J. Appl. Comput. Mech., 5(1) (2019) 174-180
DOI: 10.22055/JACM.2018.25143.1232

ISSN: 2383-4536
jacm.scu.ac.ir

M&MoCS

Shahid Chamran University of Ahvaz

Journal of
Applied and Computational Mechanics

Technical Brief

A Note on Free Vibration of a Double-beam System with Nonlinear Elastic Inner Layer

Alborz Mirzabeigy, Reza Madoliat

School of Mechanical Engineering, Iran University of Science and Technology, Narmak, Tehran, 16846, Iran

Received February 26 2018; Revised April 28 2018; Accepted for publication May 03 2018.
Corresponding author: Reza Madoliat, madoliat@iust.ac.ir

Abstract. In this note, small amplitude free vibration of a double-beam system in presence of inner layer nonlinearity is investigated. The nonlinearity is due to inner layer material and is not related to large amplitude vibration. At first, frequencies of a double-beam system with linear inner layer are studied and categorized as synchronous and asynchronous frequencies. It is revealed that the inner layer does not affect higher modes significantly and mainly affects the first frequency. Then, equation of motion in the presence of cubic nonlinearity in the inner layer is derived and transformed to the form of Duffing equation. Using an analytical solution, the effect of nonlinearity on the frequency for simply-supported and clamped boundary conditions is analyzed. Results show that the nonlinearity effect is not significant and, in small amplitude free vibration analysis of a double-beam system, the material nonlinearity of the inner layer could be neglected.

Keywords: Double-beam system, Frequency, Nonlinearity, Duffing equation, Analytical solution.

1. Introduction

Sandwich beams have wide applications in different industries, especially in the production of light weight structures which are essential elements in aircraft and ship manufacturing. Such beams are made of three layers: two faces which are thin and highly strong and an inner layer which is thick and has a light and a low stiffness. Desirable structural properties like high stiffness and low weight are achieved by combining the strong face with a thick and low-density core. A double-beam system made of two parallel beams connected together through an elastic layer is considered as an approximate model for the sandwich beam. Another important application of a double-beam system is in vibration mitigation, where an external load is applied to one of the beams as the main structure and the other beam is designed to reduce undesired vibration as an auxiliary device. The common model for the inner layer is the well-known Winkler model which considers an elastic layer as a series of closely-spaced and mutually-independent linear elastic springs in which the vertical displacement is assumed to be proportional to the contact pressure at an arbitrary point. The dynamic characteristics of the double-beam system with Winkler inner layer have been investigated in several studies [1-14]. For instance, Oniszczuk [1] investigated free vibration of double-beam systems continuously joined by a Winkler elastic layer with simply-supported boundary conditions and presented analytical formulation for the natural frequencies. De Rosa and Lippiello [4] studied the free vibration of parallel double-beams joined by a Winkler-type elastic layer by means of the differential quadrature method. The double-beam has vertical translation and rotation elastic constraints to the ends. Mirzabeigy and Madoliat [5] investigated the free vibration of a partially-connected double-beam system which was a model for damage in the inner layer. In another work, Mirzabeigy et al. [6] studied the free vibration of a double-beam system in which stiffness of Winkler inner layer varied along the length of the beam. Kozic et al. [7] applied Kerr-type or the three-parameter model for an elastic inner layer. This model was made of two Winkler layers with different stiffness values connected together through a shear layer. Vu et al. [8] considered a viscoelastic inner layer made of parallel springs and dampers and studied the forced vibration of the system. Li et al. [9] applied a semi-

Published online: January 12 2018

analytical method to study the free and forced vibration of a double-beam system having a viscoelastic inner layer. Palmeri and Adhikari [10] proposed a Galerkin-type state-space for transverse vibrations of a double-beam with a viscoelastic inner layer. Their viscoelastic model was the standard linear solid model made of a spring in parallel with Maxwell's element, where Maxwell's element is given by a spring in series with a damper.

