The Marginal Voter's Curse[*]

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Abstract

The swing voter's curse is useful for explaining patterns of voter participation, but arises because voters restrict attention to the rare event of a pivotal vote. Recent empirical evidence suggests that electoral margins influence policy outcomes, even away from the 50% threshold. If so, voters should also pay attention to the marginal impact of a vote. Adopting this assumption, we find that a *marginal voter's curse* gives voters a new reason to abstain, to avoid diluting the pool of information. The two curses have similar origins and exhibit similar patterns, but the marginal voter's curse is both stronger and more robust. In fact, the swing voter's curse turns out to be knife-edge: in large elections, a model with both pivotal and marginal considerations and a model with marginal considerations alone generate identical equilibrium behavior.

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1 Introduction

Standard models of elections restrict attention to the mechanical impact of a vote, which is that it can be *pivotal* in changing the identity of an election winner, by making or breaking a tie. Empirically, however, it seems that votes may also exert a *marginal* influence on policy outcomes, by adjusting the balance between political parties in power. Claiming a "mandate" from voters, for example, U.S. presidents who win by larger margins pursue more major policy changes (Conley, 2001). When members of Congress win reelection by larger margins, they have more partisan voting records (Faravelli, Man, and Walsh, 2015).[1] With legislative rules such as Proportional Representation, larger electoral margins shift the balance of power mechanically, by altering the composition of a legislature. Slight increases in a party's power can be important, even when that party has more than 50% of the power: in the most recent legislative session, the U.S. Republican party lacked the political strength for desired health and immigration legislation in spite of controlling both houses of Congress and the presidency, but presumably would have been successful with a larger majority. For any of these reasons, the relationship between electoral margins and policy outcomes may be as Figure 1 illustrates, where crossing the 50% threshold shifts the policy outcome discontinuously, but policies respond to electoral margins even away from this threshold. If so, then every vote has a small but direct impact on the policy outcome, since every vote slightly increases or decreases the winning party's margin of victory.

In a seminal paper, Feddersen and Pesendorfer (1996) derive an important implication of the standard pivotal voting calculus, which is that voters who lack information about available policy alternatives have a strategic incentive to abstain from voting, effectively delegating their decision to voters with superior information.[2] This is useful for explaining why voters often deliberately abstain, even in settings where voting

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[1] Fowler (2005) shows that parties that win by large margins also nominate more extreme candidates in subsequent elections. Bernhard et al. (2008) find that senators who win by larger margins moderate less in the two years before the subsequent election. Fowler (2006) also shows that bond market investors expect larger policy changes after landslide election outcomes.

[2] This relates to pivotal voting because, when a voter expects others to make an informed decision, his own vote will change the election outcome precisely when he has mistakenly voted for the inferior policy alternative.
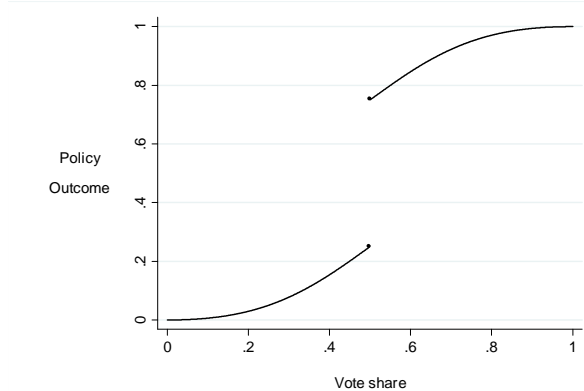
Figure 1: Mapping between vote shares and policy outcomes.

is costless, for example by casting incomplete ballots, and has been corroborated by extensive evidence that voters become more likely to vote when their information improves.[3] Since voters are only willing to rely on others' expertise when they share a common objective, that paper has also led to a resurgence of the classic *common interest* paradigm of Condorcet (1785), where elections serve to pool information rather than resolve conflicts of interest.[4] However, if voting also exerts a marginal impact on policy outcomes, as in Figure 1, then voters should take this into account, meaning that the standard pivotal voting calculus is wrong—or at least incomplete.

This paper proposes the first common interest model of voter turnout that takes both the pivotal and the marginal impact of a vote into account.[5] We include pivotal

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[3]For reviews of this empirical literature, see Triossi (2013) and McMurray (2015). In particular, turnout and roll-off are both correlated with political knowledge and with other variables associated with information, such as education and age. Lassen (2005), Banerjee et al. (2011), and Hogh and Larsen (2016) present evidence that the impact of information on voter participation is causal.

[4]For example, see Feddersen and Pesendorfer (1997, 1998, 1999), Piketty (1999), Myerson (2002), Razin (2003), Martinelli (2006), Krishna and Morgan (2011), Ahn and Oliveros (2012, 2016), Bouton and Castanheira (2012), Bhattacharya (2013), McMurray (2013, 2017a,b, 2018a,b), Ekmekci and Lauermann (2016), Osborne, Rosenthal, and Stewart (2016), Ali, Mihm, and Siga (2017), Barelli, Bhattacharya, and Siga (2017), Battaglini (2017), and Bouton, Llorente-Saguer, and Malherbe (2018).

[5]McMurray (2017a) highlights how useful a common interest paradigm can be in explaining patterns of voter behavior that are puzzling from a pure private interest perspective. With voter participation, in particular, it is common for voters to worry that they know less than others, but as noted above, such fears are only valid when there is a shared objective. The model below is not a pure common interest model; as in Feddersen and Pesendorfer (1996), the interests of some citizens are in conflict. More general forms of heterogeneity are an important direction for future work.

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voting incentives by allowing the policy outcome to jump discontinuously when one party's vote share crosses the 50% threshold, but importantly, we also allow votes to have a marginal impact on the policy outcome away from this threshold, as depicted in Figure 1. For a broad class of policy mappings, the main result is that citizens with low (though still positive) levels of information abstain, even when voting is costless. As long as a pivot remains at the 50% threshold, this is not surprising; however, abstention also occurs even when this pivot is removed entirely. In that case, voters abstain to avoid what we call the *marginal voter's curse* of nudging the policy in the wrong direction.

The marginal voter's curse arises because the impact of an individual's vote is diluted by the votes of like-minded citizens. The impact of a vote for the party that is already leading is diluted more than the impact of a vote for the trailing party. In a common interest environment, the superior party tends to be leading, which means that voting for the correct alternative/party tends to have a smaller impact on the policy outcome than voting for the inferior alternative/party has. With no information about which party is superior, therefore, the benefit of voting for either party is negative, so a voter abstains in equilibrium, even if voting is costless. Unlike the swing voter's curse, which results from low-probability, high-impact events, the marginal voter's curse is generated by high-probability, low-impact events. In spite of this contrast, however, both curses result from the same general *underdog property*, whereby an additional vote for the leading party has smaller impact than an additional vote for the losing party. The two curses thus exhibit similar comparative statics with regard to the underlying distributions of voter preferences and information.

Intuitively, it might seem that comparing nudges in one direction or the other would have less impact on voter beliefs than conditioning on an event with major impact as a pivotal vote. However, the marginal voter's curse turns out to be stronger than the swing voter's curse, in the sense that abstention is higher in a pure marginal voting model than in a pure pivotal voting model.[6] In a general model that includes

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[6]This abstention with costless voting result is the opposite of that obtained in a costly voting private value setting of Herrera, Morelli, and Palfrey (2014), where abstention is higher in a pure pivotal voting model than in a pure marginal voting model, as long as support for the two parties is not precisely balanced.

both pivotal and marginal considerations, of course, both curses operate. As the number of votes grows large, however, the importance of pivotal voting considerations also shrinks compared to marginal voting considerations. In the limit, then, even though the marginal impact of a vote is minimal, voter participation converges to the same level that would prevail if there were no discontinuity at all at the 50% threshold. In other words, a model that includes both pivotal and marginal voting considerations makes the same predictions for large elections as a model with marginal voting considerations alone. In that sense, ignoring marginal voting incentives not only fails to capture an aspect of elections that is relevant empirically; it also generates predictions that turn out to be knife-edge in a more general setting.

While this paper focuses on pure common interest voters in addition to private interest voters, a number of existing papers analyze voter participation in light of the marginal impact of voting, but in a purely private interest setting.[7] Others study participation in common interest elections, but focus only on pivotal voting.[8] In a common interest setting, Razin (2003) considers a model where political parties share voters' interests, and voluntarily adjust their policy positions to utilize information revealed by electoral margins. Introducing abstention into that model, McMurray (2017b) shows that relatively uninformed citizens abstain to avoid the "signaling voter's curse" of conveying misleading information. In contrast, the marginal voter's curse arises with a mapping from vote shares to policy outcomes that is purely mechanical. As explained below, this could reflect adjustments in the balance of power between parties that do not share voters' interests, or hold overconfident policy beliefs. Together, the swing voter's curse, signaling voter's curse, and marginal voter's curse make clear that common interest and heterogeneous expertise generate strategic abstention for a variety of institutional details.

The organization of this paper is simple. Section 2 introduces the formal model, and Section 3 analyzes equilibrium incentives for voter participation, first for elections of arbitrary size and then in the limit as the electorate grows large. For simplicity, this

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[7]For example, see Castanheira (2003), Shotts (2006), Meirowitz and Shotts (2009), Herrera, Morelli, and Palfrey (2014), Faravelli, Man, and Walsh (2015), Faravelli and Sanchez-Pages (2015), Herrera, Morelli, and Nunnari (2015) and Kartal (2015).

[8]For example, see Feddersen and Pesendorfer (1996, 1999), Krishna and Morgan (2011, 2012), and McMurray (2013).

analysis assumes that policies (aside from the discontinuity at 50%) depend linearly on the vote share. Section 3.3 then shows that the same results hold for a broad class of general policy functions, as well, such as the one pictured in Figure 1. Section 4 concludes, and proofs of theoretical results are presented in the Appendix.

2 The Model

An electorate consists of N citizens where N is finite but unknown, and follows a Poisson distribution with mean n.[9] Together, these citizens must choose a policy from an interval. There are two political parties, each with policy positions in the interval. At the beginning of the game, and with equal probability, Nature designates one of these policy positions as better for society than the other. Let A denote the party with the superior position and B denote the party with the inferior position. Letting 0 denote the inferior policy position and 1 denote the superior position, $x \in [0, 1]$ can denote any policy between the two parties' positions and also the social welfare $u(x) = x$ that will be attained if that policy is implemented.

Citizens are each independently designated as one of two types. With probability $2p$, a citizen is a *partisan*, and has a vested interest in promoting one party or the other (each with probability p), regardless of which policy position Nature designated as superior. With remaining probability $I = 1 - 2p$, a citizen is *independent* or *non-partisan*. Independents prefer to do whatever is socially optimal, evaluating policy x according to the welfare function $u(x)$ given above. From an independent's perspective, each of his fellow citizens has probability p of being a partisan supporter of the superior party A and probability p of being a partisan supporter of the inferior party B. Let a and b denote the numbers of votes cast for either party and $\lambda_+ = \frac{a}{a+b}$ and $\lambda_- = \frac{b}{a+b}$ denote the parties' vote shares (where $\lambda_+ = \lambda_- = \frac{1}{2}$ if $a = b = 0$).

