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# **STREAMING CURRENT INDUCED BY FLUID FLOW IN POROUS MEDIA**

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#### *ArticleHistory:*

Received 12 November 2017 Accepted 12 December 2017 Available online 1 January 2018 Streaming current is induced by the relative motion between a fluid and a solid surface under a fluid pressure difference. In this work, a theoretical model for the streaming current coefficient in porous media is firstly developed based on the fractal theory. The proposed model is then compared with experimental data in the literature and other models. It seems that the prediction from the proposed model is in better agreement with the experimental data than the other models. Therefore, the proposed model may be an alternative approach to predict the streaming current coefficient from the zeta potential, rock parameters and fluid properties. It is also proved that fractal theory is the alternative and useful means for studying the transport phenomenon in porous media.

**KEYWORDS**

Streaming current, zeta potential, porous media, rocks.

# **1. INTRODUCTION**

Streaming current in porous media is caused by the charge displacement as a result of an applied pressure inducing a relative motion between a fluid and a solid surface. The streaming current is directly related to the existence of an electric double layer between the fluid and the solid grain surfaces of porous media. Along with streaming potential measurements, streaming current measurements are a sensitive method to characterize the zeta [potential](https://en.wikipedia.org/wiki/Zeta_potential) of surfaces, which is very important in [colloid](https://en.wikipedia.org/wiki/Colloid) and [interface](https://en.wikipedia.org/wiki/Interface_(chemistry)) science, environmental applications, medical applications and others [1-3]. Especially, streaming potential and streaming current measurements play an important role in geophysical applications. For example, they can be used to map subsurface flow and detect subsurface flow patterns in oil reservoirs, to monitor subsurface flow in geothermal areas and volcanoes, to detect seepage through water retention structures such as dams, dikes, reservoir floors, and canals [4-6].

Normally, the modified Helmholtz–Smoluchowski equation (HS equation) is used to relate the streaming current coefficient (SCC) to the zeta potential at the solid-liquid interface, fluid relative permittivity, viscosity, microstructure parameters of porous media. However, to apply the modified HS equation, one needs to determine the effective length and the effective cross-sectional area of the porous media by either experimental measurements or semi-empirical models [7-9]. Additionally, the expression for the SCC has also been theoretically obtained from a bundle of capillary tube model [10,11]. However, to the best of our knowledge the fractal geometry theory is not yet applied to analyze the SCC in porous media. In this work, an analytical expression for the SCC in porous media is firstly developed based on the fractal theory of porous media and on the streaming current in a single capillary. The proposed model is then compared with experimental data available in the literature and other models. It is shown that the prediction from the proposed fractal model is in better agreement with the experimental data than the other models. Therefore, the proposed model may be an alternative approach to estimate the SCC from the zeta potential, rock parameters and fluid properties.

This paper includes five sections. Section 2 describes the theoretical background of streaming current as well as other models available in

literature for the SCC. Section 3 presents the theoretical development of the SCC based on the fractal theory of porous media. Section 4 contains the discussion. Conclusions are provided in the final section

# **1. THEORETICAL BACKGROUND OF STREAMING CURRENT**

## **1.1 Electrical Double Layer**



**Figure 1:** Stern model for the charge and electric potential distribution in the electric double layer at a solid-liquid interface. In this figure, the solid surface is negatively charged and the mobile counter-ions in the diffuse layer are positively charged (in most rock-water systems).

A porous medium is formed by mineral solid grains such as silicates, oxides, carbonates etc. When a solid grain surface is in contact with a liquid, it acquires a surface electric charge [12]. The surface charge repels ions in the electrolyte whose charges have the same sign as the surface charge (called the "coions") and attracts ions whose charges have the opposite sign (called the" counterions" and normally cations) in the vicinity of the electrolyte silica interface. This leads to the charge distribution known as the electric double layer (EDL) as shown in Figure 1. The EDL is made up of the Stern layer, where cations are adsorbed on

