FIVE SEPARATION AXIOMS IN QUAD SOFT NON-LINEAR STRUCTURE

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ABSTRACT

In 1999, Russian mathematician Molodtsov planted the novel notion of a soft set which can be supposed as a new mathematical approach for imprecision. Topological structures of soft set have been studied by some mathematicians in current years. M. Shabir and M. Naz presented soft topological spaces and studied some key properties of them. Soft Quad topological spaces are nothing but just Extension of soft topological spaces. When with soft single topology three more soft topologies are hugged together through mathematical discipline then the notion of soft Quad topology will automatically come in play. The central objective of this article is to publicize soft separations axioms in soft quad topological spaces. Moreover to discuss the linkage among them through two paths.

1. INTRODUCTION

Many practical problems in economics, engineering, environmental sciences, social sciences, medical sciences etc. cannot be dealt by classical methods because classical methods have inherent difficulties. The reason for these difficulties may be due to the insufficiency of the theories of parameterization tools. Numerous theories exist, for example, fuzzy set theory, intuitionistic fuzzy set theory, rough set theory, i.e., which can be considered as a mathematical tool for dealing with uncertainties [1-3]. Each of these theories has its inherent difficulties as what were pointed out by Molodtsov [4]. Molodtsov originated absolutely a new methodology for modelling uncertainties and applied successfully in directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, and so on [4]. A group researcher gave the first practical application of soft sets in decision making problems [5]. The algebraic structure of set theories dealing with uncertainty is an important problem. Many researchers have contributed towards the algebraic structure of soft set theory. Some of researchers defined soft groups and deduced their fundamental characterizations [6]. Others group of researchers introduced initial concepts of soft rings [7]. Besides, in other study, researchers defined soft semi rings and several related notions to establish a connection between soft sets and semi rings [8]. Some researchers defined soft modules and investigated their basic properties [9]. A studied soft ideal over a semi groups which characterized generalized fuzzy ideals and fuzzy ideals with thresholds of a semi groups [10]. There are two researchers introduced fuzzy soft modules and intuitionistic fuzzy soft modules and investigated some basic properties [11,12].

Shabir and Naz firstly introduced the notion of soft topological space which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces [13]. Theoretical studies of soft topological spaces have also been by some authors in [14-17]. In these studies the concept of soft point is given almost samely. Consequently, some results of classical topology are not valid in soft topological spaces. A group researcher introduced the concepts of supra topological spaces, supra closed sets, supra open sets and supra continuous mapping [18]. Many results of topological spaces remain valid in supra topological spaces, but some become false. As a generalized of soft topological spaces, the notion of supra soft topological spaces by dropping only the soft intersection condition [19]. They extend some types of different kinds of subsets of soft topological spaces using the notion of γ −operation to supra soft topological space and study the decompositions of some forms of supra soft continuity. The notion of soft bi topological space which is defined over an initial universal set X with fixed set of parameters and studied some types of soft separation axioms [20]. It is clear that a soft bi topological space is a generalization of a soft topological space as every soft bi topological space (X, η_1, η_2, E) if $\eta_1 = \eta_2 = \eta$ A. Generalized the notions of soft topological spaces such as open soft sets, closed soft sets, soft interior, and soft closure in a soft bi topological space [13]. Thus, they introduced the notions of pairwise open (closed) soft sets, pairwise soft interior operator in a soft bi topological space (X, η_1, η_2, E) and showed that the family of all pairwise open soft sets is a supra soft topology which is containing η_1, η_2 but it is not soft topology [13].

In the present study, we first give some basic ideas about soft sets and the results already studied. In addition to these, we introduce the concept of soft point to the theory of soft set. According to this definition, we investigate some important notions of soft topological spaces. Later we give separability axioms in soft Quad topological spaces and study some of their characterizations. The separability axioms are the soft separation axioms in Soft Quad topological spaces with respect to ordinary and soft points.

We expect that these results will be of greater importance for the forthcoming learning on Soft Quad topological spaces to accomplish general framework for the most practical applications and to answer the most intricate problems containing doubt in economics, engineering, medical, environment and in general.

2. PRELIMINARIES

In this section we will introduce necessary definitions and theorems for soft sets. Molodtsov defined the soft set in the following way. Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X [14].

Definition 2.1. ([4]) A pair (F, E) is called a soft set over X; where F is a mapping given by
$F : E \rightarrow P(X)$.
In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$; $F(e)$ may be considered as the set of e −elements of the soft

set$(F; E)$; or as the set of e −approximate elements of the soft set, i.e.
$(F, E) = \{(e, F(e)) : e \in E, F : E \to P(X)\}$.
After this, $SS(X)E_E$ denotes the family of all soft sets over X with a fixed set of parameter E.

Definition 2.2. ([21]) For two soft sets (F, E) and (G, E) over$X$, (F, E) is called a soft subset of (G, E) if $\forall \, e \in E$, $F(e) \subseteq G(e)$. This relationship is denoted by$(F, E) \, \widetilde{\subseteq} \, (G, E)$.
Similarly, (F, E) is called a soft superset of (G, E) if (G, E) is a soft subset of(F, E). This
Linkage is denoted by$(F, E) \, \widetilde{\supseteq} \, (G, E)$. Two soft sets (F, E) and (G, E) over X are called soft equal if (F, E) is a soft subset of (G, E) and (G, E) is a soft subset of (F, E).

Definition 2.3. ([21]) The intersection of two soft sets (F, E) and (G, E) over X is the soft set
(H, E), where $\forall e \in E$, $H(e) = F(e) \cap G(e)$. This is denoted by$(F, E) \, \widetilde{\cap} \, (G, E) = (H, E)$.

Definition 2.4. ([21]) the union of two soft sets (F, E) and (G, E) over X is the soft set (H, E),
Where $\forall e \in E, H(e) = F(e) \cup G(e)$ This is denoted by $(F, E) \, \widetilde{\cup} \, (G, E) = (H, E)$.

Definition 2.5. ([22]) A soft set (F, E) over X is said to be a null soft set denoted by φ if for all$e \in E, F(e) = \varphi$.

Definition 2.6. ([22]) A soft set (F, E) over X is said to be an absolute soft set denoted by $\widetilde{\widetilde{X}}$
If for all$e \in E$; $F(e) = X$.

