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Research Article

New Results on Stability Analysis of Uncertain Neutral-Type Lur'e Systems Derived from a Modified Lyapunov-Krasovskii Functional

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This paper is concerned with the problem of the absolute and robustly absolute stability for the uncertain neutral-type Lur'e system with time-varying delays. By introducing a modified Lyapunov-Krasovskii functional (LKF) related to a delay-product-type function and two delay-dependent matrices, some new delay-dependent robustly absolute stability criteria are proposed, which can be expressed as convex linear matrix inequality (LMI) framework. The criteria proposed in this paper are less conservative than some recent previous ones. Finally, some numerical examples are presented to show the effectiveness of the proposed approach.

1. Introduction

In many real systems, time delay is often considered as the main cause of poor performance and even instability. The stability of time-delay systems is always a hot topic for researchers. As a result, to obtain stability criteria of time-delayed systems by using the Lyapunov theorem, the main efforts are concentrated on the following several directions; one is finding an appropriate positive definite functional with a negative definite time derivative along the trajectory of system, for example, LKF with delay partitioning approach [1, 2], LKF with augmented terms [3], LKF with triple-integral and quadruple-integral terms [4, 5], and so on. The other is reducing the upper bounds of the time derivative of LKF as much as possible by developing various inequality techniques, such as Jensen inequality [6], Wirtinger-based inequality [7], auxiliary function based inequality [8], and Bessel-Legendre inequality [9]. Besides, further to increase the freedom of solving LMIs, there are some other methods, for instance, the generalized zero equality [10, 11], the one- or second-order reciprocally convex combinations [12–15], the free-weighting-matrix approach [16], and so on.

In practical engineering applications, most systems are nonlinear. As is known to all, Lur'e system, which is composed of the feedback connection of the linear dynamical system and the nonlinearity satisfying the sector-bounded condition, can represent many deterministic nonlinear systems, for example, Chua's Circuit and the Lorenz system [17]. Therefore, the study on the stability of Lur 'e systems becomes more and more popular [18–21]. Moreover, the paper [22] pointed out that many practical systems can be modeled as neutral time-delayed systems, in which not only the system states or outputs contain time delays, but also the derivative of the system states. Due to the theoretical and practical significance, the analysis of the robust stability of the time-delayed neutral-type Lur'e systems has attached great importance by many scholars [23–29], where many important robust stability criteria were given. However, the main improvement of stability criteria depends on the development of LKF and the update of inequality techniques based on linear systems. For example, recently, [29] improved the stability results of some previous ones by combining the extended double integral with Wirtinger-based inequalities technique; however, the range of delay with nonzero lower bound and the lower bound of the delay derivative are not involved; in [30], some less conservative stability criteria than some recent previous ones were derived for time-delayed Lur'e system via the second-order Bessel-Legendre inequality approach, a novel inequality technique; in [21], some improved stability criteria for time-delayed neutral-type Lur'e system were given by constructing a novel LKF consisting of a quadratic term and integral terms for the time-varying delays and the nonlinearities, and so on. Recently, C. Zhang [31] considered the effect of the LKFs while discussing the relationship between the tightness of inequalities and the conservatism of criteria for linear systems. The results illustrate the integral inequality that makes the upper bound closer to the true value does not always deduce a less conservative stability condition if the LKF is not properly constructed. Particularly, another novel LKF was proposed by C. Zhang et al. [31, 32] with delay-product-type terms $h(t)P_1$ and $(h-h(t))P_2$. Compared with the general LKF, P_1 and P_2 were just symmetrical, not always positive definite, which can lead to a less conservative stability condition by extending the freedom for checking the feasibility of stable conditions based on LMI. Recently, to fully utilize the information of delay derivative, a new LKF was constructed by W. Kwon et al. [33] with delay-dependent Lyapunov matrices $Q_1(t)$ and $Q_2(t)$. W. Kwon et al. point that the stability conditions based on an LKF with delay-dependent matrices are less conservative than those based on the LKF without delay-dependent matrices. As mentioned above, the two types of LKFs only improve one class of Lyapunov matrices, respectively, that is, only for the Lyapunov matrix P or the Lyapunov matrix Q. It is natural to wonder about whether can both classes of Lyapunov matrices be improved, simultaneously.

Inspired by the above analysis, the following ideas of reducing the conservation of the previous proposed stability criteria should be addressed:

(i) A modified LKF with the above both classes of Lyapunov matrices, that is delay-product-type and delay-dependent matrices, is constructed. Compared with the general LKFs in some previous published papers, such as [21, 28, 30], the Lyapunov matrices of the nonintegral item are just symmetrical, not always positive definite, which can extend the freedom for checking the feasibility of stable conditions based on LMI. And the delay-dependent matrices of the single-integral items are utilized, which can also further improve the utilization of time delay and its derivative information. In addition, the results proposed by [31–33] can be improved via the LKF modified in this paper due to the combination of the two types of LKFs.