the surface and are immobile due to the strong electrostatic attraction, and the diffuse layer, where the ions are mobile. The distribution of ions and the electric potential within the diffuse layer is governed by the Poisson Boltzman (PB) equation which accounts for the balance between electrostatic and thermal diffusion forces [12]. The solution to the linear PB equation in one dimension perpendicular to a broad planar interface is well-known and produces an electric potential profile that decays approximately exponentially with distance as shown in Figure 1. In the bulk liquid, the number of cations and anions is equal so that it is electrically neutral. The closest plane to the solid surface in the diffuse layer at which flow occurs is termed the shear plane or the slipping plane, and the electrical potential at this plane is called the zeta potential (ζ). The zeta potential plays an important role in determining the degree of coupling between the electric flow and the fluid flow in porous media. Most reservoir rocks have a negative surface charge and a negative zeta potential when in contact with ground water [13]. The characteristic length over which the EDL exponentially decays is known as the Debye length and is on the order of a few nanometers for typical grain electrolyte combinations. The Debye length is a measure of the diffuse layer thickness; its value depends solely on the properties of the fluid and not on the properties of the solid surface and is given by (for a symmetric, monovalent electrolyte such as NaCl) [14].

$$
\lambda_d = \sqrt{\frac{\varepsilon_o \varepsilon_r k_b T}{2000Ne^2 C_f}},
$$
\n(1)

where  $k_b$  is the Boltzmann's constant,  $\varepsilon_\theta$  is the dielectric permittivity in vacuum, *ε<sup>r</sup>* is the relative permittivity of the fluid, *T* is temperature (in K), *e* is the elementary charge, *N* is the Avogadro's number and *C<sup>f</sup>* is the electrolyte concentration (mol/L).



Pressure  $P_1 > P_2$ 

**Figure 2:** Development of streaming potential when an electrolyte is pumped through a capillary under a pressure difference  $ΔP = P<sub>1</sub> - P<sub>2</sub>$ (a porous medium is made of an array of parallel capillaries).

## **2.2 Streaming current coefficient**

## *2.2.1 Semi-empirical models*

When a pressure difference is applied across a porous medium, the relative motion happens between the pore fluid and solid grain surface. Then net ions of the diffuse layers move along with the flowing fluid at the same time. This movement of the net ions generates a convection current (called streaming current) in the porous medium as demonstrated in Figure 2 (a porous medium can be approximated as an array of capillaries).

The SCC in a porous medium is determined based on a bundle of capillary tube model as [7, 9,15,16]:

$$
C_{sc} = \frac{I_{sc}}{\Delta p} = \frac{A_e}{L_e} \cdot \frac{\varepsilon_r \varepsilon_o \zeta}{\eta}
$$
 (2)

where *Isc* is the streaming current, ∆*p* is the applied pressure difference across the porous sample;  $L_e$  and  $\,A_e\,$  are the effective length and crosssectional area of the porous sample, respectively, *η* is the dynamic viscosity of the fluid and  $\zeta$  is the zeta potential. Eq. (2) is known as the modified HS equation for streaming current coefficient in porous media [7].

The ratio of  $A_e$  /  $L_e$  can be estimated experimentally by measuring the resistance  $\mathit{R}^{\infty}$  of the porous media saturated by a highly concentrated

electrolyte solution (normally above 10-<sup>2</sup>mol/L for rock systems) with conductivity  $\ K_L^\infty$ . Since for such a high ionic strength, the surface conductivity of the saturated porous media can be considered to be negligible, one may write [7, 9].

$$
\frac{A_e}{L_e} = \frac{1}{K_L^{\infty} R^{\infty}}
$$
\n(3)

Besides Eq. (3) that is used to determine  $\left.A_{e}\right/ L_{e}$  , the ratio of  $\left.A_{e}\right/ L_{e}$ can also be estimated using empirical models available in literature that relate the apparent length *L* and the cross sectional area of the porous media *A* (external dimensions of the porous sample) to the porosity  $\phi$  . *A A*

For instance, according to, the ratio  $A_{\scriptstyle e}$  /  $L_{\scriptstyle e}$  is obtained as *L L e*  $\frac{e}{e} = \frac{A \phi^{2.5}}{2}$ 

[7-9]. Therefore, the SCC is given as

$$
C_{sc} = \frac{A\phi^{2.5}}{L} \cdot \frac{\varepsilon_r \varepsilon_o \zeta}{\eta}
$$
 (4)

#### *2.2.2 Theoretical model*

In a porous medium, the electric current density  $J_e$  (A/m<sup>2</sup>) and the fluidflow  $J_f$  (m/s) are described by the following relations [11,17,18].

