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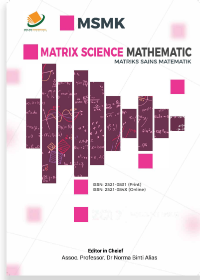

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CHARACTERIZATION OF SOFT B SEPARATION AXIOMS IN SOFT BI-TOPOLOGICAL SPACES

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ABSTRACT

The main aim of this article is to introduce Soft b separation axioms in soft bi topological spaces. We discuss soft b separation axioms in soft bi topological spaces with respect to ordinary point and soft points. Further study the behavior of soft regular b T_3 and soft normal b T_4 spaces at different angles with respect to ordinary points as well as with respect to soft points. Hereditary properties are also discussed.

KEYWORDS

Soft sets, soft points, soft bi-topological space, soft b T_i spaces ($_i$=1,2,3,4), soft b regular and soft b normal spaces.

1. INTRODUCTION

In real life condition the problems in economics, engineering, social sciences, medical science etc. We cannot beautifully use the traditional classical methods because of different types of uncertainties presented in these problems. To overcome these difficulties, some kinds of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, in which we can safely use a mathematical technique for businessing with uncertainties. But, all these theories have their inherent difficulties. To overcome these difficulties in the year 1999, Russian scientist, initiated the notion of soft set as a new mathematical technique for uncertainties, which is free form the above complications [1]. A group researcehrs, successfully applied the soft set theory in different directions, such as smoothness of functions, game theory, operation research, Riemann integration, perron integration, probability, theory of measurement and so on [2,3]. After presentation of the operations of soft sets, the properties and applications of the soft set theory have been studied increasingly [4-6]. Others group researchers discussed the linkage between soft sets and information systems [7,8]. They showed that soft sets are class of special information system. In the recent year, many interesting applications of soft sets theory have been extended by embedding the ideas of fuzzy sets industrialized soft set theory, the operations of the soft sets are redefined and in indecision making method was constructed by using their new operations [9-20].

Recently, in 2011, there are some researcher launched the study of soft Topological spaces, they beautiful defined soft Topology as a collection of τ of soft sets over X [20]. They also defined the basic conception of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axiom, soft regular and soft normal spaces and published their several behaviors. Min in scrutinized some belongings of these soft separation axioms [21]. A group researcher introduced some soft operations such as semi open soft, pre-open soft, α-open soft and β-open soft and examined their properties in detail [22]. Some of researcher introduced the concept of soft semi – separation axioms, in particular soft semi- regular spaces [23]. The concept of soft ideal was discussed for the first time [24]. They also introduced the concept of Soft local function. There concepts are discussed with a view to find new soft topological from the original one, called soft topological spaces with soft ideal (X, τ, E, I).

Application to different zone were further discussed [24-30]. The notion of super soft topological spaces was initiated for the first time [31]. They also introduced new different types of sub-sets of supra soft topological spaces and study the dealings between them in great detail. A researcher introduced the concept of semi open soft sets and studied their related properties, discussed soft separation axioms [32,33]. Mahanta introduced semi open and semi closed soft sets. [34,35]. On Soft β-Separation Axioms, Arockiarani.I, Arokialancy in generalized soft g β closed and soft gs β closed sets in soft topology are exposed [36].

In this present paper the concept of soft b T_0, soft b T_1, soft b T_2 and soft b T_3 spaces in Soft topological spaces is introduced with respect to soft points. The concept of soft b T_0, b T_1, b T_2 and soft b T_3 and soft b T_4 spaces are introduced in soft bi topological spaces with respect to ordinary as well as soft points. Many mathematicians discussed soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open set, soft α-open set and soft β-open set [37-40]. They also worked over the hereditary properties of different soft topological structures in soft topology.

In this present work hand is tried and work is encouraged over the gap that exists in soft bi-topology. Related to Soft b T_3 and soft b T_4 spaces, some Proposition in soft bi topological spaces are discussed with respect to ordinary points as well as with respect to soft points. When we talk about the distances between the points in soft topology then the concept of soft separation axioms will automatically come in play [41]. That is why these structures are catching our attentions. We hope that these results will be valuable for the future study on soft bi topological spaces to accomplish general framework for the practical applications and to solve the most intricate problems containing scruple in economics, engineering, medical, environment and in general mechanic systems of various kinds [41,42].

2. PRELIMINARIES

The following Definitions which are pre-requisites for present study.

Definition 1 [1]. Let X be an initial universe of discourse and E be a set of parameters. Let P(X) denotes the power set of X and A be a non-empty sub-set of E. A pair (F,A) is called a soft set over U, where F is a mapping given by $F: A \to P(X)$.
In other words, a set over X is a parameterized family of sub set of universe of discourse X . For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set $(F,A)$and if$e \notin A$ then $F(e) = \phi$, that is$F_{A=}\{F(e): e \in A \subseteq E, F: A \to P(X)\}$ the family of all these soft sets over X denoted by$SS(X)_A$.

Definition 2 [1]. Let$F_{A,} G_B \in SS(X)_E$ then F_A , is a soft subset of G_B denoted by$F_A \sqsubseteq G_B$, if
1. $A \subseteq B$ &
2. $F(e) \subseteq G(e), \forall \in A$

In this case F_A is said to be a soft subset of G_B and G_B is said to be a soft super set F_A, $G_B \sqsupseteq F_A$.

Definition 3 [5]. Two soft subsets F_A and G_B over a common universe of discourse set X are said to be equal if F_A is a soft subset of G_B and G_B is a soft subset ofF_A.

Definition 4 [5]. The complement of soft subset(F,A)ddenoted by $(F,A)^C$ is defined by $(F,A)^C = (F^C, A)$ $F^C \to P(X)$is a mapping given by $F^C(e) = U - F(e) \forall e \in A$ and $F^C$is called the soft complement function of F . Clearly $(F^C)^C$ is the same as F and $((F,A)^C)^C = (F,A)$.

Definition 5 [4]. The difference between two soft subset (G,E) and (G,E) over common of universe discourse X denoted by $(F,E) - (G,E)$ is the soft set(H,E) where for all $e \in E$.

Definition 6 [4]. Let (G,E) be a soft set over X and $x \in X$ We say that $x \in (F,E)$ and read as x belong to the soft set(F,E) whenever $x \in F(e) \forall e \in E$ The soft set(F,E) over X such that $F(e) = \{x\} \forall e \in E$ is called singleton soft point and denoted by $x_E, or\ (x,E)$.

