

## Dynamical Modeling and Optimization of the Roll Forming Machine based on the Particle Swarm Optimization with Negative Gradient

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**Abstract:** Dynamical modeling and optimization of mechanical system is a multi discipline complex problems. This paper deduced Lagrange equations based on 4-DOF one-sided variable cross section roll-forming system and proposed the particle swarm optimization with negative gradient. The mechanical parameters of roll-forming system are optimized by using this algorithm. Optimization results show that the dynamic performance of the roll-forming system has been effectively improved. The algorithm provides theoretic foundation for the optimization design of the roll-forming system. *Copyright © 2013 IFSA.*

**Keywords:** Dynamics; Roll-forming; Particle swarm optimization; Negative gradient.

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### 1. Introduction

In cold-roll forming, a metal sheet is fed through successive pairs of forming rolls until it is formed into some desired cross-sectional profile [1]. It has many advantages such as high production efficiency, low cost, and high bearing capacity. Roll forming process is a kind of quite complex deformation process. There have been many influential factors to be considered and solved. With the development of roll forming technology, massive outstanding achievements correlating with this have emerged up until now [2-6].

Establish the dynamics of roll forming mechanical system is the important theoretical basis of analyzed its kinetic properties, improve its performance and reasonable design every kind of parameters. However, is not a small task, precisely because of the complexity of roll forming machine.

Its dynamic model is a complicated system because they generally have many variables, many of which are nonlinear and coupled with each other.

This paper is modeled on the 4-DOF one-sided variable cross section roll-forming machine, deduced the dynamical Lagrange equations of this system. Finally, the expression of angular acceleration and angular velocity is obtained by solving the equation. The optimization became in need to satisfy the request of the dynamics and the stability requirements for this system. However, due to the complexity of the roll forming machine, it is very difficult and time-consuming to solve this kind of problem using the traditional optimization method.

Particle swarm optimization (PSO) was first introduced by Eberhart and Kennedy [7]. Particle swarm optimization is evolutionary computation technique. It came of the research on bird flock preying behavior. The particle swarm algorithm

which was characterized by its simple calculation, good robustness, and parallel calculation was adopted to many fields [8-12]. Though the particle swarm optimization has certain superiority, like other random optimization algorithms, the PSO also has the phenomenon of prematurely convergence and that has easy to be run into local optimum at a later time.

This article is intended the improved particle swarm optimization for the Dynamical equations based on 4-DOF one-sided variable cross section roll forming system. Then, based on bionics, the negative gradient is added into the formula of classical particle swarm optimization (PSO) algorithm. This method could effectively coordinate the global search and the local search ability. PSO was modified here to improve the solving precision and convergent rates of optimization procedure. This algorithm is applied to the dynamics equations of the 4-DOF one-sided variable cross section roll-forming system, used the angular acceleration of the servo motor as objective function and the structural parameters of the roll forming machine as design variables. By optimizing, we hoped to decrease the amplitude of the driven servo motor angular acceleration.

## 2. The Basic Particle Swarm Optimization Algorithm

Particle swarm algorithm was an optimization algorithm based on swarm intelligence. Compared with other optimization algorithm, particle swarm optimization has a simple structure and great universality. In the standard particle swarm algorithm, a total of  $i$  particles consist of a population which corresponds to a total of  $i$  bird consist of bird flock. Each particle equals to a feasible solution of algorithm in  $D$ -dimensional searching space. Set there are  $i$  particles in all,  $X_i = (x_{i1}, x_{i2}, \dots, x_{id})$  indicating the positions of the any particle,  $V_i = (v_{i1}, v_{i2}, \dots, v_{id})$  indicating the velocity.  $P_i = (p_{i1}, p_{i2}, \dots, p_{id})$  indicating the best previous position of the  $i^{\text{th}}$  particle, denoted as  $pbest$ .  $P_g = (p_{g1}, p_{g2}, \dots, p_{gd})$  indicating the best previous position of the population, denoted as  $gbest$ . Particle updating position and velocity by following equations:

$$v_{id}^{k+1} = \omega v_{id}^k + c_1 r_1 (p_{id}^k - x_{id}^k) + c_2 r_2 (p_{gd}^k - x_{id}^k) \quad (1)$$

$$x_{id}^{k+1} = x_{id}^k + r v_{id}^{k+1}, \quad (2)$$

where  $v_{id}^k$  is the  $i$ -th particle's velocity in  $d$  dimension at  $k$  iteration;  $\omega$  is the inertia weight, that control the particle's own original velocity and can balancing the global and local search. A larger  $\omega$

represents strong capability of global search and a smaller  $\omega$  represents strong local search ability.

$c_1$  and  $c_2$  are called weight coefficient, can track the personal best positions and the best position of the swarm respectively, they are usually set to  $c_1 = c_2 = 2$ .  $r_1$  and  $r_2$  are random number of range  $[0,1]$ ,  $r$  is constraint factor, usually set to 1. During the update process, the velocity of particles in every dimension limited by  $[-v_{\max}, v_{\max}]$ .

## 3. Particle Swarm Optimization Algorithm with Negative Gradient

Like other optimization algorithm, particle swarm optimization algorithm has its flaws as well. In this paper, in order to make particle swarm optimization algorithm can adapt to dynamic optimization problem of roll forming machine, some improvement was made based on bionics.

Steepest descent method has proposed by a noted mathematician Cauchy in 1847, also known as gradient method [13]. That is, if real-valued function  $f(x)$ , it is differentiable and defined at point  $a$ , then function  $f(x)$  fast descend in opposite directions of the gradient  $-\nabla f(a)$ . On the small neighborhood of point  $a$ , follows the direction of opposite gradient, the values of the functions

$$-\nabla f(a) = \left[ -\frac{\partial f(a)}{\partial x_1}, -\frac{\partial f(a)}{\partial x_2}, \dots, -\frac{\partial f(a)}{\partial x_d} \right]^T$$

decrease most quickly. From the point of view of bionics, the opposite directions of the gradient is the equivalent of the track at the process of the bird looking for food bird when the bird seeing food and flying to food directly.

In the initial stage of algorithm, the food is too distant and small to be seen directly. But in the later period of the optimization, with the bird close to food, the possibility of the bird seeing food would be increased. According to this idea, after added the opposite gradient term, An Improved Particle Swarm Optimization algorithm applied to the opposite gradient is proposed followed by the Basic Particle Swarm Optimization.

The updating of formula for the velocity and the position of the improved particle swarm optimization algorithm (NPSO) is as follows:

$$v_{id}^{k+1} = \omega v_{id}^k + c_1 r_1 (p_{id}^k - x_{id}^k) + c_2 r_2 (p_{gd}^k - x_{id}^k) + c_3 r_3 \left[ -\frac{\partial f(x_i^k)}{\partial x_d} \right] \quad (3)$$

$$x_{id}^{k+1} = x_{id}^k + r v_{id}^{k+1}, \quad (4)$$

where  $\omega = 0.9 - \frac{(0.9-0.4)}{i_{\max}} \times i$  is the linear decreasing inertia weight [14],  $i_{\max}$  is the maximum evolutionary generation,  $c_3 = \frac{2}{\pi} \arctan(k)$  and  $r_3$  is the random number between 0 and 1.

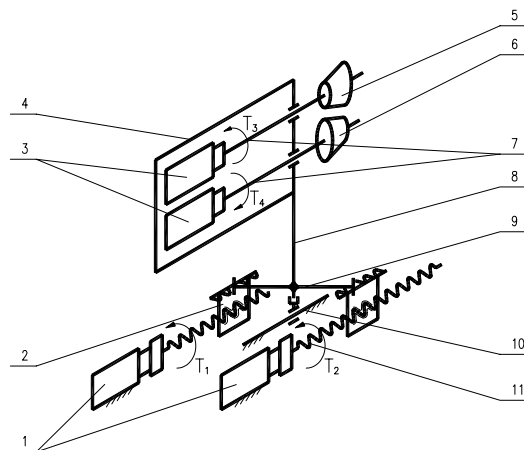
From equation (3), inertia weight decreased and the weight of negative gradient increased as the increase of iteration times. This is equivalent to bird far away from food at the beginning of the algorithm and the proportion of negative gradient is relatively lower to prevent algorithms from local optimum. Otherwise, bird approaching food slowly in the later period of the optimization. Then, the rising proportion of negative gradient can accelerate the converging speed.

