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Research Paper

Thermoelastic Vibration of Temperature-Dependent Nanobeams Due to Rectified Sine Wave Heating—A State Space Approach

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Abstract. In this study, the second type of Green and Naghdi's thermoelasticity theory is applied to present the vibration of a nanobeam subjected to rectified sine wave heating based upon the nonlocal thermoelasticity theory. Both Young's modulus and thermal conductivity are considered to be linear functions of the temperature. The Laplace transform domain is adopted to solve the governing partial differential equations using the state space approach. Numerical computations are carried out using the inverse of Laplace transforms. The effects of nonlocal parameter and angular frequency on the thermal vibration quantities are discussed. The results of all quantities are illustrated graphically and investigated.

Keywords: Green and Naghdi's theory; Nanobeam; Nonlocal thermoelasticity theory; State-space formulation; Rectified sine wave heating.

1. Introduction

Micro- and nanoscale mechanical resonators have been the topics of many important applications due to their high sensitivity, extremely high resonance frequencies, and fast response. They are widely used as ultrasensitive mass detection, scanning probe microscopes, and sensors. Sharma and Grover (2011) presented the vibration analysis of a thermoelastic thin Euler–Bernoulli beam with voids. Rezazadeh et al. (2012) presented the quality factor of thermoelastic damping of a microbeam using the modified couple stress theory. They derived the equations of coupled thermoelasticity in plane stress and plane strain conditions based on the modified couple stress theory. Guo et al. (2012) employed the dual-phase-lag heat conduction model to study thermoelastic damping of a microbeam resonator. This model is firstly presented by Tzou (1996). Lin (2014) presented the vibration analysis of beam resonator due to the thermoelastic damping and a harmonic external force. Taati et al. (2014) used the modified couple stresses and non-Fourier heat conduction to study the size-dependent explicit formulation for a coupled thermoelastic microbeam. Sharma and Kaur (2015) established the dynamic response of thermoelastic microbeam resonators under the time-varying distributed load using Lord and Shulman (1967) thermoelasticity theory. Abbas (2015) studied the thermoelastic interaction in a microscale beam under a moving heat source using Green and Naghdi theory of type III (Green and Naghdi, 1991, 1992, 1993). Abouelregal and Zenkour (2015) discussed the vibration analysis of a microbeam based upon the thermoelasticity theory with dual-phase-lags. Zenkour (2017) studied the vibration analysis of generalized thermoelastic microbeams resting on two-parameter visco-elastic foundations.

The nonlocal beam structures and their models have received increasing interest in recent years. The nonlocal theory of elasticity (Eringen, 1972, Eringen and Edelen 1972) is the mean target that used in most derivation of nanostructures. According to Eringen's theory, Eltaher et al. (2013) presented the vibration analysis of Euler–Bernoulli nanobeams using the finite element model. Zenkour and Abouelregal (2014) investigated the vibration analyses of nanobeams under sinusoidal pulse-heating with graded material in nanobeam thickness. Rahmani and Pedram (2014) investigated the vibration of functionally graded Timoshenko nanobeam based on the nonlocal elasticity. Zenkour and Abouelregal (2015) studied the vibration analysis of nanobeams under sinusoidal pulse varying heat based upon a thermoelasticity theory with one thermal relaxation. Hosseini-Hashemi et al. (2015) presented nonlinear free vibration analysis of simply-supported nanoscale Euler–Bernoulli beams incorporating different parameters according to the nonlocal elasticity theory. Zenkour and Sobhy (2015) used Eringen's nonlocal theory to present the bending analysis of nanobeams in the thermal environment. Nejad and Hadi (2016) investigated the size effects on vibration analyses of bi-directional functionally graded Euler–Bernoulli nanobeams according to Eringen's model. Ansari et al. (2016) presented the geometrically nonlinear free vibration of fractional viscoelastic Euler–Bernoulli nanobeams according to the nonlocal elasticity theory and von Kármán geometric nonlinearity.

The state-space approach is considered as the important item of the cornerstone of the modern control theory (Bahar and Hetnarski 1978, Sahmani and Ansari 2011, Youssef and Elsibai 2011, Zenkour et al. 2015). This study develops the solution of generalized thermoelastic vibration of a nanobeam resonator induced by rectified sine wave heating based on Eringen's nonlocal thermoelasticity theory. The modulus of elasticity and thermal conductivity are taken as a linear function of the reference temperature. The Laplace transform method and its inversion as well as the state space method are used to determine the field solution variables of the nanobeam analytically and numerically.

2. Formulation of the problem

The present nanobeam is represented as shown in Fig. 1 with dimensions of $L \times b \times h$. The Cartesian coordinate system (x, y, z) is used. The nanobeam is unstrained and unstressed in its equilibrium case. It is placed in a medium with the reference temperature T_0. Let us also consider that the first end of the nanobeam is loaded thermally by rectified sine wave heating. That is:

$$\Theta(0,t) = \Theta_0|\sin(\omega t)|. \tag{1}$$

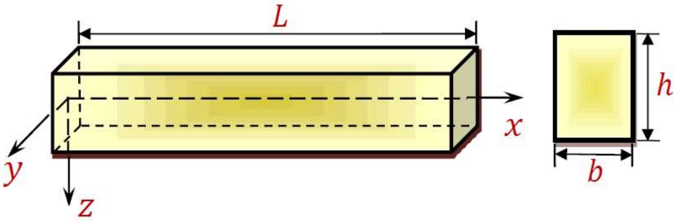

Fig. 1. The nanobeam and its coordinates.

The displacements of the present beam can be written as

$$u = -z\frac{\partial w}{\partial x}, \quad v = 0, \quad w = w(x,t), \tag{2}$$

in which w is the transverse deflection.

