Critical analysis of recent results on electron and positron elastic scattering on proton

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Abstract. Recent data on the cross section ratio for electron and positron elastic scattering on protons are discussed. A deviation from unity of this ratio constitutes a model independent signature of charge-odd contributions that arise from mechanisms beyond the one-photon approximation, as the exchange of two photons. The relevance of this issue is related to that fact that the information on the proton structure from lepton elastic (and inelastic) scattering holds only within a formalism based on the one photon exchange approximation. The present analysis shows that the deviations of the data from unity and from a previously developed theoretical approach lie within the theoretical and experimental errors. The data suggest that other reasons for explanation of the discrepancy between the electromagnetic proton form factors extracted from the experiments according to the polarized and the unpolarized methods are more likely.

1 Introduction

Three dedicated experiments were recently performed to search for two photon exchange ($2\gamma E$) contribution in elastic electron and positron scattering on the proton. The motivation was given by the discrepancy existing in electromagnetic (EM) form factors (FFs) of the proton extracted from the Akhiezer-Rekalo polarization method [1, 2], as compared to the Rosenbluth separation [3] based on the measurement of the unpolarized cross section at different angles for a fixed value of the transferred momentum square, Q^2.

The unpolarized cross section depends on the square of FFs. The magnetic contribution being enhanced by a factor of $\tau = Q^2/4M^2$ (M being the proton mass), this method is limited by the precision on the extraction of the electric FF at large Q^2.

The polarization method is based on the fact that the polarization transferred from a longitudinally polarized electron beam to a polarized proton target (or the measurement of the polarization of the recoil proton) in elastic electron proton scattering contains a term of interference between the electric and magnetic amplitudes, being more sensitive to a small electric contribution, and also to its sign. The FF ratio, G_E/G_M, is proportional to the ratio of the longitudinal to transverse polarization of the recoil proton, P_L/P_T and was systematically measured by the GEP collaboration at JLab ([4] and References therein) up to 9 GeV^2 transferred momentum. As it requires beams with high duty cycle and high polarization, large

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solid angle spectrometers and detectors as well as proton polarimeters in the GeV region, this method could be applied first in the years 2000. Meaningful data were collected up to a transferred four momentum square $Q^2 \simeq 9$ GeV^2.

The data show that, not only the precision is larger as expected, but also that the ratio, normalized to the proton magnetic moment, decreases when Q^2 increases, deviates from unity, as previously commonly accepted and,extrapolating the trend, might also cross zero and become negative.

The discrepancy between polarized and unpolarized elastic scattering experiments was scrutinized from the data of [5]. This work extends the determination of the individual FFs to the largest values of Q^2. A recent dedicated experiment at JLab [6] and reanalysis of data based on the Rosenbluth method concluded from that the Q^2 dependence of the electric and magnetic FFs was similar and well described by a dipole dependence. This result was generally accepted as consistent with QCD counting rules.

The results based on the Akhiezer-Rekalo method raised many questions on the proton structure - stimulating a large number of theoretical papers, on the used method and underlying assumptions.The presence of $2\gamma E$ was proposed as a solution of the discrepancy for the proton FFs. Indeed, such contribution induces a more complicated structure of the reaction amplitudes, therefore a larger flexibility in describing the data.

The two photon contribution was widely discussed in the literature in the 70's [7–10], and its experimental evidence was searched for. No conclusive result was found, in the limit of the experimental precision. Since that time, the $1\gamma E$ approximation was assumed *a priori*. The two (n)- photon contribution may be observed only if other mechanisms compensate the factor $Z\alpha$ ($(Z\alpha)^n$) that scales the size of the amplitude. One reason for which $2\gamma E$ may become important at large transferred momentum is that, if the transferred momentum is equally shared between the two virtual photons, the steep decreasing of FFs (calculated for $Q^2/2$) partially compensates the α-counting rule. In this context, it is expected that $2\gamma E$ becomes more important when Q^2 increases and/or when the charge Z of the target increases. Moreover, as the $2\gamma E$ amplitude contains in principle, an imaginary part, it could give some contribution to the cross section in the time-like region, TL region, due to the interference with the imaginary part of the form factors [11, 12].