Definition 7 [3]. A soft set (F,A) over X is said to be Null soft set denoted by $\overline{\emptyset}$ or $\emptyset_A$ if $\forall e \in A, F(e) = \emptyset$.

Definition 8 [3]. A soft set(F,A) over X is said to be an absolute soft denoted by $\bar{A}$ or X_A if $\forall e \in A, F(e) = X$.
Clearly, we have. $X_A^C = \emptyset_A$ and $\emptyset_A^C = X_A$.

Definition 9 [32]. Let (G,E) be a soft set over X and $e_G \in X_A$, we say that $e_G \in (F,E)$ and read as e_G belong to the soft set(F,E) whenever $e_G \in F(e) \forall e \in E$. the soft set (F,E) over X such that $F(e) = \{e_G\}$, $\forall e \in E$ is called singleton soft point and denoted by $e_G, or\ (e_G, E)$.

Definition 10 [32]. The soft set $(F,A) \in SSX_A$ is called a soft point in X_A, denoted by e_F, if for the element $e \in A, F(e) \neq \emptyset$ and $F(e^/) = \phi$ if for all $e^/ \in A - \{e\}$

Definition 11 [32]. The soft point e_F is said to be in the soft set (G,A) , denoted by $e_F \in (G,A)$ if for the element $e \in A, F(e) \subseteq G(e)$.

Definition 12 [32]. Two soft sets $(G,A), (H,A)$ in SSX_A are said to be soft disjoint, written $(G,A) \cap (H,A) = \emptyset_A$ If $G(e) \cap H(e) = \emptyset$ $\forall e \in A$.

Definition 13 [32]. The soft point $e_G, e_H \in X_A$ are disjoint, written $e_G \neq e_{H,}$ if their corresponding soft sets (G,A) and (H,A) are disjoint.

Definition 14 [5]. The union of two soft sets(F,A) and (G,B) over the common universe of discourse X is the soft set(H,C), where, $C = AUB \forall e \in C$

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in (B - A) \\ F(e), & if\ e \in A \cap B \end{cases}$$

Written as $(F,A) \cup (G,B) = (H,C)$

Definition 15 [3]. The intersection $(H.C)$ of two soft sets (F,A) and (G,B) over common universe X, denoted $(F,A) \bar{\cap} (G,B)$ is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e), \forall e \in C$.

Definition 16 [19]. Let (F,E) be a soft set over X and Y be a non-empty sub set of X. Then the sub soft set of (F,E) over Y denoted by (Y_F, E), is defined as follow $Y_{F(\alpha)} = Y \cap F(\alpha), \forall \in E$ in other words $(Y_F, E) = Y \cap (F,E)$.

Definition 17 [19]. Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X, if
1. $\emptyset,\ X \in \tau$
2. The union of any number of soft sets in τ belongs to τ
3. The intersection of any two soft sets in τ belong to τ

The triplet (X, τ, E) is called a soft topological space.

Definition 18 [36]. Let (X, τ, E) be a soft topological space over X then the member of τ are said to be soft open sets in X.

Definition 19 [36]. Let (X, τ, E) be a soft topological space over X.A soft set(F,A) over X is said to be a soft closed set in X , if its relative complement $(F,E)^C$ belong to τ.

Definition 20 [37] Let(X, τ, E)be a soft topological space and$(F,E) \subseteq SS(X)_A$ then $(F,E)$is called a b open soft set.$((F,E) \subseteq Cl(int(F,E)Uint(Cl(F,E))$
The set of all *b open soft* set is denoted by$BOS(X, \tau, E)$ or $BOS(X)$ and the set of all *b closed soft* set is denoted by$BCS(X, \tau, E)$ or $BCS(X)$

Proposition 1 [37]. Let (X, τ, E) be a soft topological space over X.If (X, τ, E) is soft b T_3-space, then for all $x \in X, x_E = (x,E)$ is b-closed soft set.

3. SOFT B-SEPARATION AXIOMS OF SOFT TOPOLOGICAL SPACES.

Definition 21 [36] .Let (X, τ, A) be a soft Topological space over X and $x, y \in X$ such that $x \neq y$ if there exist at least one soft open set (F_1, A) or (F_2, A) such that $x \in (F_1, A), y \notin (F_1, A)$ or $y \in (F_2, A), x \notin ((F_2, A)$ then (X, τ, A) is called a soft T_0 *space*.

Definition 22 [36]. Let (X, τ, A) be a soft Topological spaces over X and $x, y \in X$ such that $x \neq y$ if there exist soft open sets $(F_1, A)$and (F_2, A) such that $x \in (F_1, A), y \notin (F_1, A)$ and $y \in (F_2, A), x \notin ((F_2, A)$ then (X, τ, A) is called a soft T_1 *space*.

Definition 23 [36]. Let (X, τ, A) b e a soft Topological space over X and $x, y \in X$ such that $x \neq y$ if there exist soft open set $(F_1, A)$and (F_2, A) such that $x \in (F_1, A)$, and $y \in (F_2, A)$ and $F_1 \cap F_2 = \varphi$
Then (X, τ, A) is called soft T_2 spaces.

Definition 24 [32] .Let (X, τ, A) be a soft Topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search at least one soft open set (F_1, A) or (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$ or $e_H \in (F_2, A), e_G \notin ((F_2, A)$ then (X, τ, A) is called a soft T_0 space.

Definition 25 [32]. Let (X, τ, A) be a soft Topological spaces over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search soft open sets $(F_1, A)$and (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$ and $e_H \in (F_2, A), e_G \notin ((F_2, A)$ then (X, τ, A) is called a soft T_1 *space*.

Definition 26 [32]. Let (X, τ, A) b e a soft Topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search soft open set $(F_1, A)$and (F_2, A) such that $e_G \in (F_1, A)$, and $e_H \in (F_2, A)$ $(F_1, A) \cap (F_2, A) = \phi_A$ Then (X, τ, A) is called soft T_2 space.

Definition27 [37]. Let (X, τ, E)be a soft topological space over X and$x, y \in X$ such that $x \neq y$. If we can find soft b open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$ Then (X, τ, E) is called soft b T_0 space.

Definition 28 [37]. Let (X, τ, E) be a soft topological space over X

and $x, y \in X$ such that $x \neq y$. If we can find two soft b open sets (F, E) and (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$ Then (X, τ, E) is called soft b T_1 space.