The most standard assumption is that the policy outcome x is simply given by

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[9]This follows Myerson (1998). A known population size is unrealistic, and generates pathological equilibria, where voters play weakly dominated strategies, knowing that their votes will not be pivotal. If N is odd, the swing voter's curse also needs not arise, as a tie conveys no information about the state variable. Poisson uncertainty substantially simplifies the analysis, especially in deriving the limiting probabilities of pivotal events.

the policy position $x_w \in \{x_A, x_B\}$ of the party w who wins the election (i.e. 0 if $b > a$ and 1 if $a > b$, breaking a tie if necessary by a fair coin toss). Alternatively, the ultimate policy outcome might be a product of bargaining between the two parties.[10] If a party's bargaining power is determined by its vote share, for example, the policy outcome might be given by the weighted average $0\left(\lambda_-\right) + 1\left(\lambda_+\right) = \lambda_+$.[11] We refer to these cases as pure *pivotal voting* and *pure marginal voting*, respectively, and assume more generally that the policy outcome (and welfare) are given by the weighted average

$$x = \theta\lambda_+ + (1 - \theta) x_w \qquad (1)$$

of these two extremes. As in Figure 1, policy then shifts discontinuously when one party's vote share crosses the 50% threshold, but even away from this threshold, changes in one party's vote share push the policy outcome marginally in that party's direction.[12]

The optimal policy cannot be observed directly, but independent voters observe private signals s_i that are informative of Nature's choice.[13] These signals are of heterogeneous quality, reflecting the fact that citizens differ in their expertise on the issue at hand. Specifically, each citizen is endowed with information quality $q_i \in \mathcal{Q} = [0, 1]$, drawn independently according to a common distribution F which, for simplicity, is continuous and has full support. Conditional on $q_i = q$, a citizen's signal correctly identifies the party whose policy position is truly superior with the

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[10]McMurray (2017b) assumes parties to be like independent voters, favoring policies as close as possible to the state variable. With no need to bargain, the winning side voluntarily calibrates its policy position to account for the strength of evidence for and against its side. Here, parties are better thought of as partisan voters, favoring extreme policies but for lack of bargaining power. Alternatively, the same specification applies if parties share independent voters' interests but hold overconfident beliefs, as in McMurray (2018a).

[11]This is the welfare outcome if independent voters are risk neutral, and parties implement their preferred policies 0 and 1 with probabilities λ_- and λ_+. It could also result from a bargaining model with alternating offers, where vote shares determine the probability of being able to offer the next proposal. Alternatively, it could result from probabilistic voting across independent legislative districts, as in Levy and Razin (2015).

[12]The specification in (1) constrains the policy function to be linear in the vote share (with slope θ). Section 3.3 extends this to more general functional forms, such as that illustrated in Figure 1.

[13]Partisans could receive signals as well, of course, but would ignore them in equilibrium.

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following probability.

$$\Pr(s_i = A|q) = \frac{1}{2}(1+q) \tag{2}$$

With complementary probability, a citizen mistakes the inferior party for the superior party.

$$\Pr(s_i = B|q) = \frac{1}{2}(1-q) \tag{3}$$

With this specification, q_i can be interpreted as the correlation coefficient between a voter's private opinion and the truth. That is, a signal with $q_i = 1$ perfectly reveals Nature's choice while a signal with $q_i = 0$ is completely uninformative. An independent can vote (at no cost) for the party that he perceives to be superior, or can abstain.[14] Let $\sigma : \mathcal{Q} \to [0,1]$ denote a (mixed) participation strategy, where $\sigma(q)$ denotes the probability of voting for an individual with expertise $q \in \mathcal{Q}$, and let Σ denote the set of such strategies. If he votes for his signal, a voter's posterior belief that he has correctly voted for the superior party or mistakenly voted for the inferior party are given simply by the right-hand sides of (2) and (3), respectively, by Bayes' rule.

As an example of political decisions that might fit the structure outlined here, consider any division of funding between two programs with a common objective, such as using education money either to increase teacher salaries or to reduce class sizes—or, at higher levels, using tax revenue to strengthen either the military or the social safety net. At either level, some voters simply have a vested interest in one program or the other, and vote accordingly, but others support whichever policy they believe will be truly best for the group. Interior policies represent various possible compromises that partially fund both programs, and can result from bargaining between parties with different levels of power, but if truth were known, independent voters would prefer to fully fund whichever program is more productive over any such compromise, and may vote with an eye toward increasing the bargaining power of the party whose policy proposal seems superior.

Given a participation strategy, the probabilities with which a citizen votes for

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[14] A strategy of voting against one's signal could be allowed but would not be used in equilibrium.

party A and party B, respectively, are given by the following.

$$v_+ = p + I \int_0^1 \sigma(q) \frac{1}{2}(1+q)\, dF(q) \tag{4}$$

$$v_- = p + I \int_0^1 \sigma(q) \frac{1}{2}(1-q)\, dF(q) \tag{5}$$

These include the probability p of favoring either party for partisan reasons, as well as the probabilities of voting as an independent with any level of expertise. Together, (4) and (5) also determine the level $v = v_+ + v_-$ of voter turnout.

If every citizen follows the same participation strategy, (4) and (5) can be interpreted as the expected vote shares of the superior and inferior parties, respectively. By the decomposition property of Poisson random variables (Myerson 1998), the numbers a and b of votes for the superior and inferior parties, respectively, are independent Poisson random variables with means $n_+ = nv_+$ and $n_- = nv_-$. Thus, the probability of exactly a votes for the superior party and b votes for the inferior party is the product

$$\Pr(a, b) = \frac{e^{-n_+} n_+^a}{a!} \frac{e^{-n_-} n_-^b}{b!} \tag{6}$$

of Poisson probabilities. Similarly, the expected total number of votes can be written as nv.

By the environmental equivalence property of Poisson games (Myerson 1998), an individual from within the game reinterprets a and b as the numbers of correct and incorrect votes cast by his peers; by voting himself, he might add one to either total. When there are a votes for the superior party and b votes for the inferior party, the change in utility $\Delta_+ x(a, b)$ from contributing one additional vote for the superior party and the change in utility $\Delta_- x(a, b)$ from adding one vote for the inferior party are given by the following.

$$\Delta_+ x(a, b) = x(a+1, b) - x(a, b) \tag{7}$$

$$\Delta_- x(a, b) = x(a, b+1) - x(a, b) \tag{8}$$

The magnitudes of these utility changes depend on the numbers of votes cast for either

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side by a citizen's peers; averaging over all possible voting outcomes, the expected benefit of voting is given by

$$\Delta Eu\left(q\right) = E_{a,b}\left[\frac{1}{2}\left(1+q\right)\Delta_{+}x\left(a,b\right) + \frac{1}{2}\left(1-q\right)\Delta_{-}x\left(a,b\right)\right] \tag{9}$$

which depends on a citizen's expertise q. Implicitly, the expectation in (9) depends on the voting strategy adopted by a citizen's peers. If his peers all follow the strategy $\sigma \in \Sigma$, a citizen's best response is to vote if his q is such that (9) is positive and to abstain otherwise. A strategy σ^{*} that is its own best response constitutes a (symmetric) Bayesian Nash equilibrium of the game. Section 3 now analyzes the properties of such equilibria, first generically and then for large electorates, and then extends the model to more general relationships between vote totals and the policy outcome.

3 Equilibrium Analysis

3.1 Finite elections

The potential benefit of voting lies in a voter's ability to bring the policy outcome closer to the policy that is truly optimal. The potential damage of voting lies in the possibility that the voter will accidentally push the policy outcome away from what is optimal. Whether the net expected benefit of voting (9) is positive or negative therefore depends on how confident a voter is that his vote will push the policy outcome in the right direction, and this confidence increases with a voter's expertise q. Accordingly, best response voting follows a *quality threshold strategy*, defined in Definition 1, meaning simply that sufficiently expert voters vote, while those who lack expertise abstain. As Proposition 1 now states, this characterizes equilibrium, as well, and a standard fixed point argument on the interval of possible thresholds guarantees equilibrium existence.

Definition 1 $\sigma_{\tau} \in \Sigma$ *is a quality threshold strategy (with quality threshold τ) if* $\sigma_{\tau}\left(q\right) = 1_{q \geq \tau}$.

Proposition 1 *If $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium then it is a quality threshold strategy, with quality threshold $\tau^* > 0$. Moreover, such an equilibrium exists.*

Definition 1 allows the possibility that $\tau = 0$, meaning that all citizens vote. In equilibrium, however, Proposition 1 states that $\tau^* > 0$, meaning that the fraction $F(\tau^*)$ of independent voters who abstain is positive. For the case of $\theta = 0$, the logic for this is the swing voter's curse, familiar from Feddersen and Pesendorfer (1996) and McMurray (2013): the party whose policy position is truly superior is more likely to win by one vote than to lose by one vote, so one additional vote for this party is less likely to be pivotal than one additional vote for the opposing party. Since a mistake is more likely to have impact than a correct vote, a voter who has no information strictly prefers to abstain. By continuity, voters with near-zero expertise prefer to abstain, as well.

For the case of $\theta = 1$, the impact of a vote is entirely marginal, so pivotal events no longer matter. In that case, the swing voter's curse does not arise. Nevertheless, Proposition 1 states that $\tau^* > 0$ in that case as well, implying that a positive fraction of the electorate still abstain. They do so to avoid the *marginal voter's curse* of pushing the policy outcome in the wrong direction. For intermediate values of θ, both curses operate.

Like the swing voter's curse, the marginal voter's curse arises because the damage a voter will inflict if he is in error exceeds the benefit his vote will generate if his private opinion is correct, so an uninformed voter—and, by continuity, a poorly informed voter—prefers to abstain. The key observation is that the impact of a vote gets diluted when others vote the same way. This means that one additional vote for the losing side has greater impact on the margin of victory than one additional vote for the winning side has. If one alternative receives three out of five votes, for example, or a 60% vote share, then an additional vote for the winning party increases this vote share by seven percentage points (i.e. to 67%, or four out of six) but an additional vote for the losing party decreases the winning party's vote share by ten percentage points (i.e. to 50%, or three out of six). As before, this matters because the party whose policy position is truly superior is more likely to be ahead than behind. Thus, one additional vote for the inferior party should have greater impact than an additional

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vote for the superior party.

The result that independent voters each receive informative private signals but not all report their signals in equilibrium implies that valuable information is lost. Intuitively, this may seem to justify efforts to increase voter participation, for example by punishing non-voters with stigma or fines. However, McLennan (1998) shows that, in common interest environments such as this, whatever is socially optimal is also individually optimal, implying that equilibrium abstention in this setting actually improves welfare. To see how it can be welfare improving to throw away signals, note that citizens actually have not one but two pieces of private information: their signal realization s_i and their expertise q_i. In an ideal electoral system, all signal realizations would be utilized, but would be weighted according to their underlying expertise. Here, however, votes that are cast are all weighted equally. Whether the impact of a vote is pivotal or marginal, abstention provides a crude mechanism whereby citizens can transfer weight from the lowest quality signals to those that reflect better expertise.