$$
J_e = -\sigma_r \frac{\Delta V}{L} - L_{ek} \frac{\Delta p}{L}
$$
 (5)

$$
J_f = -L_{ek} \frac{\Delta V}{L} - \frac{k}{\eta} \frac{\Delta p}{L}
$$
 (6)

where *L* and  $\sigma_r$  are the length and the macroscopic conductivity of the fluid saturated porous sample,  $\Delta V\,$  and  $\Delta\!p\,$  are the electrical potential difference and pressure difference across the sample,  $k$  is the bulk permeability, and  $\emph{L}_{e\bar{k}}\,$  is the electrokinetic coupling coefficient. The first term in Eq. (5) is the Ohm's law and the second term in Eq. (6) is the Darcy's law. The coupling coefficient is the same in Eq. (5) and Eq. (6) because the coupling coefficients must satisfy the Onsager's reciprocal relation in the steady state. According to a study, the coupling coefficient is given by

$$
L_{ek} = -\frac{\phi}{\tau} \cdot \frac{\varepsilon_r \varepsilon_o \zeta}{\eta}
$$
 (7)

Where  $\phi$  is porosity,  $\tau$  is tortuosity of the porous medium [18].

From Eq. (5), it is possible to notice that even if no electrical potential difference is applied (  $\Delta V\,$  = 0), then simply the presence of a pressure difference can produce a streaming current with the electric current density

$$
J_e = -L_{ek} \frac{\Delta p}{L} = \frac{\phi}{\tau} \cdot \frac{\varepsilon_r \varepsilon_o \zeta}{\eta} \cdot \frac{\Delta p}{L}
$$
 (8)

Therefore, the streaming current in the porous medium is given by

$$
I_{sc} = J_e A = \frac{\phi}{\tau} \cdot \frac{\varepsilon_r \varepsilon_o \zeta}{\eta} \cdot \frac{\Delta p}{L} A
$$
 (9)

where *A* is the cross-sectional area of the porous sample.

By definition, the SCC in a porous sample is determined as

$$
C_{sc} = \frac{I_{sc}}{\Delta p} = \frac{\phi.A}{\tau.L} \cdot \frac{\varepsilon_r \varepsilon_o \zeta}{\eta}
$$
 (10)

The streaming potential coefficient *Csp* is defined as the ratio of  $\Delta V/\Delta p\;$  when the total electric current density in Eq. (5) is equal to zero (the streaming current due to  $\Delta p$  is balanced by the conduction current due to  $\,\Delta V$  ). Therefore, the following is obtained

$$
C_{sp} = \frac{\Delta V}{\Delta p} = -\frac{L_{ek}}{\sigma_r} = \frac{\phi}{\tau \sigma_r} \cdot \frac{\varepsilon_r \varepsilon_o \zeta}{\eta} = \frac{L.C_{sc}}{A.\sigma_r}
$$
(11)

Eq. (11) shows the relation between the streaming potential coefficient and the streaming current coefficient for the same fluid saturated porous medium.

# **2. THEORETICAL DEVELOPMENT FROM THE FRACTAL THEORY OF POROUS MEDIA**



**Figure 3:** A porous medium composed of a large number of tortuous capillaries with random radius.

It has been shown that many natural porous media usually have extremely complicated and disordered pore structure with pore sizes extending over several orders of magnitude and their pore spaces have the statistical selfsimilarity and fractal characters [19-22]. Fractal models provide an alternative and useful means for studying the transport phenomenon and analyzing the macroscopic transport properties of porous media. To derive the streaming current coefficient in porous media, a representative elementary volume (REV) of a cylinder of radius *r*rev and length *L*rev is considered [22]. The pores are assumed to be circular capillary tubes with radii varying from a minimum pore radius  $r_{\text{min}}$  to a maximum pore radius *r*max (0 < *r*min < *r*max < *r*rev). A porous medium is assumed to be made up of an array of tortuous capillaries with different sizes (see Figure 3). The cumulative size-distribution of pores is assumed to obey the following fractal law [20-22]:

$$
N(r) = \left(\frac{r_{\text{max}}}{r}\right)^D, \tag{12}
$$

where *N* is the number of capillaries (whose radius  $\geq r$ ) in porous media, *D* is the fractal dimension for pore space,  $0 < D < 2$  in two-dimensional space and  $0 < D < 3$  in three dimensional space [20-22]. As there are numerous capillaries in porous media, Eq. (12) can be considered as a continuous function of the radius.

