EPJ Web of Conferences **204**, 06010 (2019) https://doi.org/10.1051/epjconf/201920406010
Baldin ISHEPP XXIV

Closed system of equations for description of the $e^+e^-\gamma$ plasma generated from vacuum by strong electric field

S.A. Smolyansky[1,*], *A.M.* Fedotov[2], *A.D.* Panferov[1], and *S.O.* Pirogov[1]

[1]Saratov State University, RU - 410026 Saratov, Russia
[2]National Research Nuclear University (MEPhI), RU - 115409 Moscow, Russia

Abstract. We develop a self-consistent kinetic description of a $e^+e^-\gamma$ plasma, generated from vacuum in a focal spot of counterpropagating laser pulses. Our model assumes purely time-dependent external (laser) field, but properly takes into account the semiclassical internal (plasma) field, as well as quantum radiation. While nonperturbative kinetic description of e^+e^--pair production from vacuum and the simplest variant of backreaction problem have been previously addressed, quantum radiation is included in such a model for the first time. To achieve this goal we derived coupled kinetic equations for the electron, positron, and photon plasma species and the Maxwell equation for the internal electric field. Photon subsystem is included systematically using the BBGKY chain, which we truncate at the second order of perturbation theory by taking into account the annihilation and radiation channels. An important application of our results would be consideration of laser field depletion due to cascade production beyond the locally constant field approximation.

1 Introduction

Investigation of nonlinear effects in the strong field QED (effectiveness of the electron-positron plasma (EPP) production from vacuum in different models of the semiclassical electromagnetic field, radiation from EPP and other observable signatures, cascade processes, spin polarization, etc.) is important for development of the kinetic approaches based on both the quasiparticle [1, 2] and Wigner [3] representation. A step forward in this direction has been made in the present paper: here we demonstrate an extension of the kinetic theory (e.g., [1, 2] and reviews [4, 5]) by means of taking into account additionally interaction of EPP with the photon reservoir. The result is the self-consistent system of kinetic equations (KE's) including one for EPP production from vacuum under the combined action of a linear-polarized time-dependent quasiclassical electric field and the quantized electromagnetic field (vacuum photoeffect), and the photon KE associated with the hard photon radiation in EPP. To complete the picture we add the Maxwell equation governing the inner plasma field (back reaction).

2 Statement of the problem

Characteristic property of the problem is nonperturbative character of description of EPP creation from vacuum. In the framework of the quasiparticle representation this description

*e-mail: smol@sgu.ru

is possible in a spatial homogeneous semiclassical external field, which is assumed below, for the sake of simplicity, linear-polarized. Thus, the vector potential of the total acting electric field in the Hamiltonian gauge is $A^\mu(t) = (0, 0, 0, A(t) = A_{ex}(t) + A_{in}(t))$, where $A_{ex}(t)$ and $A_{in}(t)$ correspond to external and inner fields. On this background, emission and absorption of the hard photons are considered in the framework of perturbation theory with a small parameter $E_q/E_c \ll 1$, where E_q is the characteristic strength field amplitude of the quantized field and $E_c = m^2/e$ is the critical field. The fluctuating quantized field with the vector potential $\hat{A}^\mu(x)$ is considered as spatially inhomogeneous with an arbitrary polarization.

In the quasiparticle representation the total Hamiltonian can be splitted as follows:

$$H(t) = H_q(t) + H_{pol}(t) + H_{ph} + H_{int}(t), \tag{1}$$

where $H_q(t)$ corresponds to the EPP quasiparticle subsystem in the total semiclassical field $A(t)$:

$$H_q(t) = \int [dp]\omega(\mathbf{p},t)[a^+_\alpha(\mathbf{p},t)a_\alpha(\mathbf{p},t) + b^+_\alpha(\mathbf{p},t)b_\alpha(\mathbf{p},t)], \tag{2}$$

and is diagonal. Here we denote, for the sake of brevity, $[dp] = d^3p/(2\pi)^3$, $\omega(\mathbf{p},t) = \sqrt{\epsilon_\perp^2 + P^2}$ is quasienergy, $\epsilon_\perp = \sqrt{P_\perp^2 + m^2}$ is transversal energy, and $P = p_\parallel - eA(t)$ is longitudinal quasimomentum. The next part of the Hamiltonian,

$$H_{pol} = \frac{i}{2}\int [dp]\lambda(\mathbf{p},t)[a^+_\alpha(\mathbf{p},t)b^+_\alpha(-\mathbf{p},t) - b_\alpha(-\mathbf{p},t)a_\alpha(\mathbf{p},t)], \tag{3}$$

where $\lambda(\mathbf{p},t) = eE(t)\epsilon_\perp(\mathbf{p})/\omega^2(\mathbf{p},t)$ and the field strength, $E(t) = -\dot{A}(t)$, describes spontaneous pair creation from vacuum and annihilation.

