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INTUITIONISTIC FUZZY INTERIOR IDEAL OF SEMIGROUP BASED ON INTUITIONISTIC FUZZY POINT

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ABSTRACT

The intuitionistic fuzzification of the notion of an interior ideal in ordered semigroups is considered. The purpose of this study is to introduce the notion of $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideals and $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideals of semigroups. The important milestone of the present paper is to link ordinary intuitionistic fuzzy interior ideals, $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideals and $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideals. Moreover, semigroups are characterized by the properties of these new notions.

1. INTRODUCTION

Fuzzy set theory introduced by Zadeh is a useful tool to describe situations in which the data are imprecise or vague and handle such situations by attributing a degree to which a certain object belongs to a set [1]. Just after the establishment of fuzzy set theory, it has developed in several directions and is discovering applications in a wide variety of fields. Rosenfeld used this idea to develop the theory of fuzzy groups [2]. Fuzzy set theory provides a natural framework for generalization of the basic notions of algebra. But there is no means to incorporate the hesitation or uncertainty in the membership degrees. Atanassov introduced the concept of intuitionistic fuzzy sets in 1986, which constitute an extension of a fuzzy set theory [3]. Since then, many researchers have investigated this topic such as intuitionistic fuzzy group [4-6]. Intuitionistic fuzzy sets give both a membership degree and a non-membership degree, where sum of membership degree and non-membership degree needs to be less or equal to 1. Pu and Liu introduced the notions of "belongings" and "quasi-coincidence" of fuzzy point and fuzzy set [7]. Bhakat and Das introduced $(\in,\in\vee q)$-fuzzy subgroup as a generalization of Rosenfeld's fuzzy subgroups by using the combine notions of "belongings ($\in$)" and "quasi-coincidence (q)" of fuzzy point and fuzzy set [8]. In semigroup Jun and Song gave the concept of an (α,β)-fuzzy interior ideal, which is a generalization of a fuzzy interior ideal by using the idea of quasi-coincidence of a fuzzy point with a fuzzy set [9]. In Davvaz and Khan discussed some characterizations of regular ordered semigroups in terms of (α,β)-fuzzy generalized bi-ideals, where $\alpha,\beta\in\{\in,q,\ \in\vee q,\ \in\wedge q\}$ and $\alpha\neq\in\wedge q$ [10]. Some authors have investigated similar type of generalizations of other algebraic structures. Kazanci and Yamak introduced the concept of a generalized fuzzy bi-ideal in semigroups and gave some properties of fuzzy bi-ideals in terms of $(\in,\in\vee q)$-fuzzy bi-ideals [11]. A group of scientists characterized regular semigroups by the properties of $(\in,\in\vee q)$-fuzzy ideals, bi-ideals and quasi-ideals [12]. In a research, author has introduced more generalized forms of (α,β)-fuzzy ideals and defined $(\in,\in\vee q_k)$-fuzzy ideals of semigroups [13]. Kazanci and Yamak [11] gave $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy bi-ideals of a semigroup. Another researcher introduced $(\in,\in\vee q)$-fuzzy interior ideals of ordered semigroups and investigated some characterizations of ordered semigroups in terms of $(\in,\in\vee q)$-fuzzy interior ideals [14]. The notion of $(\overline{\in},\overline{\in}\vee\overline{q})$-fuzzy interior ideal of semigroup is introduced in [15]. A studied of $(\overline{\in},\overline{\in}\vee\overline{q}_k)$-fuzzy ideals, generalized bi-ideals and quasi-ideals of a semigroup and characterized regular semigroups by the properties of these ideals [16].

Kim and Jun introduced the notion of an intuitionistic fuzzy interior ideal of a semigroup S and gave its characterization [5]. In a study conducted by some researcher redefined (α,β)-intuitionistic fuzzy subgroups [6]. The readers are referred to for further reading regarding (α,β)-fuzzy subsets and their generalization [17-24].

In this paper, we introduce the notion of $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideals of a semigroup and discuss some interesting results and investigate the relationships among ordinary intuitionistic fuzzy interior ideals, $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideals and $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideals of semigroups.

2. BASIC DEFINITIONS AND PRELIMINARIES

In what follows S will represent a semigroup unless otherwise stated. A non-empty subset A of S is called a subsemigroup of S if $A^2\subseteq A$.