There are two different sources considered for nonlinear vibration of structures. One of them is the large amplitude motion which yields nonlinear terms in the equation of motion. Another source is the nonlinear behavior of materials. Several studies have been conducted on the nonlinear vibration of a single beam from the macro to the micro scale. There are three studies regarding nonlinear vibration in double-beam systems. Bochicchio et al. [12] applied the nonlinear model of Woinowsky-Krieger for beams in which the inner layer was considered as the linear elastic Winkler model. Koziol [13] studied the dynamic behavior of a double-beam system with viscoelastic inner layer under the action of a moving load. In this paper, the double system was resting on a nonlinear foundation. Rahman and Lee [14] studied vibration of a double-beam system with elastic Winkler inner layer under harmonic excitation. They assumed one of the beams would undergo a large amplitude due to excitation while the other would remain linear with a small amplitude of vibration.

As stated, different problems related to the vibration of a double-beam system have been studied. There is no report about the effect of a nonlinear inner layer. In this study, the effect of a nonlinear Winkler inner layer on free vibration with small amplitude was investigated. At first, the well-known solutions for frequencies of double-beam systems were applied and it was revealed that the elastic inner layer mainly affected the fundamental frequency. Then the equation of motion in presence of cubic nonlinearity in the elastic inner layer was derived using the first mode of vibration. The governing equations of motion were transformed into Duffing equations and the analytical solution was applied to study the effect of nonlinearity in the elastic layer.

2. Methodology

Consider a double-beam system as depicted in Fig 1. The two beams are identical and simply-supported boundary conditions have been considered. The beam's Young modulus is E, the density is ρ, the width of rectangular cross-section is w and the height is h. Also, the area and the cross-sectional moment of inertia are A and I, respectively. The stiffness of the linear Winkler inner layer is k_L. This problem has been handled in different papers [1-3] and the synchronous frequencies have been derived as follows:

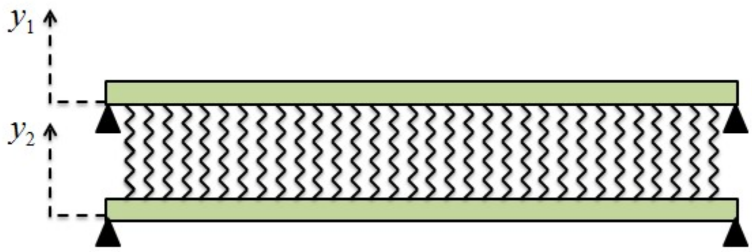

Fig. 1. A schematic of double-beam system with elastic inner layer.

$$\omega_i^S = \sqrt{\frac{EI\,\Omega_i^4}{\rho A L^4}} \tag{1}$$

In synchronous motions, the beams' motions have the same amplitude (because the beams are identical) and direction. Therefore, the potential energy is not stored in the inner layer. The asynchronous frequencies are as follows:

$$\omega_i^{As} = \sqrt{\frac{EI\,\Omega_i^4}{\rho A L^4} + \frac{2k_L}{\rho A}} \tag{2}$$

As can be seen, due to contrary motion direction of the beams, the effects of the inner layer are revealed in frequency. In the case of simply-supported boundary conditions, we have:

$$\Omega_i = i \times \pi \tag{3}$$

In order to investigate the effect of the inner layer on frequencies of the system, we consider the following geometric and physical properties:

$$L = 2m,\, w = 0.1m,\, h = 0.05m,\, E = 70Gpa,\; \rho = 2700\,kg/m^3 \tag{4}$$

To better show the effect of the inner layer, we define increase in frequency due to asynchronous motion rather than synchronous motion for the ith mode (IFAS) as follows:

$$IFAS_i = \frac{\omega_i^{As} - \omega_i^S}{\omega_i^S} \times 100 \tag{5}$$

In Table 1, the effect of different values of linear Winkler stiffness on natural frequencies has been investigated. As seen, the inner layer mostly affects the first asynchronous frequency and as the mode number is increased, the effect of the inner layer becomes smaller. Due to the great effect of inner layer on first frequency and the negligible effect on higher modes, we considered only the first mode in analysis of the double-beam system in presence of inner layer nonlinearity.

Table 1. Effect of linear elastic inner layer on frequencies of the double-beam system.