The presence of a sizable $2\gamma E$ contribution was recently suggested to explain discrepancies between two experiments on elastic electron-deuteron scattering at Jefferson Laboratory (JLab) [13, 14] [15]. The differences in the cross sections, at similar Q^2 values, but at different incident beam energies and electron scattering angles, were not increasing with Q^2, pointing instead to a systematic shift of the hall C spectrometer position (a shift of 0.3° of the central angle was indeed found).

Several model calculations of the hadronic $2\gamma E$ contribution in ep elastic scattering attempted to reconcile the FFs measurements, with quantitatively different results, since the assumptions and the physical reasons for an enhancement of this term differ essentially from one model to another [16–21]. Some of the suggested parametrization are based on the ansatz that the $2\gamma E$ amplitude would be real and linear in the ϵ variable (where $\epsilon = [1 + 2(1 + \tau)\tan^2(\theta/2)]^{-1}$ is the linear polarization of the virtual photon). Let us note that these hypotheses contradict the nature of $2\gamma E$. Indeed, model independent statements based on symmetry properties of the strong and electromagnetic interactions [22–24] can be made on the $2\gamma E$ contribution to the observables in ep elastic scattering:

- the matrix element in the one-photon exchange approximation is fully defined by two real (electric and magnetic) FFs, functions only of Q^2, whereas, for $2\gamma E$, three new amplitudes are present. They are complex functions of two variables (E, θ) or (Q^2, ϵ), where E (θ) is the energy (angle) of the scattered electron in the laboratory (Lab) system. Analyzing the spin structure of $2\gamma E$ amplitude, we may write these functions as two real FFs depending only on Q^2 and three more complex functions (of the order of α), that contain only the terms arising from the $2\gamma E$ contribution;

- nonlinearities arise in the Rosenbluth fit, i.e., in the unpolarized (reduced) cross section versus ϵ at fixed Q^2;
- due to the charge-odd (C-odd) terms, a non vanishing charge–asymmetry should be observed in the cross section for $e^{\pm}p$ scattering, as well as parity-odd polarization observables, as a vector polarization of the outgoing proton normal to the scattering plane, in the scattering with longitudinally polarized electrons.

A reanalysis of the Rosenbluth data in terms of the squared FF ratio R^2 has been proposed in [25]. A problem of renormalization of the low ϵ points was pointed out in the former analysis of Rosenbluth data, in particular in [5]. A correction of the low ϵ points changes dramatically the slope of the Rosenbluth fits, and therefore the FF ratio. In general, the discrepancy between unpolarized and polarized experiments has been critically reviewed. In spite of the large errors affecting G_E, some of the data show indeed a decrease of the ratio, already reported in the literature [25]. Up to 3-4 GeV^2 in some cases, the difference may be resolved by a proper calculation of radiative corrections [26, 27]. Large correlations between the parameters of the Rosenbluth fit, when radiative corrections are especially sizable, were pointed out in [28].

Both methods, the recoil polarization method from Akhiezer-Rekalo and the unpolarized cross section from Rosenbluth, assume the one photon exchange approximation. An exact calculation of $2\gamma E$ for ep scattering is not possible, as it requires the knowledge of the Q^2 dependence of the properties of the excited proton intermediate state. An exact calculation of the box diagram can be done in QED, putting an upper constrain on ep scattering [29]. Moreover, in [30], based on sum rules developed in QED, it was shown that the 'elastic' and inelastic contributions are mutually canceled, and that only the point-like $2\gamma E$ should be taken into account in calculating the hard $2\gamma E$ contribution. This is also in agreement with some model calculations that find corrections with opposite signs for elastic nucleon and Δ or $N*(1535)$ excitation [31, 32].

Precise measurements were proposed at VEPP-3, (Novosibirsk) [33, 34], at CLAS (JLab) [35], and at Olympus (DESY) [36], stimulated by calculations predicting a sizable effect (up to 10% at small ϵ for Q^2=2.64 GeV^2 [37]) for the ratio of the cross sections for electron and positron elastic scattering on the proton already at low Q^2. The final results of these experiments have now been published and the purpose of this contribution is to discuss them at the light of the radiative correction issue, at the light of the calculation of [38] and updating the study of [39].