Differentiating Eq. (12) with respect to *r*, one obtains

$$
-dN = Dr_{\text{max}}^Dr^{-D-1}dr\,,\tag{13}
$$

where -*dN* represents the number of pores from the radius *r* to the radius  $r + dr$ . The minus (-) in Eq. (13) implied that the number of pores decreases with the increase of pore size.

The streaming current in a capillary of radius *r* due to transport of charge in the electric double layer by the fluid under a fluid pressure difference (Δ*Prev*) across the REV is given by

$$
i_{s}(r) = \frac{\pi r^{2} \cdot \mathcal{Z}_{o} \zeta}{\eta} \cdot \frac{\Delta P_{rev}}{L_{\tau}} = \frac{\pi r^{2} \cdot \mathcal{Z}_{o} \zeta}{\eta} \cdot \frac{\Delta P_{rev}}{\tau \cdot L_{rev}} \tag{14}
$$

where  $\, L_{_{\tau}}\,$  is the real length of the tortuous capillary related to the length of the representative elementary volume  $L_{\text{rev}}$  as  $\overline{L}_{\tau} = \tau . L_{\text{rev}}$  [23]. Note that the model assumes a single value of  $\tau$  for all capillaries and this value is considered as a mean tortuosity value of all capillary sizes [22].

The streaming current through the representative elementary volume of the porous medium is the sum of the streaming currents over all individual capillaries and is given by

$$
I_s = \int_{r_{\min}}^{r_{\max}} i_s(r)(-dN). \tag{15}
$$

Substituting Eq. (13) and Eq. (14) into Eq. (15), the following is obtained

$$
I_s = \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{\pi \mathcal{L}_{\phi} \zeta}{\eta} \cdot \frac{\Delta P}{\tau L_{rev}} Dr_{\text{max}}^D r^{1-D} dr = \frac{\pi \mathcal{L}_{\phi} \zeta}{\eta} \cdot \frac{\Delta P_{rev}}{\tau L_{rev}} \cdot \frac{D}{2-D} r_{\text{max}}^D (r_{\text{max}}^{2-D} - r_{\text{min}}^{2-D}).
$$
 (16)

The streaming current density is given by

$$
J_{s} = \frac{I_{s}}{A_{rev}} = \frac{I_{s}}{\pi \frac{r^{2}}{rev}} = \frac{0}{\zeta \varepsilon \varepsilon} \cdot \frac{\Delta P_{rev}}{L_{rev}} \cdot \frac{D}{2 - D} \cdot \frac{r_{\text{max}}^{D}}{r_{rev}^{2}} (r_{\text{max}}^{2 - D} - r_{\text{min}}^{2 - D}), \tag{17}
$$
  
 $r \qquad \tau \eta$ .

where  $A_{rev} = \pi r_{rev}^2$  $A_{_{rev}} = \pi \! r_{_{rev}}^2$  is the cross sectional area of the REV perpendicular to the flow direction.

Additionally, the porosity  $\phi$  of the REV can be calculated from the definition [22].

$$
\phi = \frac{V_{pore}}{V_{rev}} = \frac{\int_{r_{\text{min}}}^{r_{\text{max}}} \pi r^2 L_r(-dN)}{\pi r_{rev}^2 L_{rev}} = \frac{\int_{r_{\text{min}}}^{r_{\text{max}}} \pi r^2 \tau L_{rev}(-dN)}{\pi r_{rev}^2 L_{rev}} = \frac{\int_{r_{\text{min}}}^{r_{\text{max}}} r^2 \tau.(-dN)}{r_{rev}^2}
$$

$$
= \frac{\tau.D}{(2-D)} \frac{r_{\text{max}}^D}{r_{rev}^2} (r_{\text{max}}^{2-D} - r_{\text{min}}^{2-D}) \qquad (18)
$$

Combining Eq. (17) and Eq. (18), one obtains

$$
J_s = \frac{\mathcal{E}_o \zeta}{\eta} \cdot \frac{\phi \Delta P_{rev}}{\tau^2 L_{rev}} \tag{19}
$$

The total streaming current through a cross-sectional area of the porous medium is

$$
I_{s-total} = J_s.A = \frac{\mathcal{E}_o \zeta}{\eta} \cdot \frac{A \phi \Delta P_{rev}}{\tau^2 \cdot L_{rev}}
$$
(20)

where *A* is the apparent cross-sectional area of the porous media.