Definition 29 [37]. Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$. If we can find two soft b open soft sets such that $x \in (F, E)$and $y \in (G, E)$ moreover$(F, E) \cap (G, E) = \phi$ Then (X, τ, E) is called a soft b T_2 space.

Definition 30 [37]. Let (X, τ, E) be a soft topological space (G, E) and b closed soft set in X and $x \in X$ such that$x \notin (G, E)$. If there occurs soft b open sets (F_1, E) and (F_2, E) such that $x \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$. Then (X, τ, E) is called soft b regular spaces. A soft b regular T_1 *Space*.is called soft b T_3space.

Definition 31 [37]. Let (X, τ, E) be a *soft topological* space and $(F, E), (G, E)$ be soft b closed sets in X such that $(F, E) \cap (G, E) = \varphi$. If there occurs b open soft sets (F_1, E) and(F_2, E) such that $(F, E) \subseteq (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$ then (X, τ, E) is called a soft b normal space. A soft b normal T_1 *space* is called soft b T_4 *space*.

4. SOFT B-SEPARATION AXIOMS OF SOFT BI-TOPOLOGICAL SPACES

Let X is an initial set and E be the non-empty set of parameters. In soft bi topological space over the soft set X is introduced [36]. Soft separation axioms in soft bi topological spaces were introduced by Basavaraj and Ittanagi [36]. In this section we introduced the concept of soft b T_3 and b T_4 spaces in soft bi topological spaces with respect to ordinary as well as soft points and some of its basic properties are studied and applied to different results in this section.

Definition 32 [36]. Let(X, τ_1, E) and (X, τ_2, E) be two different soft topologies on X. Then (X, τ_1, τ_2, E) is called a *soft bi topological space.* The two soft topologies$(X, \tau_1, E)$and(X, τ_2, E) are independently satisfy the axioms of soft topology. The members ofτ_1 are called τ_1 *soft open* set. And complement ofτ_1. Soft open set is called τ_1 *soft closed* set. Similarly, the member of τ_2 are called τ_2 *soft open* sets and the complement of τ_2 soft open sets are called τ_2 *soft closed* set.

Definition 33 [36]. Let (X, τ_1, τ_2, E)be a soft topological space over X and Y be a non-empty subset of X. Then$\tau_{1Y} = \{(Y_E, E): (F, E) \in \tau_1)\}$ and $\tau_{2Y} = \{(G_E, E): (G, E) \in \tau_2)\}$ are said to be the relative topological on Y. Then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is called relative soft bi-topological space of (X, τ_1, τ_2, E).

4.1 Soft b-Separation axioms with respect to ordinary points

In this section we introduced soft b separation axioms in soft bi topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 34. In a *soft bi topological* space (X, τ_1, τ_2, E)

1) τ_1 said to be *soft b* T_0 space with respect to τ_2 if for each pair of distinct points $x, y \in X$ there exists τ_1 soft b open set (F, E) and a τ_2 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ similarly, τ_2 is said to be *soft b* T_0 space with respect to τ_1 if for each pair of distinct points $x, y \in X$ there exists τ_2 *soft b open set* (F, E) and a τ_1 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$.*Soft bi topological spaces* (X, τ_1, τ_2, E) is said to be *pair wise soft b* T_0 space if τ_1 is *soft b* T_0 *space* with respect to τ_2 and τ_2 is *soft b* T_0 space with respect to τ_1.

2) τ_1 is said to be soft b T_1 space with respect to τ_2 if for each pair of distinct points $x, y \in X$ there exists a τ_1 *soft b open set* (F, E) and τ_2 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Similarly, τ_2 is said to be soft b T_1 space with respect to τ_1 if for each pair of distinct points $x, y \in X$ there exist a τ_2 soft b open set (F, E) and a τ_1 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (G, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. *Soft bi topological spaces* (X, τ_1, τ_2, E) is said to be *pair wise soft b* T_1 space if τ_1 is *soft b* T_1 *space* with respect to $\tau_2$and τ_2 is *soft b* T_1 space with respect to τ_1.

3)τ_1 is said to be soft b T_2 space with respect to τ_2 if for each pair of distinct points $x, y \in X$ there exists a τ_1 soft b open set (F, E) and a τ_2 soft b open set (G, E) such that $x \in (F, E)$ and $y \in (G, E)$, $(F, E) \cap (G, E) = \phi$. Similarly, τ_2 is said to be *soft b* T_2 space with respect to τ_1 if for each pair of distinct points $x, y \in X$ there exists a τ_2 *soft b open* set (F, E) and a τ_1 *soft b open* set (G, E) such that $x \in (F, E)$ and $y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. The soft bi topological space (X, τ_1, τ_2, E) is said to be pair wise soft b T_2 space if τ_1 is soft b T_2 space with respect to τ_2 and τ_2 is soft b T_2 space with respect toτ_1.

Definition 35. In a soft bi topological space (X, τ_1, τ_2, E)
1) τ_1 is said to be soft b T_3 space with respect to τ_2 if τ_1 is soft b T_1 space with respect to τ_2 and for each pair of distinct points $x, y \in X$,there exists a τ_1 b closed soft set (G, E) such that $x \notin (G, E)$, τ_1 soft b open set (F_1, E) and τ_2 soft b open set (F_2, E) such that $x \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \emptyset$. Similarly, τ_2 is said to be soft b T_3 space with respect to τ_1 if τ_2 is soft b T_1 space with respect to τ_1 and for each pair of distinct points $x, y \in X$ there exists a τ_2 soft b closed set (G, E) such that $x \notin (G, E)$, τ_2 soft b open set (F_1, E) amd τ_1 soft b open set (F_2, E) such that $x \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. (X, τ_1, τ_2, E) is said to be pair wise soft b T_3 space if τ_1 is soft b T_3 space with respect to τ_2 and τ_2 is soft b T_3 space with respect to τ_1.
2) τ_1 is said to be soft b T_4 space with respect to τ_2 if τ_1 is soft b T_1 space with respect to τ_2 , there exists a τ_1 soft b closed set (F_1, E) and τ_2 soft b closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \emptyset$ also there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft τ_1 b open set, (G_1, E) is soft τ_2 b open set such that $(F_1, E) \subseteq (F_3, E)$, $(F_2, E) \subseteq (G_1, E)$. Similarly, τ_2 is said to be soft b T_4 space with respect to τ_1 if τ_2 is soft b T_1 space with respect to τ_1, there exists τ_2 soft b closed set (F_1, E) and τ_1 soft b closed set (F_2, E) such that $(F_1, E) \cap (F_2, E) = \phi$ also there exists (F_3, E) and (G_1, E) such that (F_3, E) is soft τ_2 b open set, (G_1, E) is soft τ_1 b soft set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Thus, (X, τ_1, τ_2, E) is said to be pair wise soft b T_4 space if τ_1 is soft b T_4 space with respect to τ_2 and τ_2 is soft b T_4 space with respect to τ_1.