The constitutive equations are given according to Eringen's nonlocal elasticity theory as

$$[1 - (e_0 a)^2 \nabla^2]\sigma_{kl} = \tau_{kl}. \tag{3}$$

So, nonlocal constitutive equations of the present nanobeam are reduced to the following equation:

$$\sigma_x - \xi\frac{\partial^2 \sigma_x}{\partial x^2} = -E\left(z\frac{\partial^2 w}{\partial x^2} + \alpha_T \theta\right). \tag{4}$$

The effect of temperature-dependent properties of material is considered here. Both Young's modulus and thermal conductivity are temperature-dependent. The other elastic and thermal parameters may be keeping constant. Modulus of elasticity E and the corresponding stress-temperature modulus γ as well as the thermal conductivity K^* are given in terms of θ as (Allam et al. 2010)

$$\{E, \gamma, K^*\} = \{E_0, \gamma_0, K_0\} f(\theta), \tag{5}$$

where E_0, γ_0, and K_0 are considered to be constants. For temperature-independent material properties, one puts $f(\theta) = 1$. Here, let us consider

$$f(\theta) = 1 - \alpha\theta. \tag{6}$$

The approximating function $f(\theta)$, taking into consideration the condition $|\theta/T_0| \ll 1$, may be obtained in the following form:

$$f(\theta) = f(T_0) \approx 1 - \alpha T_0. \tag{7}$$

The equation of motion of Euler-Bernoulli nanobeams is represented by

$$\frac{\partial^2 M}{\partial x^2} = \rho A \frac{\partial^2 w}{\partial t^2}, \tag{8}$$

The flexure moment of cross-section according to the nonlocal Euler-Bernoulli theory gives

$$M(x,t) - \xi \frac{\partial^2 M}{\partial x^2} = -E_0 I(1 - \alpha T_0)\left(\frac{\partial^2 w}{\partial x^2} + \alpha_T M_T\right). \tag{9}$$

The thermal bending moment is given by

$$M_T = \frac{12}{h^3}\int_{-h/2}^{h/2} \theta(x,z,t) z\mathrm{d}z. \tag{10}$$

The substitution of Eq. (9) into Eq. (8) represents the dynamic equation of nanobeam as

$$\left[\frac{\partial^4}{\partial x^4} + \frac{\rho A}{E_0 I(1-\alpha T_0)}\frac{\partial^2}{\partial t^2}\left(1 - \xi\frac{\partial^2}{\partial x^2}\right)\right] w + \alpha_T \frac{\partial^2 M_T}{\partial x^2} = 0. \tag{11}$$

The final form of nonlocal flexure moment of nanobeam is expressed as

$$M(x,t) = \xi\rho A \frac{\partial^2 w}{\partial t^2} - E_0 I(1 - \alpha T_0)\left(\frac{\partial^2 w}{\partial x^2} + \alpha_T M_T\right). \tag{12}$$

Now, the thermal conduction equation for the present nanobeam without considering the heat source ($Q = 0$) according to Euler–Bernoulli assumption is expressed as

$$K_0 \nabla^2 \theta = \rho C_E \frac{\partial^2 \theta}{\partial t^2} - \gamma T_0 z \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial x^2}\right). \tag{13}$$

3. Solution of the problem

Let us assumed here that the temperature increment change θ is varying in the form of a sinusoidal function of z along the thickness direction of nanobeam as

$$\theta(x,z,t) = \Theta(x,t)\sin\left(\frac{\pi}{h} z\right). \tag{14}$$

Therefore, Eqs. (11) and (12) become

$$\left[\frac{\partial^4}{\partial x^4} + \frac{\rho A}{E_0 I(1-\alpha T_0)}\frac{\partial^2}{\partial t^2}\left(1 - \xi\frac{\partial^2}{\partial x^2}\right)\right] w + \frac{24\alpha_T}{\pi^2 h}\frac{\partial^2 \Theta}{\partial x^2} = 0, \tag{15}$$

$$M(x,t) = \xi\rho A \frac{\partial^2 w}{\partial t^2} - E_0 I(1 - \alpha T_0)\left(\frac{\partial^2 w}{\partial x^2} + \frac{24\ {}_0\alpha_T}{\pi^2 h}\Theta\right). \tag{16}$$

In addition, the thermal conduction equation may be obtained in a revised form through-the-thickness of nanobeam as

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\pi^2}{h^2}\right)\Theta = \frac{\rho C_E}{K_0}\frac{\partial^2 \theta}{\partial t^2} - \frac{\gamma T_0 \pi^2 h}{24 K_0}\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial x^2}\right). \tag{17}$$

The following dimensionless quantities may be used here

$$\begin{gathered}\{u', w', x', L', z', h', b'\} = \eta c\{u, w, x, L, z, h, b\}, \quad \{t', t_0'\} = \eta c^2\{t, t_0\}, \\ \xi' = \eta^2 c^2 \xi, \quad \Theta' = \frac{\Theta}{T_0}, \quad M' = \frac{M}{\eta c E_0 I}, \quad \eta = \frac{\rho C_E}{K_0}, \quad c = \sqrt{\frac{E_0}{\rho}}.\end{gathered} \tag{18}$$

Therefore, the dimensionless forms of equation of motion and the heat conduction equation as well as the flexural moment may be simplified, after dropping the primes for convenience, as

$$\frac{\partial^4 w}{\partial x^4} + A_1\frac{\partial^2 w}{\partial t^2} - A_3\frac{\partial^4 w}{\partial t^2 \partial x^2} + A_2\frac{\partial^2 \Theta}{\partial x^2} = 0, \tag{19}$$

$$\frac{\partial^2 \Theta}{\partial x^2} - A_4\Theta = A_6\frac{\partial^2 \theta}{\partial t^2} - A_5\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial x^2}\right), \tag{20}$$

$$M(x,t) = A_3\frac{\partial^2 w}{\partial t^2} - \frac{\partial^2 w}{\partial x^2} - A_2\Theta, \tag{21}$$

where

$$A_1 = \frac{12}{h^2(1-\alpha T_0)}, \quad A_2 = \frac{24 T_0 \alpha_T}{\pi^2 h}, \quad A_3 = \xi A_1,$$
$$A_4 = \frac{\pi^2}{h^2}, \quad A_5 = \frac{\gamma c^2 \pi^2 h}{24 \ _0}, \quad A_6 = \eta c^2. \tag{22}$$

The initial and boundary conditions should be considered here to solve the governing equations. Firstly, the initial conditions may be given by

$$w(x,t)|_{t=0} = \left.\frac{\partial w(x,t)}{\partial t}\right|_{t=0} = 0, \quad \Theta(x,t)|_{t=0} = \left.\frac{\partial \Theta(x,t)}{\partial t}\right|_{t=0} = 0. \tag{23}$$

