approx$$
$$(m_{1t}/m_1) \cdot [(1 + y^2) \cdot chY + y \cdot shY] \approx (m_{1t}/m_1) \cdot (1 + y^2) \cdot chY,$$
$$(u_B u_1) = (m_{1t}/m_1) \cdot ch(Y - y) = (m_{1t}/m_1) \cdot (chY \cdot chy - shY \cdot shy) \approx$$
$$(m_{1t}/m_1) \cdot [(1 + y^2) \cdot chY - y \cdot shY] \approx (m_{1t}/m_1) \cdot (1 + y^2) \cdot chY.$$

Thus,

$$(u_A u_1) \approx (u_B u_1) \approx (m_{1t}/m_1) \cdot (1 + y^2) \cdot chY.$$

And in this case:

$$\Phi = \Phi_A = \Phi_B = [(m_1/m_0) \cdot (u_A u_1) + M/m_0]/[(u_A u_B) - 1] \approx$$
$$[(m_1/m_0) \cdot (m_{1t}/m_1) \cdot (1 + y^2) \cdot chY + M/m_0]/[ch2Y - 1] =$$
$$(1/m_0)[m_{1t} \cdot (1 + y^2) \cdot chY + M] \cdot [1/(2sh^2 Y)],$$
$$\Phi_M = (M^2 - m_1^2)/(4m_0^2 sh^2 Y).$$

Thus at $y \ll 1$

$$N = 1 + [1 + (\Phi_M/\Phi^2)]^{1/2}\Phi,$$

where

$$\Phi \approx (1/m_0)[m_{1t} \cdot (1 + y^2) \cdot chY + M] \cdot [1/(2sh^2 Y)],$$
$$\Phi_M = (M^2 - m_1^2)/(4m_0^2 sh^2 Y).$$

The result of our calculations using this formula for Au + Au collisions at $s^{1/2} = 2.42$ GeV is shown in Fig. 1. The prediction of the yield dependence of pions in Au + Au interactions on their rapidity is given.

Since this equation $(u_A u_1)$ does not depend on m_A, it is valid for any hadrons and nuclei:

$$(u_A u_1) = (P_A/m_A)(P_1/m_1) = E_A \cdot E_1/m_A \cdot m_1 - \vec{p_A} \cdot \vec{p_1}/m_A \cdot m_1 =$$
$$m_A \cdot chY \cdot m_{1t} \cdot chy/m_A \cdot m_1 + m_A \cdot shY \cdot m_{1t} \cdot shy/m_A \cdot m_1 =$$
$$(m_{1t}/m_1) \cdot (chY \cdot chy + shY \cdot shy) = (m_{1t}/m_1) \cdot ch(Y + y).$$

Therefore, we conclude that our approach is also valid for projectile π mesons.

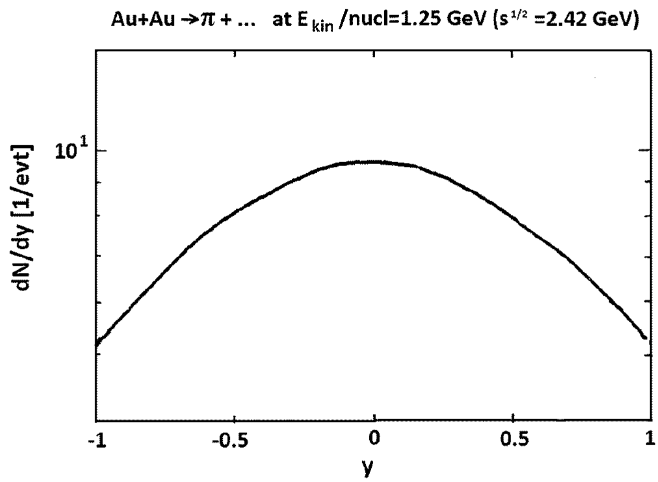

Figure 1. Distribution of secondary pions in Au + Au collisions as function of the pion rapidity

3 Self-similarity parameter in the central rapidity region

In the mid-rapidity region ($y = 0$, y is the rapidity of particle 1) the analytical form for Π was found in [3].