		$k_L = 0.5\times10^5$		$k_L = 10^5$		$k_L = 5\times10^5$	
	synchronous	asynchronous	IFAS	asynchronous	IFAS	asynchronous	IFAS
$i=1$	181.33694	200.72492	10.7%	218.39849	20.4%	327.04305	80.4%
$i=2$	725.34774	730.43601	0.7%	735.48906	1.4%	774.72797	6.8%
$i=3$	1632.0324	1634.3002	0.14%	1636.5649	0.28%	1654.5706	1.4%

3. Free vibration with nonlinear inner layer

Consider the double-beam system investigated in Section 2. In this section, the nonlinear behavior was considered for the Winkler inner layer as cubic nonlinearity which is shown by k_N . As mentioned earlier, the amplitude of motion is small and the nonlinear behavior of beams can be neglected. By assuming that the length of the beam is at least 10 times larger than its height, the thin or Euler-Bernoulli beam theory is applied to modeling the beams. Deflections of upper and lower beams are denoted by $y_1(x,t)$ and $y_2(x,t)$, respectively. To derive the governing equations of motion, the potential and kinetic energies of the system are required. The potential energy of the system has two parts: one part from bending in beams and another part from the energy stored in the elastic inner layer. The potential energy of such a system from bending in beams is as follows:

$$\Pi_{Bending} = \frac{1}{2}\int_0^L EI\,(\frac{\partial^2 y_1(x,t)}{\partial x^2})^2 dx + \frac{1}{2}\int_0^L EI\,(\frac{\partial^2 y_2(x,t)}{\partial x^2})^2 dx \tag{6}$$

The potential energy stored in the inner layer is as follows:

$$\Pi_{Inner\ Layer} = \frac{1}{2}\int_0^L k_L\,(y_1(x,t)-y_2(x,t))^2 dx + \frac{1}{2}\int_0^L k_N\,(y_1(x,t)-y_2(x,t))^4 dx \tag{7}$$

The kinetic energy of the system is as follows:

$$T = \frac{1}{2}\int_0^L \rho A\,(\frac{\partial y_1(x,t)}{\partial t})^2 dx + \frac{1}{2}\int_0^L \rho A\,(\frac{\partial y_2(x,t)}{\partial t})^2 dx \tag{8}$$

Using the Lagrangian of the system and invoking Hamilton's principle, we have:

$$\delta\int_{t_1}^{t_2} (T - \Pi_{Bending} - \Pi_{Inner\ Layer})dt = 0 \tag{9}$$

Substituting Eqs. (6-8) into Eq. (9) and performing the necessary algebra, the following partial differential equations are obtain for motion:

$$EI\frac{\partial^4 y_1(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y_1(x,t)}{\partial t^2} + k_L(y_1(x,t)-y_2(x,t)) + k_{NL}(y_1(x,t)-y_2(x,t))^3 = 0 \tag{10}$$

$$EI\frac{\partial^4 y_2(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y_2(x,t)}{\partial t^2} + k_L(y_2(x,t)-y_1(x,t)) + k_{NL}(y_2(x,t)-y_1(x,t))^3 = 0 \tag{11}$$

The governing equations of motion in Eqs. (10) and (11) can be solved using the Bernoulli-Fourier method [1-3] by assuming solutions as follows:

$$y_1(x,t) = \varphi(x)T_1(t),\ y_2(x,t) = \varphi(x)T_2(t) \tag{12}$$

where $T_1(t)$ and $T_2(t)$ denote unknown time functions, and $\varphi(x)$ is the known first mode shape function. Substituting Eq. (12) into Eqs. (10) and (11) yields:

$$EI\frac{d^4\varphi(x)}{dx^4}T_1(t) + \rho A\,\varphi(x)\frac{d^2T_1(t)}{dt^2} + k_L\,\varphi(x)(T_1(t)-T_2(t)) + k_{NL}\,\varphi^3(x)(T_1(t)-T_2(t))^3 = 0 \tag{13}$$

$$EI\frac{d^4\varphi(x)}{dx^4}T_2(t) + \rho A\,\varphi(x)\frac{d^2T_2(t)}{dt^2} + k_L\,\varphi(x)(T_2(t)-T_1(t)) + k_{NL}\,\varphi^3(x)(T_2(t)-T_1(t))^3 = 0 \tag{14}$$

By adopting the weighted residual Bubnov–Galerkin method, we have:

$$\int_0^L (EI\frac{d^4\varphi(x)}{dx^4}T_1(t)+\rho A\,\varphi(x)\frac{d^2T_1(t)}{dt^2}+k_L\,\varphi(x)(T_1(t)-T_2(t))+k_{NL}\,\varphi^3(x)(T_1(t)-T_2(t))^3)\varphi(x)dx=0 \tag{15}$$

$$\int_0^L (EI\frac{d^4\varphi(x)}{dx^4}T_2(t)+\rho A\,\varphi(x)\frac{d^2T_2(t)}{dt^2}+k_L\,\varphi(x)(T_2(t)-T_1(t))+k_{NL}\,\varphi^3(x)(T_2(t)-T_1(t))^3)\varphi(x)dx=0 \tag{16}$$