2 Scrutinizing the experimental results

The VEPP experiment, was performed at the VEPP-3 storage ring, Novosibirsk [33, 34]. The $e^{\pm}p$ cross section was measured for two beam energies, 1.6 and 1 GeV and different lepton scattering angles, spanning such ϵ, Q^2 kinematical ranges: $0.272 < \epsilon < 0.932$, and $0.298 < Q^2 < 1.0332$ GeV/c^2. A dedicated calculation of first order radiative corrections was developed and implemented [40].

The OLYMPUS experiment, was performed at the DORIS storage ring at DESY, using 2.01 GeV electron and positron beams impinging on an internal hydrogen gas target [36]. Twenty values of the ratio R were measured in the range: $0.456 < \epsilon < 0.978$, and $0.165 < Q^2 < 2.038$ GeV/c^2. Most of these values lie within $|R| < 1.02$ with a mild tendency to increase at large Q^2 and/or small ϵ. Four options of radiative corrections were implemented, following Mo-Tsai [41], or Maximon-Tjon [42] at first order or including high orders by exponentiation. The difference among these options induces at most a difference of 1.5 % in the extracted ratios.The statistical error is evaluated to be < 1 % and the systematical error is < 1.5 %, the largest source being attributed to the selection of the elastic events.

The CLAS experiment [35] published a list of 19 points, for two Q^2 values, 0.85 and 1.45 GeV^2 and several ϵ values in the range $0.39 < \epsilon < 0.91$. The electron and positron beams where produced by converting a photon beam into $e^\pm$ pairs, which explains partly the largest uncertainty of these data compared to the two previous experiments. Overlapping kinematics reduce the set to 12 independent data points, for comparison with the other data and with the calculations. The data were radiatively corrected following [43], which is a first order calculation developed to be implemented in Monte Carlo programs for inelastic scattering. It is based on similar approximations as [41].

Assuming one photon exchange, the unpolarized elastic cross section $d\sigma_{el}$ for lepton-hadron elastic scattering in the Born approximation can be expressed in terms of two structure functions, A and B, which depend on the momentum squared of the transferred photon, Q^2, only:

$$d\sigma_{el}(e^\pm h \to e^\pm h) = d\sigma_{Mott}\left[A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2}\right], \tag{1}$$

where $d\sigma_{Mott}$ is the cross section for point-like particles. This is a very general expressions that holds for any hadron of spin S, in the one photon exchange approximation. The two structure functions, at their turn, depend on $2S+1$ electromagnetic form factors. In the Born approximation, the elastic cross section is identical for positrons and electrons.

The elastic cross section is the measured cross section, $d\sigma_{meas}$ after applying corrections that take into account photon radiation from the charged particles, $\delta^\pm$. More precisely:

$$d\sigma^\pm_{meas} = d\sigma_{el}(1+\delta^\pm), \quad d\sigma_{el} = \frac{d\sigma^\pm_{meas}}{(1+\delta^\pm)}, \tag{2}$$

where $\delta^\pm$, contains charge-even and charge-odd terms (that change sign for $e^\pm$ beams). The sign + (−) stands for positrons (electrons)): $\delta^\pm = \mp\delta_{odd} + \delta_{even}$. One can write the odd term δ_{odd} as the sum of a "hard" (2γ) and a "soft" (s) contributions:

$$\delta_{odd} = \delta_{2\gamma} + \delta_s. \tag{3}$$

In the experimental data only δ_s was included in radiative corrections, although the splitting (3) may be different. Different approximations are implemented and affect these corrections, see [40–44]. As an example we illustrate the difference of $\delta^\pm$ from some first order calculations in Fig. 1, for electron (left) and for positron (right) scattering, as a function of ϵ for Q^2=1 GeV^2. The soft corrections depend from the inelasticity parameter ΔE taken here as 1% of the scattered energy, E'. The difference among the calculations is of the order of few percent, depending on the kinematics. Note that a larger value, $\Delta E \simeq 0.03E'$, is closer to the typical experimental cut, but a smaller value enhances the effect and is taken here for illustration. Fig. 1 shows the effects of the order of percent deriving from the procedures of applying radiative corrections to the data.