A non-empty subset A of a semigroup S is called an interior ideal of S if:

i. $A^2 \subseteq A$,
ii. $SAS \subseteq A$.

An intuitionistic fuzzy subset (briefly IFS) A in a non-empty set X is an object having the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle \mid x\in X\}$, where the function $\mu_A : X\to[0,1]$ and $\gamma_A : X\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for all $x\in X$. For the sack of simplicity, we shall use the symbol $A=\langle x,\mu_A,\gamma_A\rangle$ for the intuitionistic fuzzy set $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle \mid x\in X\}$.
Let $A=\langle x,\mu_A,\gamma_A\rangle$ be an IFS of S, then A is called an intuitionistic fuzzy subsemigroup of S if for all $x,y\in S$

$$(\mu_A(xy)\ge\min\{\mu_A(x),\mu_A(y)\}$$

And

$$\gamma_A(xy)\le\max\{\gamma_A(x),\gamma_A(y)\}.$$

2.1 Definition

Let $A=\langle x,\mu_A,\gamma_A\rangle$ be an intuitionistic fuzzy subset of a semigroup S. Then A is called an intuitionistic fuzzy interior ideal of S if for all $x,y,a\in S$ the following conditions hold;

(C_1) $\mu_A(xy)\ge\min\{\mu_A(x),\mu_A(y)\}$ and $\gamma_A(xy)\le\max\{\gamma_A(x),\gamma_A(y)\}$.
(C_2) $\mu_A(xay)\ge\mu_A(a)$ and $\gamma_A(xay)\le\gamma_A(a)$.

The μ_A-level cut and γ_A-level cut of an intuitionistic fuzzy subset $A=\langle x,\mu_A,\gamma_A\rangle$ of a semigroup S respectively are denoted and defined as $U(\mu_A;t)=\{x\in S\mid\mu_A(x)\ge t\}$ and $L(\gamma_A;s)=\{x\in S\mid\gamma_A(x)\le s\}$, where $t\in(0,1]$ and $s\in[0,1)$.

The (μ_A,γ_A)-level (t,s)-cut is defined as

$$C_{(t,s)}(A)=\{x\in S\mid\mu_A(x)\ge t \text{ and } \gamma_A(x)\le s\}.$$

It is clear that $C_{(t,s)}(A)=U(\mu_A;t)\cup L(\gamma_A;s)$.

2.2 Theorem [3]

Let $A=\langle x,\mu_A,\gamma_A\rangle$ be an intuitionistic fuzzy subset of S. Then $A=\langle x,\mu_A,\gamma_A\rangle$ is an intuitionistic fuzzy interior ideal of S it and only if $(\forall t\in(0,1],s\in[0,1))C_{(t,s)}(A)(\ne\phi)$ is an interior ideal of S.

2.3 Definition

Let x be a point of a non-empty set X. If $t\in(0,1]$, $s\in[0,1)$ are two real numbers such that $0<t+s\le 1$, then the IFS of the form $[x;(t,s)]=[x;x_t,1-x_{1-s}]$ is called an intuitionistic fuzzy point (IFP for short) in X, where t (resp. s) is the degree of membership (resp. non-membership) of $[x;(t,s)]$ and $x\in X$ is the support of $[x;(t,s)]$.

2.4 Definition

Consider an IFP $[x;(t,s)]$ in S, an IFS $A=\langle x,\mu_A,\gamma_A\rangle$ and $\alpha\in\{\in,q,\in\vee q\}$, we define $[x;(t,s)]\alpha A$ as follows;
(i). By $[x;(t,s)]\in A$ (resp. $[x;(t,s)]qA$) we mean $\mu_A(x)\ge t$ and $\gamma_A\le s$ (resp. $\mu_A(x)+t>1$ and $\gamma_A+s<1$) and say that $[x;(t,s)]$ belongs to (resp. quasi-coincident with) an IFS $A=\langle x,\mu_A,\gamma_A\rangle$.
(ii). $[x;(t,s)]\in\vee qA$ (resp. $[x;(t,s)]\in\wedge qA$) means that $[x;(t,s)]\in A$ or $[x;(t,s)]qA$ (resp. $[x;(t,s)]\in A$ and $[x;(t,s)]qA$).
(iii). By $[x;(t,s)]\overline{\alpha}A$ we mean that $[x;(t,s)]\alpha A$ do not hold.