3.2   Large elections

The analysis above applies for elections of arbitrary size. In most elections, however, n is quite large. Accordingly, Proposition 2, below, characterizes behavior of voters in the limit as $n \to \infty$. Not surprisingly, the quality threshold structure of voting persists. Moreover, the limiting threshold τ_∞^* is unique. This means that, if multiple equilibria exist in finite elections (a possibility which Proposition 1 does not exclude), they all converge when n is large.[15]   A unique quality threshold also translates into a unique level of turnout $1 - F(\tau_\infty^*)$. Parts 1 and 2 of Proposition 2 state further that the unique limiting threshold lies strictly between 0 and 1 (for all but one parameter combination, as explained below), implying that the fractions of independent voters who vote and abstain both remain substantial, no matter how large the electorate grows.

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[15]For pure pivotal voting, uniqueness in the limit requires that the density of expertise be sufficiently spread out, as McMurray (2013) explains. A sufficient condition for this is that f is log-concave, meaning that $\ln(f)$ is concave. This condition is unnecessary for the cases of $\theta > 0$.

A unique limit facilitates meaningful comparative static comparisons for changes in model parameters. Part 3 of Proposition 2 states that adding partisans to the electorate reduces the limiting participation threshold for independents. This increases participation among independents and, since partisans always vote, increases participation overall. Part 4 considers an overall improvement in voter information, in terms of the monotone likelihood ratio property (MLRP), as defined in Definition 2, and states that such an improvement raises the limiting equilibrium quality threshold. By themselves, lifting voters above the limiting quality threshold would increase turnout, but raising the threshold would lower turnout; when both occur, the net affect is ambiguous.[16]

Definition 2 *Let $\mathcal{F}$ denote the set of distribution functions on $[0, 1]$. If $F \in \mathcal{F}$ and $G \in \mathcal{F}$ have densities f and g then $F <_{MLRP} G$ if $\frac{g(x)}{f(x)}$ increases in x. A function $h : \mathcal{F} \to \mathcal{R}$ is increasing if $F <_{MLRP} G$ implies that $h(F) < h(G)$.*

Proposition 2 *If f is log-concave then there exists a unique quality threshold τ_∞^* such that $\lim_{n \to \infty} \tau_n^* = \tau_\infty^*$ for any sequence (τ_n^*) of equilibrium quality thresholds. Moreover, τ_∞^* satisfies the following (for all (p, θ, F), unless otherwise specified).*

(1) $\tau_\infty^* > 0$

(2) $\tau_\infty^* < 1$ if and only if $(p, \theta) \neq (0, 1)$

(3) τ_∞^* decreases with p

(4) τ_∞^* increases with F

The logic behind Proposition 2 is that, for any quality threshold τ, the proof of Proposition 1 defines another quality threshold $\tau_n^{br}(\tau)$ that characterizes its best response. As n grows large, realized vote shares converge to their expectations, and $\tau_n^{br}(\tau)$ converges to a unique limit, $\tau_\infty^{br}(\tau)$. Given the continuity of utility, a sequence of equilibrium thresholds τ_n^* must converge to a fixed point of τ_∞^{br}. For the case of pure pivotal voting, the proof that τ_∞^{br} has exactly one fixed point follows McMurray

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[16]All of the parts of this proposition are also proven by Feddersen and Pesendorfer (1996) and McMurray (2013), but only for the case of $\theta = 0$. Extending to $\theta > 0$ is useful because it shows that the empirical applications of those papers remain valid even if the full impact of a vote is not limited to pivotal events. The proof below is also more intuitive.

(2013). For pure marginal voting, the proof of Proposition 2 rewrites the fixed point condition $\tau_\infty^{br}(\tau) = \tau$ as equation (??), which is equivalent to the following.

$$\lambda_+(\tau) = \frac{v_+(\tau)}{v_+(\tau) + v_-(\tau)} = \frac{1}{2}(1 + \tau) \tag{10}$$

A key observation is that equation (10) is also the first-order condition for maximizing $\lambda_+(\tau)$. Once the left- and right-hand sides of (10) cross, therefore, the former decreases and the latter increases in τ, preventing additional intersections. Figure 2 illustrates this for a particular levels of partisanship, and for a uniform distribution of expertise.

The result that equating the left- and right-hand sides of (10) also maximizes λ_+ is intuitive in light of McLennan's (1998) observation that, in a common interest setting such as this, whatever is socially optimal is also individually optimal, and can therefore prevail in equilibrium. After all, when voters follow the quality threshold strategy σ_τ, $\lambda_+(\tau)$ can be interpreted as the fraction of voters who correctly vote for the superior party. The right-hand side of (10) gives the posterior belief of a voter with expertise $q = \tau$ exactly at the threshold, which is also the probability that this voter will vote correctly for the superior party. When τ is so low that the marginal voter has a lower probability of voting correctly than the average voter has, the marginal voter prefers to abstain in response. When τ is so high that the marginal voter is more likely to vote correctly than an average voter, he strictly prefers to vote. In equilibrium, of course, the marginal voter must be indifferent between voting and abstaining.[17]

With pure pivotal voting, the proof of Proposition 2 rewrites the fixed point condition $\tau_\infty^{br}(\tau) = \tau$ as equation (??), which is equivalent to the following.

$$\frac{\sqrt{v_+(\tau)}}{\sqrt{v_+(\tau)} + \sqrt{v_-(\tau)}} = \frac{1}{2}(1 + \tau) \tag{11}$$

Like (10), the right-hand side of (11) gives the posterior belief of the marginal voter—

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[17]This logic for why equating average and marginal probabilities maximizes the average is the same reasoning behind the familiar result in industrial organization, that average costs are minimized when they equal marginal costs.
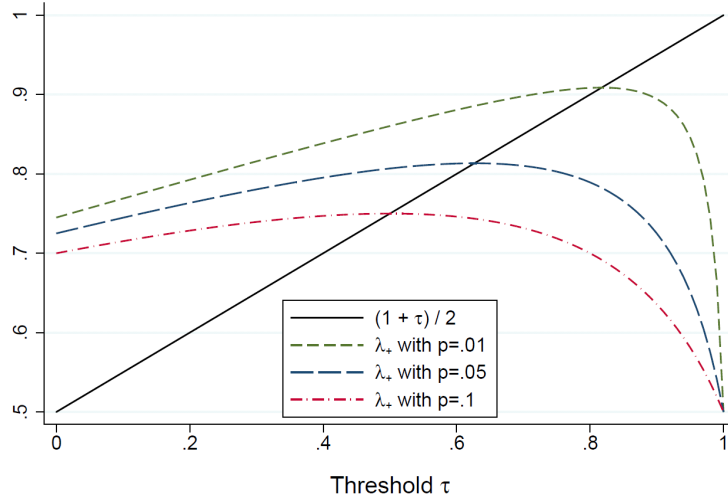
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Figure 2: Left- and right-hand side of equation (10) for different levels of partisanship.

that is, the strength of evidence of his own private information. From Myerson (2000), the left-hand side of (11) can be interpreted as the limit of the probability $\Pr\left(\mathcal{P}_-|\mathcal{P}\right)$ that a vote for the inferior party is pivotal (event $\mathcal{P}_-$), given that either a vote for the inferior party or a vote for the superior party is pivotal (event $\mathcal{P}$). Since his peers are more likely to elect the superior party by one vote than the inferior party by one vote, a mistaken vote is more likely to be pivotal than a correct vote, and the left-hand side of (11) can be interpreted as the strength of evidence that a voter is mistaken, conditional on his vote being pivotal in one direction or the other. If the probability of a mistake, conditional on being pivotal, is greater than the probability of a voter's own signal being correct (i.e., if the left-hand side of (11) exceeds the right-hand side) then a voter should abstain; otherwise (i.e., if the right-hand side exceeds the left-hand side), he should vote with his signal.[18]

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[18]Equation (11) can be written equivalently as follows.

$$\frac{1}{2}\left(1-\tau\right)\frac{\sqrt{v_+\left(\tau\right)}}{\sqrt{v_+\left(\tau\right)}+\sqrt{v_-\left(\tau\right)}}\left|-1\right| = \frac{1}{2}\left(1+\tau\right)\frac{\sqrt{v_-\left(\tau\right)}}{\sqrt{v_+\left(\tau\right)}+\sqrt{v_-\left(\tau\right)}}\left|1\right|$$

The left-hand side of this expression is the cost of voting in a large election: the marginal probability $\frac{1}{2}\left(1+\tau\right)$ of a mistaken opinion, times the limiting probability $\frac{\sqrt{v_+(\tau)}}{\sqrt{v_+(\tau)}+\sqrt{v_-(\tau)}}$ of being pivotal in the wrong direction (conditional on being pivotal at all), times the magnitude $\left|-1\right|$ of the utility impact

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The result that equilibrium behavior under pure marginal voting asymptotically maximizes the marginal voting objective raises the question of whether equilibrium behavior under pure pivotal voting also asymptotically maximizes the pure pivotal objective. Indeed, this turns out to be the case, which existing literature on common interest elections seems not to have noted: for large n, Myerson (2002) shows that the probability with which the superior party wins a majority election is of order $e^{-n\left(\sqrt{v_+}-\sqrt{v_-}\right)^2}$, and the first-order condition for maximizing this quantity is none other than (11). Thus, equilibrium behavior under marginal voting serves to maximize the superior party's margin of victory, and equilibrium behavior under pivotal voting serves to maximize the superior party's probability of winning. Whether the impact of a vote is pivotal or marginal or some hybrid of the two, the location of the equilibrium quality threshold trades off the quality and the quantity of private information reported by voters: a high threshold aggregates the signals that are best informed, while a low threshold aggregates a larger number of signals. The logic behind Part 3 of Proposition 2 is simply that, as the electorate becomes more partisan, there is a greater need for a large quantity of independent votes, to make sure that the electoral decision is made by independents, not partisans.