The quantized radiation field in a plane wave basis and its interaction with the EPP subsystem are described by the Hamiltonians

$$H_{ph} = \sum_r \int [dk]kA^{(+)}_r(\mathbf{k},t)A^{(-)}_r(\mathbf{k},t), \tag{4}$$

$$\begin{aligned} H_{int}(t) = e(2\pi)^{-3/2}\sum_{\alpha\beta}\int d^3p_1 d^3p_2 \frac{d^3k}{\sqrt{2k}}\delta(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{k}) \times \\ \times :\Big([\bar{u}u]^r_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t)a^+_\alpha(\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t) + \\ +[\bar{u}v]^r_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t)a^+_\alpha(\mathbf{p}_1,t)b^+_\beta(-\mathbf{p}_2,t) + \\ +[\bar{v}u]^r_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t)b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t) + \\ +[\bar{v}v]^r_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t)b_\alpha(-\mathbf{p}_1,t)b^+_\beta(-\mathbf{p}_2,t)\Big)A_r(\mathbf{k},t): \ . \end{aligned} \tag{5}$$

The quasiparticle spinor basis that diagonalizes the Hamiltonian (2) can be found in the explicit form [6]

$$\begin{aligned} u^+_1(\mathbf{p},t) &= B(\mathbf{p})[\omega_+, 0, P^3, P_-], \\ u^+_2(\mathbf{p},t) &= B(\mathbf{p})[0, \omega_+, P_+, -P^3], \\ v^+_1(-\mathbf{p},t) &= B(\mathbf{p})[-P^3, -P_-, \omega_+, 0], \\ v^+_2(-\mathbf{p},t) &= B(\mathbf{p})[-P_+, P^3, 0, \omega_+], \end{aligned} \tag{6}$$

where $B(\mathbf{p}) = (2\omega\omega_+)^{-1/2}$, $\omega_+ = \omega + m$ and $P_\pm = P^1 \pm iP^2$. The convolutions (vertex functions) of spinors ξ_α and η_α from the set (6) are defined by

$$[\bar{\xi}\eta]^r_{\beta\alpha}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t) = \bar{\xi}_\alpha(\mathbf{p}_1,t)\gamma^\mu\eta_\beta(\mathbf{p}_2,t)e^r_\mu(\mathbf{k}), \tag{7}$$

$e^r_\mu(\mathbf{k})$ $(r = 1, 2)$ are the unit photon polarization vectors, and $A_r(\mathbf{k}, t) = A_r^{(+)}(\mathbf{k}, t) + A_r^{(-)}(\mathbf{k}, t)$.

3 Kinetic equations for $e^-e^+\gamma$-plasma

Let us introduce the distribution functions of EPP in the quasiparticle representation

$$f(\mathbf{p}, t) = \frac{1}{2}\langle a^+_\alpha(\mathbf{p}, t) a_\alpha(\mathbf{p}, t)\rangle = \frac{1}{2}\langle b^+_\alpha(-\mathbf{p}, t) b_\alpha(-\mathbf{p}, t)\rangle \tag{8}$$

(the last equality is a consequence of the electroneutrality condition) and the photon one

$$F(\mathbf{k}, t) = \frac{1}{2}\langle A^-_r(\mathbf{k}, t) A^+_r(\mathbf{k}, t)\rangle. \tag{9}$$

The first equations of the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) chains for these distribution functions contain the $e^-e^+\gamma$-correlators of the type

$$\begin{gathered}\langle a^+_\alpha(\mathbf{p}_1, t) a_\beta(\mathbf{p}_2, t) A^{(\pm)}_r(\mathbf{k}, t)\rangle, \\ \langle b_\alpha(-\mathbf{p}_1, t) a_\beta(\mathbf{p}_2, t) A^{(\pm)}_r(\mathbf{k}, t)\rangle.\end{gathered} \tag{10}$$

Independent equations of motion are written for these mixed correlation functions. We neglect the higher-order correlations in the usual way by expressing the higher level correlators as products of the principle ones, resulting in decoupling of the type:

$$\begin{aligned}\langle a^+_\alpha(\mathbf{p}_1, t) a_\beta(\mathbf{p}_2, t) A^{(-)}_r(\mathbf{k}, t) A^{(+)}_{r'}(\mathbf{k}', t)\rangle \approx \\ \langle a^+_\alpha(\mathbf{p}_1, t) a_\beta(\mathbf{p}_2, t)\rangle\langle A^{(-)}_r(\mathbf{k}, t) A^{(+)}_{r'}(\mathbf{k}', t)\rangle \approx \\ f(\mathbf{p}_1, t)\delta_{\alpha\beta}\delta(\mathbf{p}_1 - \mathbf{p}_2)[1 + F(\mathbf{k}, t)]\delta_{rr'}\delta(\mathbf{k} - \mathbf{k}')\end{aligned} \tag{11}$$

and so on. The latter equality is a consequence of spatial homogeneity of the system.