Proposition 2. Let (Y, τ_Y, E) be a soft sub space of a soft topological space (X, τ, E) and $(F, E) \in SS(X)$ then
1. If (F, E) is b *open soft* set in Y and $Y \in \tau$, then $(F, E) \in \tau$
2. (F, E) is b *open soft* set in Y if and only if $(F, E) = Y \cap (G, E)$ for some $(G, E) \in \tau$.
3. (F, E) is b *closed soft* set in Y if and only if $(F, E) = Y \cap (H, E)$ for some (H, E) *is* τ *soft b close* set.

Proof. 1) Let (F, E) be a soft open set in$Y$, then there does exists a soft b open set (G, E) in X such that$(F, E) = Y \cap (G, E)$. Now, if $Y \in \tau$ then $Y \cap (G, E) \in \tau$ by the third condition of the definition of a soft topological space and hence$(F, E) \in \tau$.
2) Fallows from the definition of a soft subspace.
3) If (F, E) is soft b closed in Y then we have $(F, E) = Y \setminus (G, E)$, for some $(G, E) \in \tau_Y$.Now,$(G, E) = Y \cap (H, E)$ for some soft b open set $(H, E) \in \tau$.for any$\alpha \in E$. $F(\alpha) = Y(\alpha) \setminus G(\alpha)$ =$Y\setminus$G(α) = $Y \setminus (Y(\alpha) \cap H(\alpha))$ =$Y \setminus (Y \cap H(\alpha))$ = $Y \setminus H(\alpha)$ = $Y \cap (X \setminus H(\alpha))$ =$Y \cap (H(\alpha))^C$=$Y(\alpha) \cap (H(\alpha))^C$.Thus $(F, E) = Y \cap (H, E)^/$is soft b closed in X as$(H, E) \in \tau$. Conversely, suppose that $(F, E) = Y \cap (G, E)$ for some soft b closed set (G, E) in X. This qualifies us to say that$(G, E)^/ \in \tau$.Now, if $(G, E) = (X, E) \setminus (H, E)$ where (H, E) is soft b open set in τ then for any $\alpha \in E$ $F(\alpha) = Y(\alpha) \cap G(\alpha)$ $= Y \cap G(\alpha)$ =Y∩ $(X(\alpha) \setminus H(\alpha)$ =$Y \cap (X(\alpha) \setminus H(\alpha))$ =$Y \cap (X \setminus H(\alpha))$ $Y \setminus H(\alpha)$ =$Y \setminus (Y \cap H(\alpha))$ =$Y(\alpha) \setminus (Y(\alpha) \cap H(\alpha))$ .Thus $(F, E) = Y \setminus (Y \cap (H, E))$.Since$(H, E) \in \tau$, So$(Y \cap (H, E) \in \tau$. So $(Y \cap (H, E) \in \tau_Y$ and hence (F, E) is soft b closed in Y.

Proposition 3. Let (X, τ_1, τ_2, E) be a soft bi topological space over X. Then, if (X, τ_1, E) and (X, τ_2, E) are soft b T_3 space then (X, τ_1, τ_2, E) is a pair wise soft b T_2 space.
Proof: Suppose (X, τ_1, E) is a soft b T_3 space with respect to (X, τ_2, E) then according to definition for $x, y \in X, x \neq y$,by using Theorem 1, (y, E) is soft b closed set in τ_2 and $x \notin (y, E)$ there exists a τ_1 soft b open set (F, E) and a τ_2 soft b open set (G, E) such that $x \in (F, E)$, $y \in (y, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence τ_1 is soft b T_2 space with respect toτ_2. Similarly, if (X, τ_2, E) is a soft b T_3 space with respect to (X, τ_1, E) then according to definition for $x, y \in X, x \neq y$, by using Theorem 1, (x, E) is b closed soft set in τ_1 and $y \notin (x, E)$ there exists a τ_2 soft b open set (F, E) and a τ_1 soft b open set (G, E) such that $y \in (F, E)$, $x \in (x, E) \subseteq (G, E)$ and $(F_1, E) \cap (F_2, E) = \phi$ .Hence τ_2 is soft b T_2 space.this implies that (X, τ_1, τ_2, E) is a pair wise soft b T_2 space.

Proposition 4. Let (X, τ_1, τ_2, E) be a soft bi topological space over X. if (X, τ_1, E) and (X, τ_2, E) are soft b T_3 space then (X, τ_1, τ_2, E) is a pair wise soft b T_3 space.
Proof: Suppose (X, τ_1, E) is a soft b T_3 space with respect to (X, τ_2, E) then according to definition for $x, y \in X, x \neq y$ there exists a τ_1 soft b open set (F, E) and a τ_2 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each τ_1 b closed soft set (G_1, E) such that $x \notin (G_1, E)$ there exists a τ_1 soft b open set $F_1, E)$ and τ_2 soft b open set $F_2, E)$ such that $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Similarly, (X, τ_2, E) is a soft b T_3 space with respect to(X, τ_1, E). So according to definition for $x, y \in X, x \neq y$ there exists τ_2 soft b open set (F, E) and a τ_1 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each point $x \in X$ and each τ_2 b closed soft set (G_1, E) such that $x \notin (G_1, E)$ there exists a τ_2 soft

b open set (F_1, E) and a $\tau_1$soft b open set (F_2, E) such that $x \in (F_1, E), (G_1, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$. Hence (X, τ_1, τ_2, E) is pair wise soft b T_3 space.

Proposition 5.If (X, τ_1, τ_2, E) be a soft bi topological space over X if (X, τ_1, E) and (X, τ_2, E) are soft b T_4 space then (X, τ_1, τ_2, E) is pair wise soft b T_4 space.