With pure pivotal voting, the quantity of information still matters when there are no partisans, as McMurray (2013) explains, because the expected outcome is a vote share higher than 50% for the superior party, and additional votes reduce the variance around this expectation, ensuring that a vote share below 50% doesn't occur by mistake. With marginal voting, it is still valuable to ensure that the realized vote

of changing the policy outcome from good to bad. The right-hand side is the benefit of voting: the marginal probability $\frac{1}{2}\left(1-\tau\right)$ of a mistaken opinion, times the limiting probability $\frac{\sqrt{v_-(\tau)}}{\sqrt{v_+(\tau)}+\sqrt{v_-(\tau)}}$ of being pivotal in the right direction (conditional on being pivotal at all), times the magnitude $|1|$ of the utility impact of changing the policy outcome from bad to good. Thus, the limiting equilibrium condition can be seen simply as equating the cost and benefit for the marginal voter. Equation (10) can be similarly rewritten as equating costs and benefits for the marginal voter.

$$\frac{1}{2}\left(1-\tau\right)\left(1\right)\frac{v_+\left(\tau\right)}{v_+\left(\tau\right)+v_-\left(\tau\right)} = \frac{1}{2}\left(1+\tau\right)\left(1\right)\frac{v_-\left(\tau\right)}{v_+\left(\tau\right)+v_-\left(\tau\right)}$$

With marginal voting, however, the probability of influencing the electoral outcome is (1), and the (relative) utility impact of a correct vote and a mistaken vote are given by $\frac{v_-(\tau)}{v_+(\tau)+v_-(\tau)}$ and $\frac{v_+(\tau)}{v_+(\tau)+v_-(\tau)}$, respectively.

share is not too far below its expectation, but shrinking the variance also ensures that the realized vote share is not too far *above* its expectation, either, and this is undesirable. With linear utility, these positive and negative outcomes exactly cancel out. Thus, quantity is not particularly of value. In very small elections, poorly informed voters participate just to ensure that *somebody* votes, as n grows large, a voter who was previously right at the participation threshold now abstains, to avoid casting the noisiest vote. As voters become increasingly selective on quality, voter exit prompts more voter exit, and an unraveling occurs. In the limit, Part 2 of Proposition 2 states that the equilibrium participation threshold approaches the upper bound of the distribution of expertise, so that everyone abstains except a vanishing fraction of the most elite voters, who are most nearly infallible.[19] In this way, voters ensure that the superior party will not only win, but win with as large a margin as possible, which is what matters when the policy outcome responds to the marginal impact of a vote. These results are illustrated in Figure 3, which displays participation rates for independent voters under different parameter configurations, assuming a uniform distribution of expertise. Part 4 of Proposition 2 follows from similar reasoning: as others' information improves, an individual becomes more inclined to abstain, in deference to those who know more.[20]

Intuitively, it might seem that conditioning on the event a pivotal vote should have a much greater impact on behavior than conditioning on the marginal impact of a nudge in one direction or the other—especially in large elections, where a pivotal vote is such a special event, and where the magnitude of the nudge is vanishingly small. If so, abstention should be much higher—and turnout much lower—under pure pivotal voting than under pure marginal voting, and in large elections, the swing voter's curse should dominate voters' participation decisions. To the contrary, however, Proposition 3 now states that it is the marginal voter's curse that is stronger, in the sense that abstention is higher for $\theta = 1$ than for $\theta = 0$, for any level of partisanship.[21]

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[19]Log-concavity is not required for that result.

[20]Since improved information lifts some non-voters above the participation threshold but leads some voters to abstain in deference to now-more reliable peers, the net effect on voter turnout is ambiguous.

[21]If f is not log-concave then a solution to (11) need not be unique, but Proposition 3 holds for any limit point $\tau_\infty^* (p, 0, F)$ of a sequence of equilibrium thresholds.
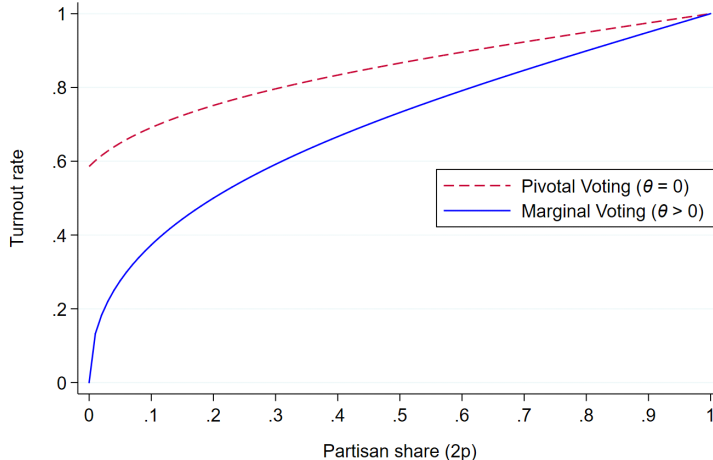
Figure 3: Turnout among independent voters as a function of the partisan share $(2p)$ when the distribution F of expertise is uniform.

Moreover, intermediate values of θ generate equilibrium behavior that converges in large elections to be identical to the case of pure marginal voting. In that sense, both curses operate in equilibrium, but as the electorate grows large, participation and abstention are determined entirely by the marginal voter's curse. The swing voter's curse can then be seen as a knife-edge result, in that *any* non-zero weight on the marginal impact of a vote snaps the equilibrium abruptly away from the case of $\theta = 0$.

Proposition 3 $\tau_\infty^* (p, 0, F) < \tau_\infty^* (p, \theta, F) = \tau_\infty^* (p, 1, F)$ *for any* (p, θ, F) *with* $\theta > 0$.

Mathematically, Proposition 3 follows because, for any τ, the left-hand side of (10) exceeds the left-hand side of (11), so equation (10) yields a higher fixed point. Intuitively, the reason that abstention is higher for pure marginal voting than for pure pivotal voting is that mistakes are more costly, because of the dilution problem described above, and the need to vote as unanimously as possible. With pure pivotal voting, a single mistake can be remedied by a single correct vote for the party with the superior policy position. The same is not true when margins matter, because vote shares become diluted, so a vote for the majority party has a lower impact on policy than a vote for the minority. As a simple illustration of this, suppose that the

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superior party received three out of five votes, or a 60% vote share. One additional vote for the opposite party reduces this vote share to 50% (three out of six), and an additional vote of support brings it back up, but only to 57% (four out of seven). Thus, it takes more than one vote to compensate for one mistaken vote. In that sense, mistakes are more permanent when a vote has a marginal impact on policy than when it doesn't, and voters work harder to avoid them. The marginal impact of a vote decays linearly as the number of voters grows large, but the probability of casting a pivotal vote decays exponentially, so in large elections, marginal considerations dominate. This is why intermediate values of θ generate equilibrium behavior that is identical in the limit to the case of pure marginal voting, $\theta = 1$.

Propositions 2 and 3 analyze how changes in model parameters impact the equilibrium threshold and voter participation. Proposition 4 now analyzes how such changes impact welfare, which can be measured by a single independent voter's utility, since partisan interests are zero-sum and are balanced by assumption. Pure pivotal voting in large elections perfectly implements the superior policy, and this does not depend on the distribution of expertise or the size of partisan shares. With pure marginal voting—or intermediate values of θ, which are asymptotically equivalent—welfare is lower, unless there are no partisans.[22] In general, welfare improves as voter information improves, and decreases with the expected partisan share p.

Proposition 4 Let $u^*_\infty = \lim_{n \to \infty} E\left[u\left(x^*_n\right)\right]$. If $\theta = 0$ then $u^*_\infty = 1$. If $\theta > 0$ then u^*_∞ increases in F and decreases in p, with $u^*_\infty = \begin{cases} 1 & \text{if } p = 0 \\ \frac{1}{2} & \text{if } p = \frac{1}{2} \end{cases}$.

That improving voter information improves welfare is intuitive. That partisans have a negative impact under pure marginal voting but no impact at all under pure pivotal voting relates again to the dilution principle described above. With pivotal voting, an additional A partisan and an additional B partisan simply nullify each other's votes, leaving independents to wield the same influence as before. With marginal voting, however, adding equal numbers of partisan votes on either side dilutes

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[22]Even with no partisans, the result that $u^*_\infty = 1$ for $\theta > 0$ relies on the assumption that q has full support; more generally, asymptotic welfare equals the posterior $\frac{1}{2}\left(1 + q_{max}\right)$ of the best informed members of the electorate.

the impact of non-partisan votes. When three out of five independents make the right decision and there are no partisans, for example, the superior party receives 60% of the votes. With one partisan on each side, this drops to 57% (or four out of seven); with two partisans on each side, it drops to 56% (or five out of nine). The more partisans there are, the more difficult it becomes for the electorate to be united in the direction of truth.

3.3 General Policy Functions

Using the parameter θ, the policy function described in Section 2 can be written as a single function $x = \psi(a, b)$ of vote totals a and b,

$$x = \psi(a, b) = \begin{cases} \frac{a}{a+b}\theta + 0(1-\theta) & \text{if } a < b \\ \frac{a}{a+b}\theta + \frac{1}{2}(1-\theta) & \text{if } a = b \\ \frac{a}{a+b}\theta + 1(1-\theta) & \text{if } a > b \end{cases} \tag{12}$$

which includes pure pivotal voting, pure marginal voting, and mixtures as special cases. This particular policy specification is special, however, in that the marginal voting component is simply linear in the vote share $\lambda_+ = \frac{a}{a+b}$. The purpose of this section is to show that the results above hold for much more general functional forms, as well, such as that pictured in Figure 1. Specifically, Proposition 5 below states a sufficient condition for a positive fraction of the electorate to abstain in equilibrium, which is that the policy function ψ satisfies Conditions 1 through 3.

Condition 1 (Monotonicity) $\psi(a, b)$ *increases in a and decreases in b.*

Condition 2 (Symmetry) *For any $a, b \in \mathbb{Z}_+$, $\psi(b, a) = 1 - \psi(a, b)$.*

Condition 3 (Underdog property) $|\psi(a, b+1) - \psi(a, b)| \quad - |\psi(a+1, b) - \psi(a, b)|$ *has the same sign as $a - b$.*

Monotonicity merely states that A and B votes push the policy outcome toward 1 and 0 and therefore increase and decrease utility, respectively. Symmetry implies that reversing the numbers of votes that each party receives exactly reverses the parties'

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power, for example implying that $\psi(a,b) = \frac{1}{2}$ when $a = b$. The underdog property states that the impact of one additional vote for the party that has fewer votes is greater than the impact of one additional vote for the party with a majority. This property is satisfied by (12), including for the extreme cases of pure pivotal or pure marginal voting, but also holds for a much broader class of policy functions. If ψ is any monotonic and symmetric function of the vote share $\lambda_+ = \frac{a}{a+b}$, for example, then Condition 3 holds if ψ is *S-shaped*—that is, convex for vote shares in $\left[0, \frac{1}{2}\right]$ and concave for vote shares in $\left[\frac{1}{2}, 1\right]$—meaning that the vote share has a diminishing marginal impact on the majority party's power. "Contest" functions of the form

$$\psi(a,b) = \frac{a^z}{a^z + b^z} = \left[1 + \left(\frac{1}{\lambda_+} - 1\right)^z\right]^{-1}$$

satisfy Condition 3, as well, and are S-shaped for $z > 1$ but an inverted S-shape (i.e. concave and then convex) for $z < 1$.

Proposition 5 *If $\psi : \mathbb{Z}_+^2 \to [0,1]$ satisfies Conditions 1 through 3 then $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium only if it is a quality threshold strategy σ_{τ^*} with $\tau^* > 0$. Moreover, such an equilibrium exists.*

Fundamentally, the logic of Proposition 5 is the same as the logic of Proposition 1: a voter with no private information is equally likely to make the policy outcome better or worse, but the underdog property implies that an additional vote for the trailing party will have greater impact than an additional vote for the leader, and when others vote informatively, the trailing party is likely to be inferior.