Finally, the nonlinear equations of motion in terms of unknown time functions are derived as follows:

$$\ddot{T}_1+\alpha T_1+\beta(T_1-T_2)+\lambda(T_1-T_2)^3=0 \tag{17}$$

$$\ddot{T}_2+\alpha T_2+\beta(T_2-T_1)+\lambda(T_2-T_1)^3=0 \tag{18}$$

where

$$\alpha=\frac{\int_0^L EI\frac{d^4\varphi(x)}{dx^4}\varphi(x)dx}{\int_0^L \rho A\,\varphi^2(x)dx},\qquad \beta=\frac{\int_0^L k_L\,\varphi^2(x)dx}{\int_0^L \rho A\,\varphi^2(x)dx}=\frac{k_L}{\rho S},\qquad \lambda=\frac{\int_0^L k_{NL}\,\varphi^4(x)dx}{\int_0^L \rho A\,\varphi^2(x)dx} \tag{19}$$

In the case of simply-supported boundary conditions, the fundamental mode function is:

$$\varphi(x)=\sin(\pi\frac{x}{L}) \tag{20}$$

Consequently, the other parameters are calculated as follows:

$$\alpha=\frac{\pi^4EI}{\rho AL^4},\qquad \lambda=\frac{3k_{NL}}{4\rho A} \tag{21}$$

In the case of clamped boundary conditions, the fundamental mode function is:

$$\varphi(x)=(\cos(\Omega x)-\cosh(\Omega x))-\frac{\cos(\Omega)-\cosh(\Omega)}{\sin(\Omega)-\sinh(\Omega)}(\sin(\Omega x)-\sinh(\Omega x)),\Omega=4.7300 \tag{22}$$

As a result, the other parameters are calculated as follows:

$$\alpha=\frac{4.73^4EI}{\rho AL^4},\qquad \lambda=\frac{1.8519k_{NL}}{\rho A} \tag{23}$$

It is assumed that the midpoint of the beams is subjected to an initial displacement and zero initial velocity. Therefore, the initial conditions of nonlinear differential equations in Eqs. (17) and (18) become:

$$T_1(0)=C_1,\dot{T}_1(0)=0,T_2(0)=C_2,\dot{T}_2(0)=0 \tag{24}$$

To derive the analytical solution for Eqs. (17) and(18), intermediate variables are introduced as follows [15-18]:

$$T_1:=v,\qquad T_2-T_1:=u \tag{25}$$

by applying the introduced variables, Eqs. (17) and (18) are transformed into:

$$\ddot{v}+\alpha v-\beta u-\lambda u^3=0 \tag{26}$$

$$\ddot{u}+\ddot{v}+\alpha u+\alpha v+\beta u+\lambda u^3=0 \tag{27}$$

by combining Eqs. (26) and (27), we have:

$$\ddot{u}+(\alpha+2\beta)u+2\lambda u^3=0,\quad u(0)=T_2(0)-T_1(0)=C_2-C_1=C,\ \dot{u}(0)=0 \tag{28}$$

Eq. (28) is the well-known nonlinear Duffing equation with cubic nonlinearity. Several methods have been proposed and applied to analytically solving nonlinear differential equations [19-29]. For nonlinear Duffing equation in the following form:

$$\ddot{q}+\vartheta_1 q+\vartheta_2 q^3=0,\ q(0)=A,\dot{q}(0)=0 \tag{29}$$

different methods have been applied to derive the analytical amplitude-frequency relationship for Eq. (29). One relation was obtained using energy balance method based on Galerkin-Petrov (EGP) approach as follows [18]:

$$\omega_{EGP}=\sqrt{\vartheta_1+\frac{7}{10}\vartheta_2A^2} \tag{30}$$

Another relation was obtained using homotopy perturbation method (HPM) as follows [30]:

$$\omega_{HPM}=\sqrt{\vartheta_1+\frac{3}{4}\vartheta_2A^2} \tag{31}$$

When the amplitude is not large, the solutions in Eqs. (30) and (31) yield good accuracy in comparison with exact solutions from integration. By analogy between Eq. (28) and Eq. (29), frequency-amplitude relationships for Eq. (30) are obtain as follows:

$$\omega_{EGP} = \sqrt{\alpha + 2\beta + \frac{7}{5}\lambda C^2} \tag{32}$$

$$\omega_{HPM} = \sqrt{\alpha + 2\beta + \frac{3}{2}\lambda C^2} \tag{33}$$

By substituting the obtained values for α, β, λ into Eqs. (32) and (33), the nonlinear frequencies of the double-beam system with simply-supported boundary conditions are derived as follows:

$$\omega_{EGP} = \sqrt{\frac{\pi^4 E I}{\rho A L^4} + \frac{2k_L}{\rho A} + \frac{21k_{NL}}{20\rho A}C^2} \tag{34}$$

$$\omega_{HPM} = \sqrt{\frac{\pi^4 E I}{\rho A L^4} + \frac{2k_L}{\rho A} + \frac{9k_{NL}}{8\rho A}C^2} \tag{35}$$

In order to compare the two analytical nonlinear frequencies of the system, linear stiffness of the inner layer is assumed to be $k_L = 10^5$. As stated before, the amplitude of motion is not large. Considering the length of beams ($L = 2m$), the maximum value of amplitude will be 0.2. The results are presented in Table 2. As can be seen, the results are very similar and both methods can be used to study the system. Therefore, we used the homotopy perturbation solution.

Table 2. Comparison between different solutions for nonlinear frequency in the case of simply-supported boundary conditions.

C	$k_N = 0.5k_L$		$k_N = 0.75k_L$		$k_N = k_L$	
	ω_{EGP}	ω_{HPM}	ω_{EGP}	ω_{HPM}	ω_{EGP}	ω_{HPM}
0.05	218.42074	218.422333	218.43187	218.434255	218.44299	218.446177
0.1	218.48750	218.493857	218.53199	218.541527	218.57647	218.589186
0.2	218.75432	218.779719	218.93202	218.970087	219.10958	219.160289

In the case of clamped boundary conditions, the nonlinear frequency using homotopy perturbation method was derived as follows:

$$\omega = \sqrt{\frac{4.73^4 E I}{\rho A L^4} + \frac{2k_L}{\rho A} + \frac{2.7778k_{NL}}{\rho A}C^2} \tag{36}$$

To investigate the effect of inner layer nonlinearity on the asynchronous frequency, we defined increase in frequency due to inner layer nonlinearity (IFLN) as follows:

$$IFLN = \frac{\omega - \omega_{C=0}}{\omega_{C=0}} \tag{37}$$

For numerical analysis, the values of parameters in Eq. (4) were considered and the linear stiffness of the inner layer was assumed to be $k_L = 10^5$ and the maximum value of amplitude was considered to be 0.2. In Fig. 2, the effects of inner layer nonlinearity and the amplitude of motion on the frequency of system with simply-supported boundary conditions are depicted using IFLN. Moreover, Fig. 3 shows the effects of inner layer nonlinearity and the amplitude of motion on the frequency of system with clamped boundary conditions using IFLN. It is obvious that the inner layer nonlinearity in the form of cubic nonlinear term does not greatly affect the free vibration frequency of a double-beam system when the amplitude is not large.

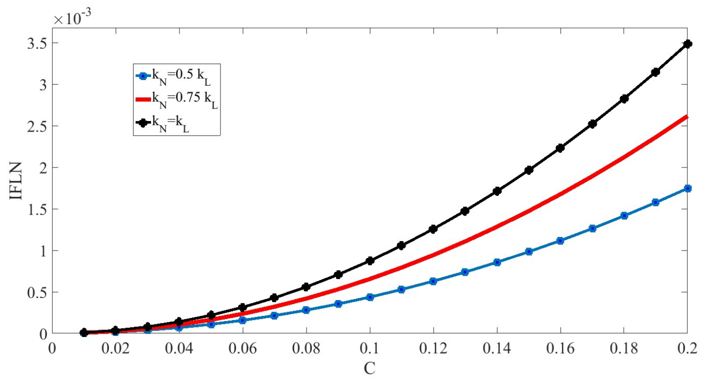

Fig. 2. Effect of nonlinearity in inner layer on frequency of double-beam system with simply-supported boundary conditions