A deviation from unity of the ratio:

$$R^{meas} = \frac{d\sigma^{meas}(e^+p \to e^+p)}{d\sigma^{meas}(e^-p \to e^-p)} = \frac{1+\delta_{even} - \delta_{2\gamma} - \delta_s}{1+\delta_{even} + \delta_{2\gamma} + \delta_s} \tag{4}$$

is a clear signature of (soft and hard) charge-odd contributions to the cross section.

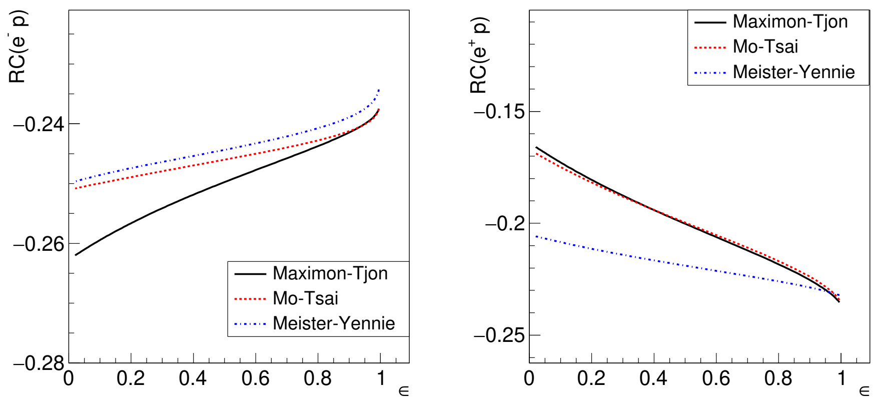

Figure 1. Radiative correction factor as a function of ϵ for e^-p (left) and e^+p (right), from [42] (solid black line), [41] (dashed red line), [44] (dash-dotted blue line) for Q^2=1 GeV^2 and $\Delta E = 0.01E'$

A C-odd effect is enhanced in the ratio of $e^+p \to e^+p$ over $e^-p \to e^-p$ cross sections, R, with respect to the asymmetry, A^{odd}:

$$A^{odd} = \frac{d\sigma(e^+p \to e^+p) - d\sigma(e^-p \to e^-p)}{d\sigma(e^+p \to e^+p) + d\sigma(e^-p \to e^-p)} = \frac{\delta_{odd}}{1+\delta_{even}} = \frac{R-1}{R+1}, \quad R = \frac{1+A_{odd}}{1-A_{odd}}. \tag{5}$$

By correcting the data for the well-known and assumption-free contributions of the vertex-type corrections δ_{even} and soft contributions δ_s, R^{meas} from Eq. 4 reduces to

$$R_{2\gamma} \simeq \frac{1-\delta_{2\gamma}}{1+\delta_{2\gamma}}, \tag{6}$$

where $\delta_{2\gamma}$ is the contribution of hard virtual two-photon exchange.

In the analysis of the experimental data, the radiative correction codes are embedded in the Monte Carlo used to analyze the data, and it is not straightforward to unfold the effects from the acceptance and the efficiency of the setup. Note that the ΔE term is by far the most sizable among the odd terms, becoming larger when the inelasticity cut is smaller. At the elastic peak it becomes infinite.

The ratio $R_{2\gamma}$ is shown in Fig. 2 as a function of ϵ (left) and Q^2 (right), with the corresponding linear fit. Most of the data deviate from unity by less than 2%. A slight increase with decreasing ϵ is seen. The authors of the VEPP experiment, [33], point out a significant $2\gamma E$ effect increasing with Q^2, but this is not confirmed by the OLYMPUS data. Unfortunately the VEPP results, that are the most precise, lack an absolute normalization.

