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TRANSMISSION AND DIFFRACTION OF IMPULSE WAVES
IN FOAM MEDIA WITH CAVITIES

О.А. Мікуліч, В.І. Шваб'юк. **Поширення та дифракція імпульсних хвиль у пінистих середовищах з порожнинами.** Широке застосування у будівництві пінистих матеріалів, що дають можливість значно здешевити та полегшити конструкції, зумовлюють інтерес до розвитку методів дослідження напруженого стану таких матеріалів за дії різноманітних динамічних навантажень, що обумовлені технологічними і механічними впливами. Дослідження поширення хвиль, що виникає за дії таких впливів, дасть можливість більш точно оцінити міцність таких елементів конструкцій та ефективність їх використання. Метою роботи є розробка методики дослідження поширення та дифракції пружних імпульсів у пінистих матеріалах, послаблених тунельними порожнинами довільного перерізу. Для розв'язання задачі використано метод граничних інтегральних рівнянь сумісно з перетворенням Фур'є за часом, що дало можливість отримати інтегральні рівняння у комплексному вигляді для псевдоконтинууму Коссера. З використанням розробленого підходу досліджено поширення та дифракцію слабких ударних хвиль на тунельних порожнинах у пінистих середовищах на основі аналізу полів динамічних та радіальних напружень.
Ключові слова: пінисті середовища, нестаціонарна задача, дифракція хвиль

O. Mikulich, V. Shvabjuk. **Transmission and diffraction of impulse waves in foam media with cavities**. Widespread use of foam materials in construction, which makes it possible to significantly reduce the cost and facilitate construction, caused increasing interest in the development of methods studying the stress state of such materials during action of various dynamic loads triggered by technological and mechanical influences. Research of transmission of waves that arises from the effects of such influences will give an opportunity to more accurately assess the strength of such structural elements and the effectiveness of their use. The aim of the work is to develop a method for studying the transmission and diffraction of elastic impulses in foam materials with tunnel cavities of an arbitrary cross-section. In order to solve the problem the boundary integral equations method was used together with the time Fourier transform, which made it possible to obtain integral equations in a complex form for the Cosserat pseudo-continuum. Using the developed approach the research of transmission and diffraction of weak shock waves on tunnel cavities in foam media was carried out based on the analysis of fields of dynamic and radial stresses.
Keywords: foam media, non-stationary problem, wave diffraction

Introduction. Recently foam materials have been widely used in construction, which makes it possible not only to cheapen and facilitate construction, but also they have good thermal and vibration insulation characteristics. This explains significant interest in development of research methods of stress state of such materials due to the effects of various dynamic loads caused by technological and mechanical influences. Research of waves transmission processes that arise from the action of dynamic loads enables more accurate strength evaluation of such elements' structures and the effectiveness of their use.

Works devoted to the experimental study of dimensional effects in polymer foams with open and closed cells within the Cosserat moment theory of elasticity are well-known [1 − 3]. Polymer foams have shown significant dimensional effects at torsion and bending: the studied samples are more rigid than expected according to classical elasticity [3]. Such dimensional effects are taken into account on the basis of the moment (micropolar) theory of elasticity, which allows the points to rotate and also transmit and include distributed moments (moment stresses). On the basis of experimental studies it has been shown that foam with cell size of 0.4 mm has isotropic characteristics.

Research of the stressed state of foam media for a plane and spatial stress state should be carried out on the basis of the equations of the Cosserat moment continuum taking into account the rotational-shear deformation [4].

Numerical methods for studying the transmission of wave processes in media, taking into account the moment stress [5], are described.

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A lot of papers have been devoted to building analytic solutions for specific classes of dynamic and static problems for the Cosserat moment continuum [6].

An approach based on the combined use of the boundary integral equations method and time Fourier transform is developed, which makes it possible to investigate the dynamic stress state of media with microstructure weakened by tunnel cavities of arbitrary cross-section during the action of dynamic [7] and impulse [8] load , which is applied to the boundary of the cavity. The integral equations constructed within the framework of the Cosserat pseudo-continuum take into account the influence of rotational-shear deformations that appear in the media due to the action of dynamic loading.