The substantive assumption reflected in the underdog property is crowding out. That is, the impact of an individual's vote is smaller, the more people there are voting with him. To see this, rewrite the case of pure marginal voting as follows,

$$\psi(a,b) = \frac{1}{2} + \frac{1}{2}\frac{a-b}{a+b} \tag{13}$$

thereby making clear that deviations from $\frac{1}{2}$ are proportional to the electoral margin, which is proportional to the vote differential but inversely proportional to the total

number of votes. Next, consider an alternative policy function,

$$\psi(a,b) = \frac{1}{2} + \frac{1}{2}\frac{a-b}{a+b}\frac{a+b}{N} = \frac{1}{2} + \frac{1}{2}\frac{a-b}{N} \tag{14}$$

where deviations from $\frac{1}{2}$ are in proportion to the electoral margin $\frac{a-b}{a+b}$, but also to the turnout rate $\frac{a+b}{N}$.[23] The product of these is proportional to the vote differential but inversely proportional to the total number N of voters and non-voters combined, which remains constant no matter how many votes are cast. Equations (13) and (14) thus have similar structure, but the latter does not satisfy Condition 3. With a constant marginal impact, voters no longer have any reason to abstain. Which type of function more closely matches a particular electoral setting is an open question. De jure, we are not aware of any electoral rules that are explicit functions of turnout; de facto, however, it may well be that mandates are stronger when turnout is higher.[24]

In discussing the shape of ψ, this section has maintained the assumption of linear utility. With more exotic utility functions, Condition 3 would have to be augmented to require that the difference $|u[\psi(a,b+1)] - u[\psi(a,b)]| - |u[\psi(a+1,b)] - u[\psi(a,b)]|$ in *utility* have the same sign as $a - b$. In that sense, the underdog condition is as much a restriction on preferences as it is on the mapping from vote shares to policy outcomes. In contrast, the swing voter's curse does not impose any restriction on utility, except that policy 1 is preferred to policy 0, because when voting is purely pivotal, only these two policy outcomes are possible, so the shape of u over intermediate policy outcomes is irrelevant.

4   Conclusion

That voters should focus on the rare event of a pivotal vote is often viewed as the central hallmark of rationality in models of elections. In common interest settings,

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[23]We thank an anonymous referee for this example.

[24]McMurray (2017b) models mandates as a Bayesian reaction by candidates to voter information. In that setting, electoral margins indeed imply stronger mandates when turnout is higher, because adding signals in the same proportion as existing signals strengthens a candidate's beliefs, leading her to put less weight on her prior. Abstention then still occurs, however, because of the signaling voter's curse, as voters try to manipulate the message that is being sent.

this has been shown to have dramatic consequences for voting behavior, including the swing voter's curse, which has been useful for explaining patterns of voter participation. Embracing the common interest paradigm but assuming, in light of recent evidence, that margins of victory matter even away from the 50% threshold, this paper has discovered a new strategic incentive for abstention, the marginal voter's curse. The two curses exhibit similar patterns, and both are manifestations of the same underdog property, whereby votes from like-minded voters crowd out an individual's influence on the election outcome. In large elections, however, the marginal voter's curse is more severe, in that abstention is higher with pure marginal voting than with pure pivotal voting. It is also more robust, in that marginal and pivotal considerations together generate the same behavior as marginal considerations alone. These predictions are confirmed empirically in the laboratory experiments of Herrera, Llorente-Saguer, and McMurray (2018).

In legislative elections, Proportional Representation is a common alternative to majority rule. With PR, changes to a party's vote share can matter even away from the 50% threshold, as Section 1 notes, so that the standard pivotal voting calculus, and therefore the swing voter's curse, do not directly apply. Existing literature often models PR as above, equating the policy outcome to a party's vote share. This does not perfectly match the institutional details of PR, where number of legislative seats is finite, but does capture the ideal that the composition of the legislature should match the electorate as closely as possible. To the extent that the model of Section 2 accurately approximates PR, therefore, the marginal voter's curse predicts that strategic abstention should occur under PR, just as it does under majority rule. This is useful because, empirically, Sobbrio and Navarra (2010) find that poorly informed voters in either system are more likely to abstain.[25] Partial ballots seem just as prevalent under PR as they are in majority rule. In the 2011 Peruvian national elections, for example, 12% of those who went to the polls failed to cast valid votes in the Presidential election (the first round of a runoff system), but larger fractions, namely 23% and 39% respectively, failed to vote in the PR elections for Congress and for the Andean Parliament.[26] Just as in majoritarian settings, a lack of information

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[25]See also Riambau (2018).

[26]See the webpage of the Oficina Nacional de Procesos Electorales (http://www.web.onpe.gob.pe).

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seems an intuitive rationale for such selective abstention.

Section 2 assumes that voters are equally likely to be A partisan or B partisan, that the two parties are ex-ante equally likely to be superior, that private signals are equally informative in either case, and that utility in the two states is symmetric. Such symmetry keeps the analysis tractable, but asymmetries are relevant to many applications, and so should be explored in future work. The model above also assumes that truth is binary: the two parties' policy positions are the best policy and worst policy available. Given that assumption, it is not surprising that pivotal voting is superior to marginal voting, as it guarantees one of these extremes. In many applications, however, the optimal policy may not be either extreme, but rather a compromise between the two.[27] In such situations, the welfare ranking in Proposition 4 may well be reversed. To explore this possibility, future work should seek extend the present model to additional truth states.[28] In addition to these directions, future work should study richer forms of preference heterogeneity. The present model mixes private and common values, but independents are perfectly unified, and partisans do not care at all about the true state of the world; more generally, all voters might have objectives that put some weight on their own interests, and some weight on the common good.[29]

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Peru's experience is more informative than that of many other countries that use PR, because elsewhere, elections at different levels of government are often held at different times or have different rules for elligibility.

[27]In the examples of Section 2, a mix of defense and domestic spending may be more useful than fully funding one priority or the other.

[28]This substantially complicates the analysis, because, before voting himself, a voter who believes that policy needs to move in a particular direction has to first forecast whether the votes of his peers might push it so far in the desired direction that he actually now wants to pull it back.

[29]As McMurray (2017a) discusses, voters might also be subject to informational impediments, not modeled here, that lead them to persist in beliefs that they know to be unpopular.

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Online Appendix

Online Appendix to the paper: "The Marginal Voter's Curse" by H. Herrera, A. Llorente-Saguer, and J. McMurray, published at the *Economic Journal.* This version: November 2018. This file contains the proofs of the lemmas and propositions in the paper.

Proof of Proposition 1. We establish this proposition first for the case of pure marginal voting, $\theta = 1$, then for the case of pure pivotal voting, $\theta = 0$, and finally for the general case of $\theta \in (0, 1)$. In all three cases, the first step is to show that the best response σ^{br} to any voting strategy is a quality threshold strategy. If $\theta = 1$ then the policy outcome $x = \lambda_+$ is simply the vote share of the party with the superior policy position. Changes in utility (7) and (8) from an additional vote for the superior party and from an additional vote for the inferior party can then be written as follows,

$$\Delta_+ x(a, b) = \frac{a+1}{a+b+1} - \frac{a}{a+b} = \Delta\lambda_+ \tag{15}$$

$$\Delta_- x(a, b) = \frac{a}{a+b+1} - \frac{a}{a+b} = -\Delta\lambda_- \tag{16}$$

in terms of the increases $\Delta\lambda_+ = \frac{a+1}{a+b+1} - \frac{a}{a+b}$ and $\Delta\lambda_- = \frac{b+1}{a+b+1} - \frac{b}{a+b}$ in these vote shares that an additional correct vote or an additional incorrect vote cause, respectively.

Since $\Delta\lambda_+$ and $\Delta\lambda_-$ are both positive, (9) is increasing in q, and is positive for all q above the following threshold.

$$\tau^{br} = \frac{E_{a,b}(\Delta\lambda_-) - E_{a,b}(\Delta\lambda_+)}{E_{a,b}(\Delta\lambda_-) + E_{a,b}(\Delta\lambda_+)} \tag{17}$$

In other words, the best response to σ is a quality threshold strategy, with quality threshold given by (17): a voter votes if his expertise exceeds τ^{br}, and abstains otherwise. In particular, if his peers follow a quality threshold strategy with arbitrary quality threshold τ then a voter's best response is another quality threshold strategy, with quality threshold τ^{br}. Accordingly, (17) can be reinterpreted as an implicit function from the compact interval $[0, 1]$ of possible thresholds into itself. The continuity of (4) through (9) and $\Delta\lambda_+$ and $\Delta\lambda_-$ imply that $\tau^{br}(\tau)$ is continuous in τ, so a fixed point $\tau^* = \tau^{br}(\tau^*)$ exists by Brouwer's theorem, and characterizes a quality threshold strategy $\sigma^* = \sigma_{\tau^*}$ that is its own best response, thus constituting a Bayesian Nash equilibrium.

The denominator of (17) is positive, and the numerator can be rewritten as follows.

$$
\begin{aligned}
E_{a,b}\left(\Delta\lambda_{-}\right) - E_{a,b}\left(\Delta\lambda_{+}\right) &= E_{a,b}\left(\frac{b-a}{a+b+1} - \frac{b-a}{a+b}\right) \\
&= \sum_{b=0}^{\infty}\sum_{a=b}^{\infty}\left(\frac{b-a}{a+b+1} - \frac{b-a}{a+b}\right)\left[\Pr\left(a,b\right) - \Pr\left(b,a\right)\right]
\end{aligned}
$$

The term in parentheses is positive, and from (6), the difference in brackets can be written as follows.

$$
\begin{aligned}
\frac{e^{-n_{+}}n_{+}^{a}}{a!}\frac{e^{-n_{-}}n_{-}^{b}}{b!} - \frac{e^{-n_{+}}n_{+}^{b}}{b!}\frac{e^{-n_{-}}n_{-}^{a}}{a!} &= \frac{e^{-nv}n^{a+b}}{a!b!}\left(v_{+}^{a}v_{-}^{b} - v_{+}^{b}v_{-}^{a}\right) \\
&= \frac{e^{-nv}n^{a+b}}{a!b!}v_{+}^{b}v_{-}^{b}\left(v_{+}^{a-b} - v_{-}^{a-b}\right)
\end{aligned}
$$

For any strategy in which a positive fraction of the electorate votes, (4) and (5) make clear that $v_{+} > v_{-}$, implying that this expression and therefore the numerator of (17) are strictly positive, as well. That $\tau^{br} > 0$ for any best response implies that $\tau^{*} > 0$ in equilibrium, as well, as claimed.