Such kind of procedure results in a closed system of KE's for the $e^-e^+\gamma$-plasma.

3.1 Modified KE for EPP production from vacuum

The modified KE has the following form:

$$\dot{f}(\mathbf{p}, t) = I(\mathbf{p}, t) + C^{ann}(\mathbf{p}, t) + C^{em}(\mathbf{p}, t), \tag{12}$$

where the source term

$$I(\mathbf{p}, t) = \frac{1}{2}\lambda(\mathbf{p}, t)\int^t dt'\lambda(\mathbf{p}, t')[1 - 2f(\mathbf{p}, t')]\cos\left(2\int_{t'}^t d\tau\omega(\mathbf{p}, \tau)\right) \tag{13}$$

describes the EPP creation from vacuum in a semiclassical linear-polarized electric field [1–5]. The collision integrals correspond to the one-photon annihilation (pair production) and one-photon emission (absorption) processes, correspondingly:

$$\begin{aligned}C^{ann}(\mathbf{p}, t) &= \int[dp_1][dk]\int^t dt'\delta(\mathbf{p} - \mathbf{p}_1 - \mathbf{k})K^{ann}(\mathbf{p}, \mathbf{p}_1, \mathbf{k}; t, t')\{f(\mathbf{p}, t')\times \\ &\times\ f(\mathbf{p}_1, t')[1 + F(\mathbf{k}, t')] - [1 - f(\mathbf{p}, t')][1 - f(\mathbf{p}_1, t')]F(\mathbf{k}, t')\},\end{aligned} \tag{14}$$

$$\begin{aligned}C^{em}(\mathbf{p}, t) &= \int[dp_1][dk]\int^t dt'\delta(\mathbf{p} - \mathbf{p}_1 - \mathbf{k})K^{em}(\mathbf{p}, \mathbf{p}_1, \mathbf{k}; t, t')\{f(\mathbf{p}, t')\times \\ &\times\ [1 - f(\mathbf{p}_1, t')][1 + F(\mathbf{k}, t')] - f(\mathbf{p}, t')[1 - f(\mathbf{p}_1, t')]F(\mathbf{k}, t')\}.\end{aligned} \tag{15}$$

with the kernels

$$K^{ann}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t') = (2\pi)^3\frac{e^2}{2k}Re\{[\overline{u}v]^{r+}_{\alpha\beta}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t)[\overline{u}v]^{r}_{\alpha\beta}(\mathbf{p},\mathbf{p}_1,-\mathbf{k};t')e^{-i\theta^{(+,-)}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t')}\}, \quad (16)$$

$$K^{em}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t') = (2\pi)^3\frac{e^2}{2k}Re\{[\overline{u}u]^{r+}_{\alpha\beta}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t)[\overline{u}u]^{r}_{\alpha\beta}(\mathbf{p},\mathbf{p}_1,-\mathbf{k};t')e^{-i\theta^{(-,-)}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t')}\} \quad (17)$$

and the phases

$$\theta^{(\pm)}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t') = \int_{t'}^{t} d\tau[\omega(\mathbf{p},\tau)\pm\omega(\mathbf{p}_1,\tau)-k]. \quad (18)$$

The collision integrals (14), (15) admits interpretation in terms of "gain - loss".