Fig. 1. Open-cell polymer foam

Aim. The aim of the work is to develop a method for studying the transmission and diffraction of weak shock waves on tunnel cavities of arbitrary cross-section in foam media.

Materials and methods. Let us consider an infinite isotropic polymer foam medium (Fig. 1), which is weakened by a tunnel cavity of an arbitrary cross-section (Fig. 2), whose boundary is marked by L [3]. Cartesian coordinate system $Ox_1x_2x_3$ will be placed in the gravity center of the medium.

Fig. 2. Foam media model

In order to study the transmission and diffraction of elastic impulse waves in a tunnel cavity, we shall analyse the distribution of dynamic radial and circular stresses in a foam polymeric medium.

During the analysis of deformations in a foam medium it is evident that under the action of a load each microparticle of the medium carries not only translational, but also rotational displacement [9]. Each micro-rotation of the particles of the medium is associated with their translational displacement. In addition, microparticles of the foam medium can not complete micro-rotation without displacement. This is due to the structure of the foam material (Fig. 3).

Therefore the vectors of macro- and micro-rotations coincide for foam materials:

$$\varpi = \frac{1}{2} rot\,\mathbf{u}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{1}$$

where $\mathbf{u}$ – gravity center displacement vector;

ϖ – rotation vector.

It should be noted that $\mathbf{U}$ and ϖ are continuous functions.

The equation of foam media motion can be described on the basis of the Cosserat pseudo-continuum equations in the form of [5, 6]

$$(\lambda + 2\mu)\,grad\,div\,\mathbf{u} + rot\,rot\left(\frac{B}{4}\Delta\mathbf{u} - \mu\mathbf{u}\right) = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{2}$$

where $\mathbf{u}(\mathbf{x},t) = \{u_j(\mathbf{x},t)\}$, $j = 1, 2$ – arbitrary point displacement vector $\mathbf{x} = \{x_1,\ x_2\}$;

ρ – material density;

λ, μ – Lame constants;

Δ – Laplace operator;

B – constant that corresponds to microstructure of material (B=$\gamma + \varepsilon$);

t – time.

The boundary conditions of the problem are formulated the following way

a

$$\sigma_n\big|_L = \varphi_1(\mathbf{x}, t), \ \tau_{sn}\big|_L = \varphi_2(\mathbf{x}, t), \tag{3}$$
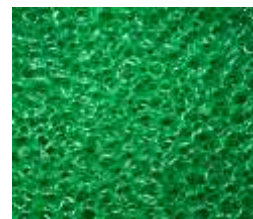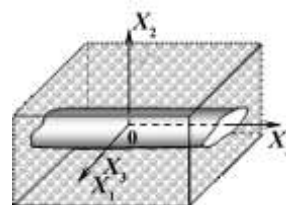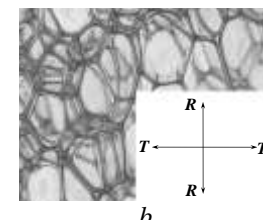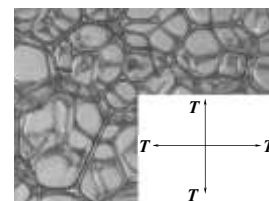
b

Fig. 3. Deformation analysis in foam media

where $\varphi_2(\mathbf{x}, t)$, $\varphi_2(\mathbf{x}, t)$ – the known at cavity boundary of function, determined by incident elastic wave potential [10];

$\vec{n}$ – normal to cavity boundary.

In order to solve the problem the approach [7, 8] based on joint application of the boundary integral equations method and time Fourier transform is used [11]

$$\tilde{f}(x, \omega) = \int_{-\infty}^{\infty} f(x, t)e^{-i\omega t}dt, \quad f(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(x, \omega)e^{i\omega t}d\omega, \tag{4}$$

where ω – cyclic frequency.