4. Solution of the problem in Laplace transform domain

The Laplace transform may be applied here to simplify and solve the governing equations. Equations (19)-(21) may be expressed in the Laplace transform space by the following form:

$$\left(\frac{\mathrm{d}^4}{\mathrm{d}x^4} - A_3 s^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} + A_1 s^2\right) \bar{w} = -A_2 \frac{\mathrm{d}^2 \bar{\Theta}}{\mathrm{d}x^2}, \tag{24}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - B_1\right) \bar{\Theta} = -B_2 \frac{\mathrm{d}^2 \bar{w}}{\mathrm{d}x^2}, \tag{25}$$

$$\bar{M}(x,s) = -\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - A_3 s^2\right) \bar{w} - A_2 \bar{\Theta}, \tag{26}$$

in which the over bar symbol represents its Laplace transform. In addition,

$$B_1 = A_4 + A_6 s^2, \quad B_2 = A_5 s^2. \tag{27}$$

Now, let us consider a new function as follows:

$$\frac{\mathrm{d}^2 \bar{w}}{\mathrm{d}x^2} = \bar{\phi}. \tag{28}$$

Substituting in Eqs. (24) and (25), then one obtains

$$\frac{\mathrm{d}^2 \bar{\phi}}{\mathrm{d}x^2} = -B_3 \bar{w} - B_4 \bar{\Theta} + B_5 \bar{\phi}, \tag{29}$$

$$\frac{\mathrm{d}^2 \bar{\Theta}}{\mathrm{d}x^2} = B_1 \bar{\Theta} - B_2 \bar{\phi}, \tag{30}$$

where

$$B_3 = A_1 s^2, \quad B_4 = B_1 A_2, \quad B_5 = A_3 s^2 + B_2 A_2. \tag{31}$$

5. State-space approach

Here, we try to solve the present problem by using the method of direct integration by means of the matrix exponential (Bahar and Hetnarski 1978). Choosing as state variables the functions

$$\bar{V}_1 = \bar{w}, \quad \bar{V}_2 = \bar{\Theta}, \quad \bar{V}_3 = \bar{\phi}, \quad \bar{V}_4 = \frac{\mathrm{d}\bar{V}_1}{\mathrm{d}x}, \quad \bar{V}_5 = \frac{\mathrm{d}\bar{V}_2}{\mathrm{d}x}, \quad \bar{V}_6 = \frac{\mathrm{d}\bar{V}_3}{\mathrm{d}x}, \tag{32}$$

then, Eqs. (28)-(30) can be expressed in the matrix form as

$$\frac{\mathrm{d}\bar{V}(x,s)}{\mathrm{d}x} = \bar{V}'(x,s) = A(s)\bar{V}(x,s), \tag{33}$$

or

$$\begin{Bmatrix} \bar{V}_1' \\ \bar{V}_2' \\ \bar{V}_3' \\ \bar{V}_4' \\ \bar{V}_5' \\ \bar{V}_6' \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & B_1 & -B_2 & 0 & 0 & 0 \\ -B_3 & -B_4 & B_5 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \\ \bar{V}_5 \\ \bar{V}_6 \end{Bmatrix}. \tag{34}$$

The general solution of the above-mentioned system of equations can be obtained as

$$\bar{V}(x,s) = \mathrm{e}^{A(s)x} \bar{V}(0,s), \tag{35}$$

where $\mathrm{e}^{A(s)x}$ represents the transfer matrix in the exponential form and

$$\bar{V}(0,s) = \{\bar{w}, \bar{\Theta}, \bar{\phi}, \bar{w}', \bar{\Theta}', \bar{\phi}'\}^t(0,s). \tag{36}$$

The characteristic equation corresponding to $A(s)$ may be expressed as

$$k^6 - lk^4 + mk^2 - n = 0, \tag{37}$$

where k represents a characteristic root of $A(s)$ and

$$l = B_1 + B_5, \quad m = B_1 B_5 - B_2 B_4 + B_3, \quad n = B_1 B_3, \tag{38}$$

and its roots, namely k_1, k_2, and k_3, satisfy

$$k_1^2 + k_2^2 + k_3^2 = l, \quad k_1^2 k_2^2 + k_2^2 k_3^2 + k_3^2 k_1^2 = m, \quad k_1^2 k_2^2 k_3^2 = n. \tag{39}$$

In view of Caley-Hamilton theorem, $A(s)$ should satisfy its own characteristic equation in the matrix sense. Then, it follows that

$$A^6 - lA^4 + mA^2 - n = 0. \tag{40}$$

The above-mentioned equation shows that A^6 and all higher powers of $A(s)$ can be expressed as linear combinations of A^5, A^4, A^3, A^2, A, and I (a unit matrix of order six). Therefore, Taylor's series expansion of $\mathrm{e}^{A(s)x}$ can be expressed by

$$\mathrm{e}^{A(s)x} = a_0 I + a_1 A + a_2 A^2 + a_3 A^3 + a_4 A^4 + a_5 A^5 = L(x,s), \tag{41}$$

where $a_j(x,s)$ are undetermined parameters. Replacing A in Eq. (41) with $\mathrm{k_i}$ of Eq. (37) yields

$$\mathrm{e}^{\pm k_i x} = a_0 I + a_1 k_i + a_2 k_i^2 + a_3 k_i^3 + a_4 k_i^4 + a_5 k_i^5, \quad i = 1,2,3. \tag{42}$$

Appendix A show that how the above-mentioned equations are solved to get the parameters $a_j(x,s)$. Therefore, the exponential matrix is obtained in the form

$$\mathrm{e}^{A(s)x} = L(x,s) = \left[L_{ij}(x,s)\right], \quad i,j = 1,2,\dots,6, \tag{43}$$

in which the elements $\left[L_{ij}(x,s)\right]$ are given in Laplace transform domain.

6. Applications

We consider the case of a nanobeam whose edges are either clamped or simply supported.

Set I: Simply-supported (SS) nanobeams:

In this case we have the following boundary conditions:

$$w(x,t)|_{x=0,L} = 0, \quad \left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=0,L} = 0. \tag{44}$$

Now, Eqs. (1) and (43) may be simplified in Laplace transform domain as

$$\bar{w}(x,s)|_{x=0} = 0, \quad \left.\frac{\mathrm{d}^2 \bar{w}(x,s)}{\mathrm{d}x^2}\right|_{x=0} = 0, \quad \bar{\phi}(x,s)\big|_{x=0} = 0. \tag{45}$$

$$\bar{\Theta}(x,s)|_{x=0} = \Theta_0 \left(\frac{\omega\,\mathrm{co}\ \left(\frac{\pi s}{2\omega}\right)}{s^2+\omega^2}\right) = \bar{G}(s). \tag{46}$$

Moreover, we can get at the other end of nanobeam x = L as

$$\bar{w}(L,s) = \bar{\Theta}(L,s) = \bar{\phi}(L,s) = 0. \tag{47}$$

Therefore, one gets $\bar{w}'(0,s)$, $\bar{\Theta}'(0,s)$, and $\bar{\phi}'(0,s)$ as

$$\begin{Bmatrix} \bar{w}'(0,s) \\ \bar{\Theta}'(0,s) \\ \bar{\phi}'(0,s) \end{Bmatrix} = -\bar{G}(s) \begin{bmatrix} L_{14}(L,s) & L_{15}(L,s) & L_{16}(L,s) \\ L_{24}(L,s) & L_{25}(L,s) & L_{26}(L,s) \\ L_{34}(L,s) & L_{35}(L,s) & L_{36}(L,s) \end{bmatrix}^{-1} \begin{Bmatrix} L_{12}(L,s) \\ L_{22}(L,s) \\ L_{32}(L,s) \end{Bmatrix}. \tag{48}$$