#### 4. Dynamical Analysis of Roll Forming Machine

One-sided variable cross section roll-forming system has 4 degrees of freedom. The dynamic equations of the mechanical system can be represented with the Lagrange equations [15].

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = Q_k \quad (k = 1, 2, \dots, N), \quad (5)$$

where  $L$  is the system's kinetics,  $q_k$  is a generalized coordinate;  $Q_k$  is the generalized force. The Lagrange equations provide a separate set of equations that have the same freedom as the system. The sketch of One-sided variable cross section roll-forming system is as shown in Fig. 1.



**Fig. 1.** Sketch Map of One-pass One-sided Variable Cross Section Roll-forming Machine. 1- servo motor, 2- gear rack, 3- servo motor, 4- roller frame 5- upper roller group, 6- down roller group, 7- roller axis 8- vertical turning axis of the frame, 9- gear, 10- sliding guide, 11- ball screw.

Ball screw 11 driven by servo motor 1 led gear rack 2 moving in a straight line. On the inside of the gear rack 2 has rack. Both sides of racks can simultaneously mesh with gear 9. If both sides of racks move at same speed and direction, gear 9 together with frame 4 will be driven back or forward. If both sides of racks move at same speed and reverse direction, the gear 9 and frame 4 will be driven to rotate around vertical axis of the gear. If the racks will move at different speed and direction, gear 9 and frame 4 will be driven to move along transversal direction, and rotate around the axis of the gear at the same time. Meanwhile, servo motor 3 drives both the upper and lower rolls to rotate around its axis.

Set the rotation angle, angular velocity, and torque of servo motors 3 are respectively expressed as  $\varphi_i$ ,  $\omega_i$  and  $T_i (i=1,2,3,4)$ . The direction of the torque is shown in Fig. 1. The pitch of the ball screw is  $h$ , speed ratio of reducer that connects the motor shaft and the ball screw is  $i$ , the pitch radius of gear is  $r$ . As the relationship shown in Fig. 2, in the left side, straight-line velocity of the rack is  $v_j = \frac{h}{2\pi} \dot{\varphi}_j (j=1,2)$ , translational velocity of the gear is:  $v_y = \frac{1}{2}(v_1 + v_2) = \frac{h}{4\pi i}(\dot{\varphi}_1 + \dot{\varphi}_2)$ , rotation speed of gear around the axis is:  $\omega_z = \frac{1}{2r}(v_1 - v_2) = \frac{h}{4\pi r i}(\dot{\varphi}_1 - \dot{\varphi}_2)$ .

$J_{dji} (i = 1, 2, 3, 4)$  is the moment of inertia of servo motor;  $J_{jsi} (i = 1, 2, 3, 4)$  is the moment of inertia of reducer connected with servo motor;  $J_{sgj} (j = 1, 2)$  is the moment of inertia of ball screw;  $J_{cl}$  is the moment of inertia of gear;  $m_{zz}$  is the mass of rotating spindle;  $J_{zz}$  is the moment of inertia of rotating spindle;  $m_{zs}, m_{ys}$  is the mass of gear rack;  $m_{xj}$  is the mass of rotating frame assembly;  $J_{xj}$  is the moment of inertia of frame assembly;  $J_{uz}, J_{lz}$  is the moment of inertia of upper and lower roller axle;  $J_{ug}, J_{lg}$  is the moment of inertia of upper and lower roller.