In the sphere of Fourier-images the motion equation (2) of the moment Cosserat pseudo-continuum will have the following form

$$(\lambda + \mu)\partial_j\tilde{\theta} + \frac{B}{4}\Delta\partial_j\tilde{\theta} - \frac{B}{4}\Delta\Delta\tilde{u}_j + \mu\Delta\tilde{u}_j + \omega^2\tilde{u}_j = 0, \tag{5}$$

where $\tilde{u}_j$ – displacement Fourier-images; that are calculated on the basis of the dependencies (4);

$\tilde{\theta} = \partial_j\tilde{u}_j$.

The Fourier-images of boundary conditions (3) are indicated as follows

$$\tilde{\sigma}_n\big|_L = \tilde{\varphi}_1(\mathbf{x}, t), \quad \tilde{\tau}_{sn}\big|_L = \tilde{\varphi}_2(\mathbf{x}, t), \tag{6}$$

where $\tilde{\varphi}_2(\mathbf{x}, t)$, $\tilde{\varphi}_2(\mathbf{x}, t)$ – images of functions given at cavity boundary [10].

According to [7, 8] the potential image of the general displacement solution of the first main problem is assumed as [12]

$$\tilde{u}_i(\mathbf{x}, \omega) = \int_L p_j(\mathbf{x}^0, \omega) \cdot U_{ji}^*(\mathbf{x}, \mathbf{x}^0, \omega)\, ds, \tag{7}$$

where p_1, p_2 – unknown complex potential functions;

U_{ij}^* – fundamental tensor of dominant functions chosen in the form of $U_{jk}^* = U_{jk}^{*\,kl} + U_{jk}^{*\,m}$;

$U_{jk}^{*\,kl}$ – displacement fundamental tensor of classic elasticity theory [10];

$U_{jk}^{*\,m}$ – fundamental tensor, that takes into account effect of rotational-shear deformations in the Cosserat pseudo-continuum [7].

Integration along the boundary is performed by variables x_1^0, x_2^0, while $\mathbf{x}^0 = \{x_1^0, \ x_2^0\}$.

In order to fulfill the image conditions (2) on the cavity boundary let us calculate stress according to the formulas similar to the Hooke law in the moment Cosserat continuum [4]

$$\begin{aligned} \sigma_{kl} &= \lambda e_{rr}\delta_{kl} + (2G + \alpha)e_{kl} + \alpha \in_{klm} (r_m - \varpi_m); \\ \mu_{kl} &= \beta\varpi_{r,r}\delta_{kl} + \gamma\varpi_{k,l} + \varepsilon\varpi_{l,k}, \end{aligned} \tag{8}$$

where σ_{kl} – force stress;

μ_{kl} – couple stress;

$e_{kl} = (\tilde{u}_{k,l} + \tilde{u}_{l,k})/2$;

$\in_{klm}$ – Levi-Civita symbol;

$r_k = \in_{klm} \tilde{u}_{m,l}/2$;

ϖ – micro-rotation, that is determined taking into account (1);

$\alpha, \beta, \gamma, \varepsilon$ – elastic constants of material in the Cosserat continuum.

After completing the above transformations we obtain integral expression of the following form:

$$\tilde{\sigma}_n = \int_L (f_j(\mathbf{x}, \mathbf{x}^0)p_j)ds; \quad \tilde{\tau}_{sn} = \int_L (g_j(\mathbf{x}, \mathbf{x}^0)p_j)ds, \tag{9}$$

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where f_j, g_j – known functions containing modified Bessel functions.