For the case of $\theta = 0$, or pure pivotal voting, the policy outcome is a random variable x_{w} that equals 0 if $a < b$, 1 if $a > b$, and 0 or 1 with equal probability if $a = b$. A single vote for the superior party therefore increases that party's probability of winning by the following amount,

$$
\Pr\left(\mathcal{P}_{+}\right) = \frac{1}{2}\Pr\left(a = b\right) + \frac{1}{2}\Pr\left(a = b + 1\right) \tag{18}
$$

which is the standard probability of being *pivotal* (event $\mathcal{P}_{+}$). Similarly, the probability with which a vote for the inferior party is pivotal (event $\mathcal{P}_{-}$) is given by the following.

$$
\Pr\left(\mathcal{P}_{-}\right) = \frac{1}{2}\Pr\left(a = b\right) + \frac{1}{2}\Pr\left(b = a + 1\right) \tag{19}
$$

A pivotal vote for the party with the superior policy position increases utility from zero to one (a change of 1) and a pivotal vote for the inferior party decreases utility from one to zero (a change of -1). Outside of these pivotal events, a vote does not change the policy outcome, and so does not impact utility; accordingly, the expected benefit (9) of voting reduces to the following.

$$
\Delta Eu\left(q\right) = \frac{1}{2}\left(1 + q\right)\Pr\left(\mathcal{P}_{+}\right) - \frac{1}{2}\left(1 - q\right)\Pr\left(\mathcal{P}_{-}\right) \tag{20}
$$

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Since pivot probabilities are positive, (20) increases in q, and is positive if and only if q exceeds the following threshold.

$$\tau^{br} = \frac{\Pr(\mathcal{P}_-) - \Pr(\mathcal{P}_+)}{\Pr(\mathcal{P}_-) + \Pr(\mathcal{P}_+)} \tag{21}$$

In other words, the best response to σ is a quality threshold strategy, with quality threshold given by (21): a voter votes if his expertise exceeds τ^{br}, and abstains otherwise. In particular, if his peers follow a quality threshold strategy with arbitrary quality threshold τ then a voter's best response is another quality threshold strategy, with quality threshold τ^{br}. Accordingly, (17) can be reinterpreted as an implicit function from the compact interval $[0, 1]$ of possible thresholds into itself. The continuity of (4) through (9) and (18) through (20) imply that $\tau^{br}(\tau)$ is continuous in τ, so a fixed point $\tau^* = \tau^{br}(\tau^*)$ exists by Brouwer's theorem, and characterizes a quality threshold strategy $\sigma^* = \sigma_{\tau^*}$ that is its own best response, thus constituting a Bayesian Nash equilibrium.

The denominator of (21) is positive, and from (18) and (19), the numerator is proportional to the following.

$$\Pr(a = b+1) - \Pr(b = a+1) = \sum_{k=0}^{\infty} [\Pr(k+1, k) - \Pr(k, k+1)]$$

From (6), the bracketed expression can be rewritten as follows.

$$\frac{e^{-n_+} n_+^{k+1}}{(k+1)!} \frac{e^{-n_-} n_-^k}{k!} - \frac{e^{-n_+} n_+^k}{k!} \frac{e^{-n_-} n_-^{k+1}}{(k+1)!} = \frac{e^{-n_+ - n_-} n_+^k n_-^k}{k!(k+1)!} (n_+ - n_-)$$
$$= \frac{e^{-n_+ - n_-} n_+^k n_-^k}{k!(k+1)!} n (v_+ - v_-)$$

As noted above, $v_+ > v_-$ for any strategy in which a positive fraction of the electorate votes, so the final difference in parentheses is positive, implying that the entire expression is positive, as is the numerator of (21). That $\tau^{br} > 0$ for any best response implies that $\tau^* > 0$ in equilibrium, as well, as claimed.

If θ is strictly between 0 and 1 then the electoral rule is a hybrid of marginal voting and pivotal voting then the expected benefit of voting is merely the weighted average of the expected benefits above.

$$\Delta Eu(q) = \frac{1}{2}(1+q)[\theta(\Delta\lambda_+) + (1-\theta)\Pr(\mathcal{P}_+)] - \frac{1}{2}(1-q)[\theta(\Delta\lambda_-) + (1-\theta)\Pr(\mathcal{P}_-)]$$

As in the cases of $\theta = 0$ and $\theta = 1$, this difference increases in q, and is positive if

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and only if q exceeds the following threshold.

$$\tau^{br} = \frac{\theta\left(\Delta\lambda_- - \Delta\lambda_+\right) + (1-\theta)\left[\Pr\left(\mathcal{P}_-\right) - \Pr\left(\mathcal{P}_+\right)\right]}{\theta\left(\Delta\lambda_+ + \Delta\lambda_-\right) + (1-\theta)\left[\Pr\left(\mathcal{P}_+\right) + \Pr\left(\mathcal{P}_-\right)\right]} \tag{22}$$

In other words, the best response to σ is again a quality threshold strategy, this time with quality threshold given by (22). In particular, the best response to a quality threshold strategy with arbitrary quality threshold τ is another quality threshold strategy, with quality threshold τ^{br}, so (17) can be reinterpreted as an implicit function from the compact interval $[0,1]$ of possible thresholds into itself. The continuity of (4) through (20) (and of $\Delta\lambda_+$ and $\Delta\lambda_-$) imply that $\tau^{br}(\tau)$ is continuous in τ, so a fixed point $\tau^* = \tau^{br}(\tau^*)$ exists by Brouwer's theorem, and characterizes a quality threshold strategy $\sigma^* = \sigma_{\tau^*}$ that is its own best response, thus constituting a Bayesian Nash equilibrium. Clearly, (22) lies between (17) and (21), which are both positive for any τ. This implies that $\tau^{br}(\tau)$ is positive, and therefore that the fixed point τ^* is positive, as well. ∎

Lemma 1 *If voting follows a quality threshold strategy σ_τ with quality threshold $\tau < 1$ then the following hold for any n.*

$$E_{a,b}\left(\Delta\lambda_+\right) = \frac{v_-}{nv^2} + \frac{n\left(v_+^2 - v_-^2\right) - 2v_-}{2nv^2}e^{-nv} \tag{23}$$

$$E_{a,b}\left(\Delta\lambda_-\right) = \frac{v_+}{nv^2} + \frac{n\left(v_-^2 - v_+^2\right) - 2v_+}{2nv^2}e^{-nv} \tag{24}$$

Proof of Lemma 1. The expected vote share of the superior party can be written as follows,

$$
\begin{aligned}
E_{a,b}\left[\lambda_+(a,b)\right] &= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty} e^{-nv}\frac{n_+^a\, n_-^b}{a!\, b!}\lambda_+(a,b) \\
&= \frac{1}{2}e^{-nv} + e^{-nv}\sum_{a=1}^{\infty}\sum_{b=0}^{\infty}\frac{n_+^a\, n_-^b}{a!\, b!}\left(\frac{a}{a+b}\right) \\
&= \frac{1}{2}e^{-nv} + e^{-nv}\sum_{a=1}^{\infty}\left[\frac{1}{(a-1)!}\frac{n_+^a}{n_-^a}\sum_{b=0}^{\infty}\frac{n_-^{a+b}}{b!\,(a+b)}\right]
\end{aligned}
$$

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where the second equality follows because $\lambda_+ (0,0) = \frac{1}{2}$. Differentiating and integrating the inner summand as follows,

$$
\begin{aligned}
\sum_{b=0}^{\infty} \frac{n_-^{a+b}}{b! \, (a+b)} &= \sum_{b=0}^{\infty} \int_0^{n_-} \frac{d}{dt} \left(\frac{t^{a+b}}{b!} \frac{1}{a+b} \right) dt \\
&= \int_0^{n_-} \sum_{b=0}^{\infty} \left(\frac{t^{a+b-1}}{b!} \right) dt \\
&= \int_0^{n_-} t^{a-1} e^t \, dt
\end{aligned}
$$

this reduces further to the following.

$$
\begin{aligned}
E_{a,b} \left[\lambda_+ (a,b) \right] &= \frac{1}{2} e^{-nv} + e^{-nv} \frac{n_+}{n_-} \int_0^{n_-} \sum_{a=1}^{\infty} \frac{\left(\frac{n_+}{n_-} t \right)^{a-1}}{(a-1)!} e^t \, dt \\
&= \frac{1}{2} e^{-nv} + e^{-nv} \frac{v_+}{v_-} \int_0^{n_-} e^{\left(\frac{v_+}{v_-} t \right)} e^t \, dt \\
&= \frac{1}{2} e^{-nv} + e^{-nv} \frac{v_+}{v_-} \int_0^{n_-} e^{\frac{v}{v_-} t} \, dt \\
&= \frac{1}{2} e^{-nv} + e^{-nv} \frac{v_+}{v} \left(e^{nv} - 1 \right) \\
&= \frac{v_+}{v} + \frac{v_- - v_+}{2v} e^{-nv} \qquad\qquad (25)
\end{aligned}
$$

If a citizen votes for the party with the superior platform, this increases the expected vote share to the following.

$$
\begin{aligned}
E_{a,b} \left[\lambda_+ (a+1,b) \right] &= \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} e^{-nv} \left(\frac{n_+^a}{a!} \right) \left(\frac{n_-^b}{b!} \right) \frac{a+1}{a+b+1} \\
&= e^{-nv} \sum_{a=0}^{\infty} \left[\frac{a+1}{a!} \frac{n_+^a}{n_-^{a+1}} \sum_{b=0}^{\infty} \frac{n_-^{a+b+1}}{b! \, (a+b+1)} \right]
\end{aligned}
$$

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Differentiating and integrating as before, this reduces further as follows.

$$
= \; e^{-nv} \sum_{a=0}^{\infty} \frac{a+1}{a!} \frac{n_+^a}{n_-^{a+1}} \int_0^{n_-} t^a e^t \, dt
$$

$$
= \; e^{-nv} \int_0^{n_-} \sum_{a=0}^{\infty} \left(\frac{a}{a!} \frac{n_+^a}{n_-^{a+1}} t^a + \frac{1}{a!} \frac{n_+^a}{n_-^{a+1}} t^a \right) e^t \, dt.
$$

$$
= \; e^{-nv} \int_0^{n_-} \left[\frac{n_+}{n_-^2} t \sum_{a=1}^{\infty} \frac{\left(\frac{n_+}{n_-} t \right)^{a-1}}{(a-1)!} + \frac{1}{n_-} \sum_{a=0}^{\infty} \frac{\left(\frac{n_+}{n_-} t \right)^a}{a!} \right] e^t \, dt
$$

$$
= \; e^{-nv} \int_0^{n_-} \left[\frac{n_+}{n_-^2} t e^{\left(\frac{n_+}{n_-} t \right)} + \frac{1}{n_-} e^{\left(\frac{n_+}{n_-} t \right)} \right] e^t \, dt
$$

$$
= \; \frac{n_+}{n_-^2} e^{-nv} \int_0^{n_-} t e^{\frac{v}{v_-} t} \, dt + \frac{1}{n_-} e^{-nv} \int_0^{n_-} e^{\frac{v}{v_-} t} \, dt
$$

Integrating by parts, this reduces to the following.

$$
\frac{n_+}{n_-^2} e^{-nv} \left(\frac{v_-}{v} e^{nv} - 0 - \frac{v_-}{v} \int_0^{n_-} e^{\frac{v}{v_-} t} \right) + \frac{1}{nv} e^{-nv} \left(e^{nv} - 1 \right)
$$

$$
= \; \frac{v_+}{nv_-^2} e^{-nv} \left[\frac{nv_-^2 e^{nv}}{v} - \frac{v_-^2}{v^2} \left(e^{nv} - 1 \right) \right] + \frac{1}{nv} \left(1 - e^{-nv} \right)
$$

$$
= \; \left(\frac{v_+}{v} - \frac{v_+}{nv^2} + \frac{1}{nv} \right) + \left(\frac{v_+}{nv^2} - \frac{1}{nv} \right) e^{-nv}
$$

$$
= \; \frac{nvv_+ + v_-}{nv^2} - \frac{v_-}{nv^2} e^{-nv} \tag{26}
$$

The difference $E_{a,b}(\Delta\lambda_+)$ between (25) and (26) is then given by (23). A symmetric derivation shows $E_{a,b}(\Delta\lambda_-)$ to be equal to (24). ■