Finally, the final solutions in the Laplace transform domain may be given after some simplifications as

$$\begin{aligned} \bar{w}(x,s) = {} & \frac{G(s)(B_1-k_1^2)(B_1-k_2^2)(B_1-k_3^2)}{B_1 B_2}\left[\frac{\sinh(k_1(L-x))}{(k_1^2-k_2^2)(k_1^2-k_3^2)\sinh(k_1 L)}\right. \\ & \left. + \frac{\sinh(k_2(L-x))}{(k_2^2-k_1^2)(k_2^2-k_3^2)\sinh(k_2 L)} + \frac{\sinh(k_3(L-x))}{(k_3^2-k_2^2)(k_3^2-k_1^2)\sinh(k_3 L)}\right], \end{aligned} \tag{49}$$

$$\bar{\theta}(x,z,s) = -\frac{G(s)(B_1-k_1^2)(B_1-k_2^2)(B_1-k_3^2)\sin\left(\frac{\pi z}{h}\right)}{B_1}\left[\frac{k_1^2 \sinh(k_1(L-x))}{(k_1^2-B_1)(k_1^2-k_2^2)(k_1^2-k_3^2)\sinh(k_1 L)}\right.$$

$$+\frac{k_2^2\sinh(k_2(L-x))}{(k_2^2-B_1)(k_2^2-k_1^2)(k_2^2-k_3^2)\sinh(k_2L)}+\frac{k_3^2\sinh(k_3(L-x))}{(k_3^2-B_1)(k_3^2-k_2^2)(k_3^2-k_1^2)\sinh(k_3L)}\Bigg]. \tag{50}$$

Additional quantities may be obtained with the aid of the above-mentioned expressions of $\bar{w}(x,s)$ and $\bar{\theta}(x,s)$.

Set II: Clamped-clamped (CC) nanobeams:

For a nanobeam clamped at both ends, the boundary conditions for the lateral vibration and displacements are

$$w(x,t)|_{x=0,L}=0, \quad u(x,t)_{x=0,L}=0. \tag{51}$$

Now, Eqs. (1) and (43) may be simplified in the Laplace transform domain as

$$\bar{w}(x,s)|_{x=0,L}=0, \quad \left.\frac{\mathrm{d}\bar{w}(x,s)}{\mathrm{d}x}\right|_{x=0,L}=0 \tag{52}$$

In the same way and using the previous technique, after substituting in the boundary conditions (52) and (46), we obtain expressions for $\bar{w}(x,s)$ and $\bar{\theta}(x,z,s)$ in terms of x, z, and s.

Finally, the final form of all quantities are given in the time domain after using the inversion of Laplace transform (see Tzou 1996).

7. Numerical results and discussions

The lateral vibration and temperature as well as other corresponding quantities such as the axial displacement and the bending moment may be graphically illustrated in x direction in terms of z and t. The nanobeam is made of Silicon to analyze the thermoelastic coupling effect. The material parameters of such material are given by

$$E_0=169\text{ GPa},\quad \rho=2330\text{ kg/m}^3,\quad C_E=713\text{ J/(kg K)},\quad T_0=293\text{ K},$$
$$\alpha_T=2.59\times10^{-6}(1/\text{K}),\quad \nu=0.22,\quad K_0=156\text{ W/(mK)}.$$

Calculations are made for fixed aspect ratios of nanobeam $L/h=10$ and $b/h=0.5$. Moreover, some parameter are fixed as $L=1$, $z=h/6$ and $t=0.12$. Here, the dimensionless nonlocal parameter $\bar{\xi}$ ($\bar{\xi}=10^6\xi$) is used. This parameter should be less than 4 $(\mu\text{m})^2$. It is assumed in all figures, except otherwise stated, that $\bar{\xi}=1$, $\omega=3$, and $F_0=0.9$ (temperature-dependent). The following numerical results are divided into two categories:

Three cases are numerically considered. The first case is concerned with the effect of nonlocal parameter $\bar{\xi}$ on the response of all quantities. In Figs. 2-5, different values of $\bar{\xi}$ are considered. The first is $\bar{\xi}=0$ to represent the original local theory. The other nonzero values, $\bar{\xi}=1$ and $\bar{\xi}=3$, indicate the nonlocal thermoelasticity theory. It is observed that:

1. Figure 2 shows the distribution of transverse vibrations w of both SS and CC nanobeams that begin at ends with zero values (i.e. vanishes) and satisfies the both boundary conditions at $x=0$ and L. The deflections of SS nanobeam are more than three times of those of CC nanobeam.

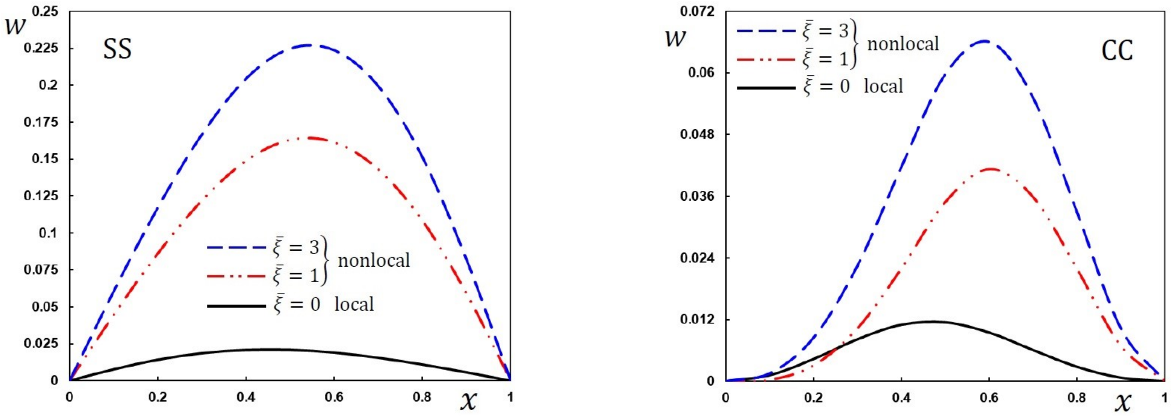

Fig. 2. The transverse deflection distributions versus the axial direction with different values of the nonlocal parameter $\bar{\xi}$.