Definition 2.7. ([23]) The soft set $(F, A) \in SS(X)_A$ is called a soft point in X_A , denoted by e_F , if for the element $e \in A, F(e) \neq \emptyset$ and $F(e^/) = \phi$ if for all $e^/ \in A - \{e\}$

Definition 2.8. ([22]) the complement of a soft set (F, E); denoted by $(F, E)^c$, is defined as $(F, E)^c = (F^c; E)$; where $F^c : E \to P(X)$ is a mapping given by$F^c (e) = X \setminus F (e)$; $\forall e \in E$ and F^c is called the soft complement function of F:

Definition 2.9. ([22]) Let Y be a non-empty subset of$X$, then $\widetilde{\widetilde{Y}}$ denotes the soft set (Y, E) over X for which $(e) = Y$, for all $\in E$.

Definition 2.10. ([22]) Let (F, E) be a soft set over X and Y be a non-empty subset of X. Then
The sub soft set of (F, E) over Y denoted by $(^Y F, E)$, is defined as follows
$^Y F (e) = Y \cap F (e)$, for all $\forall e \in E$
In other words $(^Y F, E) = \widetilde{\widetilde{Y}} \cap F (e)$

Definition 2.11. ([22]) Let $\mathfrak{I}$ be the collection of soft sets over X, then $\mathfrak{I}$ is said to be a soft
Topology on X if
(1) $\varphi, \widetilde{X} \in \mathfrak{I}$
(2) The union of any number of soft sets in $\mathfrak{I}$ belongs to $\mathfrak{I}$
(3) The intersection of any two soft sets in $\mathfrak{I}$ belongs to$\mathfrak{I}$.
The triplet (X,$\mathfrak{I}$, E) is called a soft topological space over X.

Proposition 2.12. ([22]) Let $(X, \mathfrak{I}, E)$ be a soft topological space over$X$. Then the collection
$\mathfrak{I}_e = \{F(e); (F, E) \in \mathfrak{I}\}$ For each$e \in E$, defines a topology on X.

Definition 2.13. ([22]) Let $(X, \mathfrak{I}, E)$ be a soft topological space over X and (F, E) be a soft set over. Then the soft closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of(F, E). Clearly $\overline{(F, E)}$ is the smallest soft closed set over X which contains(F, E).

Proposition 1. ([22]) Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is soft T_3-space, then for all $x \in X, x_E = (x, E)$ is-closed soft set.

Proposition 2. ([22]) Let (Y, τ_Y, E) be a soft sub space of a soft topological space (X, τ, E) and $(F, E) \in SS(X)$ then
1. If (F, E) is soft *open* set in Y and $Y \in \tau$, then $(F, E) \in \tau$
2. (F, E) is soft *open soft* set in Y if and only if $(F, E) = Y \cap (G, E)$ for some $(G, E) \in \tau$.
3. (F, E) is soft *closed soft* set in Y if and only if $(F, E) = Y \cap (H, E)$ for some (H, E) *is τ soft closed.*

3. SOME BASIC RESULTS IN QUAD SOFT TOPOLOGY WITH RESPECT TO CRISP POINTS

Definition 3.1. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$be a quad soft topological space over$X$, where $(X, \tau_1, E), (X, \tau_2, E)$, (X, τ_3, E) and (X, τ_4, E)be four different soft topologies on$X$. Then a sub set (F, E) is said to be quad-open (in short hand q-open) if $(F, E) \subseteq \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and its complement is said to be soft q-closed.

Definition 3.2. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$be a quad soft topological space over$X$, (G, E) be a soft set over X and$x \in X$. Then x is said to be a soft q interior point of (G, E) if there exists soft q-open set (F, E) such that$x \in (F, E) \subseteq (G, E)$.

Definition 3.3. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$be a soft quad topological space over$X$, (G, E) be a soft set over X and $x \in X$. Then (G, E) is said to be a soft q neighborhood of x if there exists soft q-open set (F, E) such that$x \in (F, E) \subseteq (G, E)$.

Proposition 3. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$be a soft quad topological space over$X$, (G, E) be a soft set over X and$x \in X$. If x is a soft interior point of (G, E) then x is an interior point of $G(\alpha)$ in$(\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)_{(\alpha)}$ for each $\alpha \in E$.

Proof. For any$\alpha \in E, G(\alpha) \subseteq X$. If $x \in X$ is a soft interior point of (G, E) then there exists soft q-open set (F, E) such that$x \in (F, E) \subseteq (G, E)$. This means that$x \in F(\alpha) \subseteq G(\alpha)$. As$F(\alpha) \in (X, \tau_1, \tau_2, \tau_3, \tau_4, E)$so $F(\alpha)$ is open set in $(\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)_{(\alpha)}$ and$x \in F(\alpha)$. This implies that x is an interior point of $G(\alpha)$ in$(\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)_{(\alpha)}$

Proposition 4. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$be a soft quad topological space over$X$, then each $x \in X$ has a soft q neighborhood;
If (F, E) and (G, E) are soft q neighborhoods of some$x \in X$, then $(F, E) \cap (G, E)$ is also a soft neighborhood of$x$.
If (F, E) is a soft q neighborhood of $x \in X$ and$(F, E) \subseteq (G, E)$, then (G, E) is also a soft q neighborhood of$x \in X$.

Proof. (1) For any$x \in X$, x $\in \widetilde{X}$ and since$\widetilde{X} \in (\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)$, so$x \in X \subseteq \widetilde{X}$.
Thus $\widetilde{X}$ is a soft neighborhood of$x$.
Let (F, E) and (G, E) be the soft neighborhoods of $x \in X$, then there exist soft q open sets (F_1, E),,$(F_2, E) \in (\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)$ such that

$$x \in (F_1, E) \subseteq (F, E). \text{ And } x \in (F_2, E) \subseteq (G, E)$$

Now $x \in (F_1, E)$ and $x \in (F_2, E)$ implies that $(F_1, E) \cap (F_2, E)$ and$(F_1, E) \cap (F_2, E) \in (\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)$. So we have $x \in (F_1, E) \cap (F_2, E) \subseteq (F, E) \cap (G, E)$.

Thus $(F, E) \cap (G, E)$ is a soft neighborhood of$x$.
Let (F, E) be a soft q neighborhood of $x \in X$ and$(F, E) \subseteq (G, E)$. By definition there exists a soft q open set (F_1, E) such that $x \in (F_1, E) \subseteq (F, E) \subseteq (G, E)$.

$$\text{Thus } x \in (F_1, E) \subseteq (G, E)$$

Hence (G, E) is a soft q neighborhood of$x$.

Proposition 5. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$be a soft quad topological space over$X$.Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft $q - T_1$ space iff each soft point is a soft q closed set.