2.5 Definition

For a non-empty subset A of S the characteristic function $\chi_A=<x,\mu_{\chi_A}(x),\gamma_{\chi_A}(x)>$ of A is defined by

$$\mu_{\chi_A}(x): S\to[0,1], x\mapsto\mu_{\chi_A}(x):=\begin{cases}1 & \text{if } x\in A,\\ 0 & \text{if } x\notin A,\end{cases}$$
$$\gamma_{\chi_A}(x): S\to[0,1], x\mapsto\gamma_{\chi_A}(x):=\begin{cases}0 & \text{if } x\in A,\\ 1 & \text{if } x\notin A.\end{cases}$$

2.6 Proposition

A non-empty subset A of S is an intuitionistic fuzzy interior ideal of S if and only if the characteristic function $\chi_A=<x,\mu_{\chi_A}(x),\gamma_{\chi_A}(x)>$ of A is an intuitionistic fuzzy interior ideal of S.

3. $(\overline{\in},\overline{\in}\vee\overline{q})$-INTUITIONISTIC FUZZY INTERIOR IDEALS

In this section, we introduce $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideals of a semigroup S.

3.1 Definition

An intuitionistic fuzzy subset $A=<x,\mu_A(x),\gamma_A(x)>$ of S is called an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideal of S, if for all $x,y,a\in S$, $t,t_1,t_2\in(0,1]$ and $s,s_1,s_2\in[0,1)$, satisfies the following conditions:
(C_3). $[xy;\min\{t_1,t_2\},\max\{s_1,s_2\}]\overline{\in}A\to[x;(t_1,s_1)]\overline{\in}\vee\overline{q}A$ or $[y;(t_2,s_2)]\overline{\in}\vee\overline{q}A,$
(C_4). $[xay;(t,s)]\overline{\in}A\to[a;(t,s)]\overline{\in}\vee\overline{q}A.$

3.2 Example

Consider a semigroup $S=\{a,b,c,d,e\}$ with the following multiplication table:

Table 1: Multiplication of $S=\{a,b,c,d,e\}$

.	a	b	c	d	e
a	a	d	a	d	d
b	a	b	a	d	d
c	a	d	c	d	e
d	a	d	a	d	d
e	a	d	c	d	e

Define an IFS $A=<x,\mu_A,\gamma_A>$ by

$$\mu_A : S\to[0,1]\mid\mu_A(x)=\begin{cases}0.60 \text{ if } x=a,\\ 0.40 \text{ if } x=b,\\ 0.30 \text{ if } x=c,\\ 0.60 \text{ if } x=d,\\ 0.20 \text{ if } x=e,\end{cases}$$

and

$$\gamma_A : S\to[0,1]\mid\gamma_A(x)=\begin{cases}0.30 \text{ if } x=a,\\ 0.40 \text{ if } x=b,\\ 0.60 \text{ if } x=c,\\ 0.30 \text{ if } x=d,\\ 0.50 \text{ if } x=e.\end{cases}$$

Then $A=<x,\mu_A,\gamma_A>$ is an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideals of S for all $t,t_1,t_2\in(0,1]$ and $s,s_1,s_2\in[0,1)$.

3.3 Lemma

An intuitionistic fuzzy subset $A=<x,\mu_A(x),\gamma_A(x)>$ of S is an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideal of S it and only if for all $x,y,a\in S$, the following hold.

(C_5). $\max\{\mu_A(xy),0.5\}\geq\min\{\mu_A(x),\mu_A(y)\}$ and $\min\{\gamma_A(xy),0.5\}\leq\max\{\gamma_A(x),\gamma_A(y)\}$,

(C_6). $\max\{\mu_A(xay),0.5\}\geq\mu_A(a)$ and $\min\{\gamma_A(xay),0.5\}\leq\gamma_A(a)$.

Proof

(C_3) $\Rightarrow$ (C_5). If

$$\max\{\mu_A(ab),0.5\}<t=\min\{\mu_A(a),\mu_A(b)\}$$

and

$$\min\{\gamma_A(ab),0.5\}>s=\max\{\gamma_A(a),\gamma_A(b)\}$$

for some $a,b\in S$ and $t\in(0.5,1]$, $s\in[0,0.5)$. Then $[ab;(t,s)]\overline{\in}A$ but $[a;(t,s)]\in A$, $[b;(t,s)]\in A$, also $[a;(t,s)]qA$ $[b;(t,s)]qA$, a contradiction by (C_3).