Proof: Suppose (X, τ_1, E) is soft b T_4 space with respect to (X, τ_2, E).So according to definition for $x, y \in X, x \neq y$ there exist a τ_1 soft b open set (F, E) and a τ_2 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ each τ_1 soft b closed set (F_1, E) and a τ_2 soft b closed set $F_2, E)$ such that $(F_1, E) \cap (F_2, E) = \phi$. There exist (F_3, E) and (G_1, E) such that (F_3, E) is soft τ_2 b open set (G_1, E) is soft τ_1 b open set $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and$(F_3, E) \cap (G_1, E) = \phi$. Similarly, τ_2 is soft b T_4 space with respect to τ_1 so according to definition for $x, y \in X, x \neq y$ there exists a τ_2 soft b open set (F, E) and a τ_1 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ and for each τ_2 soft b closed set (F_1, E) and τ_1 soft b closed set $F_2, E)$ such that $(F_1, E) \cap (F_2, E) = \phi$.there exists (F_3, E) and(G_1, E) such that (F_3, E) is soft τ_2 b open set (G_1, E) is soft τ_1 b open set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$ hence (X, τ_1, τ_2, E) is pair wise soft b T_4 space.

Proposition 6. Let (X, τ_1, τ_2, E) be a soft bi topological space over X and Y be a non-empty subset of X. if $(X, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_3 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_3 space.

Proof: First we prove that $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_1 space.

Let $x, y \in X, x \neq y$ if (X, τ_1, τ_2, E) is pair wise space then this implies that (X, τ_1, τ_2, E) is pair wise soft τ_1 space. So there exists τ_1 soft b open (F, E) and τ_2 soft b open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$ now $x \in Y \; and \; x \notin (G, E)$. Hence $x \in Y \cap (F, E) = (Y_F, E)$ then $y \notin Y \cap (\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$.

Therefore, $y \notin Y \cap (F, E) = (Y_F, E)$. Now $y \in Y$ and $y \in (G, E)$hence $y \in Y \cap (G, E) = (G_Y, E)$ where$(G, E) \in \tau_2$. Consider $x \notin (G, E)$ this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. There fore $x \notin Y \cap (G, E) = (G_Y, E)$ thus $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_1 space.

Now we prove that (X, τ_1, τ_2, E) is pair wise soft b T_3 space then (X, τ_1, τ_2, E) is pair wise soft b regular space.

Let $y \in Y$ and (G, E) be a soft b closed set in Y such that $y \notin (G, E)$ where $(G, E) \in \tau_1$ then $(G, E) = (Y, E) \cap (F, E)$ for some soft b closed set in τ_1.hence $y \notin (Y, E) \cap (F, E)$ but $y \in (Y, E)$, so $y \notin (F, E)$ since (X, τ_1, τ_2, E) is soft b T_3 space

(X, τ_1, τ_2, E) is soft b regular space so there exists τ_1 soft b open set (F_1, E) and τ_2 soft b open set $F_2, E)$ such that

$$y \in (F_1, E), (G, E) \subseteq (F_2, E)$$
$$(F_1, E)(F_2, E) = \phi$$

Take $(G_1, E) = (Y, E) \cap (F_2, E)$ then $(G_1, E), (G_2, E)$ are soft b open set in Y such that

$$y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$$
$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$$
$$(G_1, E) \cap (G_2, E) = \phi$$

There fore τ_{1Y} is soft b regular space with respect to τ_{2Y}.similarly, Let $y \in Y$ and (G, E) be a soft b closed sub set in Y. such that $y \notin (G, E)$, where $(G, E) \in \tau_2$ then $(G, E) = (Y, E) \cap (F, E)$ where (F, E) is some soft b closed set in τ_2. $y \notin (y, E) \cap (F, E)$ But $y \in (Y, E)$ so $y \notin (F, E)$ since (X, τ_1, τ_2, E) is soft b regular space so there exists τ_2 soft b open set (F_1, E) and τ_1 soft b open set(F_2, E). Such that

$$y \in (F_1, E), (G, E) \subseteq (F_2, E)$$
$$(F_1, E) \cap (F_2, E) = \phi$$

Take $(G_1, E) = (Y, E) \cap (F_1, E)$

$$(G_1, E) = (Y, E) \cap (F_1, E)$$

Then (G_1, E) and (G_2, E) are soft b open set in Y such that

$$y \in (G_1, E), (G, E) \subseteq (Y, E) \cap (F_2, E) = (G_2, E)$$
$$(G_1, E) \cap (G_2, E) \subseteq (F_1, E) \cap (F_2, E) = \phi$$

There fore τ_{2Y} is soft b regular space with respect toτ_{1Y}..

$\Rightarrow (X, \tau_1, \tau_2, E)$ is pair wise soft b T_3 space.

Proposition 7. Let (X, τ_1, τ_2, E) be a soft bi topological space over X and Y be a soft b closed sub space of X. if (X, τ_1, τ_2, E) is pair wise soft b T_4 space then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_4 space.

Proof: Since (X, τ_1, τ_2, E) is pair wise soft b T_4 space so this implies that (X, τ_1, τ_2, E) is pair wise soft b T_1 space as proved above.

We prove (X, τ_1, τ_2, E) is pair wise soft b normal space.

Let $(G_1, E), (G_2, E)$ be soft b closed sets in Y such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Then $(G_1, E) = (Y, E) \cap (F_1, E)$

And $(G_2, E) = (Y, E) \cap (F_2, E)$

For some soft b closed sets such that (F_1, E) is soft b closed set in τ_1 (F_2, E) is soft b closed set in τ_2.