(ii) The double integral items of the modified LKF in this paper are decomposed into two subintervals, that is $[0,h(t)]$ and $[h(t),h]$, instead of being considered directly in [33], which further make full use of the information of time-varying delays $h(t)$, $h-h(t)$ and their derivative $\dot{h}(t)$. And the quadratic generalized free-weighting matrix inequality (QGFMI) technique can be used fully in each subinterval, which can further reduce the conservatism of the stability conditions.

(iii) To deal with the delay-derivative-dependent single-integral items feasibly, another double integral items of $V_4(t)$ are also added to the LKF under the above two subintervals, instead of introducing a positive integral item, which is actually difficult to estimate, to the derivative of the LKF like [33].

(iv) Indeed, the main result of [33] was not LMI due to the terms with $h^2(t)$ even $h^3(t)$. The matrix inequalities of the stability criteria proposed in this paper are converted to LMIs via the properties of quadratic functions application, which can be solved easily by Matlab LMI-toolbox. In conclusion, it is interesting and still challenging problem to address the above issues, which offers motivation to derive less conservative stability criteria for the time-delayed neutral-type Lur'e systems.

This paper mainly analyzes and studies the stability of uncertain neutral-type Lur'e systems with mixed time-varying delays. Some less conservative delay-dependent absolute stability criteria and robust absolute stability criteria than some previous ones are derived via a modified LKF application. In the end, four popular numerical examples are given to illustrate that this method improves some existing methods and achieves good results in stability. The structure of this paper is as follows: *Section 1* describes the research background and research topic status and defines the scope of the study of this article; *Section 2* describes the main research questions, including some necessary definitions, assumptions, and lemmas; *Section 3* presents the main results, including theorems and corollaries; in *Section 4* the discussions and simulations based on numerical examples are given; *Section 5* summarizes the whole thesis.

Notation. $P>0$ (<0) represents a positive (negative) definite matrix. I and 0 represent an identity matrix and a zero matrix with the corresponding dimensions, respectively. $*$ denotes the symmetric terms in a block matrix and $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix. e_i $(i=1,\ldots,m)$ are block entry matrices with $e_2^T=[0\ \ I\ \ \underbrace{0\cdots 0}_{m-2}]$, where m is the dimension of the vector ξ. $F_{[\alpha(t),\beta(t)]}$ denotes F is the function of $\alpha(t)$ and $\beta(t)$. $\mathrm{sym}\{B\}=B+B^T$.

2. Problem Formulation

Consider the following neutral-type Lur'e system with mixed time-varying delays:

$$\begin{aligned}\dot{x}(t)-C\dot{x}(t-\tau(t))=&\,[A+\Delta A(t)]\,x(t)\\&+[A_1+\Delta A_1(t)]\,x(t-h(t))\\&+[B+\Delta B(t)]\,f(\sigma(t)),\end{aligned}$$

$$\begin{aligned}
&\sigma(t) = H^T x(t), \quad \forall t \geq 0, \\
&x(s) = \varphi(s), \\
&\dot{x}(s) = \dot{\varphi}(s), \\
&\qquad s \in [-\max(h_2, \tau), 0], \ h(t) \in \mathbf{C.1} \\
&\qquad s \in [-\max(h, \tau), 0], \ h(t) \in \mathbf{C.2},
\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $\sigma(t) \in \mathbb{R}^m$ are the state and output vectors of the system, respectively. A, A_1, B, C, and H are real constant matrices with appropriate dimensions; $\varphi(s)$ is an $\mathbb{R}^n$-valued continuous initial functional specified on $[-\max(h,\tau), 0]$ or $[-\max(h_2,\tau), 0]$ with known positive scalars h, h_2, and τ. $f(\sigma(t)) \in \mathbb{R}^m$ is the nonlinear functional in the feedback path. The time-varying delays $h(t)$ and $\tau(t)$ are continuous-time functional and satisfy the following two types of conditions:

C. 1.

$$\begin{aligned}
&0 \leq \tau(t) \leq \tau, \\
&\dot{\tau}(t) \leq \tau_d < 1, \\
&h_1 \leq h(t) \leq h_2, \\
&\mu_1 \leq \dot{h}(t) \leq \mu_2 \\
&\qquad \forall t \geq 0,
\end{aligned} \tag{2}$$

C. 2.

$$\begin{aligned}
&0 \leq \tau(t) \leq \tau, \\
&\dot{\tau}(t) \leq \tau_d < 1, \\
&0 \leq h(t) \leq h, \\
&\mu_1 \leq \dot{h}(t) \leq \mu_2, \\
&\qquad \forall t \geq 0,
\end{aligned} \tag{3}$$

where $\tau \geq 0$, $\tau_d < 1$, $h_1 \geq 0$, $h_2 \geq 0$, $h \geq 0$, μ_1 and $\mu_2 < 1$ are constants.