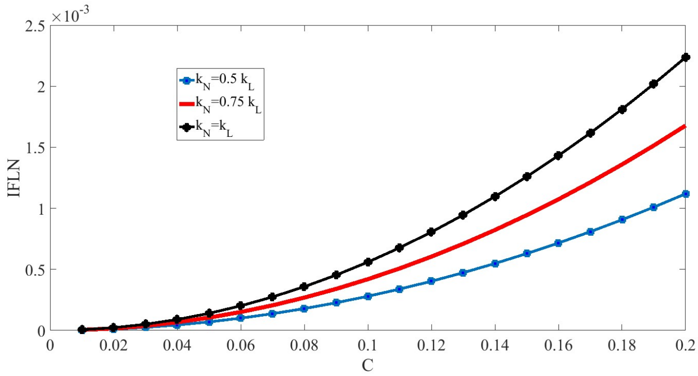

Fig. 3. Effect of nonlinearity in inner layer on frequency of double-beam system with clamped boundary conditions.

4. Conclusion

In this note, the effect of nonlinearity in an inner elastic layer on the frequency of a double-beam system in small amplitude vibration was investigated. At first, by making a distinction between synchronous and asynchronous motions of beams, the effect of linear inner layer was investigated and the results showed that the inner layer mainly affected the first mode asynchronous frequency, having little effect on higher mode frequency. Then, the equations of motion by considering cubic nonlinearity in the inner layer and small amplitude vibration were derived. By introducing intermediate variables, the equations of motion were transformed to the well-known Duffing equation. Using the amplitude-frequency relationship of Duffing equation, the effect of inner layer nonlinearity on frequency in small amplitude free vibration was investigated. The results showed that the effect is not significant, although this effect is larger for simply-supported boundary conditions than the clamped boundary conditions. If the amplitude of motion becomes large, the inner layer nonlinearity has obvious effects on frequency. We conclude that in small amplitude free vibration of a double-beam system, if the inner layer nonlinearity is neglected, the calculated frequency is valid with high accuracy.

Conflict of Interest

The authors declare no conflict of interest.

References

[1] Oniszczuk, Z., Free transverse vibrations of elastically connected simply supported double-beam complex system. *Journal of sound and vibration,* 232(2), 2000, 387-403.

[2] Zhang, Y.Q., Lu, Y., Wang, S.L., Liu, X., Vibration and buckling of a double-beam system under compressive axial loading. *Journal of Sound and Vibration*, 318(1-2), 2008, 341-352.

[3] Mirzabeigy, A., Dabbagh, V., Madoliat, R., Explicit formulation for natural frequencies of double-beam system with arbitrary boundary conditions. *Journal of Mechanical Science and Technology*, 31(2), 2017, 515-521.

[4] De Rosa, M.A., Lippiello, M., Non-classical boundary conditions and DQM for double-beams. *Mechanics Research Communications*, 34(7-8), 2007, 538-544.

[5] Mirzabeigy, A., Madoliat, R., Free vibration analysis of partially connected parallel beams with elastically restrained ends. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 230(16), 2016, 2851-2864.

[6] Mirzabeigy, A., Madoliat, R., Vahabi, M., Free vibration analysis of two parallel beams connected together through variable stiffness elastic layer with elastically restrained ends. *Advances in Structural Engineering*, 20(3), 2017, 275-287.

[7] Kozić, P., Pavlović, R., Karličić, D., The flexural vibration and buckling of the elastically connected parallel-beams with a Kerr-type layer in between. *Mechanics Research Communications*, 56, 2014, 83-89.

[8] Vu, H.V., Ordonez, A.M., Karnopp, B.H., Vibration of a double-beam system. *Journal of Sound and Vibration* 229(4), 2000, 807-822.

[9] Li, Y.X., Hu, Z.J., Sun, L.Z., Dynamical behavior of a double-beam system interconnected by a viscoelastic layer. *International Journal of Mechanical Sciences*, 105, 2016, 291-303.

[10] Palmeri, A., Adhikari, S., A Galerkin-type state-space approach for transverse vibrations of slender double-beam systems with viscoelastic inner layer. *Journal of Sound and Vibration*, 330(26), 2011, 6372-6386.