Proof. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft T_1space and (x, E) be an arbitrary soft point. We show that $(x, E)^c$ is a soft q open set. Let $(y, E) \in (x, E)^c$ such that$(x, E) \neq (y, E)$. Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft T_1space, there exists q soft open set (G, E) such that$(y, E) \in (G, E), (x, E) \notin (G, E)$.Then $(y, E) \in (G, E) \subseteq (x, E)^c$.This implies that $(x, E)^c$is a q soft open set. Equivalently, (x, E) is a q soft closed set. Suppose that for each (x, E) is a q soft closed set. Then automatically $(x, E)^c$ is a q soft open set. Let$(x, E) \neq (y, E)$. Thus $(y, E) \in (x_e, E)^c$ and$(x, E) \in (x, E)^c$. In a similar fashion $(y, E)^c$ is a q soft open set such that $(x, E) \in (y, E)^c$.and$(y, E) \notin (y, E)^c$. Therefore $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a soft $q - T_1$ space

4. SOME BASIC RESULTS IN QUAD SOFT TOPOLOGY WITH RESPECT TO SOFT POINTS

Definition 4.1. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$be a quad soft topological space over$X$, where $(X, \tau_1, E), (X, \tau_2, E)$, (X, τ_3, E) and (X, τ_4, E)be four different soft topologies on$X$. Then a sub set (F, E) is said to be quad-open (in short

hand q-open) if $(F, E) \subseteq \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and its complement is said to be soft q-closed.

Definition 4.2. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a quad soft topological space over X, (G, E) be a soft set over X and $x_e \in X$. Then x is said to be a soft q interior point of (G, E) if there exists soft q-open set (F, E) such that $x_e \in (F, E) \subseteq (G, E)$.

Definition 4.3. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X, (G, E) be a soft set over X and $x_e \in X$. Then (G, E) is said to be a soft q neighborhood of x if there exists soft q-open set (F, E) such that $x_e \in (F, E) \subseteq (G, E)$.

Proposition 6. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X, (G, E) be a soft set over X and $x_e \in X$. If x is a soft interior point of (G, E) then x is an interior point of $G(\alpha)$ in $(\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)_{(\alpha)}$ for each $\alpha \in E$.

Proof. For any $\alpha \in E$, $G(\alpha) \subseteq X$. If $x \in X$ is a soft interior point of (G, E) then there exists soft q-open set (F, E) such that $x \in (F, E) \subseteq (G, E)$. This means that $x \in F(\alpha) \subseteq G(\alpha)$. As $F(\alpha) \in (X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ so $F(\alpha)$ is open set in $(\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)_{(\alpha)}$ and $x \in F(\alpha)$. This implies that x is an interior point of $G(\alpha)$ in $(\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)_{(\alpha)}$

Proposition 7. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X, then each $x_e \in X$ has a soft q neighborhood;

If (F, E) and (G, E) are soft q neighborhoods of some $x_e \in X$, then $(F, E) \cap (G, E)$ is also a soft neighborhood of x.

If (F, E) is a soft q neighborhood of $x_e \in X$ and $(F, E) \subseteq (G, E)$, then (G, E) is also a soft q neighborhood of $x_e \in X$.

Proof. (1) For any $x_e \in X$, $x_e \in \tilde{X}$ and since $\tilde{X} \in (\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)$, so $x_e \in X \subseteq \tilde{X}$.

Thus $\tilde{X}$ is a soft neighborhood of x_e.

Let (F, E) and (G, E) be the soft neighborhoods of $x_e \in X$, then there exist soft q open sets $(F_1, E), (F_2, E) \in (\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)$ such that

$$x_e \in (F_1, E) \subseteq (F, E) \text{ And } x_e \in (F_2, E) \subseteq (G, E)$$

Now $x_e \in (F_1, E)$ and $x_e \in (F_2, E)$ implies that $(F_1, E) \cap (F_2, E)$ and $(F_1, E) \cap (F_2, E) \in (\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4)$. So we have $x_e \in (F_1, E) \cap (F_2, E) \subseteq (F, E) \cap (G, E)$.

Thus $(F, E) \cap (G, E)$ is a soft neighborhood of x.
Let (F, E) be a soft q neighborhood of $x_e \in X$ and $(F, E) \subseteq (G, E)$ .So by definition there exists a soft q open set (F_1, E) such that $x_e \in (F_1, E) \subseteq (F, E) \subseteq (G, E)$.

Thus $x_e \in (F_1, E) \subseteq (G, E)$

Hence (G, E) is a soft q neighborhood of x_e.

Proposition 8. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft $q - T_1$ space iff each soft point is a soft q closed set.

Proof. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft T_1 space and (x_e, E) be an arbitrary soft point. We show that $(x_e, E)^c$ is a soft q open set. Let $(y_{e'}, E) \in (x_e, E)^c$ such that $(x_e, E) \neq (y_{e'}, E)$. Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft T_1 space, there exists q soft open set (G, E) such that $(y_{e'}, E) \in (G, E)$, $(x_e, E) \notin (G, E)$. Then $(y_{e'}, E) \in (G, E) \subseteq (x_e, E)^c$. This implies that $(x_e, E)^c$ is a q soft open set. Equivalently, (x_e, E) is a q soft closed set. Suppose that for each (x_e, E) is a q soft closed set. Then automatically $(x_e, E)^c$ is a q soft open set. Let $(x_e, E) \neq (y_{e'}, E)$. Thus $(y_{e'}, E) \in (x_e, E)^c$ and $(x_e, E) \in (x_e, E)^c$. In a similar fashion $(y_{e'}, E)^c$ is a q soft open set such that $(x_e, E) \in (y_{e'}, E)^c$ and $(y_{e'}, E) \notin (y_{e'}, E)^c$. Therefore $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a soft $q - T_1$ space

5. SOFT SEPARATION AXIOMS IN QUAD SOFT TOPOLGICAL SPACES

In this section we inaugurated soft separation axioms in soft quad topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 5.1. Let (X, τ_1, E), (X, τ_2, E) , (X, τ_3, E) and (X, τ_4, E) be four different soft topologies on X. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a *soft quad topological space.* The soft four topologies (X, τ_1, E), $(X, \tau_2, E), (X, \tau_3, E)$ and (X, τ_4, E) are independently satisfying the axioms of soft topology. The members of τ_1 are called τ_1 *soft open* set. And complement of τ_1 Soft open set is called τ_1 *soft closed*. Similarly, the member of τ_2 are called τ_2 *soft open* sets and the complement of τ_2 soft

open sets are called τ_2 *soft closed* set. The members of τ_3 are called τ_3 *soft open* set. And complement of τ_3 Soft open set is called τ_3 *soft closed* set and the members of τ_4 are called τ_4 *soft open* set. And complement of τ_4 Soft open set is called τ_4 *soft closed* set.