In this case N_A and N_B are equal to each other: $N_A = N_B = N$.

$$N = [1 + (1 + \Phi_M/\Phi^2)^{1/2}]\Phi, \tag{9}$$

where

$$\Phi = (m_{1t} chY + M)/(2m_0 sh^2 Y), \tag{10}$$

$$\Phi_M = (M^2 - m_1^2)/(4m_0^2 \cdot sh^2 Y). \tag{11}$$

Here m_{1t} is the transverse mass of the particle 1, $m_{1t} = (m_1^2 + p^2)^{1/2}$, Y - rapidity of interacting nuclei. And then

$$\Pi = N \cdot chY. \tag{12}$$

For baryons we have the following:

$$\Pi_b = (m_{1t} chY - m_1) chY/(m_0 sh^2 Y),$$

and for antibaryons –

$$\Pi_a = (m_{1t} chY + m_1) chY/(m_0 sh^2 Y).$$

The results of calculations for the ratio of the antiproton cross section to the proton one after integration of over dm_{1t} are in good agreement with the experimental data [4, 5].

Taking into account the quark and gluon contributions we will obtain the following expression for the inclusive cross-section of hadron production in the central rapidity region [5]:

$$E(d^3\sigma/dp^3) = [\phi_q(y = 0, p_t) + \phi_g(y = 0, p_t) \cdot (1 - \sigma_{nd}/g(s/s_0)^{\Delta})] \cdot g \cdot (s/s_0)^{\Delta}. \tag{13}$$

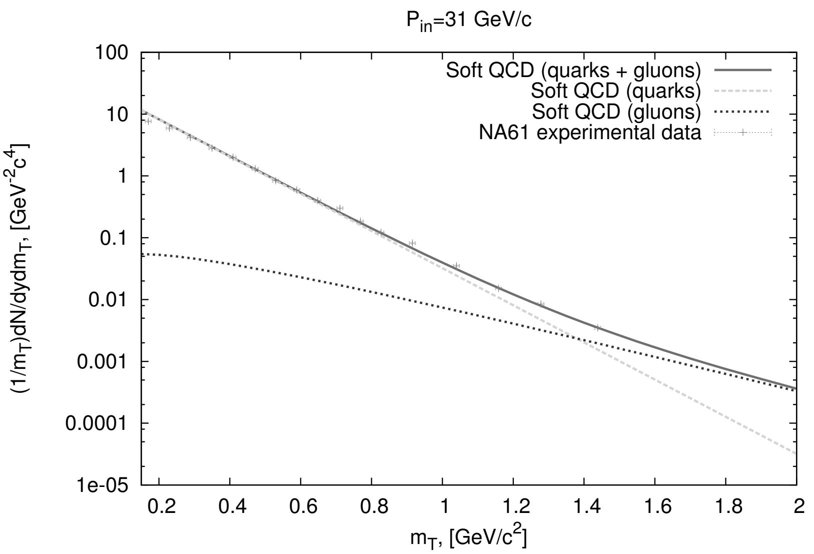

Figure 2. Results of the calculations of the inclusive cross-section of hadron production in pp collisions as function of transverse mass at the initial momenta P_{in} = 31 GeV/c. They are compared to the NA61 experimental data from [6]

In formula (13) we apply the following symbols: σ_n - cross-section of hadron production by means of the n-pomeron exchange; $\phi = \phi(\Pi)$; g - constant ($\sim$ 20 mbarn), which is calculated within the "quasi-eikonal" approximation; S_0 = 1 GeV^2; $\Delta = [\alpha_p(0)-1] \sim 0.08$, where $\alpha_p(0)$ is the subcritical pomeron intercept.