The weighted average of the ratio $R_{2\gamma}$ for all data and for the individual data set, to be compared to unity for no $2\gamma E$ contribution, is shown in Tab. 1. The compatibility with a constant $R_{2\gamma} = 1$ is indicated by $\chi^2/N(1)$. One may see that a deviation of about 3σ is visible for the VEPP data, where χ^2/N is much larger than 1. The average for this data set is larger than unity, whereas it is smaller for the CLAS and OLYMPUS data. Adding a parameter decreases the χ^2/N, that falls below unity for these two last sets of data. The results of the linear fit are also reported in Tab. 1. The fact that $\chi^2/N \simeq 2.0$ for all data sets is much larger than for each individual set shows the large difference between the VEPP data compared to the two others data sets.

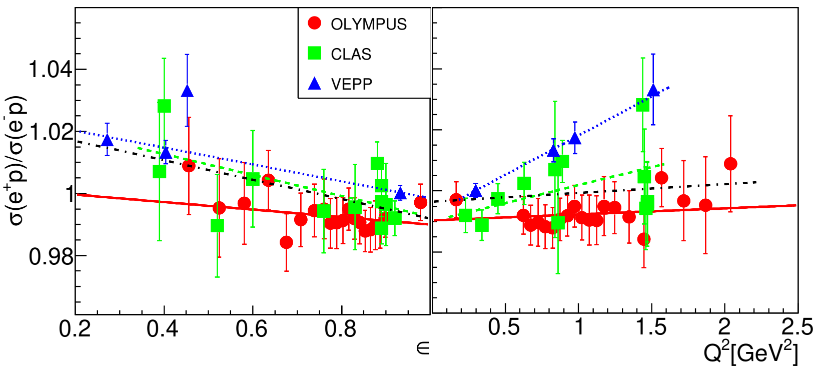

Figure 2. Radiatively corrected ratio of positron to electron cross sections $R_{2\gamma} = \sigma(e^+p)/\sigma(e^-p)$ with the corresponding linear fits as a function of ϵ (left) and Q^2 (right) from OLYMPUS [36] (red circles and red solid line), CLAS [35] (green squares and green dashed line) and VEPP-3 [33] (blue triangles and blue dotted line). The black dash-dotted line corresponds to the global linear fit

Table 1. Weighted average of the ratio $R_{2\gamma}$ for all data and for the individual data sets (the OLYMPUS data corresponding to the set (a) of [36]), to be compared to unity for no $2\gamma e$ contribution. The compatibility with a constant $R_{2\gamma} = 1$ is indicated by χ^2 (1). The results from linear fits in ϵ and Q^2 are also given

		All data	OLYMPUS	CLAS	VEPP
Experiment					
$< R_{2\gamma} >$		0.999± 0.001	0.999± 0.001	0.997 ± 0.002	1.006± 0.002
$\chi^2/N(1)$		69.3/35=1.98	19/19=1.00	12.1/11=1.1	23.7/3=7.9
$R_{2\gamma} = a_0 + a_1\epsilon$	a_0	1.023± 0.005	1.002± 0.014	1.026± 0.018	1.026 ± 0.005
	a_1	-0.031± 0.006	-0.012± 0.017	-0.034± 0.020	-0.027 ± 0.007
χ^2/N		38.6/34= 1.13	5.44/18= 0.3	9.72/10 =0.97	3.08/2 =1.5
$R_{2\gamma} = b_0 + b_1 Q^2$	b_0	0.981±0.004	0.990± 0.005	0.990± 0.004	0.992 ± 0.004
	b_1	0.014± 0.003	0.002± 0.005	0.011± 0.006	0.026 ± 0.006
χ^2/N		68.3/34= 2.0	5.74/18=0.32	8.4/10=0.8	0.05/2=0.02