And $(F_1, E) \cap (F_2, E) = \phi$

From **Proposition 2.** Since, Y is soft b closed sub set of X then $(G_1, E), (G_2, E)$ are soft b closed sets in X such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since (X, τ_1, τ_2, E) is pair wise soft b normal space. So there exists soft b open sets (H_1, E) and (H_2, E) such that

(H_1, E) is soft b open set in τ_1 and (H_2, E) is soft b open set in τ_2 such that

$$(G_1, E) \subseteq (H_1, E)$$
$$(G_2, E) \subseteq (H_2, E)$$
$$(H_1, E) \cap (H_2, E) = \phi$$

Since $(G_1, E), (G_2, E) \subseteq (Y, E)$

Then $(G_1, E) \subseteq (Y, E) \cap (H_1, E)$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And $[(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$

Where $(Y, E) \cap (H_1, E)$ and $(Y, E) \cap (H_2, E)$ are soft b open sets in Y there fore τ_{1Y} is soft b normal space with respect to τ_{2Y}. Similarly, let $(G_1, E), (G_2, E)$ be soft b closed sub set in Y such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Then $(G_1, E) = (Y, E) \cap (F_1, E)$

And $(G_2, E) = (Y, E) \cap (F_2, E)$

For some soft b closed sets such that (F_1, E) is soft b closed set in τ_2 (F_2, E) soft b closed set in τ_1 and

$$(F_1, E)(F_2, E) = \phi$$

From **Proposition 2.** Since, Y is soft b closed sub set in X then $(G_1, E), (G_2, E)$ are soft b closed sets in X such that

$$(G_1, E) \cap (G_2, E) = \phi$$

Since (X, τ_1, τ_2, E) is pair wise soft b normal space so there exists soft b open sets (H_1, E) and (H_2, E)

Such that (H_1, E) is soft b open set is τ_2 and (H_2, E) is soft b open set in τ_1 such that

$$(G_1, E) \subseteq (H_1, E)$$
$$(G_2, E) \subseteq (H_2, E)$$
$$(H_1, E) \cap (H_2, E) = \phi$$

Since $(G_1, E), (G_2, E) \subseteq (Y, E)$

Then $(G_1, E) \subseteq (Y, E) \cap (H_1, E)$

$$(G_2, E) \subseteq (Y, E) \cap (H_2, E)$$

And $[(Y, E) \cap (H_1, E)] \cap [(Y, E) \cap (H_2, E)] = \phi$

Where $(Y, E) \cap (H_1, E)$ and $(Y, E) \cap (H_2, E)$ are soft b open sets in Y there fore τ_{2Y} is soft b normal space with respect to τ_{1Y}.

$\Rightarrow (Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_4 space.

4.2 Soft b-Separation axioms with respect to ordinary points

In this section, we introduced soft b separation axioms in soft topology and in soft bi topology with respect to soft points. With the application of these soft b separation axioms different result are discussed.

Definition 36. Let (X, τ, A) be a soft Topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen at least one soft b open set (F_1, A) OR (F_2, A) such that $e_G \in (F_1, A)$, $e_H \notin (F_1, A)$ or $e_H \in (F_2, A)$, $e_G \notin ((F_2, A)$ then (X, τ, A) is called a *soft b* T_0 *space.*

Definition 37. Let (X, τ, A) be a soft Topological spaces over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen *soft b open sets* $(F_1, A)$and (F_2, A) such that $e_G \in (F_1, A)$, $e_H \notin (F_1, A)$ and $e_H \in (F_2, A)$, $e_G \notin ((F_2, A)$ then (X, τ, A) is called soft b T_1 *space.*

Definition 38. Let (X, τ, A) b e a *soft Topological* space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen *soft b open sets* $(F_1, A)$and (F_2, A) such that $e_G \in (F_1, A)$, and $e_H \in (F_2, A)$

$(F_1, A) \cap (F_2, A) = \phi_A$ Then (X, τ, A) is called soft b T_2 *space.*

Definition 39. Let (X, τ, E) be a soft topological space (G, E) be b closed soft set in X and $e_G \in X_A$ such that $e_G \notin (G, E)$. If there occurs soft b open sets (F_1, E) and (F_2, E) such that $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$. Then (X, τ, E) is called soft b regular spaces. A soft b regular T_1 *Space* is called soft b T_3space.

Definition 40. In a *soft bi topological* space (X, τ_1, τ_2, E)

1) τ_1 said to be *soft b* T_0 space with respect to τ_2 if for each pair of distinct points $e_G, e_H \in X_A$ there happens τ_1 soft b open set (F, E) and a τ_2 soft b open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (G, E)$, Similarly, τ_2 is said to be *soft b* T_0 space with respect to τ_1 if for each pair of distinct points $e_G, e_H \in X_A$ there happens τ_2 *soft b open set* (F, E) and a τ_1 soft b open set (G, E) such that $e_G \in (F, E)$ and $e_H \notin (F, E)$ or $e_H \in (G, E)$ and $e_G \notin (G, E)$. *Soft bi topological spaces* (X, τ_1, τ_2, E) is said to be *pair wise soft b* T_0 space if τ_1 is *soft b* T_0 *space* with respect to τ_2 and τ_2 is soft b T_0 spaces with respect toτ_1.

2) τ_1 is said to be soft b T_1 space with respect to τ_2 if for each pair of

distinct points $e_G, e_H \in X_A$there happens a τ_1 *soft b open set* $(F.E)$ and τ_2 soft b open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (G,E)$ and $e_H \in (G,E)$ and $e_G \notin (G,E)$.Similarly, τ_1 is said to be soft b T_1 space with respect to τ_1 if for each pair of distinct points $e_G, e_H \in X_A$there exist a τ_2 soft b open set (F,E) and a τ_1 soft b open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (G,E)$ and $e_H \in (G,E)$ and $e_G \notin (G,E)$. *Soft bi topological spaces* (X, τ_1, τ_2, E) is said to be *pair wise soft b* T_1 space if τ_1 is *soft b* T_1 *space* with respect to τ_2 and τ_2 is soft b T_1 spaces with respect toτ_1.

3) τ_1 is said to be soft b T_2 space with respect to τ_2, if for each pair of distinct points $e_G, e_H \in X_A$ there happens a τ_1 soft b open set (F,E) and a τ_2 soft b open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (G,E)$ and $e_H \in (G,E)$ and $e_G \notin (G,E)$ and $(F,E) \cap (G,E) = \phi$. Similarly, τ_2 is aid to be *soft b* T_2 space with respect to τ_1 if for each pair of distinct points $e_G, e_G \in X_A$ there happens a τ_2 *soft b open* set (F,E) and a τ_1 *soft b open* set (G,E) such that $e_G \in (F,E)$ and $e_G \in (G,E)$ and $(F,E) \cap (G,E) = \phi$. The soft bi topological space (X, τ_1, τ_2, E) is said to be pair wise soft b T space if τ_1 is soft b T_2 space with respect to τ_2 and τ_2 is soft b T_2 space with respect toτ_1.