For a uniform sample of homogeneous media, one has

$$
\frac{\Delta P_{rev}}{L_{rev}} = \frac{\Delta P}{L}
$$
 (21)

where *L* is the apparent length of the porous media, Δ*P* is the fluid pressure difference across the porous media. Eq. (20) is now written as

$$
I_{s-total} = \frac{\mathcal{E}_o \zeta}{\eta} \cdot \frac{A \phi \Delta P}{\tau^2 \cdot L}
$$
 (22)

Therefore, the streaming current coefficient is obtained as

$$
C_{sc} = \frac{I_{s-total}}{\Delta P} = \frac{\mathcal{E}_o \zeta}{\eta} \cdot \frac{A \phi}{\tau^2 L}.
$$
 (23)

Eq. (23) indicates that the SCC obtained in this work using the fractal theory for porous media is also related to the zeta potential, the fluid relative permittivity, the viscosity, the cross-sectional area and the length of the porous sample in the similar way to Eq. (10) that is obtained from a bundle of capillary tube model. The main difference between those equations is the term of  $\tau$  in the denominator of Eq. (23) due to taking into account of the tortuosity in our work.

# **3. DISCUSSION**

To examine the proposed model for the SCC, the experimental data reported in [24] for ten cylindrical sandstone samples (25 mm in diameter and around 20 mm in length) saturated by six different salinities (0.02, 0.05, 0.1, 0.2, 0.4 and 0.6 mol/l NaCl solutions) are used. Parameters of the samples of sandstone are reported in a study and re-shown in Table 1 in

which the tortuosity is obtained from the relation  $\tau = F.\phi$  [24,25].

The measured zeta potential at the different electrolyte concentrations reported in a study using Eq. (2) and Eq. (3) is re-shown in Table 2 [24]. From the measured zeta potential, the diameter, the length, the porosity and the tortuosity of the rock samples, the SCC is calculated using Eq. (23)

and is shown in Table 3 (  ${\cal E}_r$  is taken as 80,  ${\cal E}_o\,$  is taken as 8.854 F/m and  $\eta$  is taken as 0.001 Pa.s [26].

Sample ID Density $\rho_{s}$		Length	Porosity	Formation Factor F	Tortuosity	
		$L$ (mm)	(percent) $\phi$	(no units)	$\tau = F.\phi$ (no units)	
	(g/cm <sup>3</sup> )					
D <sub>1</sub>	2.61	21	30.6	9.13	2.79	
D <sub>2</sub>	2.61	20	30.2	7.87	2.37	
D <sub>3</sub>	2.61	23	30.9	8.42	2.60	
D4	2.63	23	32.1	8.64	2.77	
D5	2.65	22	29.8	8.32	2.47	
D6	2.62	23	31.0	8.49	2.63	
D7	2.65	22	29.4	8.15	2.39	
D <sub>8</sub>	2.61	16	31.0	11.79	3.65	
D <sub>9</sub>	2.65	18	29.3	9.30	2.72	
D <sub>10</sub>	2.62	18	31.5	8.79	2.76	

**Table 1:** The parameters of sandstone samples reported in [10].

**Table 2:** The magnitude of the zeta potential (mV) at different electrolyte concentrations (mol/l) reported in [10].

Sample ID	$C_f = 0.02$	$C_f = 0.05$	$C_f = 0.1$	$C_f = 0.2$	$C_f = 0.4$	$C_f = 0.6$
D1	62.20	48.62	40.97	35.35	32.76	18.66
D <sub>2</sub>	92.92	57.06	51.86	48.96	35.60	20.80
D <sub>3</sub>	65.14	41.02	37.16	31.11	26.19	19.54
D <sub>4</sub>	71.54	60.89	46.76	39.00	32.50	22.86
D <sub>5</sub>	88.20	72.70	60.86	33.53	32.23	21.69
D6	67.58	46.19	27.12	27.07	28.79	21.71
D <sub>7</sub>	105.45	62.67	53.56	44.87	31.28	26.39
D <sub>8</sub>	240.33	144.79	90.45	63.32	47.98	25.55
D <sub>9</sub>	124.02	76.47	57.30	50.17	36.14	21.55
D <sub>10</sub>	135.44	63.87	51.75	38.04	34.75	22.55