The asymmetry including soft and hard 2γ contributions at first order in α as calculated in [38], A^K_{odd} is:

$$A^K_{odd} = \frac{d\sigma^{e^+p} - d\sigma^{e^-p}}{d\sigma^{e^+p} + d\sigma^{e^-p}} = \frac{2\alpha}{\pi(1+\delta_{even})}\left[\ln\frac{1}{\rho}\ln\frac{(2\Delta E)^2}{ME} - \frac{5}{2}\ln^2\rho + \ln x\ln\rho + \mathrm{Li}_2\left(1-\frac{1}{\rho x}\right) - \mathrm{Li}_2\left(1-\frac{\rho}{x}\right)\right]. \quad (7)$$

$$\rho = \left(1 - \frac{Q^2}{s}\right)^{-1} = 1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}, \ x = \frac{\sqrt{1+\tau}+\sqrt{\tau}}{\sqrt{1+\tau}-\sqrt{\tau}}.$$

It was compared to the data existing at that time in [39]. The results were given for an inelasticity cut $\Delta E/E = 0.03$, that is consistent with most experiments. Let us stress however that our result for the hard 2γ contribution does not depend on this term and on the cut.

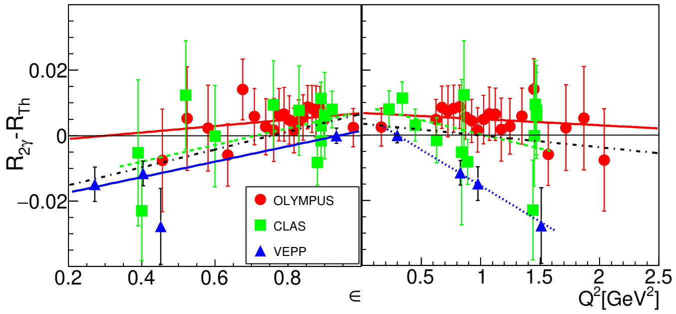

Figure 3. Point to point difference between the calculation from Eq. (7), [38] and the data for the ratio R, with the corresponding linear fits as a function of ϵ (left) and Q^2 (right) from OLYMPUS [36] (red circles and red solid line), CLAS [35] (green squares and green dashed line) and VEPP-3 [33] (blue triangles and blue dotted line), after removing the odd corrections as in Eq. (8) with δ_M from [42]. The black dash-dotted line corresponds to the global linear fit

The term containing ΔE gives the largest contribution to the asymmetry and has a large ϵ dependence.

As radiative corrections applied to the data may differ from one paper to another by some finite expression (which depends on kinematical invariants), in order to be less sensitive to model corrections, we consider the total odd contribution from [38] and remove the odd correction from the calculations used in the data. This means that we have to proceed from R^{meas} to $R_{2\gamma}$:

$$R^K_{2\gamma} = \frac{1 - A^K_{odd}(1 + \delta_{even}) + \delta_M}{1 + A^K_{odd}(1 + \delta_{even}) - \delta_M}, \tag{8}$$

where δ_M can be calculated from [41], or from the corresponding correction of [42].

We calculate the asymmetry A^K_{odd} from Eq. (7), then the ratio $R^K_{2\gamma}$ from Eq. (8) to be compared to the data. The ratio depends on two variables, Q^2 and ϵ. First we study the Q^2 and ϵ dependence separately, then, in order to have all the data and the calculation in a plot, we consider the absolute difference between each data point and Eq. (7), calculated for the corresponding values of the two variables, Q^2 and ϵ.

The difference point by point from the experimental value and the calculation is considered here. The calculation from Eq. (7), [38] is plotted after removing the odd corrections as in Eq. (8) with δ_M from [42] (Fig. 3) or from [41] as illustrated in Fig. 4.

The point by point difference between data and calculation shows in most cases a difference below 1%, what is beyond the theoretical and experimental precisions, with a slight ϵ and Q^2 dependencies and it is consistent with zero within the errors. A Q^2-dependent discrepancy appears in the data from VEPP, but they have a different sign than the other experiments. The average ratio is compatible with one, within the error, except for the VEPP data. The linear fit finds a mild positive slope for the CLAS and VEPP data, and an intercept compatible with unity at percent level. The two parameter fit may exceed in some cases the precision of the data, as $\chi^2 \ll 1$).