[11] Stojanović, V., Kozić, P., Pavlović, R., Janevski, G., Effect of rotary inertia and shear on vibration and buckling of a double beam system under compressive axial loading. *Archive of Applied Mechanics*, 81(12), 2011, 1993-2005.

[12] Bochicchio, I., Giorgi, C., Vuk, E., Buckling and nonlinear dynamics of elastically coupled double-beam systems. *International Journal of Non-Linear Mechanics*, 85, 2016, 161-173.

[13] Koziol, P., Wavelet approximation of Adomian's decomposition applied to the nonlinear problem of a double-beam response subject to a series of moving loads. *Journal of Theoretical and Applied Mechanics*, 52(3), 2014, 687-697.

[14] Rahman, M.S., Lee, Y.Y., New modified multi-level residue harmonic balance method for solving nonlinearly vibrating double-beam problem. *Journal of Sound and Vibration*, 406, 2017, 295-327.

[15] Cveticanin, L., Vibrations of a coupled two-degree-of-freedom system. *Journal of Sound and Vibration*, 247(2), 2001, 279-292.

[16] Cveticanin, L., The motion of a two-mass system with non-linear connection, *Journal of Sound and Vibration*, 252 (2), 2002, 361-369.

[17] Lai, S.K., Lim, C.W., Nonlinear vibration of a two-mass system with nonlinear stiffnesses. *Nonlinear Dynamics*, 49(1-2), 2007, 233-249.

[18] Mirzabeigy, A., Madoliat, R. Free Vibration Analysis of a Conservative Two-Mass System with General Odd Type Nonlinear Connection. *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences*, 88(1), 2018, 145-156.

[19] He, J.H., A coupling method of a homotopy technique and a perturbation technique for non-linear problems. *International Journal of Non-Linear Mechanics*, 35(1), 2000, 37-43.

[20] He, J.H., Preliminary report on the energy balance for nonlinear oscillations. *Mechanics Research Communications*, 29(2-3), 2002, 107-111.

[21] Sedighi, H.M., Shirazi, K.H., Noghrehabadi, A., Application of recent powerful analytical approaches on the non-linear vibration of cantilever beams. *International Journal of Nonlinear Sciences and Numerical Simulation*, 13(7-8), 2012, 487-494.

[22] Sedighi, H.M., Shirazi, K.H., Attarzadeh, M.A., A study on the quintic nonlinear beam vibrations using asymptotic approximate approaches. *Acta Astronautica*, 91, 2013, 245-250.

[23] Mirzabeigy, A., Yazdi, M.K., Yildirim, A., Nonlinear dynamics of a particle on a rotating parabola via the analytic and semi-analytic approaches. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, 13(1), 2013, 38-43.

[24] Khan, Y., Mirzabeigy, A., Improved accuracy of He’s energy balance method for analysis of conservative nonlinear oscillator. *Neural Computing and Applications*, 25(3-4), 2014, 889-895.

[25] Sedighi, H.M., Size-dependent dynamic pull-in instability of vibrating electrically actuated microbeams based on the strain gradient elasticity theory. *Acta Astronautica*, 95, 2014, 111-123.

[26] Joubari, M., Mehdi, M., Ganji, D.D., Javanian Jouybari, H., Determination of Periodic Solution for Tapered Beams with Modified Iteration Perturbation Method. *Journal of Applied and Computational Mechanics*, 1(1), 2015, 44-51.

[27] El-Naggar, A.M., Ismail, G., Periodic Solutions of the Duffing Harmonic Oscillator by He's Energy Balance Method. *Journal of Applied and Computational Mechanics*, 2(1), 2016, 35-41.

[28] Khan, Y., Mirzabeigy, A., Arjmand, H., Nonlinear oscillation of the bifilar pendulum: an analytical approximation. *Multidiscipline Modeling in Materials and Structures*, 13(2), 2017, 297-307.

[29] Sedighi, H.M., Daneshmand, H.M., Zare, J., The influence of dispersion forces on the dynamic pull-in behavior of vibrating nano-cantilever based NEMS including fringing field effect. *Archives of Civil and Mechanical Engineering*, 14(4), 2014, 766-775.

[30] Mirzabeigy, A., Madoliat, R., Large amplitude free vibration of axially loaded beams resting on variable elastic foundation. *Alexandria Engineering Journal*, 55(2), 2016, 1107-1114.