2. As shown in Fig. 3, it is observed that the temperature θ of SS nanobeam is no longer increasing and has its maximum near the first edge for the local theory and at the mid-axis for the nonlocal theory. However, the temperature θ of CC nanobeam is no longer increasing and has its maximum at $x\cong0.4$ for the local theory and at $x\cong0.6$ for the nonlocal theory. The temperature for both SS and CC nanobeams may be vanished at the second edge of the nanobeam.
3. The value of the axial displacement u of SS nanobeam is increasing through the axial distance of the nanobeam. The local theory gives the highest displacements compared with the nonlocal theory in the range $0\le x<0.56$ and vice versa

otherwise as given in Fig.4. It is to be noted that the distribution of axial displacement u of CC nanobeam satisfies boundary conditions that begin zero values (i.e. vanishes).

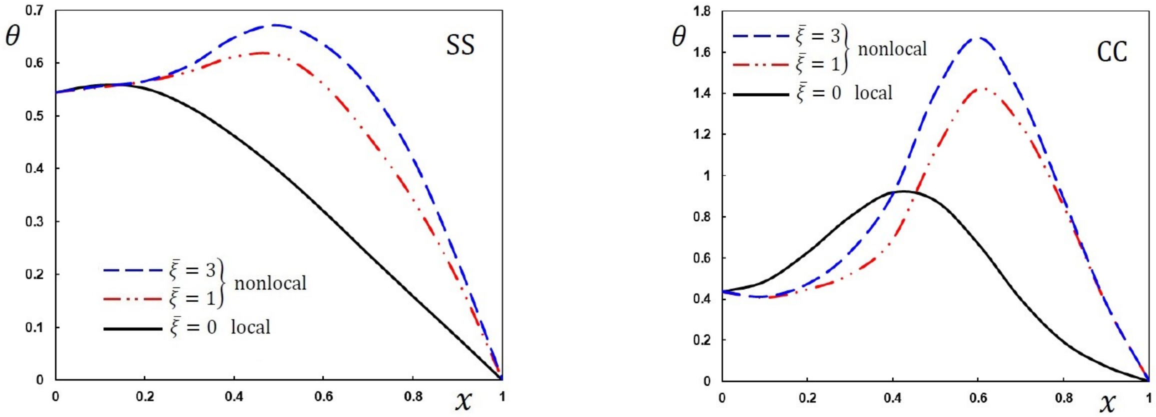

Fig. 3. The temperature distributions versus the axial direction with different values of the nonlocal parameter $\bar{\xi}$.

4. In Fig. 5, the bending moment M of SS nanobeam is no longer decreasing for the nonlocal theory and has its minimum near the second edge of the nanobeam (at $x \cong 0.7$). However, it is decreasing through the axial direction to get its minimum at the second edge for the local theory. The bending moment M of CC nanobeam propagates systematically for the local theory and non-systematically for the nonlocal theory. The maximum bending moments occur at the second end of CC nanobeam using the nonlocal theory.

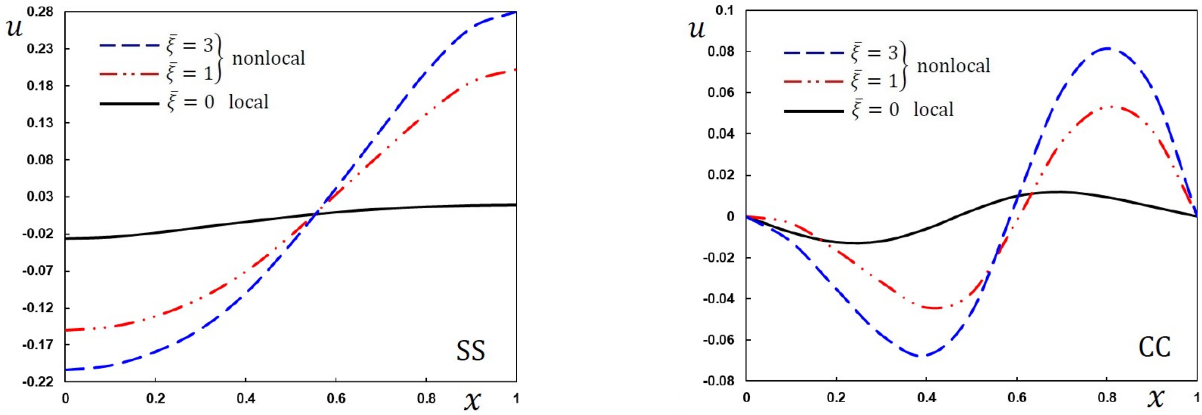

Fig. 4. The displacement distributions versus the axial direction with different values of the nonlocal parameter $\bar{\xi}$.

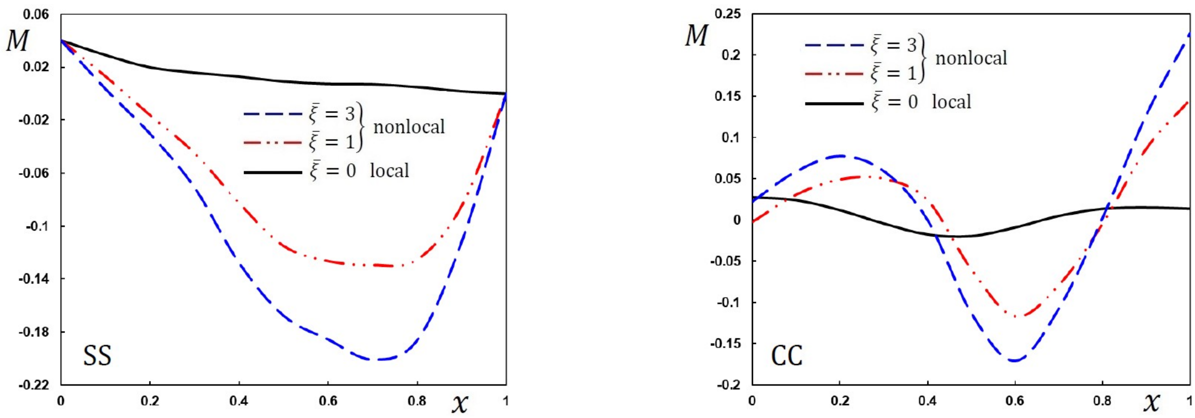

Fig. 5. The flexure moment distributions versus the axial direction with different values of the nonlocal parameter $\bar{\xi}$.

5. The sensitivity of the field quantities to the variation of the nonlocal parameter is obviously appeared in Figs. 2-5. The bending moment M decreases as $\bar{\xi}$ increases while both the lateral vibration w and the temperature θ increase for both SS and CC nanobeams.