(C_5) $\Rightarrow$ (C_3). If $[ab;\min\{t_1,t_2\},\max\{s_1,s_2\}]\overline{\in}A$, then $\mu_A(ab)<\min\{t_1,t_2\}$ and $\gamma_A(ab)>\max\{s_1,s_2\}$. We consider the following two cases:
Case I: If

$$\mu_A(ab)\geq\min\{\mu_A(a),\mu_A(b)\}$$

And

$$\gamma_A(ab)\leq\max\{\gamma_A(a),\gamma_A(b)\},$$

then,

$$\min\{\mu_A(a),\mu_A(b)\}<\min\{t_1,t_2\}$$

And

$$\max\{\gamma_A(a),\gamma_A(b)\}>\max\{s_1,s_2\},$$

it follows that $\mu_A(a)<t_1$, $\gamma_A(a)>s_1$ or $\mu_A(b)<t_2$, $\gamma_A(b)>s_2$. Consequently $[a;(t_1,s_1)]\overline{\in}A$ or $[b;(t_2,s_2)]\overline{\in}A$.
Case II: If

$$\mu_A(ab)<\min\{\mu_A(a),\mu_A(b)\},$$

And

$$\gamma_A(ab)>\max\{\gamma_A(a),\gamma_A(b)\},$$

then, by (C_5) we have

$$0.5\geq\min\{\mu_A(a),\mu_A(b)\},\ 0.5\leq\max\{\gamma_A(a),\gamma_A(b)\}.$$

Let $[a;(t_1,s_1)]\in A$ or $[b;(t_2,s_2)]\in A$, then,

$$0.5\geq\mu_A(a)\geq t_1,\ 0.5\leq\gamma_A(a)\leq s_1$$

Or

$$0.5\geq\mu_A(b)\geq t_2,\ 0.5\leq\gamma_A(b)\leq s_2.$$

It follows that $[a;(t_1,s_1)]\overline{q}A$ or $[b;(t_2,s_2)]\overline{q}A$. Thus, in both case we see that $[a;(t_1,s_1)]\overline{\in}\vee\overline{q}A$ or $[b;(t_2,s_2)]\overline{\in}\vee\overline{q}A$.

(C_4) $\Rightarrow$ (C_6) Let us consider

$$\max\{\mu_A(xay),0.5\}<t=\mu_A(a),$$

And

$$\min\{\gamma_A(xay),0.5\}>s=\gamma_A(a),$$

for some $x,y,a\in S$ and $t\in(0.5,1]$, $s\in[0,0.5)$, then $[xay;(t,s)]\overline{\in}A$ but $[a;(t,s)]\in A$, also $[a;(t,s)]qA$, a contradiction by (C_4).

(C_6) $\Rightarrow$ (C_4). Let $[xay;(t,s)]\overline{\in}A$, then $\mu_A(xay)<t$ and $\gamma_A(xay)>s$ for some $x,y,a\in S$. We consider the following two cases:

Case I: If $\mu_A(xay)\geq\mu_A(a)$, $\gamma_A(xay)\leq\gamma_A(a)$, then $\mu_A(a)<t$, $\gamma_A(a)>s$, it follows that $[a;(t,s)]\overline{\in}A$.

Case II: If $\mu_A(xay)<\mu_A(a)$, $\gamma_A(xay)>\gamma_A(a)$, then by ($C_6$) we have $0.5\geq\mu_A(a)$, $0.5\leq\gamma_A(a)$. Let $[a;(t,s)]\in A$, then $0.5\geq\mu_A(a)\geq t$, $0.5\leq\gamma_A(a)\leq s$. It follows that $[a;(t,s)]\overline{q}A$. Thus, in both case we see that $[a;(t,s)]\overline{\in}\vee\overline{q}A$.

3.4 Definition

An intuitionistic fuzzy subset $A=<x,\mu_A(x),\gamma_A(x)>$ of S is called an $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideal of S, if for all $x,y,a\in S$, $t,t_1,t_2\in(0,1]$ and $s,s_1,s_2\in[0,1)$, satisfies the following conditions:

(C_7). $[xy;\min\{t_1,t_2\},\max\{s_1,s_2\}]\overline{\in}A\rightarrow[x;(t_1,s_1)]\overline{\in}A$ or $[y;(t_2,s_2)]\overline{\in}A$,

(C_8). $[xay;(t,s)]\overline{\in}A\rightarrow[a;(t,s)]\overline{\in}A$.