Definition 41. In a soft bi topological space (X, τ_1, τ_2, E)
1) τ_1 is said to be soft b T_3 space with respect to τ_2 if τ_1 is soft b T_1 space with respect to τ_2 and for each pair of distinct points $e_G, e_H \in X_A$,there exists a τ_1
b closed soft set (G,E) such that $e_G \notin (G,E)$, τ_1 soft b open set (F_1,E) and τ_2 soft b open set (F_2,E) such that $e_G \in (F_1,E), (G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \emptyset$. Similarly, τ_2 is said to be soft b T_3 space with respect to τ_1 if τ_2 is soft b T_1 space with respect to τ_1 and for each pair of distinct points $e_G, e_H \in X_A$ there exists a τ_2 soft b closed set (G,E) such that $e_G \notin (G,E)$, τ_2 soft b open set (F_1,E) amd τ_1 soft b open set (F_2,E) such that $e_G \in (F_1,E), (G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. (X, τ_1, τ_2, E) is said to be pair wise soft b T_3 space if τ_1 is soft b T_3 space with respect to τ_2 and τ_2 is soft b T_3 space with respect to τ_1.
2) τ_1 is said to be soft b T_4 space with respect to τ_2 if τ_1 is soft b T_1 space with respect to τ_2 , there exists a τ_1 soft b closed set (F_1,E) and τ_2 soft b closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \emptyset$ also there exists (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_1 b open set, (G_1,E) is soft τ_2 b open set such that $(F_1,E) \subseteq (F_3,E)$, $(F_2,E) \subseteq (G_1,E)$. Similarly, τ_2 is said to be soft b T_4 space with respect to τ_1 if τ_2 is soft b T_1 space with respect to τ_1, there exists τ_2 soft b closed set (F_1,E) and τ_1 soft b closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$. Also there exists (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_2 b open set, (G_1,E) is soft τ_1 b soft set such that $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$. Thus, (X, τ_1, τ_2, E) is said to be pair wise soft b T_4 space if τ_1 is soft b T_4 space with respect to τ_2 and τ_2 is soft b T_4 space with respect to τ_1.

Proposition 8. Let (X, τ, E) be a soft topological space over X.If (X, τ, E) is soft b T_3-space, then for all $e_G \in X_E$ $e_G = (e_G, E)$ is b-closed soft set.

Proof: We want to prove that e_G is b-closed soft set, which is sufficient to prove that e_G^c is b-open soft set for all $e_H \in \{e_G\}^c$.Since (X, τ, E) is soft b T_3-space, then there exists soft b set sets$(F,E)_{e_H}$ and (G,E) such that $e_{H_E} \subseteq (F,E)_{e_H}$ and $e_{G_E} \cap (F,E)_{e_H} = \phi$ and $e_{G_E} \subseteq (G,E)$ and $e_{H_E} \cap (G,E) = \phi$.It follows that, $\cup_{e_H \in (e_G)^c (F,E)_{e_H} \subseteq e_{G_E}^c}$ Now, we want to prove that $e_{G_E}^c \subseteq \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$.Let $\cup_{e_H \in (e_G)^c} (F,E)_{e_H} = (H,E)$.Where $H(e) = \cup_{e_H \in (e_G)^c (F(e)_{e_H}}$ for all $e \in E$.Since $e_{G_E}^c(e) = (e_G)^c$for all $e \in E$ from Definition 9, so, for all $e_H \in \{e_G\}^c$and $e \in E$ $e_{G_E}^c(e) = \{e_G\}^c = \cup_{e_H \in (e_G)^c} \{e_H\} = \cup_{e_H \in (e_G)^c\, F(e)_{e_H} = H(e)}$.Thus, $e_{G_E}^c \subseteq \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$ from Definition 2, and so, $e_{G_E}^c = \cup_{e_H \in (e_G)^c} (F,E)_{e_H}$.
This means that, $e_{G_E}^c$ is soft b-open set for all $e_H \in \{e_G\}^c$.Therefore, e_{G_E} is b-closed soft set.

Proposition 9. Let (X, τ_1, τ_2, E) be a soft bi topological space over X. Then, if (X, τ_1, E) and (X, τ_2, E) are soft b T_3 space, then (X, τ_1, τ_2, E) is a pair wise soft b T_2 space.

Proof: Suppose (X, τ_1, E) is a soft b T_3 space with respect to (X, τ_2, E), then according to definition for, $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, ($e_H$, E) is soft b closed set in τ_2 and $e_G \notin (e_H, E)$ there exists a τ_1 soft b open set (F,E) and a τ_2 soft b open set (G,E) such that $e_G \in (F,E)$, $e_H \in (y,E) \subseteq (G,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence, τ_1 is soft b T_2 space with respect to τ_2. Similarly, if (X, τ_2, E) is a soft b T_3 space with respect to (X, τ_1, E), then according to definition for , $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, ($e_G$, E) is b closed soft set in τ_1 and $y \notin (x,E)$ there exists a τ_2 soft b open set (F,E) and a τ_1 soft b open set (G,E) such that $e_H \in (F,E)$, $e_G \in (x,E) \subseteq (G,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence, τ_2 is a soft b T_2 space. Thus (X, τ_1, τ_2, E) is a pair wise soft b T_2 space.

Proposition 10. Let (X, τ_1, τ_2, E) be a soft bi topological space over X. if (X, τ_1, E) and (X, τ_2, E) are soft b T_3 space then (X, τ_1, τ_2, E) is a pair wise soft b T_3 space.

Proof: Suppose (X, τ_1, E) is a soft b T_3 space with respect to (X, τ_2, E) then according to definition for $e_G, e_H \in X_A$ $e_G \neq e_H$ there happens a τ_1 soft b open set (F,E) and a τ_2 soft b open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each point $e_G \in X_A$and each τ_1 b closed soft set (G_1,E) such that $e_G \notin (G_1,E)$ there happens a τ_1 soft b open set (F_1,E) and τ_2 soft b open set (F_2,E) such that $e_G \in (F_1,E), (G_1,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$.Similarly,$(X, \tau_2, E)$ is a soft β_3 space with respect to(X, τ_1, E). So according to definition for$e_G, e_H \in X_A, e_G \neq e_H$ there exists τ_2 soft b open set (F,E) and a τ_1 soft b open set (G,E) such that $e_H \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each point $e_G \in X_A$ and each τ_2 b closed soft set (G_1,E) such that $e_G \notin (G_1,E)$ there exists a τ_2 soft b open set (F_1,E) and a τ_1 soft b open set (F_2,E) such that $e_G \in (F_1,E), (G_1,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Hence (X, τ_1, τ_2, E) is pair wise soft b T_3 space.