Definition 5.2. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and $x, y \in X$ such that $x \neq y$
If we can find soft q-open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ *then* $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_0 space.

Definition 5.3. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and $x, y \in X$ such that $x \neq y$ If we can find two soft q-open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$ then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_1 space.

Definition 5.4. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and $x, y \in X$ such that $x \neq y$. If we can find two q- open soft sets such that $x \in (F, E)$ and $y \in (G, E)$ moreover $(F, E) \cap (G, E) = \phi$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a soft qT_2 space.

Definition 5.5. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft topological space (G, E) be q-closed soft set in X and $x \in X_A$ such that $x \notin (G, E)$. If there occurs soft q-open sets (F_1, E) and (F_2, E) such that $x \in (F_1, E)$, $(G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft q-regular spaces. A soft q-regular qT_1 *Space* is called soft qT_3 space.
Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a soft q- regular spaces. A soft q-regular T_1 *Space*.is called soft qT_3 space.

Definition 5.6. $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a *soft quad topological* space $(F_1, E), (G, E)$ be closed soft sets in X such that
$(F, E) \cap (G, E) = \varphi$ If there exists q- open soft sets (F_1, E) and (F_2, E) such that $(F, E) \subseteq (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a q-soft normal space. A soft q-normal qT_1 *Space* is called soft qT_4 *Space*.

Definition 5.7. Let (X, τ, A) be a soft Topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen at least one soft semi open set (F_1, A) or (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$ or $e_H \in (F_2, A), e_G \notin ((F_2, A)$ then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a *soft qT_0 space.*
Definition 5.8. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft Topological spaces over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen *soft q-open sets* (F_1, A) and (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$ and $e_H \in (F_2, A), e_G \notin ((F_2, A)$ then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_1 space.
Definition 5.9. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a *soft Topological* space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen *soft q-open sets* (F_1, A) and (F_2, A) such that $e_G \in (F_1, A)$, and $e_H \in (F_2, A)$
$(F_1, A) \cap (F_2, A) = \phi_A$ .Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft qT_2 space
Definition 5.10. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft topological space (G, E) be q-closed soft set in X and $e_G \in X_A$ such that $e_G \notin (G, E)$. If there occurs soft q-open sets (F_1, E) and (F_2, E) such that $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called soft q-regular spaces. A soft q- regular qT_1 *Space* is called soft qT_3 space.

5. SOF AXIOMS WITH RESPECT TO CRISP POINTS

In this section separation axioms in quad soft topological structures are examined with respect to ordinary points and different related results are also examined [24].

Definition 6.1. In a *soft quad topological* space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$
1) $\tau_1 \cup \tau_2$ is said to be *soft T_0 space* with respect to $\tau_3 \cup \tau_4$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_1 \cup \tau_2$ soft open set (F, E) and a $\tau_3 \cup \tau_4$ soft open set (G, E) such that $x \in (F, E)$ and $y \notin (G, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be *soft T_0* space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_3 \cup \tau_4$ *soft open set* (F, E) and $\tau_1 \cup \tau_2$ soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. *Soft quad topological spaces* $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be *pair wise soft T_0* space if $\tau_1 \cup \tau_2$ is *soft T_0 space* with respect to $\tau_3 \cup \tau_4$ and to $\tau_3 \cup \tau_4$ and is *soft T_0* space with respect to $\tau_1 \cup \tau_2$.
2) $\tau_1 \cup \tau_2$ is said to be soft T_1 space with respect to $\tau_3 \cup \tau_4$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_1 \cup \tau_2$ *soft open set* (F, E) and to $\tau_3 \cup \tau_4$ soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft T_1 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $x, y \in X$ such that $x \neq y$ there exists $\tau_3 \cup \tau_4$ soft open set (F, E) and a to $\tau_1 \cup \tau_2$ soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. *Soft quad topological spaces* $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be *pair wise soft T_1* space if $\tau_1 \cup \tau_2$ is *soft T_1 space* with respect to $\tau_3 \cup \tau_4$ and to $\tau_3 \cup \tau_4$ is *soft T_1* space with respect to $\tau_1 \cup \tau_2$.

3) $\tau_1 \cup \tau_2$ is said to be soft T_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_1 \cup \tau_2$ soft open set (F, E) and a $\tau_3 \cup \tau_4$ soft open set (G, E) such that $x \in (F, E)$ and $y \in (G, E)$, $(F, E) \cap (G, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is said to be *soft* T_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_3 \cup \tau_4$ *soft open* set (F, E) and a $\tau_1 \cup \tau_2$ *soft open* set (G, E) such that $x \in (F, E)$, $y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. The soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft T_2 space if $\tau_1 \cup \tau_2$ is soft T_2 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft T_2 space with respect to $\tau_1 \cup \tau_2$.

Definition 6.2. In a soft quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$
1) $\tau_1 \cup \tau_2$ is said to be soft T_3 space with respect to a $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft T_1 space with respect to $\tau_3 \cup \tau_4$ and for each pair of points $x, y \in X$ such that $x \neq y$ there exists $\tau_1 \cup \tau_2$
Soft closed set (G, E) such that $x \notin (G, E)$, a $\tau_1 \cup \tau_2$ soft open set (F_1, E) and $\tau_3 \cup \tau_4$ soft open set (F_2, E) such that $x \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \emptyset$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft T_3 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft T_1 space with respect to $\tau_1 \cup \tau_2$ and for each pair of points $x, y \in X$ such that $x \neq y$ there exists a $\tau_3 \cup \tau_4$ soft closed set (G, E) such that $x \notin (G, E)$, $\tau_3 \cup \tau_4$ soft open set (F_1, E) and $\tau_1 \cup \tau_2$ soft open set (F_2, E) such that $x \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is said to be pair wise soft T_3 space if $\tau_1 \cup \tau_2$ is soft T_3 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft T_3 space with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft T_4 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft T_1 space with respect to $\tau_3 \cup \tau_4$, there exists a $\tau_1 \cup \tau_2$ soft closed set (F_1, E) and $\tau_3 \cup \tau_4$ soft closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \emptyset$. Also there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_1 \cup \tau_2$ open set, (G_1, E) is soft $\tau_3 \cup \tau_4$ open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft T_4 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft T_1 space with respect to $\tau_1 \cup \tau_2$, there exists $\tau_3 \cup \tau_4$ soft closed set (F_1, E) and $\tau_1 \cup \tau_2$ soft closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. Also there exist (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ open set, (G_1, E) is soft $\tau_1 \cup \tau_2$ open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Thus, (X, τ_1, τ_2, E) is said to be pair wise soft T_4 space if $\tau_1 \cup \tau_2$ is soft T_4 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft T_4 space with respect to $\tau_1 \cup \tau_2$.