3.5 Example

Consider a semigroup $S=\{0,a,b,c\}$ with the following multiplication table:

.	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	a
c	0	0	a	b

Define an IFS $A=<x,\mu_A,\gamma_A>$ by

$$\mu_A:S\rightarrow[0,1]\,|\,\mu_A(x)=\begin{cases}0.30\text{ if }x=0,\\0.20\text{ if }x=a,\\0.40\text{ if }x=b,\\0.20\text{ if }x=c,\end{cases}$$

And

$$\gamma_A:S\rightarrow[0,1]\,|\,\gamma_A(x)=\begin{cases}0.50\text{ if }x=0,\\0.60\text{ if }x=a,\\0.30\text{ if }x=b,\\0.40\text{ if }x=c.\end{cases}$$

Then $A=<x,\mu_A,\gamma_A>$ is an $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideals of S for all $t,t_1,t_2\in(0,1]$ and $s,s_1,s_2\in[0,1)$.

3.6 Theorem

Let $A=<x,\mu_A(x),\gamma_A(x)>$ be an intuitionistic fuzzy subset of S. Then A is an intuitionistic fuzzy interior ideal of S if A is an $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideal of S.

Proof

Let A is an intuitionistic fuzzy interior ideal of S and $[xy;\min\{t_1,t_2\},\max\{s_1,s_2\}]\overline{\in}A$, then $\mu_A(xy)<\min\{t_1,t_2\}$, $\gamma_A(xy)>\max\{s_1,s_2\}$. By ($C_1$) we see that

$$\min\{\mu_A(x),\mu_A(y)\}\leq\mu_A(xy)<\min\{t_1,t_2\},$$

And

$$\max\{\gamma_A(x),\gamma_A(y)\}\geq\gamma_A(xy)>\max\{s_1,s_2\}.$$

Follows that $[x;\min\{t_1,t_2\},\max\{s_1,s_2\}]\,\overline{\in}\,A$ or $[y;\min\{t_1,t_2\},\max\{s_1,s_2\}]\,\overline{\in}\,A$.

Finally, if $[xay;(t,s)]\,\overline{\in}\,A$, then $\mu_A(xay)<t$ and $\gamma_A(xay)>s$ and by (C_2) we have

$$\mu_A(a)\leq\mu_A(xay)<t,$$

And

$$\gamma_A(a)\geq\gamma_A(xay)>s.$$

This shows that $[a;(t,s)]\,\overline{\in}\,A$.

From the Theorem (3.6) it is clear that every intuitionistic fuzzy interior ideal is an $(\in,\in)$-intuitionistic fuzzy interior ideal and it is obvious that every $(\in,\in)$-intuitionistic fuzzy interior ideal of S is an $(\in,\in\vee q)$-intuitionistic fuzzy interior ideal of S.

In the next theorem, we give a condition for an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideal of S to be an $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideal of S.

3.7 Theorem

Let $A=\langle x,\mu_A,\gamma_A\rangle$ be an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideal of S. If $\mu_A(x)>0.5$ and $\gamma_A(x)<0.5$ for all $x\in S$, then $A=\langle x,\mu_A,\gamma_A\rangle$ is an $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideal of S.

Proof

Let $A=\langle x,\mu_A,\gamma_A\rangle$ be an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideal of S and $\mu_A(x)>0.5$ and $\gamma_A(x)<0.5$ for all $x\in S$. Let $t_1,t_2\in(0,1]$, $s_1,s_2\in[0,1)$ and consider $[xy;\min\{t_1,t_2\},\max\{s_1,s_2\}]\,\overline{\in}\,A$ then $\mu_A(xy)<\min\{t_1,t_2\}$, $\gamma_A(xy)>\max\{s_1,s_2\}$. By ($C_5$) we have

$$\min\{\mu_A(x),\mu_A(y)\}\leq\max\{\mu_A(xy),0.5\}<\max\{\min\{t_1,t_2\},0.5\},$$
$$=\begin{cases}\min\{t_1,t_2\} & \text{if } \min\{t_1,t_2\}\geq 0.5,\\ 0.5 & \text{if } \min\{t_1,t_2\}<0.5,\end{cases}$$

and

$$\max\{\gamma_A(x),\gamma_A(y)\}\geq\min\{\gamma_A(xy),0.5\}>\min\{\max\{s_1,s_2\},0.5\},$$
$$=\begin{cases}0.5 & \text{if } \max\{s_1,s_2\}\geq 0.5,\\ \max\{s_1,s_2\} & \text{if } \max\{s_1,s_2\}<0.5.\end{cases}$$