Having singled out irregular components of subintegral functions and having carried out the boundary transition on the basis of the Plemel-Sokhotsky formulas in dependences (9), we obtain a system of integral equations for identifying unknown on the boundary functions

$$\frac{\mathrm{Re}\,(q)}{2} + \mathbf{v.p.}\int_L (f_1(\mathbf{x},\mathbf{x}^0)q d\zeta + f_2(\mathbf{x},\mathbf{x}^0)\overline{q}\,d\overline{\zeta}) = \tilde{\varphi}_1(\mathbf{x},\omega);$$

$$\frac{\vartheta_1\,\mathrm{Im}\,(q)}{2} + \mathbf{v.p.}\int_L (g_j(\mathbf{x},\mathbf{x}^0)q d\zeta + g_j(\mathbf{x},\mathbf{x}^0)\overline{q}\,d\overline{\zeta}) = \tilde{\varphi}_2(\mathbf{x},\omega),$$

(10)

where $pds = -iqd\zeta$ – unknown function, $p = p_1 + ip_2$, $\zeta = x_1^0 + ix_2^0$;

$\vartheta_1 = 1 - B\omega^2/(4\rho)$ for the case of plane deformation.

Here the integrals are understood in terms of principal value.

The system of integral equations (10) was solved numerically using the method [10], which is based on consistent use of the mechanical quadratures and collocation method.

After identifying unknown functions at the boundary, the calculation of circular stress images on the boundary of the cavity and images of radial stresses in the medium was carried out numerically on the basis of the dependences obtained from the formulas (8):

$$\tilde{\sigma}_\theta = \vartheta_2\,\mathrm{Re}(q) + \mathbf{v.p.}\int_L (h_j(\mathbf{x},\mathbf{x}^0)pds), \quad \tilde{\sigma}_r = \mathbf{v.p.}\int_L w_j(\mathbf{x},\mathbf{x}^0)p_j ds,$$

(11)

where h_j, w_j – known functions;

$\vartheta_2 = -(1+\nu)/\nu$ for the case of plane deformation.

The calculation of the originals obtained on the basis of the formulas (11) of stresses was carried out using the inverse discrete Fourier transform [11] based on the Cooley-Tukey algorithm [12].

Results. On the basis of the developed approach the transmission and diffraction of a weak shock wave in foam media with a tunnel cavity of a circular and elliptic cross-section was studied. The potential of an incident elastic wave was given in the following form [10]

$$\varphi(\mathbf{x}, t) = \begin{cases} \varphi_0 f(\mathbf{x}/a - c_1 t/a), t \ge 0 \\ 0, \qquad\qquad t < 0 \end{cases} \quad \psi(\mathbf{x},\, t) = 0,$$

where φ_0 – constant;

a – arbitrary characteristic size;

$f(\tau) = p_*\tau^{n_*}e^{-\alpha_*\tau}$, $\tau > 0$, $n_* \ge 0$, – impulse modulation with time;

$\tau = t \cdot c_1/a$ – non-dimensional time parameter;

Let us carry out numerical calculations for the foam with open cells size of 0.4 mm, density $\rho = 30\,\mathrm{kg/cm^3}$, Young's modulus $E = 81\,\mathrm{kPa}$, Poisson's ratio $\nu = 0.3$; size factor $l = 1.6\,\mathrm{mm}$ [3].

In order to prove the validity of the developed approach, let us move the center of the cavity on a distance of $8a$ from the origin of coordinates. In accordance with the basic principles of wave mechanics, until a wave reaches the corresponding cross-section the value of the dynamic radial stresses must be zero.

The results of numerical calculations of relative radial stresses for the cross-sections separated from the origin of coordinates by the distance $3a$ (curve 1, point A_1), $6a$ (curve 2, point A_2), $11a$ (curve 3, point A_3), $12a$ (curve 4, point A_4) are shown in Fig. 4 for cases of pulse duration $\tau = 1$ ($\alpha_* = 10$, $p_* = 185$, $n_* = 2$) – Fig. 4, a, $\tau = 2$ ($\alpha_* = 2$, $p_* = 30$, $n_* = 2$) – Fig. 4, b and $\tau = 8$ ($\alpha_* = 1$, $p_* = 1.85$, $n_* = 2$) – Fig. 4, c respectively. During calculations it was assumed that $\sigma_0 = 1\,\mathrm{kPa}$.