Proposition 9. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. Then, if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft T_3 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft T_2 space.

Proof: Suppose (X, τ_1, τ_2, E) is a soft T_3 space with respect to (X, τ_3, τ_4, E) then according to definition for $x, y \in X$, which distinct, by using Proposition 1, (Y, E) is soft closed set in $\tau_3 \cup \tau_4$ and $x \notin (Y, E)$ there exists a $\tau_1 \cup \tau_2$ soft β open set (F, E) and a $\tau_3 \cup \tau_4$ soft open set (G, E) such that $x \in (F, E), y \in (Y, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $\tau_1 \cup \tau_2$ is soft T_2 space with respect to $\tau_3 \cup \tau_4$. Similarly, if (X, τ_3, τ_4, E) is a soft T_3 space with respect to (X, τ_1, τ_2, E) then according to definition for $x, y \in X, x \neq y$, by using Theorem 2, (x, E) is closed soft set in $\tau_1 \cup \tau_2$ and $y \notin (x, E)$ there exists a $\tau_3 \cup \tau_4$ soft open set (F, E) and a $\tau_1 \cup \tau_2$ soft open set (G, E) such that $y \in (F, E)$, $x \in (x, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $\tau_3 \cup \tau_4$ is soft T_2 space. This implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft T_2 space.

Proposition 10. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft T_3 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft T_3 space.

Proof: Suppose (X, τ_1, τ_2, E) is a soft T_3 space with respect to (X, τ_3, τ_4, E) then according to definition for $x, y \in X, x \neq y$ there exists a (X, τ_1, τ_2, E) soft open set (F, E) and a (X, τ_3, τ_4, E) soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each (X, τ_1, τ_2, E) closed soft set (G_1, E) such that $x \notin (G_1, E)$ there exists a (X, τ_1, τ_2, E) soft open set (F_1, E) and (X, τ_3, τ_4, E) soft open set (F_2, E) such that $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly, to (X, τ_3, τ_4, E) is a soft T_3 space with respect to (X, τ_1, τ_2, E) So according to definition for $x, y \in X, x \neq y$ there exists a (X, τ_3, τ_4, E) soft open set (F, E) and a (X, τ_1, τ_2, E) soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each (X, τ_3, τ_4, E) closed soft set (G_1, E) such that $x \notin (G_1, E)$ there exists (X, τ_3, τ_4, E) soft β open set (F_1, E) and (X, τ_1, τ_2, E) soft open set (F_2, E) such that $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft T_3 space.

Proposition 11. If $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft T_4 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft T_4 space.

Proof: Suppose (X, τ_1, τ_2, E) is soft T_4 space with respect to

(X, τ_3, τ_4, E). So according to definition for $x, y \in X, x \neq y$ there exist a (X, τ_1, τ_2, E) soft open set (F, E) and a (X, τ_3, τ_4, E) soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ each (X, τ_1, τ_2, E) soft closed set (F_1, E) and a (X, τ_3, τ_4, E) soft closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. There exist (F_3, E) and (G_1, E) such that (F_3, E) is soft (X, τ_3, τ_4, E) and soft open set (G_1, E) is soft (X, τ_1, τ_2, E) open set $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Similarly, (X, τ_3, τ_4, E) is soft T_4 space with respect to (X, τ_1, τ_2, E) is so according to definition for $x, y \in X, x \neq y$ there exists a (X, τ_3, τ_4, E) soft semi open set (F, E) and a (X, τ_1, τ_2, E) soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each (X, τ_3, τ_4, E) soft closed set (F_1, E) and (X, τ_1, τ_2, E) soft closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$.there exists soft open sets (F_3, E) and (G_1, E) such that (F_3, E) is soft (X, τ_3, τ_4, E) β open set (G_1, E) is soft (X, τ_1, τ_2, E) open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Hence $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft T_4 space.

Proposition 12. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and Y be a non-empty subset of X. if $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_3 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_3 space.

Proof: First we prove that $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_1 space. Let $x, y \in X, x \neq y$ if $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise space then this implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft space. So there exists (X, τ_1, τ_2, E) soft open (F, E) and (X, τ_3, τ_4, E) soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ now $x \in Y$ and $x \notin (G, E)$. Hence $x \in Y \cap (F, E) = (Y_F, E)$ then $y \notin Y \cap (\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$. Therefore, $y \notin Y \cap (F, E) = (Y_F, E)$. Now $y \in Y$ and $y \in (G, E)$. Hence $y \in Y \cap (G, E) = (G_Y, E)$ where $(G, E) \in (X, \tau_3, \tau_4, E)$. Consider $x \notin (G, E)$ this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. There fore $x \notin Y \cap (G, E) = (G_Y, E)$ thus $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_1 space.
Now we prove that $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_3 space then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft regular space.
Let $y \in Y$ and (G, E) be a soft closed set in Y such that $y \notin (G, E)$ where $(G, E) \in (X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ then $(G, E) = (Y, E) \cap (F, E)$ for some soft closed set in $(X, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$. Hence $y \notin (Y, E) \cap (F, E)$ but $y \in (Y, E)$, so $y \notin (F, E)$ since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft T_3 space (X, τ_3, τ_4, E) is soft regular space so there exists (X, τ_1, τ_2, E) soft open set (F_1, E) and (X, τ_3, τ_4, E) soft open set (F_2, E) such that
$$y \in (F_1, E), (G, E) \subseteq (F_2, E)$$
$$(F_1, E)(F_2, E) = \phi$$
Take $(G_1, E) = (Y, E) \cap (F_2, E)$ then $(G_1, E), (G_2, E)$ are soft open set in Y such that
$$y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E)$$
$$= (G_2, E)$$
$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$$
$$(G_1, E) \cap (G_2, E) = \phi$$
There fore $\tau_{1Y} \cup \tau_{2Y}$ is soft regular space with respect to $\tau_{3Y} \cup \tau_{4Y}$. Similarly, Let $y \in Y$ and (G, E) be a soft closed sub set in Y such that $y \notin (G, E)$, where $(G, E) \in (X, \tau_3, \tau_4, E)$ then $(G, E) = (Y, E) \cap (F, E)$ where (F, E) is some soft closed set in (X, τ_3, τ_4, E). $y \notin (Y, E) \cap (F, E)$ But $y \in (Y, E)$ so $y \notin (F, E)$ since (X, τ_1, τ_2, E) is soft regular space so there exists (X, τ_3, τ_4, E) soft β open set (F_1, E) and (X, τ_1, τ_2, E) soft open set (F_2, E). Such that
$$y \in (F_1, E), (G, E) \subseteq (F_2, E)$$
$$(F_1, E) \cap (F_2, E) = \phi$$
Take
$$(G_1, E) = (Y, E) \cap (F_1, E)$$
$$(G_1, E) = (Y, E) \cap (F_1, E)$$
Then (G_1, E) and (G_2, E) are soft open set in Y such that
$$y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$$
$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$$
There fore $\tau_{3Y} \cup \tau_{4Y}$ is soft regular space with respect $\tau_{1Y} \cup \tau_{2Y}$
$\Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_3 space.