This implies $\mu_A(x)<\min\{t_1,t_2\}$, $\gamma_A(x)>\max\{s_1,s_2\}$ or $\mu_A(y)<\min\{t_1,t_2\}$, $\gamma_A(y)>\max\{s_1,s_2\}$ i.e.,

$$[x;(\min\{t_1,t_2\},\max\{s_1,s_2\})]\,\overline{\in}\,A,$$

Or

$$[y;(\min\{t_1,t_2\},\max\{s_1,s_2\}]\,\overline{\in}\,A.$$

Let $x,y,a\in S$ with $[xay;(t,s)]\,\overline{\in}\,A$, then $\mu_A(xay)<t$, $\gamma_A(xay)>s$ and from (C_6) we see that;

$$\mu_A(a)\leq\max\{\mu_A(xay),0.5\}<\max\{t,0.5\},$$
$$=\begin{cases}t & \text{if } t\geq 0.5,\\ 0.5 & \text{if } t<0.5,\end{cases}$$

and

$$\gamma_A(a)\geq\min\{\gamma_A(xay),0.5\}>\min\{s,0.5\},$$
$$=\begin{cases}s & \text{if } s\leq 0.5,\\ 0.5 & \text{if } s>0.5.\end{cases}$$

Follows that $[a;(t,s)]\,\overline{\in}\,A$. Consequently, $A=\langle x,\mu_A,\gamma_A\rangle$ is an $(\overline{\in},\overline{\in})$-intuitionistic fuzzy interior ideal of S.

3.8 Theorem

For an intuitionistic fuzzy subset $A=\langle x,\mu_A,\gamma_A\rangle$ of S, the following are equivalent for all $t\in(0.5,1]$ and $s\in[0,0.5)$:

(C_9). A is an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideal of S.

(C_{10}). $C_{(t,s)}(A)(\neq\phi)$ is an interior ideal of S.

Proof

Assume that A is an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideal of S and $C_{(t,s)}(A)\neq\phi$. Let $x,y\in C_{(t,s)}(A)$, then $\mu_A(x)\geq t$, $\gamma_A(x)\leq s$ and $\mu_A(y)\geq t$, $\gamma_A(y)\leq s$. (C_5) implies that

$$\max\{\mu_A(xy),0.5\}\geq\min\{\mu_A(x),\mu_A(y)\}\geq\min\{t,t\}=t,$$

And

$$\min\{\gamma_A(xy),0.5\}\leq\max\{\gamma_A(x),\gamma_A(y)\}\leq\max\{s,s\}=s.$$

Thus, $xy\in C_{(t,s)}(A)$.

Next, we suppose $x,y,a\in S$ with $a\in C_{(t,s)}(A)$, then $\mu_A(a)\geq t$, $\gamma_A(a)\leq s$. From (C_6), we have

$$\max\{\mu_A(xay),0.5\}\geq\mu_A(a)\geq t,$$

And

$$\min\{\gamma_A(xay),0.5\}\leq\gamma_A(a)\leq s.$$

Thus $xay\in C_{(t,s)}(A)$. Consequently $C_{(t,s)}(A)$ is an interior ideal of S.

Conversely, let $C_{(t,s)}(A)$ is an interior ideal of S. Assume that there exist $a,b\in S$ such that

$$\max\{\mu_A(ab),0.5\}<\min\{\mu_A(a),\mu_A(b)\},$$

And

$$\min\{\gamma_A(ab),0.5\}>\max\{\gamma_A(a),\gamma_A(b)\},$$

then,

$$\max\{\mu_A(ab),0.5\}<t_0\leq\min\{\mu_A(a),\mu_A(b)\},$$

And

$$\min\{\gamma_A(ab),0.5\}>s_0\geq\max\{\gamma_A(a),\gamma_A(b)\},$$

for some $t_0\in(0.5,1]$ and $s_0\in[0,0.5)$, it follows that $a\in C_{(t_0,s_0)}(A)$ and $b\in C_{(t_0,s_0)}(A)$ but $ab\,\overline{\in}\,C_{(t_0,s_0)}(A)$, a contradiction.