Then substituted its in formula (5), the dynamics equation of one-sided variable cross section roll-forming system was obtained.

$$\begin{cases} J_{11}\ddot{\varphi}_1 + J_{12}\ddot{\varphi}_2 = T_1 + \frac{h}{4\pi r i}(M_{uv} + M_{dv}) + \frac{h}{4\pi i}(F_{ua} + F_{da}) \\ J_{22}\ddot{\varphi}_2 + J_{12}\ddot{\varphi}_1 = T_2 + \frac{h}{4\pi r i}(M_{uv} + M_{dv}) + \frac{h}{4\pi i}(F_{ua} + F_{da}) \\ J_{33}\ddot{\varphi}_3 = T_3 + \frac{M_{uh}}{i} \\ J_{44}\ddot{\varphi}_4 = T_4 + \frac{M_{dh}}{i} \end{cases} \quad (6)$$

Then angular acceleration of servo motor can be solved:

$$\begin{cases}
 \varphi_1 = \frac{\begin{bmatrix} T_1 + \frac{h}{4\pi r i} (M_{uv} + M_{dv}) - \frac{h}{4\pi i} (F_{ua} + F_{da}) & J_{12} \\ T_2 - \frac{h}{4\pi r i} (M_{uv} + M_{dv}) - \frac{h}{4\pi i} (F_{ua} + F_{da}) & J_{22} \end{bmatrix}}{\begin{bmatrix} J_{11} & J_{12} \\ J_{12} & J_{22} \end{bmatrix}} \\
 \varphi_2 = \frac{\begin{bmatrix} J_{11} & T_1 + \frac{h}{4\pi r i} (M_{uv} + M_{dv}) - \frac{h}{4\pi i} (F_{ua} + F_{da}) \\ J_{12} & T_2 - \frac{h}{4\pi r i} (M_{uv} + M_{dv}) - \frac{h}{4\pi i} (F_{ua} + F_{da}) \end{bmatrix}}{\begin{bmatrix} J_{11} & J_{12} \\ J_{12} & J_{22} \end{bmatrix}} \\
 \varphi_3 = \frac{1}{J_{33}} \left( T_3 + \frac{M_{uh}}{i} \right) \\
 \varphi_4 = \frac{1}{J_{44}} \left( T_4 + \frac{M_{dh}}{i} \right)
 \end{cases} \quad (7)$$

Among it,

$$\begin{aligned}
 J_{11} &= \frac{1}{i^2} \left[ i^2 J_{dj1} + i^2 J_{js1} + J_{sg1} + \left( \frac{h}{4\pi r} \right)^2 J_{cl} + \right. \\
 &\quad m_{zz} \left( \frac{h}{4\pi} \right)^2 + J_{zz} \left( \frac{h}{4\pi} \right)^2 + m_{zs} \left( \frac{h}{2\pi} \right)^2 + \\
 &\quad \left. m_{xj} \left( \frac{h}{4\pi} \right)^2 + J_{xj} \left( \frac{h}{4\pi r} \right)^2 \right] \\
 J_{22} &= \frac{1}{i^2} \left[ i^2 J_{dj2} + i^2 J_{js2} + J_{sg2} + \left( \frac{h}{4\pi r} \right)^2 J_{cl} + \right. \\
 &\quad m_{zz} \left( \frac{h}{4\pi} \right)^2 + J_{zz} \left( \frac{h}{4\pi r} \right)^2 + m_{ys} \left( \frac{h}{2\pi} \right)^2 + \\
 &\quad \left. m_{xj} \left( \frac{h}{4\pi} \right)^2 + J_{xj} \left( \frac{h}{4\pi r} \right)^2 \right] \\
 J_{12} &= \frac{1}{i^2} \left[ -J_{cl} \left( \frac{h}{4\pi r} \right)^2 + m_{zz} \left( \frac{h}{4\pi} \right)^2 \right. \\
 &\quad \left. J_{zz} \left( \frac{h}{4\pi r} \right)^2 + m_{xj} \left( \frac{h}{4\pi} \right)^2 - J_{xj} \left( \frac{h}{4\pi r} \right)^2 \right] \\
 J_{33} &= \frac{1}{i^2} \left[ i^2 J_{dj3} + i^2 J_{js3} + J_{ug} + J_{uz} \right] \\
 J_{44} &= \frac{1}{i^2} \left[ i^2 J_{dj4} + i^2 J_{js4} + J_{dg} + J_{dz} \right]
 \end{aligned}$$