The second case illustrated how the field quantities of both SS and CC nanobeams vary with different values of angular frequency of the thermal vibration ω. Figures 6-9 show that:

1. The increasing of ω tends to the increase of lateral deflection w and temperature θ as shown in Figs. 6 and 7. The maximum deflection occurs at $x \cong 0.5$ for SS nanobeams and at $x \cong 0.6$ for CC nanobeams. Moreover, the maximum temperature occurs at $x \cong 0.4$ for SS nanobeams and at $x \cong 0.6$ for CC nanobeams.

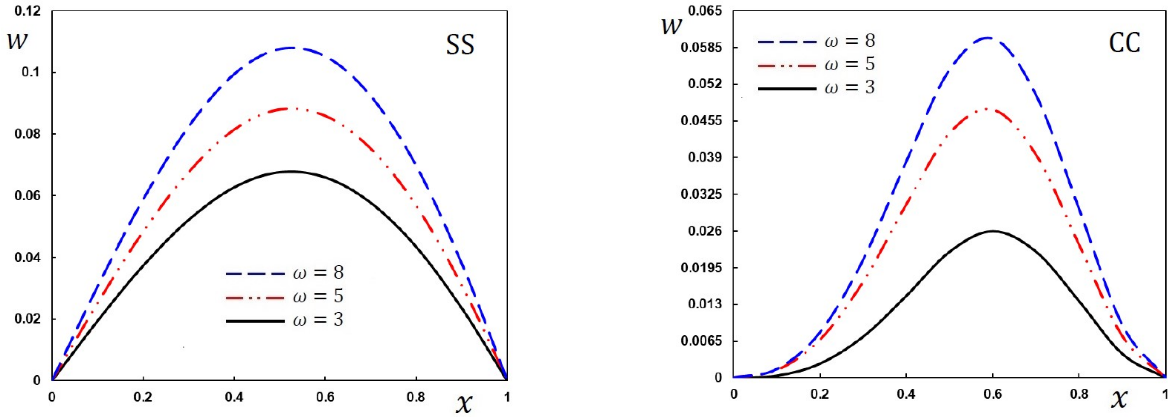

Fig. 6. The transverse deflection distribution versus the axial direction for different values of the angular frequency of thermal vibration ω.

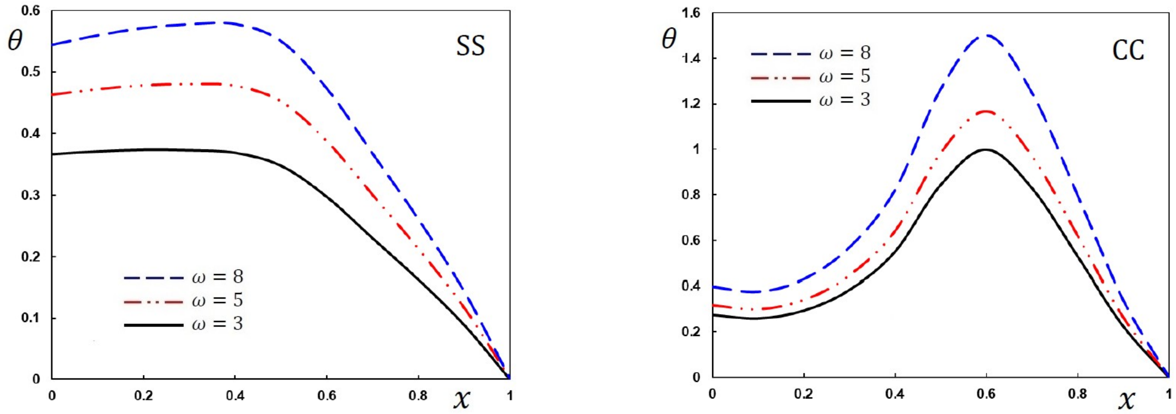

Fig. 7. The temperature distributions versus the axial direction for different values of the angular frequency of thermal vibration ω.

2. As shown in Fig. 8, the axial displacement u of SS nanobeam increases through the axial direction for all values of ω. As ω increases, the axial displacement decreases in the range $0 \leq x < 0.56$ and vice versa otherwise. Besides, the displacement u of CC nanobeam propagates semi-systematically and as ω increases u decreases in the range $0 \leq x < 0.6$ and vice versa otherwise.
3. As shown in Fig. 9, the bending moment M of SS nanobeam is no longer decreasing and has their minimums at $x \approx 0.78$. As ω increases, M increases in the range $x \leq 0.18$ and vice versa otherwise. Furthermore, the bending moment M of CC nanobeam propagates semi-systematically and has their maximum at the second end. As ω increases, M decreases in the range $0.4 \leq x < 0.8$ and vice versa otherwise.
4. It can be observed that the angular frequency of the thermal vibration parameter ω has great effects on distributions of the field quantities.

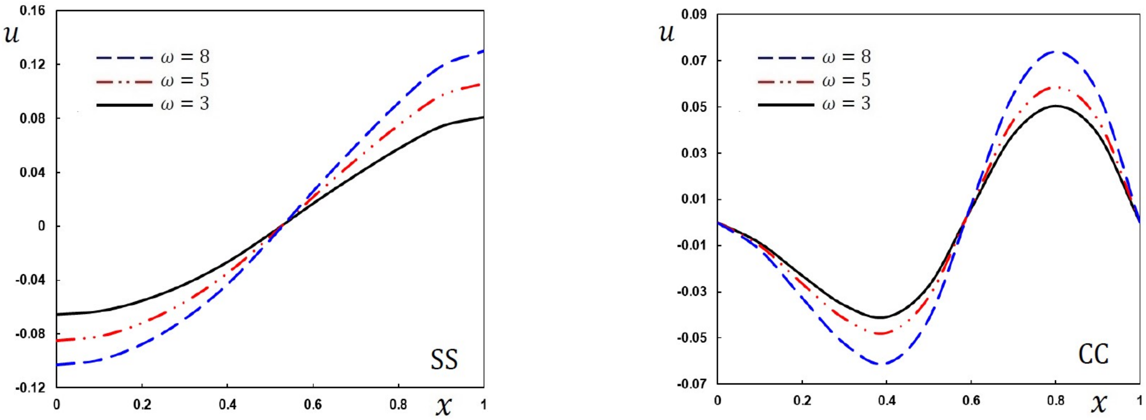

Fig. 8. The displacement distributions versus the axial direction for different values of the angular frequency of thermal vibration ω.