Based on Table 3, the variation of the SCC deduced from Eq. (23) with electrolyte concentration for various rock samples is shown in Fig. 4a. Additionally, a group researchers experimentally obtained the SCC by measuring the streaming current and the applied pressure difference for the same rocks [24]. The variation of the measured SCC reported in same study with electrolyte concentration is shown in Figure 4b [24]. The values of the SCC predicted from Eq. (10) and Eq. (4) with the same input parameters as those for Eq. (23) are shown in Fig. 4c and Fig. 4d, respectively. It is seen that the values of the SCC predicted from all models (described in Eq. (4), Eq. (10) and Eq. (23)) are in the same ranges as those measured in [24]. However, the results obtained from Eq. (23) are better agreement with the experimental than the others.

**Table 3:** The magnitude of streaming current coefficient obtained by using Eq. (23) at different electrolyte concentrations for various rock samples (in

pA/Pa).							
Sample ID	$C_f = 0.02$	$C_f = 0.05$	$C_f = 0.1$	$C_f = 0.2$	$C_f = 0.4$	$C_f = 0.6$	
D <sub>1</sub>	39.47	30.85	25.99	22.43	20.78	11.84	
D <sub>2</sub>	84.38	51.81	47.09	44.46	32.32	18.88	
D <sub>3</sub>	44.00	27.71	25.10	21.01	17.69	13.20	
D4	44.09	37.52	28.81	24.03	20.02	14.08	
D <sub>5</sub>	66.09	54.47	45.60	25.12	24.15	16.25	
D <sub>6</sub>	44.63	30.50	17.91	17.87	19.01	14.33	
D7	83.32	49.52	42.32	35.45	24.71	20.85	
D <sub>8</sub>	118.46	71.37	44.58	31.21	23.65	12.59	
D <sub>9</sub>	92.27	56.89	42.63	37.32	26.88	16.03	
D <sub>10</sub>	105.03	49.53	40.13	29.50	26.94	17.48	

Additionally, some researchers have reported the streaming potential coefficient  $C_{sc}$ , the zeta potential  $\zeta$ , the electrical conductivity of the saturated samples  $\sigma_{\scriptscriptstyle r}$  , porosity  $\phi$ , the length  ${\scriptstyle L}$  and the cross-sectional area *A* for six cylindrical Berea Sandstone rocks at seven different electrolyte concentrations [27]. The experimental data given in are used to compare the SCC predicted from Eq. (23) and the SCC deduced from Eq. (11) [27]. The variation of the SCC with the electrolyte concentration for the Berea samples obtained from Eq. (23) and Eq. (11) is shown in Figure 5. It is seen that the proposed model is also able to reproduce the similar trend to the experimentally deduced result. Therefore, the proposed model based on the fractal theory is an alternative approach to predict the SCC from the zeta potential, rock parameters and the fluid properties.



**Figure 4:** Variation of the streaming current coefficient (SCC) predicted from Eq. (23) (see Figure 4a), from experimental data obtained from (see Figure 4b), from Eq. (10) (see Figure 4c) and from Eq. (4) (see Figure 4d) with different electrolyte concentration for various rock samples [24]



**Figure 5:** Variation of the streaming current coefficient (SCC) with electrolyte concentration for six Berea samples. Figure 5a is deduced from experimental data reported in a study using Eq. (11). Figure 5b is predicted from Eq. (23) with the knowledge of *L, ϕ*, *A* (A = π*d*2/4) and *ζ* reported in [27].

## **4. CONCLUSIONS**

In this work, a model for the streaming current coefficient has been derived based on the fractal geometry theory for porous media and the capillary model. The proposed model is related to the zeta potential, the fluid relative permittivity, the viscosity, the cross-sectional area and the length of the porous sample. The model predictions are performed and compared with those based on the hydraulic radius model, the semiempirical model and experimental data available in the literature. A good agreement is found between the predictions by the models and experimental data. Furthermore, it is seen that the proposed model is also able to reproduce the similar trend to the experimentally deduced result. Therefore, the proposed model based on the fractal theory can be an alternative approach to predict the magnitude of the SCC from the zeta potential and the fluid properties. It is also proved that fractal theory is the alternative and useful means for studying the transport phenomenon in porous media.

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