Proposition 13. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X and Y be a soft closed sub space of X. if $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft T_4 space then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_4 space.

Proof: Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft T_4 space so this implies that $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft T_1 space as proved above.
We prove $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft normal space.
Let $(G_1, E), (G_2, E)$ be soft closed sets in Y such that
$$(G_1, E) \cap (G_2, E) = \phi$$
Then $\qquad (G_1, E) = (Y, E) \cap (F_1, E)$
And $\qquad (G_2, E) = (Y, E) \cap (F_2, E)$
For some soft closed sets such that (F_1, E) is soft closed set in $\tau_1 \cup \tau_2$ soft closed set (F_2, E) in $\tau_3 \cup \tau_4$.
And $\qquad (F_1, E) \cap (F_2, E) = \phi$
From Proposition 2. Since, Y is soft closed sub set of X then $(G_1, E), (G_2, E)$ are soft closed sets in X such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is pair wise soft β normal space. So there exists soft open sets (H_1, E) and (H_2, E) such that

(H_1, E) is soft open set in $\tau_1 \cup \tau_2$ and (H_2, E) is soft open set in $\tau_3 \cup \tau_4$ such that

$$(G_1, E) \subseteq (H_1, E)$$
$$(G_2, E) \subseteq (H_2, E)$$
$$(H_1, E) \cap (H_2, E) = \phi$$

Since $\qquad (G_1, E), (G_2, E) \subseteq (Y, E)$

Then $\qquad (G_1, E) \subseteq (Y, E) \cap (H_1, E)$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And $\qquad [(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$

Where $(Y, E) \cap (H_1, E)$ and $(Y, E) \cap (H_2, E)$ are soft open sets in Y there fore $\tau_{1Y} \cup \tau_{2Y}$ is soft normal space with respect to $\tau_{3Y} \cup \tau_{4Y}$. Similarly, let $(G_1, E), (G_2, E)$ be soft closed sub set in Y such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Then $\qquad (G_1, E) = (Y, E) \cap (F_1, E)$

And $\qquad (G_2, E) = (Y, E) \cap (F_2, E)$

For some soft closed sets such that (F_1, E) is soft closed set in $\tau_3 \cup \tau_4$ and (F_2, E) soft closed set in $\tau_1 \cup \tau_2$ and

$$(F_1, E)(F_2, E) = \phi$$

From Proposition 2. Since, Y is soft closed sub set in X then $(G_1, E), (G_2, E)$ are soft closed sets in X such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is pair wise soft normal space so there exists soft open sets (H_1, E) and (H_2, E)

Such that (H_1, E) is soft open set is $\tau_3 \cup \tau_4$ and (H_2, E) is soft open set in $\tau_1 \cup \tau_2$ such that

$$(G_1, E) \subseteq (H_1, E)$$
$$(G_2, E) \subseteq (H_2, E)$$
$$(H_1, E) \cap (H_2, E) = \phi$$

Since $\qquad (G_1, E), (G_2, E) \subseteq (Y, E)$

Then $\qquad (G_1, E) \subseteq (Y, E) \cap (H_1, E)$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And $\qquad [(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$

Where $(Y, E) \cap (H_1, E)$ and $(Y, E) \cap (H_2, E)$ are soft β open sets in Y there fore $\tau_{3Y} \cup \tau_{4Y}$ is soft normal space with respect to $\tau_{1Y} \cup \tau_{2Y}$

$\Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}E)$ is pair wise soft T_4 space.

6. SOFT AXIOMS WITH RESPECT TO SOFT POINTS

In this section, we brought out soft topological structures known as separation axioms in quad soft topology with respect to soft points. With the applications of these soft separation axioms different result are discussed

Definition 7.1. In a *soft quad topological* space $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$

1) $\tau_1 \cup \tau_2$ said to be *soft* T_0 space with respect to $\tau_3 \cup \tau_4$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens $\tau_1 \cup \tau_2$ soft open set (F, E) and a $\tau_3 \cup \tau_4$ soft open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (G, E)$, Similarly, $\tau_3 \cup \tau_4$ is said to be *soft* T_0 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens $\tau_3 \cup \tau_4$ *soft open set* (F, E) and a $\tau_1 \cup \tau_2$ soft open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$. *Soft quad topological spaces* $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be *pair wise soft* T_0 *space* if $\tau_1 \cup \tau_2$ is *soft T_0 space* with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft T_0 spaces with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft T_1 space with respect to $\tau_3 \cup \tau_4$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens a $\tau_1 \cup \tau_2$ *soft open set* $(F. E)$ and $\tau_3 \cup \tau_4$ soft open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (G, E)$ and $e_H \in (G, E)$ and $e_G \notin (G, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft T_1 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_H \in X_A$ there exist a $\tau_3 \cup \tau_4$ soft open set (F, E) and a $\tau_1 \cup \tau_2$ soft open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (G, E)$ and $e_H \in (G, E)$ and $e_G \notin (G, E)$. *Soft quad topological space* $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be *pair wise soft* T_1 space if $\tau_1 \cup \tau_2$ is *soft T_1 space* with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft T_1 spaces with respect to $\tau_1 \cup \tau_2$.

3) $\tau_1 \cup \tau_2$ is said to be soft T_2 space with respect to $\tau_3 \cup \tau_4$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens a $\tau_1 \cup \tau_2$ soft open set (F, E) and a $\tau_3 \cup \tau_4$ soft open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ and $e_H \in (G, E)$ and $e_G \notin (G, E)$ and $(F, E) \cap (G, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is aid to be *soft* T_2 space with respect to $\tau_1 \cup \tau_2$ if for each pair of distinct points $e_G, e_G \in X_A$ there happens a $\tau_3 \cup \tau_4$ *soft open* set (F, E) and a $\tau_1 \cup \tau_2$ *soft open* set (G, E) such that $e_G \in (F, E)$ and $e_G \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. The soft quad topological space $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be pair wise soft T_2 space if $\tau_1 \cup \tau_2$ is soft T_2 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft T_2 space with respect to $\tau_1 \cup \tau_2$.