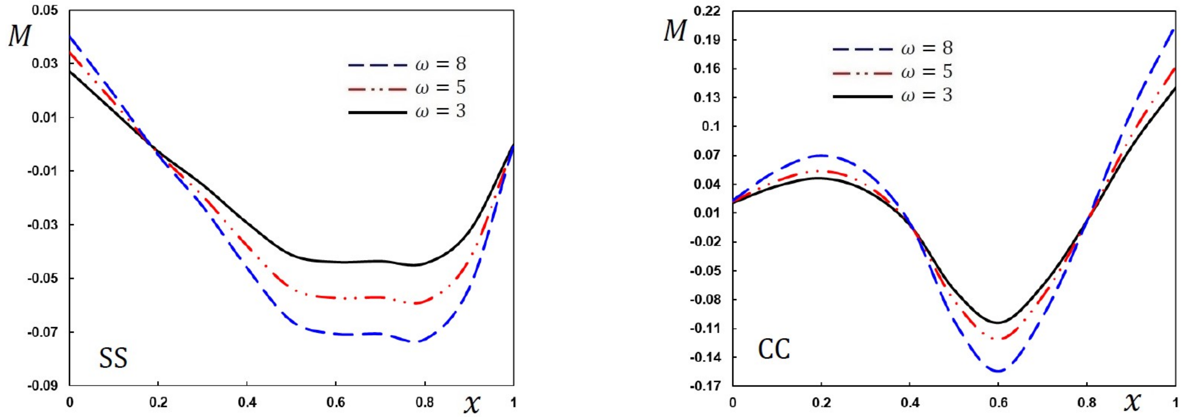

Fig. 9. The flexure moment distributions versus the axial direction for different values of the angular frequency of thermal vibration ω.

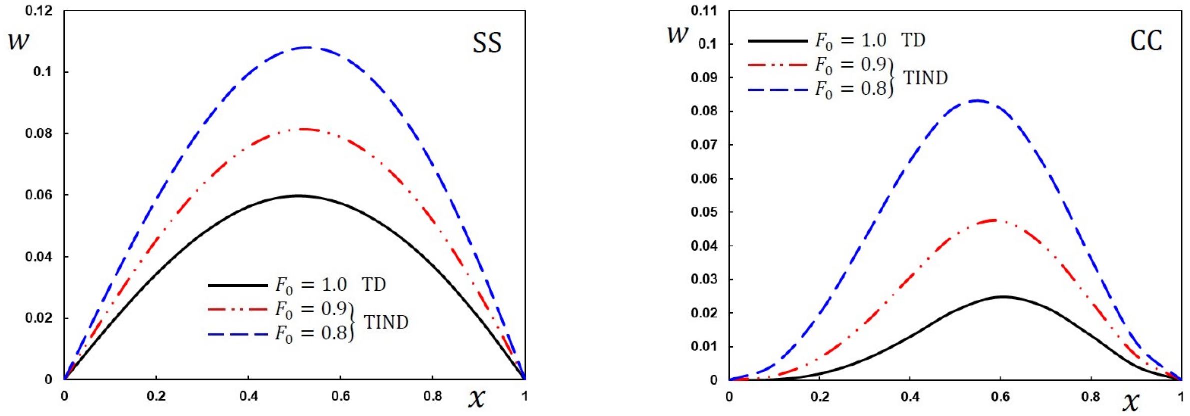

Fig. 10. The transverse deflection distributions versus the axial direction in the case of temperature-independent mechanical properties.

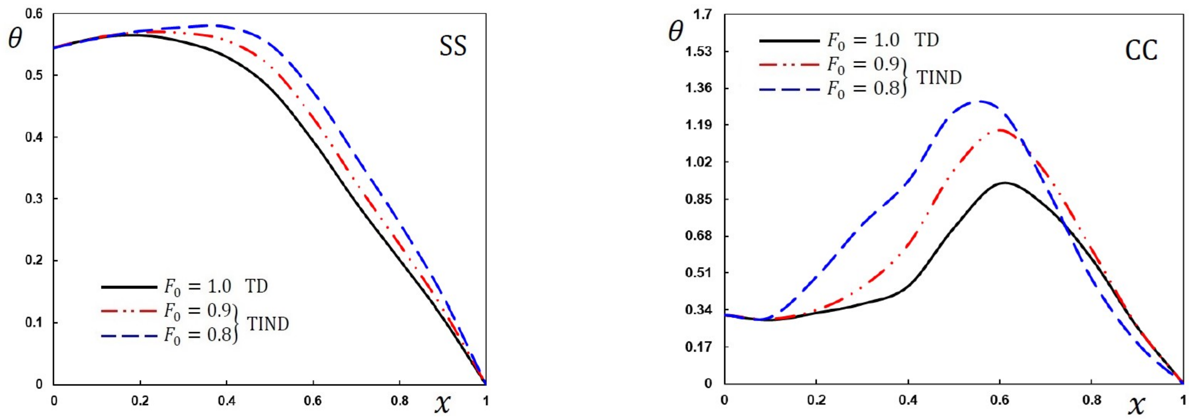

Fig. 11. The temperature distributions versus the axial direction in the case of temperature-independent mechanical properties.

Case 3 has discussed the behavior of lateral vibration, temperature, axial displacement, and bending moment with the effect of reference temperature $F_0 = 1 - \alpha T_0$. Figures 10-13 present the effect of reference temperature T_0 on the field quantities distributions when the modulus of elasticity and thermal conductivity are linear functions of the reference temperature. In the case of temperature-dependent (TD) material properties, one takes $F_0 = 0.9$ and 0.8. However, in the case of temperature-independent (TIND) material properties, one puts $T_0 = 0$ (or $F_0 = 1$). As shown in Figs. 10-13, one can see that:

1. For both SS and CC nanobeams, the increasing of F_0 causes increasing in the lateral vibration w and the temperature θ fields. The bending moment M of SS nanobeam is decreasing while this of CC nano beam is unstable.
2. The axial displacement u of SS nanobeam may be vanished near the center of the beam at $x \approx 0.56$. In this position, it is also independent of the reference temperature. Otherwise, u is strongly dependent on the reference temperature.

Moreover, the axial displacement u of CC nanobeam propagates systematically and vanishes at both ends. As F_0 increases, u increases in the range $0 \leq x < 0.5$ and decreases in the range $0.5 \leq x < 1$.

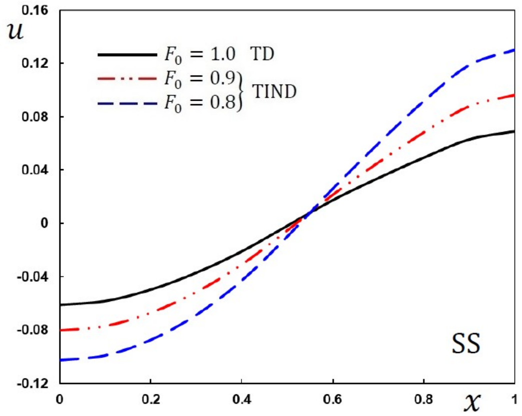

Fig. 12. The displacement distributions versus the axial direction in the case of temperature-independent mechanical properties.