3.2 KE in the photon sector

The photon sector KE has the form

$$\dot{F}(\mathbf{k},t) = S^{ann}(\mathbf{k},t) + S^{em}(\mathbf{k},t), \quad (19)$$

where the collision integrals in the annihilation and emission channels are equal to:

$$S^{ann}(\mathbf{k},t) = \int[dp_1][dp_2]\int^{t} dt'\delta(\mathbf{p}_1-\mathbf{p}_2-\mathbf{k})K^{ann}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t,t')\times \quad (20)$$
$$\times\{f(\mathbf{p}_1,t')f(\mathbf{p}_2,t')[1+F(\mathbf{k},t')]-[1-f(\mathbf{p}_1,t')][1-f(\mathbf{p}_2,t')]F(\mathbf{k},t')\},$$

$$S^{em}(\mathbf{k},t) = \int[dp_1][dp_2]\int^{t} dt'\delta(\mathbf{p}_1-\mathbf{p}_2-\mathbf{k})K^{em}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t,t')\times \quad (21)$$
$$\times\{f(\mathbf{p}_1,t')[1-f(\mathbf{p}_2,t')][1+F(\mathbf{k},t')]-f(\mathbf{p}_1,t')[1-f(\mathbf{p}_2,t')]F(\mathbf{k},t')\}$$

with the same kernels (16), (17), correspondingly. Thus, these kernels are universal for the fermion and photon sectors.

Kinetic equation (19) with the collision integrals (20), (21) is a linear integro-differential equation of the non-Markovian type with respect to the photon distribution function $F(\mathbf{k},t)$:

$$\dot{F}(\mathbf{k},t) = I(\mathbf{k},t) + \int^{t} dt'\Phi(\mathbf{k};t,t')F(\mathbf{k},t') \quad (22)$$

with the source term $I(\mathbf{k},t)$ and kernel $\Phi(\mathbf{k};t,t')$ are functionals of the EPP distribution function $f(\mathbf{p},t)$.

3.3 Maxwell equation

The plasma inner field is defined by the equation [7]

$$\dot{E}_{in}(t) = -j_{cond}(t) - j_{pol}(t). \quad (23)$$

Here the conductivity and polarization current densities read:

$$j_{cond}(t) = 2e\int[dp]\frac{P}{\omega(\mathbf{p},t)}f(\mathbf{p},t), \quad (24a)$$

$$j_{pol}(t) = e\int[dp]\frac{\epsilon_\perp}{\omega(\mathbf{p},t)}[u(\mathbf{p},t)-\frac{e\dot{E}P}{4\omega^4(\mathbf{p},t)}], \quad (24b)$$

where $u(\mathbf{p},t)$ is the polarization current function. The second term on the r.h.s. of (24b) is the counter-term arising due to renormalization.

4 Summary

The obtained system of equations (12), (19), (23) extends the well-known KE for EPP creation from vacuum [1–5] by taking into account interaction with photons radiated by EPP. We take into account both mechanisms of the second order with respect to the parameter $E/E_c \ll 1$: one-photon annihilation (and the reversal process of the vacuum photoeffect) and absorption (emission) processes. The equations can be used for detailed studies of the EPP kinetics in a strong field, including radiation from the strong field region (EPP diagnostics), cascade processes [8, 9], etc.

The work is supported by the RFBR research project 17-02-00375a.

References

[1] I. Bialynicky-Birula, P. Gornicki, and J. Rafelski, Phys. Rev. D **44**, 1825 (1991)

[2] S.M. Schmidt, D. Blaschke, G. Röepke, S.A. Smolyansky, A.V. Prozorkevich, and V.D. Toneev, Int. J. Mod. Phys. E **7**, 709 (1998)

[3] F. Hebenstreit, R. Alkofer, and H. Gies, Phys. Rev. D **82**, 105026 (2010)

[4] D.B. Blaschke, A.V. Prozorkevich, G. Röepke, C.D. Roberts, S.M. Schmidt, D.S. Shkizmanov, and S.A. Smolyansky, Eur. Phys. J. D **55**, 341 (2009)

[5] D.B. Blaschke, S.A. Smolyansky, A.D. Panferov, L. Juchnowsky, in *Proc. of the Helmholtz Int. Sum. School: Quantum Field Theory at the limits: from Strong Field to Heavy Quarks (HQ 2016), July 18-30, 2016, JINR, Dubna* (Hamburg, Germany, 2017) 1

[6] D.B. Blaschke, V.V. Dmitriev, G. Röepke, and S.A. Smolyansky, Phys. Rev. D **84**, 085028 (2011)

[7] J.C.R. Bloch, V.A. Mizerny, A.V. Prozorkevich, C.D. Roberts, S.M. Schmidt, S.A. Smolyansky, and D.V. Vinnik, Phys. Rev. D **60**, 116011 (1999)

[8] A.M. Fedotov, N.B. Narozhny, G. Mourou, and G. Korn, Phys. Rev. Lett. **105**, 080402 (2010)

[9] E.N. Nerush, I.Yu. Kostyukov, A.M. Fedotov, N.B. Narozhny, N.V. Elkina, and H. Ruhl, Phys. Rev. Lett. **106**, 035001 (2001)