Definition 7.2. In a soft quad topological space $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$

1) $\tau_1 \cup \tau_2$ is said to be soft T_3 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft T_1 space with respect to $\tau_3 \cup \tau_4$ and for each pair of distinct points $e_G, e_H \in X_A$, there exists a $\tau_1 \cup \tau_2$ closed soft set (G, E) such that $e_G \notin (G, E)$, $\tau_1 \cup \tau_2$ soft open set (F_1, E) and $\tau_3 \cup \tau_4$ soft open set (F_2, E) such that $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \emptyset$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft T_3 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft T_1 space with respect to $\tau_1 \cup \tau_2$ and for each pair of distinct points $e_G, e_H \in X_A$ there exists a $\tau_3 \cup \tau_4$ soft open set (G, E) such that $e_G \notin (G, E)$, $\tau_3 \cup \tau_4$ soft open set (F_1, E) amd $\tau_1 \cup \tau_2$ soft open set (F_2, E) such that $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be pair wise soft T_3 space if $\tau_1 \cup \tau_2$ is soft T_3 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft T_3 space with respect to $\tau_1 \cup \tau_2$.

2) $\tau_1 \cup \tau_2$ is said to be soft T_4 space with respect to $\tau_3 \cup \tau_4$ if $\tau_1 \cup \tau_2$ is soft T_1 space with respect to $\tau_3 \cup \tau_4$, there exists a $\tau_1 \cup \tau_2$ soft closed set (F_1, E) and $\tau_3 \cup \tau_4$ soft closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \emptyset$, also, there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_1 \cup \tau_2$ open set, (G_1, E) is soft $\tau_3 \cup \tau_4$ open set such that $(F_1, E) \subseteq (F_3, E)$, $(F_2, E) \subseteq (G_1, E)$. Similarly, $\tau_3 \cup \tau_4$ is said to be soft T_4 space with respect to $\tau_1 \cup \tau_2$ if $\tau_3 \cup \tau_4$ is soft T_1 space with respect to $\tau_1 \cup \tau_2$ there exists $\tau_3 \cup \tau_4$ soft closed set (F_1, E) and $\tau_1 \cup \tau_2$ soft closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. Also there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ open set, (G_1, E) is soft $\tau_1 \cup \tau_2$ soft set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Thus, $(X, \tau_1, \tau_2, \tau_1, \tau_2, E)$ is said to be pair wise soft T_4 space if $\tau_1 \cup \tau_2$ is soft T_4 space with respect to $\tau_3 \cup \tau_4$ and $\tau_3 \cup \tau_4$ is soft T_4 space with respect to $\tau_1 \cup \tau_2$.

Proposition 14. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft topological space over X. $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft T_3 space, then for all $e_G \in X_E$ $e_G = (e_G, E)$ is soft-closed set.

Proof: We want to prove that e_G is closed soft set, which is sufficient to prove that e_G^c is open soft set for all $e_H \in \{e_G\}^c$. Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is soft T_3 space, then there exists soft sets $(F, E)_{e_H}$ and (G, E) such that $e_{H_E} \subseteq (F, E)_{e_H}$ and $e_{G_E} \cap (F, E)_{e_H} = \phi$ and $e_{G_E} \subseteq (G, E)$ and $e_{H_E} \cap (G, E) = \phi$. It follows that, $\cup_{e_H \in (e_G)^c (F, E)_{e_H} \subseteq e_{G_E}^c}$ Now, we want to prove that $e_{G_E}^c \subseteq \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$. Let $\cup_{e_H \in (e_G)^c} (F, E)_{e_H} = (H, E)$. Where $H(e) = \cup_{e_H \in (e_G)^c (F(e)_{e_H}}$ for all $e \in E$. Since $e_{G_E}^c(e) = (e_G)^c$ for all $e \in E$ from Definition 9, so, for all $e_H \in \{e_G\}^c$ and $e \in E$ $e_{G_E}^c(e) = \{e_G\}^c = \cup_{e_H \in (e_G)^c} \{e_H\} = \cup_{e_H \in (e_G)^c F(e)_{e_H} = H(e)}$. Thus, $e_{G_E}^c \subseteq \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$ from Definition 2, and so, $e_{G_E}^c = \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$.

This means that, $e_{G_E}^c$ is soft open set for all $e_H \in \{e_G\}^c$. Therefore, e_{G_E} is closed soft set.

Proposition 15. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. Then, if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft T_3 space, then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft T_2 space.

Proof: Suppose if (X, τ_1, τ_2, E) is a soft T_3 space with respect to (X, τ_3, τ_4, E), then according to definition for, $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, (e_H, E) is soft closed set in (X, τ_3, τ_4, E) and $e_G \notin (e_H, E)$ there exist a (X, τ_1, τ_2, E) soft open set (F, E) and a (X, τ_3, τ_4, E) soft open set (G, E) such that $e_G \in (F, E)$, $e_H \in (y, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence, (X, τ_1, τ_2, E) is soft T_2 space with respect to (X, τ_3, τ_4, E) Similarly, if (X, τ_3, τ_4, E) is a soft T_3 space with respect to (X, τ_1, τ_2, E), then according to definition for , $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, (e_G, E) is closed soft set in (X, τ_1, τ_2, E) is and $y \notin (x, E)$ there exists a (X, τ_3, τ_4, E) soft open set (F, E) and a (X, τ_1, τ_2, E) soft open set (G, E) such that $e_H \in (F, E)$, $e_G \in (x, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence, (X, τ_3, τ_4, E) is a soft T_2 space. Thus $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft T_2 space.

Proposition 16. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ be a soft quad topological space over X. If if (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft T_3 space then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is a pair wise soft T_3 space.

Proof: Suppose (X, τ_1, τ_2, E) is a soft T_3 space with respect to (X, τ_3, τ_4, E) then according to definition for $e_G, e_H \in X_A$ $e_G \neq e_H$ there happens $\tau_1 \cup \tau_2$ soft open set (F, E) and a $\tau_3 \cup \tau_4$ soft open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ and for each point $e_G \in X_A$ and each $\tau_1 \cup \tau_2$ closed soft set (G_1, E) such that $e_G \notin (G_1, E)$ there happens a $\tau_1 \cup \tau_2$ soft open set (F_1, E) and $\tau_3 \cup \tau_4$ soft open set (F_2, E) such that $e_G \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly (X, τ_3, τ_4, E) is a soft T_3 space with respect to (X, τ_1, E). So according to definition for $e_G, e_H \in X_A$, $e_G \neq e_H$ there exists a $\tau_3 \cup \tau_4$ soft open set (F, E) and $\tau_1 \cup \tau_2$ soft open set (G, E) such that $e_H \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ and for each point $e_G \in X_A$ and each $\tau_3 \cup \tau_4$ closed soft set (G_1, E) such that $e_G \notin (G_1, E)$ there exists $\tau_3 \cup$

τ_4 soft open set (F_1, E) and $\tau_1 \cup \tau_2$ soft open set (F_2, E) such that $e_G \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ is pair wise soft T_3 space.