Proposition 11. If (X, τ_1, τ_2, E) be a soft bi topological space over X if (X, τ_1, E) and (X, τ_2, E) are soft b T_4 space then (X, τ_1, τ_2, E) is pair wise soft b T_4 space.

Proof: Suppose (X, τ_1, E) is soft b T_4 space with respect to (X, τ_2, E).So according to definition for $e_G, e_H \in X_A$, $e_G \neq e_H$ there happens a τ_1 soft b open set (F,E) and a τ_2 soft b open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ each τ_1 soft b closed set (F_1,E) and a τ_2 soft b closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$.There occurs (F_3,E) and (G_1,E) such that (F_3,E) is soft τ_2 b open set (G_1,E) is soft τ_1 b open set $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and$(F_3,E) \cap (G_1,E) = \phi$. Similarly, τ_2 is soft b T_4 space with respect to τ_1 so according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a τ_2 soft b open set (F,E) and a τ_1 soft b open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ and for each τ_2 soft b closed set (F_1,E) and τ_1 soft b closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$.there occurs (F_3,E) and(G_1,E) such that (F_3,E) is soft τ_2 b open set (G_1,E) is soft τ_1 b open set such that $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$ hence (X, τ_1, τ_2, E) is pair wise soft b T_4 space.

Proposition 12. Let (X, τ_1, τ_2, E) be a soft bi topological space over X and Y be a non-empty subset of X. if $(X, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_3 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_3 space.

Proof: first we prove that $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pair wise soft b T_1 space.
Let $e_G, e_H \in X_A, e_G \neq e_H$ if (X, τ_1, τ_2, E) is pair wise space then this implies that (X, τ_1, τ_2, E) is pair wise soft τ_1 space. So there exists τ_1 soft b open set (G,E) such that $e_G \in (F,E)$ and $e_H \notin (F,E)$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ now $e_G \in Y$ *and* $e_G \notin (G,E)$. Hence $e_G \in Y \cap (F,E) = (Y_F, E)$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$. this means that $\alpha \in E$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$.
There fore, $e_H \notin Y \cap (F,E) = (Y_F, E)$. Now $e_H \in Y$ and $e_H \in (G,E)$. Hence, $e_H \in Y \cap (G,E) = (G_Y, E)$ where $(G,E) \in \tau_2$. Consider $x \notin (G,E)$ .this means that $\alpha \in E$ then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. There fore $e_G \notin Y \cap (G,E) = (G_Y, E)$ thus $(Y, \tau_{1y}, \tau_{2y}, E)$ is pair wise soft b T_1 space.
Now, we prove that (X, τ_1, τ_2, E) is pair wise soft b T_3 space then (X, τ_1, τ_2, E) is pair wise soft b regular space.
Let $e_H \in Y$ and (G,E) be soft b closed set in Y such that $e_H \notin (G,E)$ where $(G,E) \in \tau_1$ then $(G,E) = (Y,E) \cap (F,E)$ for some soft b closed set in τ_1.hence $e_H \notin (Y,E) \cap (F,E)$ but $e_H \in (Y,E)$, so $e_H \notin (F,E)$ since (X, τ_1, τ_2, E) is soft b T_3 space
(X, τ_1, τ_2, E) is soft b regular space so there happens τ_1 soft b open set (F_1,E) and τ_2 soft b open set (F_2,E) such that

$$e_H \in (F_1,E), (G,E) \subseteq (F_2,E)$$
$$(F_1,E)(F_2,E) = \phi$$

Take $(G_1,E) = (Y,E) \cap (F_2,E)$ then $(G_1,E), (G_2,E)$ are soft b open sets in Y such that

$$e_H \in (G_1,E), (G,E) \subseteq (Y,E) \cap (F_2,E) = (G_2,E)$$
$$(G_1,E) \cap (G_2,E) \subseteq (F_1,E) \cap (F_2,E) = \phi$$
$$(G_1,E) \cap (G_2,E) = \phi$$

Therefore, τ_{1Y} is soft b regular space with respect to τ_{2Y}. Similarly, Let $e_H \in Y$ and (G,E) be a soft b closed sub set in Y such that$e_H \notin (G,E)$, where $(G,E) \in \tau_2$ then $(G,E) = (Y,E) \cap (F,E)$ where (F,E) is some soft b closed set inτ_2. $e_H \notin (Y,E) \cap (F,E)$ but $e_H \in (Y,E)$ so $e_H \notin (F,E)$ since (X, τ_1, τ_2, E) is soft b regular space so there happens τ_2 soft b open set (F_1,E) and τ_1 soft b open set (F_2,E). Such that

$$e_H \in (F_1,E), (G,E) \subseteq (F_2,E)$$
$$(F_1,E) \cap (F_2,E) = \phi$$

Take $(G_1,E) = (Y,E) \cap (F_1,E)$

$$(G_1,E) = (Y,E) \cap (F_1,E)$$

Then (G_1,E) and (G_2,E) are soft β open set in Y such that

$$e_H \in (G_1,E), (G,E) \subseteq (Y,E) \cap (F_2,E) = (G_2,E)$$
$$(G_1,E) \cap (G_2,E) \subseteq (F_1,E) \cap (F_2,E) = \phi$$

Therefore τ_{2Y} is soft b regular space.

5. CONCLUSION

Topology is the most important branch of mathematics which deals with mathematical structures. Recently, many researchers have studied the soft set theory which is initiated by Molodtsov and safely applied to many problems which contain uncertainties in our social life. Some researchers introduced and deeply studied the concept of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we have continued to study the properties of soft b separation axioms in soft bi topological spaces with respect to soft points as well as ordinary points of a soft topological spaces. We defined soft T_0, T_1, $T_2$and T_3 spaces with respect to soft points and studied their behaviors in soft bi topological spaces. We also extended these axioms to different results. These soft separation axioms would be useful for the growth of the theory of soft topology to solve complex problems, comprising doubts in economics, engineering, medical etc. We also beautifully discussed some soft transmissible properties with respect to ordinary as well as soft points. We hope that these results in this paper will help the researchers for strengthening the toolbox of soft topology. In the next study, we extend the concept of soft semi open, α- open, Pre-open and b^{**}open soft sets in soft bi topological spaces with respect to ordinary as well as soft points

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