Fig. 4 shows that when the pulse duration of elastic incident wave decreases, there is an increase in values of the maximum relative radial stresses: there is an inverse proportion on the coefficient of

proportionality, which is equal to pulse duration. Also at $\tau = 1$ the distribution of dynamic radial stresses has a more significant oscillatory character due to the influence of rotational-shear deformations.
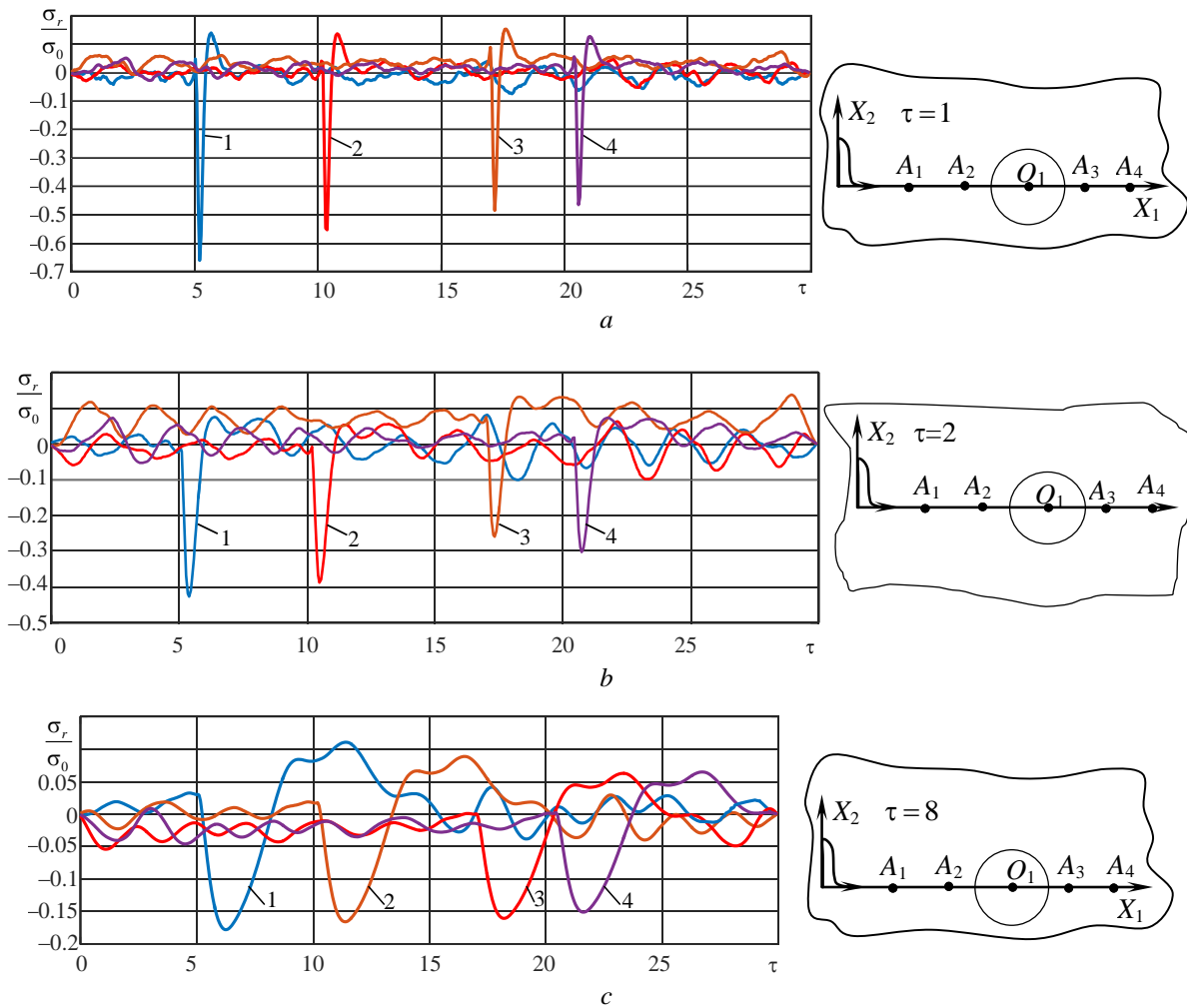

Fig. 4. Distribution of dynamic radial stresses in a foam medium with a cylindrical cavity

After the passage of the wave through the corresponding cross-section, the further stressed state of the medium is determined by the waves reflected from the boundary of the cavity.