Proposition 17. If $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ be a soft quad topological space over X. (X, τ_1, τ_2, E) and (X, τ_3, τ_4, E) are soft T_4 space then $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ is pair wise soft T_4 space.

Proof: Suppose $((X, \tau_1, \tau_2, E))$ is soft T_4 space with respect to (X, τ_3, τ_4, E). So according to definition for $e_G, e_H \in X_A$, $e_G \neq e_H$ there happens a $\tau_1 \cup \tau_2$ soft open set (F, E) and a $\tau_3 \cup \tau_4$ soft open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ each $\tau_1 \cup \tau_2$ soft closed set (F_1, E) and a $\tau_3 \cup \tau_4$ soft closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. There occurs (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ open set (G_1, E) is soft a $\tau_1 \cup \tau_2$ open set $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is soft T_4 space with respect to $\tau_1 \cup \tau_2$ so according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a $\tau_3 \cup \tau_4$ soft β open set (F, E) and a $\tau_1 \cup \tau_2$ soft β open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ and for each $\tau_3 \cup \tau_4$ soft closed set (F_1, E) and $\tau_1 \cup \tau_2$ soft closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$. there occurs (F_3, E) and (G_1, E) such that (F_3, E) is soft $\tau_3 \cup \tau_4$ open set (G_1, E) is soft $\tau_1 \cup \tau_2$ open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$ hence $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ is pair wise soft T_4 space.

Proposition 18. Let $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ be a soft quad topological space over X and Y be a non-empty subset of X. if $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_3 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_3 space.

Proof. First we prove that $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_1 space. Let $e_G, e_H \in X_A, e_G \neq e_H$ if $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ is pair wise soft space then this implies that $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ is pair wise soft $\tau_1 \cup \tau_2$ space. So there exists $\tau_1 \cup \tau_2$ soft open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$ now $e_G \in Y$ and $e_G \notin (G, E)$. Hence $e_G \in Y \cap (F, E) = (Y_F, E)$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$.

There fore, $e_H \notin Y \cap (F, E) = (Y_F, E)$. Now $e_H \in Y$ and $e_H \in (G, E)$. Hence, $e_H \in Y \cap (G, E) = (G_Y, E)$ where $(G, E) \in \tau_3 \cup \tau_4$. Consider $x \notin (G, E)$ .this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. There fore $e_G \notin Y \cap (G, E) = (G_Y, E)$ thus $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_1 space Now, we prove that $(Y, \tau_{1Y}, \tau_{2Y}, \tau_{3Y}, \tau_{4Y}, E)$ is pair wise soft T_3 space.

Let $e_H \in Y$ and (G, E) be soft closed set in Y such that $e_H \notin (G, E)$ where $(G, E) \in \tau_1 \cup \tau_2$ then $(G, E) = (Y, E) \cap (F, E)$ for some soft closed set in $\tau_1 \cup \tau_2$ hence $e_H \notin (Y, E) \cap (F, E)$ but $e_H \in (Y, E)$, so $e_H \notin (F, E)$ since $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ is soft T_3 space

$(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ is soft regular space so there happens $\tau_1 \cup \tau_2$ soft open set (F_1, E) and $\tau_3 \cup \tau_4$ soft open set (F_2, E) such that

$$e_H \in (F_1, E), (G, E) \subseteq (F_2, E)$$
$$(F_1, E)(F_2, E) = \phi$$

Take $(G_1, E) = (Y, E) \cap (F_2, E)$ then $(G_1, E), (G_2, E)$ are soft open sets in Y such that

$$e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E)$$
$$= (G_2, E)$$
$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$$
$$(G_1, E) \cap (G_2, E) = \phi$$

Therefore, (τ_{1Y}, τ_{2Y}) is soft regular space with respect to (τ_{3Y}, τ_{4Y}) Similarly, Let $e_H \in Y$ and (G, E) be a soft closed sub set in Y such that $e_H \notin (G, E)$, Where $(G, E) \in \tau_3 \cup \tau_4$ then $(G, E) = (Y, E) \cap (F, E)$ where (F, E) is some soft closed set in $\tau_3 \cup \tau_4$. $e_H \notin (Y, E) \cap (F, E)$ But $e_H \in (Y, E)$ so $e_H \notin (F, E)$ since $(X, \tau_1, \tau_{2,}\tau_3, \tau_4, E)$ is soft regular space so there happens $\tau_3 \cup \tau_4$ soft open set (F_1, E) and $\tau_1 \cup \tau_2$ soft open set (F_2, E). Such that

$$e_H \in (F_1, E), (G, E) \subseteq (F_2, E)$$
$$(F_1, E) \cap (F_2, E) = \phi$$

Take
$$(G_1, E) = (Y, E) \cap (F_1, E)$$
$$(G_1, E) = (Y, E) \cap (F_1, E)$$

Then (G_1, E) and (G_2, E) are soft open set in Y such that
$$e_H \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$$

$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$$

Therefore (τ_{3Y}, τ_{4Y}) is soft regular space.

7. CONCLUSION

A soft set with single specific topological structure is unable to accept the duty to construct the whole theory. So to make the theory rich, some superfluous structures on soft set has to be announced. It makes, it more springy to develop the soft topological spaces with its countless applications. In this respect we introduce strong topological structure known as soft quad topological structure in this article.

Topology is such supreme branch of mathematics which catches the applications of both pure and applied mathematics in an attractive way. When mathematicians talk about structures then the concept of soft topology automatically come in action. Recently, many scholars have studied the soft set theory which is coined by Molodtsov [16] and carefully applied to many difficulties which contain uncertainties in our social life. Shabir and Naz [18] familiarized and deeply studied the origin of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we continuously study the character of soft separation axioms in quad soft topological spaces with respect to soft points as well as ordinary points of a soft topological space. We introduce soft qT_0 structure, soft qT_1 structure, soft qT_2 structure, soft qT_3 and soft qT_4 structure with respect to soft and crisp points

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