### 5. Example Analysis

With one stand of roll forming machine that forming certain variable cross-sectional part as an example. Plug geometrical dimensions of roll

forming machine, counter-force of roller  $F_{ua}$  and  $F_{da}$ , torque of roller  $M_{uv}$ ,  $M_{dv}$ ,  $M_{uh}$ ,  $M_{dh}$ , torque of servo motor  $T_i (i = 1, 2, 3, 4)$  into the equation (6) and (7), we can obtain angular acceleration curves of servo motor  $\ddot{\varphi}_i (i = 1, 2, 3, 4)$ , as shown in Fig. 2.

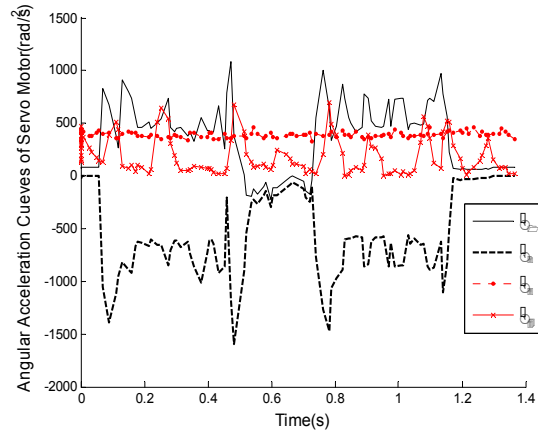


Fig. 2. Angular Acceleration Curves of Servo Motor.

Dynamic optimization problem of the one-sided variable cross section roll-forming system is how to select system parameters to minimize angular acceleration of servo motors. Design variables are: parameters of roll forming system (mass, inertia) and motor parameters

$$X = \{r, J_{cl}, m_{zz}, J_{zz}, m_{xj}, J_{xj}, J_{ug}, J_{dg}, J_{dj3}, J_{dj4}\}$$

The constraint conditions are: the dynamics equations of one-sided variable cross section roll-forming system (6) and (7), the value of design variables. The basic method to solve the multiobjective optimization problem is the evaluation function method [16]. Its basic idea of this method is to use intuitive background of the geometry and application to construct the objective function, that is translate multiobjective optimization problem into single objective optimization problem. Then use it to determine the optimum solution, Meanwhile this is the optimum solution of the multiobjective optimization problem.

According to the above conditions, applies an ideal point method for solving the model. That is found an optimum solution of single objective optimization problem  $\min(\ddot{\varphi}_i) (i = 1, 2, 3, 4)$ , because there is no design variables that make all single objective can attain superior in the meantime. Under this metric, we hope to seek a point as an evaluation function that is as close to the ideal point as possible and it is generally with Euclid distance as modulus. So, the objective function of the multi-objective Particle Swarm Optimization algorithm is:

$$fitness = \min(\sqrt{\sum_{i=1}^4 [f(X_i) - \min(\varphi_i)]^2}) \quad (8)$$

The particle swarm optimization algorithm with negative gradient is used to design the optimization of the one-sided variable cross section roll forming system and the reasonable results are achieved. Design variables and the constraint conditions are as shown in Table 2.

Set the number of particles is 50, the maximum number of iteration is 100, the optimize results is as shown in Table 2. To substitute the optimization results into equation we can get the angular acceleration of servo motor. The optimization results indicated that no significant difference existed between the two groups in the angular acceleration of servo motor 1, 2. But the values of the angular acceleration of servo motor 3, 4 decreased significantly. The angular acceleration curves of before and after the optimization is as shown in Fig. 3 and Fig. 4.