The first part of the inclusive spectrum (Soft QCD (quarks)) is related to the function $\phi_q(y = 0, \Pi)$, which is fitted by the following form:

$$\phi_q(y = 0, \Pi) = A_q \cdot \exp(-\Pi/C_q), \tag{14}$$

where A_q = 3.68 (GeV/c)^2, C_q = 0.147.

The function $\phi_q(y = 0, \Pi)$ related to the second part (Soft QCD (gluons)) of the spectrum is fitted by the following form:

$$\phi_q(y = 0, \Pi) = A_g m_{1t} \cdot \exp(-\Pi/C_g), \tag{15}$$

where A_g = 1.7249 (GeV/c)^2, C_g = 0.289. Using (13) we can calculate the inclusive cross section of hadron production as a function of the transverse mass.

For example, in Fig. 2 the calculated inclusive spectrum $(1/m_{1t}) \cdot d\sigma/(dm_{1t}dy)$ of π^- mesons produced in pp collisions at the initial momentum P_{in} = 31 GeV/c is presented versus their transverse mass m_T. It is possible to see a good agreement between the results of our calculations and experimental data of NA61 experiment [6].

In Fig. 3 we present the calculations of inclusive spectra of charged hadrons (mainly pions and kaons) produced in pp collision at $\sqrt{s}$ = 7 TeV performed by using (13) and the perturbative QCD (PQCD) within the leading order (LO) compared to the LHC data. It is possible to see a good agreement between the results of our calculations and experimental data at $p_t < 2$ GeV/c. More detailed information on the application of our method for spectrum calculations was given in [8] and in the report at the seminar [9].

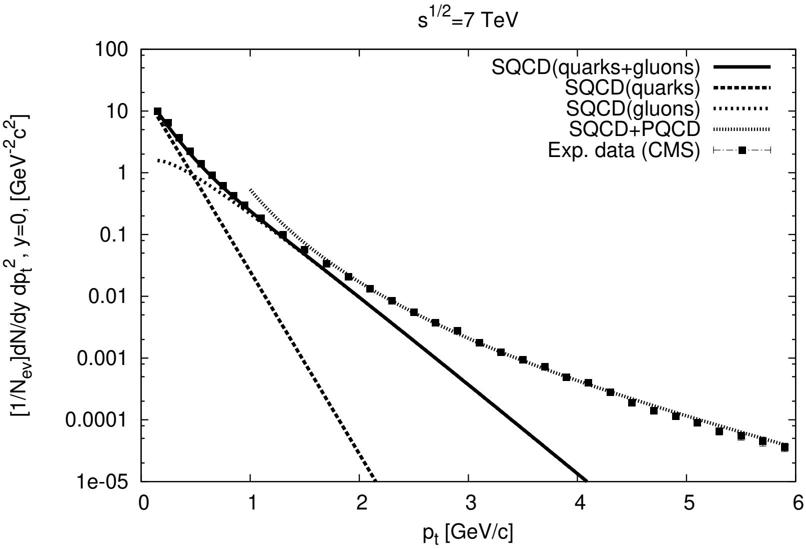

Figure 3. Results of the calculations of the inclusive cross-section of charge hadrons produced in pp collisions at the LHC energies as a function of their transverse momentum p_t at $\sqrt{s}$ = 7 TeV. The points are the LHC experimental data [7]

4 Conclusion

The approach described in this paper has shown the following:

- the inclusive spectra of the produced hadrons in hadron-hadron and nuclear-nuclear collisions can be presented as the universal function dependent of the self-similarity parameter in the analytical form;

- the experimental data are in good agreement with the results of our calculations in a wide energy range from a few GeV to a few TeV in the central rapidity region;

- the use of the self-similarity approach allows us to describe the ratio of the total yields of the particles to antiparticles produced in AA collisions as a function of the energy in the mid-rapidity region and a wide energy range.

In this article we have described the self-similarity parameter depending on the rapidity in the mid-rapidity region.

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