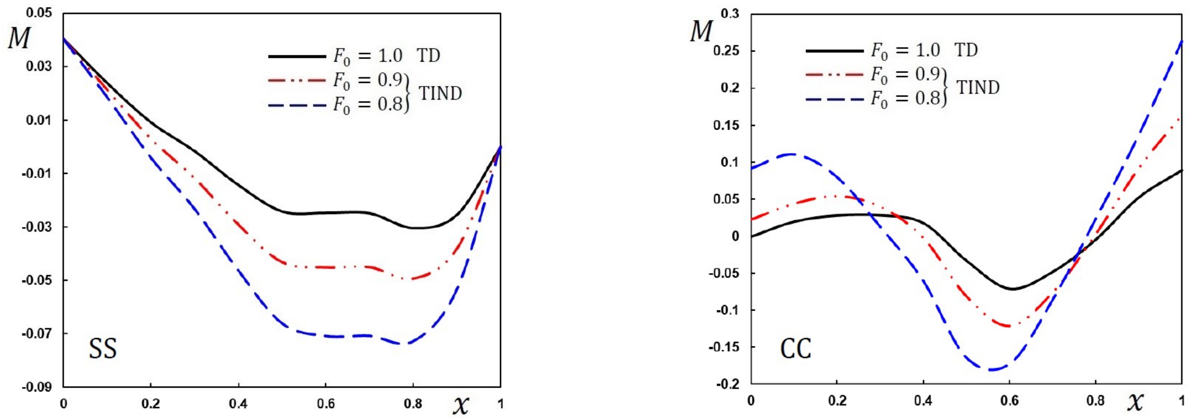

Fig. 13. The flexure moment distributions versus the axial direction in the case of temperature-independent mechanical properties.

8. Conclusions

This study presents a new model of the nonlocal generalized thermoelasticity without energy dissipation for Euler–Bernoulli nanobeams subjected to the rectified sine wave heating is constructed. Young's modulus and thermal conductivity are taken as linear functions of temperature. The effects of the nonlocal parameter and the angular frequency of thermal vibration parameter on the field variables are investigated. Numerical techniques based on the Laplace transformation has been adopted. The effects of reference temperature, nonlocal parameter, and the angular frequency of thermal vibration parameter on all quantities have been shown and presented graphically. It is found that the nonlocal parameter has significant effects on all quantities. On the other hand, the thermoelastic bending moments, displacement, and temperature have a strong dependency on the angular frequency of the thermal vibration parameter. The results of the generalized thermoelasticity theory without energy dissipation (local thermoelasticity) can be obtained as a limited case of the present study.

Conflict of Interest

The authors declare no conflict of interest.

Nomenclature

$A = bh$	area of beam cross-section
a	an internal characteristic length
b	width of nanobeam $(-b/2 \leq y \leq b/2)$
C_E	specific heat at constant strain
$E = E_0 f(\theta)$	Young's modulus
$e = \varepsilon_{kk}$	volumetric strain
e_0	a constant of the nonlocal parameter
$E_0 I$	flexural rigidity

h	thickness of nanobeam $(-h/2 \le z \le h/2)$
$I = bh^3/12$	inertia moment of beam cross-section
$K^* = K_0 f(\theta)$	thermal conductivity
$1/\eta = K^*/\rho C_E$	thermal diffusivity
K_0	thermal conductivity at reference temperature T_0
L	length of nanobeam $(0 \le x \le L)$
M	flexural moment of nanobeam
M_T	thermal moment of nanobeam
s	Laplace's variable
t	time
T_0	environment (reference) temperature
Q	heat source
u	axial displacement
w	lateral deflection
α	empirical material constant
α_t	coefficient of thermal expansion
$\alpha_T = \alpha_t/(1-2\nu)$	thermal expansion parameter
$\xi = (e_0 a)^2$	nonlocal parameter
∇^2	Laplacian
$f(\theta)$	nondimensional function of temperature
σ_{kl}	classical (Cauchy) or local stress tensor
σ_x	nonlocal normal stress
τ_{kl}	nonlocal stress tensor
$\theta = T - T_0$	thermodynamical temperature
ν	Poisson's ratio
ρ	material density
$\gamma = E\alpha_t/(1-2\nu)$	stress-temperature modulus
ω	angular frequency of thermal vibration
Θ_0	thermal constant

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Appendix A

The parameters a_j after solving Eq. (44) are represented as

$$a_0(x,s) = -\frac{k_2^2k_3^2(k_2^2-k_3^2)\cosh(k_1x)+k_1^2k_3^2(k_3^2-k_1^2)\cosh(k_2x)+k_2^2k_1^2(k_1^2-k_2^2)\cosh(k_3x)}{(k_1^2-k_2^2)(k_2^2-k_3^2)(k_3^2-k_1^2)},$$

$$a_1(x,s) = -\frac{k_2^3k_3^3(k_2^2-k_3^2)\sinh(k_1x)+k_1^3k_3^3(k_3^2-k_1^2)\sinh(k_2x)+k_2^3k_1^3(k_1^2-k_2^2)\sinh(k_3x)}{k_1k_2k_3(k_1^2-k_2^2)(k_2^2-k_3^2)(k_3^2-k_1^2)},$$

$$a_2(x,s) = \frac{(k_2^4-k_3^4)\cosh(k_1x)+(k_3^4-k_1^4)\cosh(k_2x)+(k_1^4-k_2^4)\cosh(k_3x)}{(k_1^2-k_2^2)(k_2^2-k_3^2)(k_3^2-k_1^2)},$$

$$a_3(x,s) = \frac{k_2k_3(k_2^4-k_3^4)\sinh(k_1x)+k_1k_3(k_3^4-k_1^4)\sinh(k_2x)+k_1k_2(k_1^4-k_2^4)\sinh(k_3x)}{k_1k_2k_3(k_1^2-k_2^2)(k_2^2-k_3^2)(k_3^2-k_1^2)},$$

$$a_4(x,s) = \frac{(k_2^2-k_3^2)\cosh(k_1x)+(k_3^2-k_1^2)\cosh(k_2x)+(k_1^2-k_2^2)\cosh(k_3x)}{(k_1^2-k_2^2)(k_2^2-k_3^2)(k_3^2-k_1^2)},$$

$$a_5(x,s) = -\frac{k_2k_3(k_2^2-k_3^2)\sinh(k_1x)+k_1k_3(k_3^2-k_1^2)\sinh(k_2x)+k_1k_2(k_1^2-k_2^2)\sinh(k_3x)}{k_1k_2k_3(k_1^2-k_2^2)(k_2^2-k_3^2)(k_3^2-k_1^2)}.$$