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Proof of Proposition 2. For a quality threshold strategy with arbitrary quality threshold $\tau \in [0, 1]$, (4) and (5) reduces so that expected vote shares v_+ and v_- and turnout rate v can be written as follows,

$$
\begin{aligned}
v_+ (\tau) &= p + I \int_\tau^1 \frac{1}{2} (1 + q) \, dF(q) \\
&= p + \frac{1}{2} I [1 - F(\tau)] [1 + m(\tau)] \qquad (27) \\
v_- (\tau) &= p + I \int_\tau^1 \frac{1}{2} (1 - q) \, dF(q) \\
&= p + \frac{1}{2} I [1 - F(\tau)] [1 - m(\tau)] \qquad (28) \\
v (\tau) &= 2p + I [1 - F(\tau)]
\end{aligned}
$$

where $m(\tau) = E(q|q > \tau)$ denotes the average expertise among citizens who actually vote. Using these, the *likelihood ratio* $\rho(\tau)$ of a correct vote to an incorrect vote can then be written as follows,

$$
\rho(\tau) = \frac{v_+ (\tau)}{v_- (\tau)} = \frac{K + [1 - F(\tau)] [1 + m(\tau)]}{K + [1 - F(\tau)] [1 - m(\tau)]} \qquad (29)
$$

in terms of the ratio $K = \frac{2p}{I}$ of partisan to independent voters.

Given these preliminaries, we establish the proposition as in Proposition 1, first for the case of pure marginal voting, $\theta = 1$, then for the case of pure pivotal voting, $\theta = 0$, and finally for the intermediate cases $\theta \in (0, 1)$. For $\theta = 1$, the proof of Proposition 1 shows that the best response to a quality threshold strategy with quality threshold τ is another quality threshold strategy, with quality threshold given by (17). Equivalently, τ^{br} solves the following,

$$
\frac{1 + \tau}{1 - \tau} = \frac{E_{a,b} (\Delta \lambda_-)}{E_{a,b} (\Delta \lambda_+)} = \frac{2v_+ + [n (v_+^2 - v_-^2) - 2v_-] e^{-nv}}{2v_- + [n (v_-^2 - v_+^2) - 2v_+] e^{-nv}} \qquad (30)
$$

where the arguments of v_+, v_-, and v are supressed for brevity, and where the second equality follows from technical Lemma 1.[30] As n grows large, the right-hand side of this expression converges simply to (29). Since (30) is continuous both in τ and in n, the limit τ_∞^* of any sequence (τ_n^*) of solutions to (30) must therefore solve the

---

[30]Lemma 1 only applies if $\tau < 1$; if $\tau = 1$ then $E_{a,b} (\Delta \lambda_+) = \lambda_+ (0, 0) = \frac{1}{2}$ and $E_{a,b} (\Delta \lambda_-) = \lambda_- (0, 0) = \frac{1}{2}$, so $\frac{E_{a,b}(\Delta \lambda_-)}{E_{a,b}(\Delta \lambda_+)} = 1$ and $\tau^{br} (\tau) = 0$ for any n.

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following, simpler equation.

$$\rho(\tau) = \frac{1+\tau}{1-\tau} \tag{31}$$

From (29) it is clear that the left-hand side of (31) exceeds the right-hand side at $\tau = 0$, implying that $\tau^*_\infty > 0$ as claimed in (1).

As τ increases from zero to one, the right-hand side of (31) increases from one to infinity. For any p, differentiating (27) and (28) with respect to τ yields

$$v'_+ = -\frac{I}{2}(1+\tau)f(\tau)$$

$$v'_- = -\frac{I}{2}(1-\tau)f(\tau)$$

and differentiating (29) therefore yields

$$\rho'(\tau) = \frac{v'_+(\tau)v_-(\tau) - v_+(\tau)v'_-(\tau)}{v_-(\tau)^2} = \frac{I}{2}f(\tau)\frac{-(1+\tau)v_-(\tau)+(1-\tau)v_+(\tau)}{v_-(\tau)^2}$$

which is positive if and only if the left-hand side of (31) exceeds the right-hand side. In other words, any solution to (31) uniquely maximizes $\rho(\tau)$ (and therefore $\frac{\rho(\tau)}{1+\rho(\tau)} = \frac{v_+(\tau)}{v_+(\tau)+v_-(\tau)} = \lambda_+(\tau)$). There is exactly one such solution. If $p = K = 0$ then (29) reduces to $\rho(\tau) = \frac{1+m(\tau)}{1-m(\tau)}$ and the solution to (31) requires $m(\tau) = \tau$, which is uniquely satisfied for $\tau^*_\infty = 1$. If $p > 0$ then (29) exceeds the right-hand side of (31) for $\tau = 0$ but not for $\tau = 1$, so the solution τ^*_∞ is strictly between 0 and 1. This establishes claim (2). (29) also decreases in K (and therefore in p) for all τ, implying that the solution τ^*_∞ to (31) strictly decreases in K (and therefore in p), as claimed in (3).

Claim (4) follows because $\rho(\tau)$ increases with F, as shown below. Since the right-hand side of (31) increases in τ, an increase in the left-hand side for all τ yields a higher solution τ^*_∞ than before. In other words, τ^*_∞ increases in F. To see that $\rho(\tau)$ increases with F, first rewrite (29) as follows,

$$\rho(\tau) = \frac{K + \int_\tau^1 (1+q)f(q)\,dq}{K + \int_\tau^1 (1-q)f(q)\,dq} = \frac{\int_0^1 \Gamma^+_\tau(q)f(q)\,dq}{\int_0^1 \Gamma^-_\tau(q)f(q)\,dq} \tag{32}$$

in terms of $\Gamma^+_\tau(q) = K + 1_{q \geq \tau}(1+q)$ and $\Gamma^-_\tau(q) = K + 1_{q \geq \tau}(1-q)$, where $1_{q \geq \tau}$ is the indicator function that equals one if $q \geq \tau$ and zero otherwise. For any K, $\Gamma^+_\tau(q)$ and $\Gamma^-_\tau(q)$ are non-negative and respectively increasing and decreasing functions of q. Written this way, it can be shown that ρ increases in F. To see this, write (32) for distribution functions $F <_{MLRP} G$ as ρ_F and ρ_G. The difference $\rho_G - \rho_F$ can

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then be written as follows,

$$\frac{\int_0^1 \Gamma_\tau^+(q) \, g(q) \, dq}{\int_0^1 \Gamma_\tau^-(q) \, g(q) \, dq} > \frac{\int_0^1 \Gamma_\tau^+(q) \, f(q) \, dq}{\int_0^1 \Gamma_\tau^-(q) \, f(q) \, dq}$$

which is proportional to the following,

$$\int_0^1 \int_0^1 \Gamma_\tau^+(q) \, \Gamma_\tau^-(q') \, f(q') \, g(q) \, dq dq' - \int_0^1 \int_0^1 \Gamma_\tau^+(q) \, \Gamma_\tau^-(q') \, f(q) \, g(q') \, dq dq'$$

$$= \int_0^1 \int_0^1 \Gamma_\tau^+(q) \, \Gamma_\tau^-(q') \left[f(q) \, g(q') - f(q') \, g(q) \right] dq dq'$$

$$= \iint_{q > \tau} \Gamma_\tau^+(q) \, \Gamma_\tau^-(\tilde{q}) \left[f(\tilde{q}) \, g(q) - f(q) \, g(\tilde{q}) \right] dq d\tilde{q}$$

$$+ \iint_{\tau > q} \Gamma_\tau^+(q) \, \Gamma_\tau^-(\tilde{q}) \left[f(\tilde{q}) \, g(q) - f(q) \, g(\tilde{q}) \right] dq d\tilde{q}$$

$$= \iint_{q > \tau} \left[\Gamma_\tau^+(q) \, \Gamma_\tau^-(\tilde{q}) - \Gamma_\tau^+(\tilde{q}) \, \Gamma_\tau^-(q) \right] \left[f(\tilde{q}) \, g(q) - f(q) \, g(\tilde{q}) \right] dq d\tilde{q}$$

$$= \iint_{q > \tau} \Gamma_\tau^-(q) \, \Gamma_\tau^-(\tilde{q}) \left[\frac{\Gamma_\tau^+(q)}{\Gamma_\tau^-(q)} - \frac{\Gamma_\tau^+(\tilde{q})}{\Gamma_\tau^-(\tilde{q})} \right] f(q) \, f(\tilde{q}) \left[\frac{g(q)}{f(q)} - \frac{g(\tilde{q})}{f(\tilde{q})} \right] dq d\tilde{q}$$

where the third equality follows from reversing the labels of q and $\tilde{q}$ in the second double integral. Since Γ_τ^+ and $\frac{g(q)}{f(q)}$ both increase in q and since Γ_τ^- decreases in q, this expression is positive, implying that $\rho_G > \rho_F$, as claimed.

Having established claims (1) through (4) for the case of $\theta = 1$, we turn now to the case of $\theta = 0$, following the arguments of Theorem 4 of McMurray (2013), generalized to accommodate $p > 0$. Myerson (2000) provides a useful preliminary result, which is that pivot probabilities can be written as follows (again suppressing the argument of v_+ of v_-), where $h_1(n)$ and $h_2(n)$ both approach one as n grows large.

$$\Pr(\mathcal{P}_+) = \frac{e^{-n\left(\sqrt{v_+} - \sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+ v_-}}} \frac{\sqrt{v_+} + \sqrt{v_-}}{\sqrt{v_+}} h_1(n) \tag{33}$$

$$\Pr(\mathcal{P}_-) = \frac{e^{-n\left(\sqrt{v_+} - \sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+ v_-}}} \frac{\sqrt{v_+} + \sqrt{v_-}}{\sqrt{v_-}} h_2(n) \tag{34}$$

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Using these formulas (and since $v_+(\tau) \neq v_-(\tau)$ for any quality threshold strategy), the best-response condition (21) converges to the following as n grows large.

$$\tau = \frac{\sqrt{v_+(\tau)} - \sqrt{v_-(\tau)}}{\sqrt{v_+(\tau)} + \sqrt{v_-(\tau)}}$$

Like (17), this can be written in terms of the likelihood ratio $\rho(\tau)$, as follows.

$$\rho(\tau) = \frac{v_+(\tau)}{v_-(\tau)} = \left(\frac{1+\tau}{1-\tau}\right)^2 \tag{35}$$

The limit τ_∞^* of any sequence τ_n^* of equilibrium thresholds must be a solution to this equation.