Thus it can be seen that this optimization method is efficient to improve performance of the roll forming system.

### 6. Conclusions

1) From the energy standpoint, the Lagrange equations were used to establish the dynamic equations of the one-sided variable cross section roll-forming system. Angular acceleration of servo motor is obtained by solving the equation.

2) To the optimization problem of one-sided variable cross section roll-forming system, this paper presents a particle swarm optimization algorithm with negative gradient based on bionics. The parameters of mechanical systems are optimized for this method. The optimization results indicated that amplitude of angular acceleration of the upper and lower roller servo motor enable effectively reduced. The primary experiments and theorized analysis prove the method is approving and promising.

Table 2. Optimum results of objective variables.

Design variables	Pitch radius of gear	Moment of inertia of gear	Mass of rotating spindle	Moment of inertia of rotating spindle	Mass of rotating frame assembly
	$r$ (m)	$J_{cl}$ (Kg · m <sup>2</sup> )	$m_{zz}$ (Kg)	$J_{zz}$ (Kg · m <sup>2</sup> )	$m_{xj}$ (Kg)
Before optimization	0.132	0.27	263.4	0.02	422.32
After optimization	0.2	0.4337	323.89	0.02053	541.28
Design variables	Moment of inertia of rotating frame assembly	Moment of inertia of upper roller	Moment of inertia of lower roller	Moment of inertia of upper roller servo motor	Moment of inertia of lower roller servo motor
	$J_{xj}$ (Kg · m <sup>2</sup> )	$J_{ug}$ (Kg · m <sup>2</sup> )	$J_{dg}$ (Kg · m <sup>2</sup> )	$J_{dj3}$ (Kg · m <sup>2</sup> )	$J_{dj4}$ (Kg · m <sup>2</sup> )
Before optimization	36.36	0.0326	0.031	0.0026	0.0026
After optimization	46.661	0.052584	0.042543	0.0036	0.0036

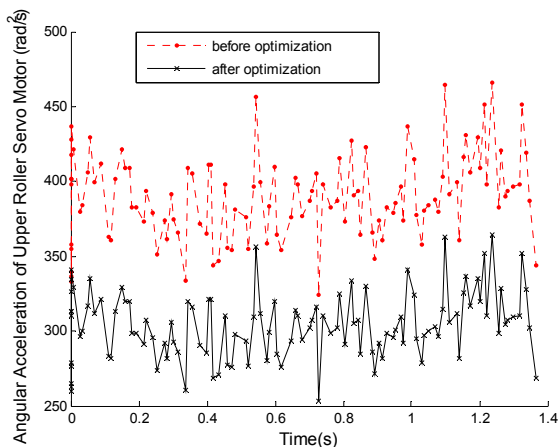


Fig. 3. Angular Acceleration of Upper Roller Servo Motor Before-and-after Optimization.

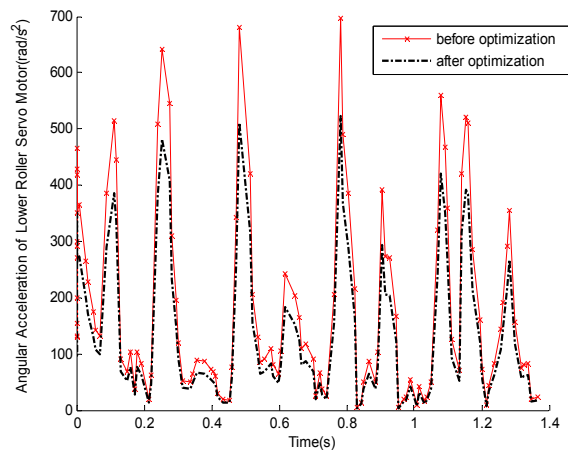


Fig. 4. Angular Acceleration of Lower Roller Servo Motor Before-and-after Optimization.

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