Numerical calculations have confirmed the implementation of basic principles of wave mechanics for foam media: relative radial stresses are practically zero until a wave reaches the corresponding cross-section.

In order to study the effect of the cross-section shape of the cavity on the dynamic stress state of the medium we shall calculate the relative radial stresses for the case of the elliptic cross-section cavities with the ratio of the semiaxis 2 (Fig. 5, *a*) and 5 (Fig. 5, *b*) for $\tau = 8$.

From Fig. 5 it is evident that the maximum values of the dynamic radial stresses in a foam medium with a tunnel elliptic cross-section cavity are 2.7 (for the ratio of semiaxis of the ellipse 2) and 4.97 (for the ratio of semiaxis of the ellipse 5) times less than the corresponding values for the case of a circular cross-section cavity. This is due to the basic properties of the diffraction phenomenon: the wave is more likely to be able to avoid the barrier in the form of an elliptical tunnel cavity, if the major semiaxis of the cross-section coincides with the direction of wave transmission.

To fully assess the stress state of the medium we investigate the distribution of dynamic circular stresses at the cavity boundary under the action of a weak shock wave at $\tau = 8$. The results of

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numerical calculations are shown in Fig. 6. for circular (a), elliptic with a ratio of semiaxes 2 (*b*) and elliptic with a ratio of semiaxes 5 (*c*) tunnel cavities in a foam medium [3]. Here θ is the polar angle.
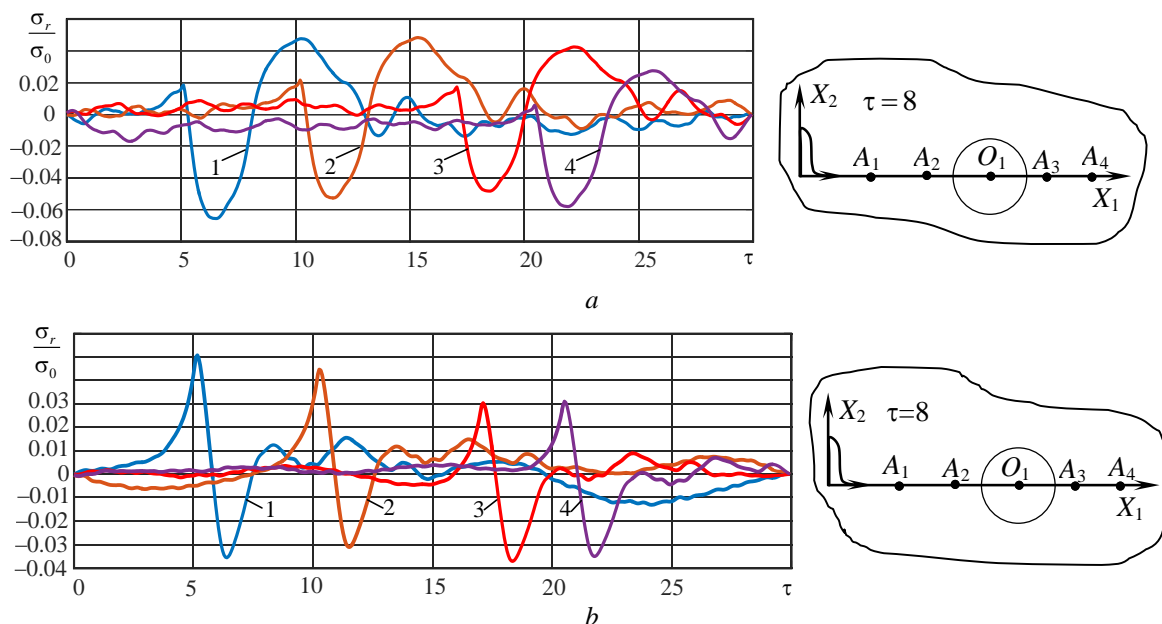