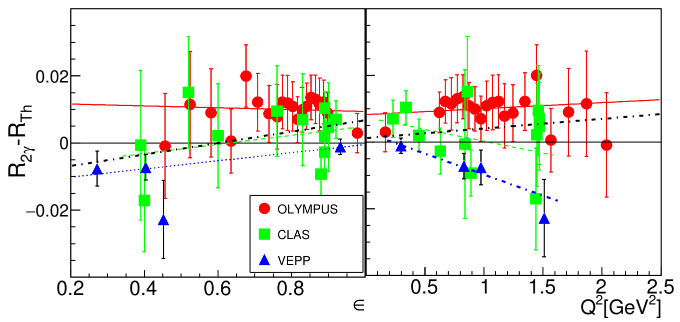

Figure 4. Same as Fig. 4, where δ_M is calculated from [41]

3 Conclusions

Radiative corrections are usually applied at first order to the measured cross sections, to recover the Born cross section. Standard calculations include the emission from the initial and final leptons, the initial and final proton and their interference as well as virtual corrections due to vacuum polarization and to the electron self energy. The $2\gamma E$ box diagram is also partly applied to cancel divergences.

In frame of the one-photon approximation, elastic electron and positron scattering on the proton should coincide. A large asymmetry between electron and positron scattering was indeed found in the experimental data, reaching 6-7%, but most of the asymmetry comes from the interference between initial and final photon emission, and it is highly reduced when the data are properly radiatively corrected.

The size of additional $2\gamma E$ contribution does not exceed the expected value from α-counting (few %), see Fig. 2. The main conclusion of the recent experiments is that (difficult) measurements at larger Q^2 are necessary: the present results are performed at $Q^2 \leq 3$ GeV^2 and do not show evident increase with Q^2. A coherent increase is seen in the most precise VEPP data, that depend, however, on a normalization between two data sets at different beam energy. Note that an effect growing with Q^2 and reaching 6% is necessary to bring in agreement the data on the ratio G_E/G_M, based on the Akhiezer-Rekalo and the Rosenbluth methods. The claim of model calculations predicting a large effect already at low Q^2 is not experimentally validated.

Overall, we point out inconsistencies in the claim of the presence of two-photon contributions. The attempts to extract FFs as real quantities, function of one variable, Q^2, in the presence of $2\gamma E$, is erroneous by principle. In presence of two-photon effects one can not extract nucleon FFs from the unpolarized cross section. The matrix element contains three amplitudes of complex nature, functions of two kinematical variables instead than two real functions of Q^2 only. Correcting the unpolarized the cross section by an assumed two photon effect and re-extracting FFs, is erroneous, as it integrates the conceptual and operative contradiction of merging the Born approximation and the two-photon effects. Advocating a large contribution of the $1\gamma - 2\gamma$ interference, would invalidate the definition of FF itself, as real function of the single variable Q^2 [22–24].

We also stress that the discrepancy between the unpolarized and polarized FF ratio experiments, that raised excitement in the discipline may not be real. Following the recent work of [25] a problem of renormalization of the low ϵ data in the previous data, in particular in

[5], was pointed out. Besides the uncertainties inherent to radiative correction calculations, problems of parameter correlations were pointed out in [28].

From the analysis of the present and all previous data, no evident effect increasing with Q^2, beyond the expectation from α-counting is found. No theoretical strong argument has been put forward to justify a large $2\gamma E$ contribution.

We do not enter here in the comparison and the merit of the existing model dependent $2\gamma E$ calculations. Let us note that, when and if a qualitative agreement may be found on reproducing the difference between polarized and unpolarized FF ratio, the agreement disappears when compared to another observable, the ϵ dependence of P_L/P_T, see Fig. 27 of [4]. Only the calculation [26], based on high order radiative corrections obtained with the structure function method [45, 46], reproduces the results both on the unpolarized cross section and on the polarization ratio. This is due to the fact that ϵ nonlinearities from this calculation are very small. This calculation, together with the work of [47], shows also that radiative corrections increase with Q^2 and induce a large ϵ and Q^2 dependence in the individual longitudinal and transverse polarized cross sections, what explains the deviation of P_L from the Born expectation (see Fig. 19 of [4]) although they essentially cancel in the ratio.

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