As before, the limiting equilibrium condition (35) can be rewritten using (29), and solved for K, as follows.

$$\frac{1 - F(\tau)}{\tau}\left[\frac{(1+\tau^2)}{2}m(\tau) - \tau\right] = K \tag{36}$$

As we show below, the left-hand side of (36) starts at $+\infty$ for $\tau = 0$ and decreases with τ until it reaches its minimum at some negative level for a unique minimizer $\underline{\tau} \in (0,1)$, then increases again to zero for $\tau = 1$. That the left-hand side is strictly decreasing when positive implies that (36) has a unique solution τ_∞^* for any $K \geq 0$, which is hence decreasing in K and therefore in p. Moreover, for $p = 0 = K$ it satisfies $0 < \tau_\infty^* < \underline{\tau} < 1$. This establishes claims (1) through (3). Claim (4) follows just as in the case of pure marginal voting: the right-hand side of (35) increases with τ, and an increase in F raises the left-hand side, thereby increasing the solution τ_∞^*.

To see that the left-hand side of (36) decreases on $[0, \underline{\tau}]$ for some $\underline{\tau} < 1$, differentiate it to yield the following.

$$\frac{1 - \tau^2}{2}[1 - F(\tau)]\left(\frac{f(\tau)}{1 - F(\tau)} - \frac{m(\tau)}{\tau^2}\right) \tag{37}$$

To obtain the above we differentiated $m(\tau) = \frac{1}{1-F(\tau)}\int_\tau^1 qf(q)\,dq$ using the Leibnitz formula, as follows.

$$m'(\tau) = \frac{-[1 - F(\tau)]\tau f(\tau) + f(\tau)\int_\tau^1 qf(q)\,dq}{[1 - F(\tau)]^2} = \frac{f(\tau)}{1 - F(\tau)}[m(\tau) - \tau]$$

It is clear from the definition of $m(\tau) = E(q|q \geq \tau)$ that it always exceeds τ. Since $f(\tau)$ is log-concave, Bagnoli and Bergstrom (2005) show further that $m'(\tau) < 1$

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and that $\frac{f(\tau)}{1-F(\tau)}$ increases in τ. This former implies that $\frac{m(\tau)}{\tau}$ decreases in τ, the latter implies that the difference in parentheses in (37) increases in τ, and is therefore negative and positive to the left and the right of some unique minimizer $\underline{\tau} \in (0,1)$, respectively.

Proposition 3 states below that τ_∞^* is the same for any $\theta \in (0,1]$. Thus, the proofs of claims (1) through (4) for the case of $\theta = 1$ apply to intermediate cases $\theta \in (0,1)$, as well. ∎

Proof of Proposition 3. Equation (22) gives the equilibrium condition for a general model with arbitrary $\theta \in [0,1]$ and finite n. According to Lemma 1, the marginal changes $\Delta\lambda_+$ and $\Delta\lambda_-$ in policy associated with additional votes for the superior and inferior parties, respectively, can be rewritten as (23) and (24). According to Myerson (2000), pivot probabilities $\Pr(\mathcal{P}_+)$ and $\Pr(\mathcal{P}_-)$ can be written as (33) and (34). (22) can thus be written as follows (once again suppressing the argument of v_+ of v_-).

$$\frac{1+\tau}{1-\tau} = \frac{\theta\left[\frac{v_+}{nv^2} + \frac{n\left(v_-^2 - v_+^2\right) - 2v_+}{2nv^2}e^{-nv}\right] + (1-\theta)\left[\frac{e^{-n\left(\sqrt{v_+} - \sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+ v_-}}}\frac{\sqrt{v_+}+\sqrt{v_-}}{\sqrt{v_-}}h_1(n)\right]}{\theta\left[\frac{v_-}{nv^2} + \frac{n\left(v_+^2 - v_-^2\right) - 2v_-}{2nv^2}e^{-nv}\right] + (1-\theta)\left[\frac{e^{-n\left(\sqrt{v_+} - \sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+ v_-}}}\frac{\sqrt{v_+}+\sqrt{v_-}}{\sqrt{v_+}}h_2(n)\right]} \quad (38)$$

For any $\theta > 0$, exponential terms vanish more quickly than other terms, so this expression converges simply to (31) in the limit as n grows large. This establishes the equality in the proposition.

For $\theta = 0$, equation (38) instead converges to (35), which is equivalent to (11), while (31) is equivalent to (10). As the proof Proposition 2 shows, the latter has a unique solution $\tau_\infty^*(p,1,F)$, which also maximizes its left-hand side. If f is not log-concave, Equation (11) may have multiple solutions. However, the right-hand sides of (10) and (11) are the same, and the left-hand side of (11) is no greater than the left-hand side of (10). For any $\tau > \tau_\infty^*(p,1,F)$, therefore, the left-hand side of (11) is smaller than the left-hand side of (10), which is smaller than the right-hand side of either equation, and is therefore not a solution to (11). ∎

Proof of Proposition 4. Section 2 equates utility $u(x) = x$ with the policy outcome, which for generic θ is given in (1). Expected utility can therefore be written as follows, in terms of the numbers a and b of votes for the superior and inferior party.

$$E[u(x)] = \theta\frac{a}{a+b} + (1-\theta)\left[\Pr(a > b) + \frac{1}{2}\Pr(a = b)\right] \quad (39)$$

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For any θ, Proposition 1 characterizes equilibrium in finite elections as a quality threshold strategy, with expected vote shares given by (27) and (28). The realized numbers a and b of votes for the superior and inferior parties are independent Poisson random variables, with respective means $nv_+(\tau)$ and $nv_-(\tau)$.

For any n, Proposition 1 characterizes equilibrium voting as a quality threshold strategy, with quality threshold τ_n^*. If $\theta = 0$ then any limit point τ_∞^* of $\{\tau_n^*\}$ must solve Equation (11). As n grows large, the ratio $\frac{a}{b}$ of actual votes for either party approaches in probability the ratio $\frac{v_+(\tau_\infty^*)}{v_-(\tau_\infty^*)} = \rho(\tau_\infty^*)$ of expected vote shares. Since $\rho(\tau) > 1$ for any τ, the probability that $\frac{a}{b} > 1$ approaches one, implying that welfare approaches $u_\infty^* = 1$. If $\theta > 0$ then $\{\tau_n^*\}$ has a unique limit point, which must solve Equation (10). In that case, $\frac{a}{a+b}$ converges in probability to $u_\infty^* = \frac{\rho(\tau_\infty^*)}{1+\rho(\tau_\infty^*)}$, which increases with $\rho(\tau_\infty^*)$. The proof of Proposition 2 shows that $\rho(\tau_\infty^*)$ increases with F and decreases with p, implying that u_∞^* increases with F and decreases with p, as well. If $p = K = 0$ then (29) reduces to $\rho(\tau) = \frac{1+m(\tau)}{1-m(\tau)}$, so a solution to (31) requires $m(\tau) = \tau$, which is true if and only if $\tau_\infty^* = 1$. If $p = \frac{1}{2}$ (so that $I = 0$) then (27) and (28) reduce to $v_+(\tau) = v_-(\tau) = p$ for any τ, so (29) reduces to $\rho(\tau) = 1$ and the unique solution to (31) is $\tau_\infty^* = 0$, implying that welfare approaches $u_\infty^* = \frac{\rho(\tau_\infty^*)}{1+\rho(\tau_\infty^*)} = \frac{1}{2}$, as claimed. ∎

Proof of Proposition 5. With a generalized policy function, the expected benefit of voting (9) can be rewritten as follows for the case of $\theta = 1$,

$$\Delta Eu(q) = E_{a,b}\left[\frac{1}{2}(1+q)\Delta_+\psi(a,b) + \frac{1}{2}(1-q)\Delta_-\psi(a,b)\right]$$

in terms of the difference in policy $\Delta_+\psi(a,b) = \psi(a+1,b) - \psi(a,b)$ induced by one additional vote for the superior party A and the policy difference $\Delta_-\psi(a,b) = \psi(a,b+1) - \psi(a,b)$ induced by one additional vote for party B. By monotonicity, these differences are positive and negative, respectively. Given the symmetry condition, the latter difference can also be written as follows,

$$\begin{aligned}
\Delta_-\psi(a,b) &= [1 - \psi(b+1,a)] - [1 - \psi(b,a)] \\
&= \psi(b,a) - \psi(b+1,a) \\
&= -\Delta_+\psi(b,a)
\end{aligned}$$

in terms of the positive impact of a correct vote when there are b votes for party A and a votes for party B.

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Since $\Delta_+\psi(a,b)$ and $\Delta_+\psi(b,a)$ are both positive (by monotonicity), (9) is positive if and only if q exceeds the following threshold,

$$\tau^{br} = \frac{E_{a,b}[\Delta_+\psi(b,a)] - E_{a,b}[\Delta_+\psi(a,b)]}{E_{a,b}[\Delta_+\psi(a,b)] + E_{a,b}[\Delta_+\psi(b,a)]} \tag{40}$$

which generalizes (17). In particular, the best response to a quality threshold strategy with quality threshold τ is another quality threshold strategy, with quality threshold τ^{br}. Since $\Pr(a,b)$ is continuous in τ, so is $\Delta Eu(q)$, and therefore $\tau^{br}(\tau)$, even if $\psi(a,b)$ is not continuous in a and b. A fixed point $\tau^* = \tau^{br}(\tau^*)$ therefore exists by Brouwer's theorem, which defines a quality threshold strategy that is its own best response. This establishes equilibrium existence.

The denominator of (40) is positive, and the numerator reduces as follows.

$$\sum_{a,b} [\Delta_+\psi(b,a) - \Delta_+\psi(a,b)] \frac{e^{-n(v_+ - v_-)} n^{a+b} v_+^a v_-^b}{a! b!}$$

$$= \sum_{a>b} [\Delta_+\psi(b,a) - \Delta_+\psi(a,b)] \frac{e^{-n(v_+ - v_-)} n^{a+b} v_+^a v_-^b}{a! b!}$$

$$+ \sum_{a<b} [\Delta_+\psi(b,a) - \Delta_+\psi(a,b)] \frac{e^{-n(v_+ - v_-)} n^{a+b} v_+^a v_-^b}{a! b!}$$

Relabeling variables in the second summation yields the following.

$$\sum_{a>b} [\Delta_+\psi(b,a) - \Delta_+\psi(a,b)] \frac{e^{-n(v_+ - v_-)} n^{a+b} v_+^a v_-^b}{a! b!}$$

$$+ \sum_{b<a} [\Delta_+\psi(a,b) - \Delta_+\psi(b,a)] \frac{e^{-n(v_+ - v_-)} n^{a+b} v_+^b v_-^a}{a! b!}$$

$$= \sum_{a>b} [\Delta_+\psi(b,a) - \Delta_+\psi(a,b)] \frac{e^{-n(v_+ - v_-)} n^{a+b} v_+^b v_-^b}{a! b!} \left(v_+^{a-b} - v_-^{a-b} \right)$$

Monotonicity and the underdog property together imply that $\Delta_+\psi(a,b) < \Delta_+\psi(b,a)$ if and only if $a > b$. This, together with the fact that $n_+ = nv_+ > nv_- = n_-$, implies that the above expression is strictly positive, and therefore that τ^{br} is positive. That the best-response threshold is strictly positive implies that any equilibrium threshold is positive as well, and a positive fraction of the electorate therefore prefer to abstain in equilibrium. ∎

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