Therefore,

$$\max\{\mu_A(ab),0.5\}\geq\min\{\mu_A(a),\mu_A(b)\},$$

And

$$\min\{\gamma_A(ab),0.5\}\leq\max\{\gamma_A(a),\gamma_A(b)\},$$

for all $x,y\in S$.

Next we consider,

$\max\{\mu_A(xay),0.5\}<\mu_A(a)$ and $\min\{\gamma_A(xay),0.5\}>\gamma_A(a)$ for some $x,y,a\in S$, then there exist $t_1\in(0.5,1]$ and $s_1\in[0,0.5)$ such that $\max\{\mu_A(xay),0.5\}<t_1\leq\mu_A(a)$ and $\min\{\gamma_A(xay),0.5\}>s_1\geq\gamma_A(a)$, it follows that $xay\,\overline{\in}\,C_{(t_1,s_1)}(A)$ but $a\in C_{(t_1,s_1)}(A)$, a contradiction. Therefore, $\max\{\mu_A(xay),0.5\}\geq\mu_A(a)$ and $\min\{\gamma_A(xay),0.5\}\leq\gamma_A(a)$ for all $x,y,a\in S$.

Consequently, A is an $(\overline{\in},\overline{\in}\vee\overline{q})$-intuitionistic fuzzy interior ideal of S.

For any IFS $A=\langle x,\mu_A,\gamma_A\rangle$ of S, $t\in(0,1]$ and $s\in[0,1)$, we consider two subsets:

$$Q_{(t,s)}(A) := \{x \in S \mid [x;(t,s)]qA\},$$

$$[A]_{(t,s)} := \{x \in S \mid [x;(t,s)] \in \vee qA\}.$$

It is obvious that $[A]_{(t,s)} = C_{(t,s)}(A) \cup Q_{(t,s)}(A)$.

3.9 Proposition

Let $A = \langle x, \mu_A, \gamma_A \rangle$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$ -intuitionistic fuzzy interior ideal of S, then $Q_{(t,s)}(A)(\neq \phi)$ is an interior ideal of S for all $t \in (0.5,1]$, $s \in [0, 0.5)$.

Proof

Consider $A = \langle x, \mu_A, \gamma_A \rangle$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$ -intuitionistic fuzzy interior ideal of S. Let $t \in (0.5,1]$ and $s \in [0, 0.5)$ such that $Q_{(t,s)}(A) \neq \phi$. Let $x, y \in Q_{(t,s)}(A)$ then $\mu_A(x) + t > 1$, $\gamma_A(x) + s < 1$ and $\mu_A(y) + t > 1$, $\gamma_A(y) + s < 1$. (C_5) implies that

$$\begin{aligned} \max\{\mu_A(xy), 0.5\} &\geq \min\{\mu_A(x), \mu_A(y)\} \\ &> \min\{1-t, 1-t\} = 1-t, \end{aligned}$$

And

$$\begin{aligned} \min\{\gamma_A(xy), 0.5\} &\leq \max\{\gamma_A(x), \gamma_A(y)\} \\ &< \max\{1-s, 1-s\} = 1-s. \end{aligned}$$

Thus $xy \in Q_{(t,s)}(A)$.

Next we suppose $x, y, a \in S$ such that $a \in Q_{(t,s)}(A)$, then $\mu_A(a) + t > 1$, $\gamma_A(a) + s < 1$. (C_6), implies that

$$\max\{\mu_A(xay), 0.5\} \geq \mu_A(a) > 1-t,$$

And

$$\min\{\gamma_A(xay), 0.5\} \leq \gamma_A(a) < 1-s.$$

Follows that $Q_{(t,s)}(A)$ is an interior ideal of S.

4. CONCLUSION

In the world of contemporary mathematics, the use of algebraic structures in computer science, control theory and fuzzy automata theory always gain the interest of researchers. Algebraic structures particularly semigroups play a key role in such applied branches. Further, the fuzzification of several subsystems of semigroups are used in various models involving uncertainties. In this paper, we introduced new types of subsystems of semigroup called $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy subsystems and characterized semigroups in terms of $(\overline{\in}, \overline{\in} \vee \overline{q})$-intuitionistic fuzzy interior ideals and $(\overline{\in}, \overline{\in})$-intuitionistic fuzzy interior ideals. Finally, interior ideals and intuitionistic fuzzy interior ideal of type $(\overline{\in}, \overline{\in} \vee \overline{q})$ are connected by intuitionistic fuzzy level subset.

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