a

b

Fig. 5. Distribution of dynamic radial stresses in a foam medium with an elliptical cavity
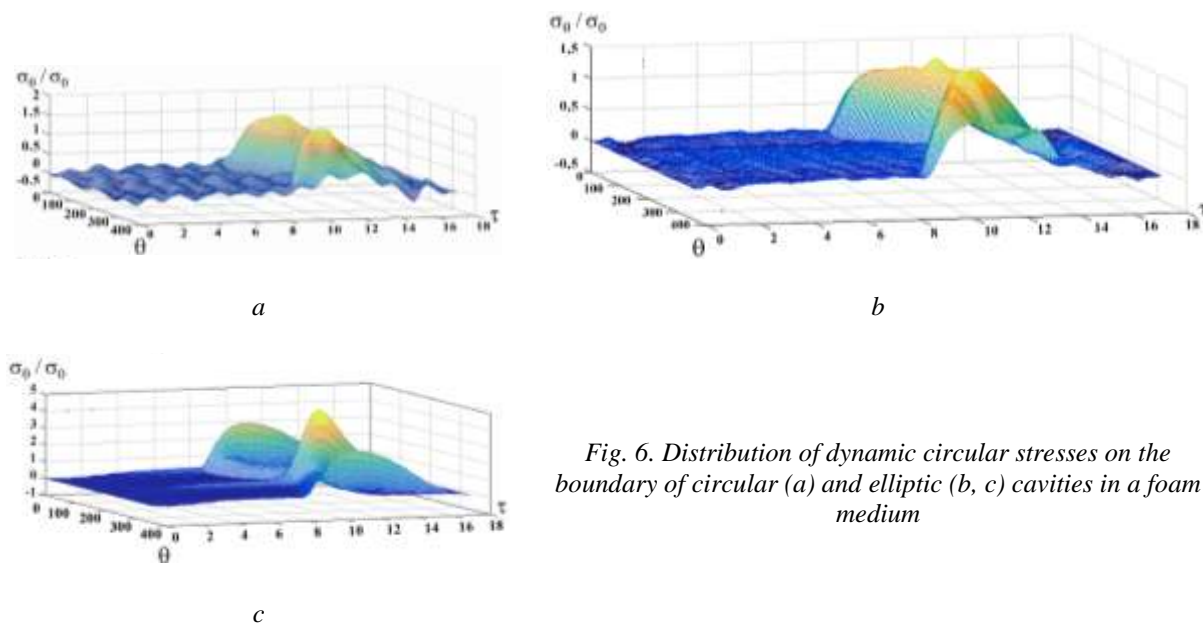

a

b

c

Fig. 6. Distribution of dynamic circular stresses on the boundary of circular (a) and elliptic (b, c) cavities in a foam medium

Fig. 6 proves that the maximum values of dynamic circular stresses for the case of a tunnel circular cross-section cavity are $1,5\sigma_0$, for the elliptic cross-section cavities with the ratio of semiaxis 2 and 5 – $1,44\sigma_0$ and $3,34\sigma_0$ respectively. During the action of a weak shock wave the maximum dynamic stresses on the boundary of circular cross-section cavity appear at vertical points, for the elliptical cross-section – at the points of the major semiaxis. Numerical calculations confirm the validity of basic principles of wave mechanics for foam media: stresses are zero until the wave reaches the cavity.

Conclusions. The developed approach based of the combined use of time Fourier transform and the boundary integral equations method enables the investigation of transmission and diffraction of

elastic impulse waves on tunnel cavities in a foam medium. The use of the equations of the moment Cosserat continuum made it possible to take into account the effect of rotational-shear deformations that take place in the medium due to the action of dynamic load.

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