The nonlinear functional $f(\sigma(t))$ in the feedback path is given by

$$f(\sigma(t)) = [f_1(\sigma_1(t)) \ \ f_2(\sigma_2(t)) \ \ \cdots \ \ f_m(\sigma_m(t))]^T \tag{4}$$

satisfying the finite sector condition:

$$\begin{aligned}
f_i(\sigma_i(t)) \in \boldsymbol{K}_{[0,k_i]} = \{&f_i(\sigma_i(t)) \mid f_i(0) = 0, \ 0 \\
&< \sigma_i(t) f_i(\sigma_i(t)) \leq k_i \sigma_i(t)^2, \ \sigma_i(t) \neq 0\}
\end{aligned} \tag{5}$$

or the infinite sector condition:

$$\begin{aligned}
f_i(\sigma_i(t)) \in \boldsymbol{K}_{[0,\infty)} = \{&f_i(\sigma_i(t)) \mid f_i(0) \\
&= 0, \ \ \sigma_i(t) f_i(\sigma_i(t)) > 0, \ \sigma_i(t) \neq 0\},
\end{aligned} \tag{6}$$

where $K = \text{diag}\{k_1, k_2, \ldots, k_m\}$.

$\Delta A(t)$, $\Delta A_1(t)$, and $\Delta B(t)$ denote real-valued matrix functions representing parameter uncertainties, which are assumed to satisfy

$$[\Delta A(t) \ \ \Delta B(t) \ \ \Delta A_1(t)] = DF(t) [E_a \ \ E_b \ \ E_{a1}], \tag{7}$$

where D, E_a, E_b, and E_{a1} are known constant matrices with appropriate dimensions, and $F(t)$ is an unknown matrix with Lebesgue-measurable elements and satisfies

$$F^T(t) F(t) \leq I, \quad \forall t \geq 0. \tag{8}$$

This paper mainly analyzes and studies the stability of uncertain neutral-type Lur'e system (1) under conditions (2), (3), (5), (6), (7), and (8) based on Lyapunov stability theory. For neutral-type systems, the assumption that $\rho(C) < 1$ [41] is required, where $\rho(\cdot)$ denotes the spectral radius of C. To obtain the main results of this paper, the following definition and lemmas are important.

Definition 1 (robustly absolute stability). The uncertain neutral-type Lur'e system described by (1) is said to be robustly absolutely stable in the sector $[0, K]$ (or $[0, \infty)$), if the system is asymptotically stable for any nonlinear function $f(\sigma(t))$ satisfying (5) (or (6)) and all admissible uncertainties.

Lemma 2 (see [15]). *For given vectors α_1, α_2 and positive real scalars λ satisfying $0 < \lambda < 1$, symmetric positive definite matrix $R_1, R_2 \in \mathbb{R}^{n\times n}$, and any matrix $U_{01}, U_{02} \in \mathbb{R}^{n\times n}$, the following inequality holds*

$$\begin{aligned}
\frac{\alpha_1^T R_1 \alpha_1}{\lambda} + \frac{\alpha_2^T R_2 \alpha_2}{1-\lambda} \geq &\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}^T \\
&\cdot \begin{bmatrix} R_1 + (1-\lambda) T_1 & (1-\lambda) U_{01} + \lambda U_{02} \\ * & R_2 + \lambda T_2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix},
\end{aligned} \tag{9}$$

where $T_1 = R_1 - U_{02} R_2^{-1} U_{02}^T$, $T_2 = R_2 - U_{01}^T R_1^{-1} U_{01}$.

Lemma 3 (QGFMI [33]). *For any given matrices X, Y, a positive definite matrix R and a continuous differentiable function $\{\omega(s) \mid s \in [a,b]\}$, the following inequality holds*

$$\begin{aligned}
-\int_a^b \omega^T(s) R \omega(s)\, ds \leq &\begin{bmatrix} \eta_0 \\ \eta_1 \end{bmatrix}^T \\
&\cdot \begin{bmatrix} (b-a) X R^{-1} X^T & X [I \ \ 0] \\ * & \dfrac{b-a}{3} Y R^{-1} Y^T + \text{sym}\{Y [-I \ \ 2I]\} \end{bmatrix} \begin{bmatrix} \eta_0 \\ \eta_1 \end{bmatrix},
\end{aligned} \tag{10}$$

where η_0 is an any vector, and $\eta_1^T = [\int_a^b \omega^T(s) ds \ \ (1/(b-a)) \int_a^b \int_\theta^b \omega^T(s) ds\, d\theta]$.

Lemma 4 (see [42]). *For a given quadratic function $l(s) = a_2 s^2 + a_1 s + a_0$, where $a_i \in \boldsymbol{R}$ $(i = 0, 1, 2)$, $h_{12} = h_2 - h_1$, if the following inequalities hold*

$$\begin{aligned}
&(i) \ \ l(h_1) < 0; \\
&(ii) \ \ l(h_2) < 0; \\
&(iii) \ -h_{12}^2 a_2 + l(h_1) < 0,
\end{aligned} \tag{11}$$

one has $l(s) < 0$, for all $s \in [h_1, h_2]$.

$$
\begin{aligned}
&h_{1t} = h(t) - h_1,\ h_{2t} = h_2 - h(t),\ h_{12} = h_2 - h_1,\\
&h_d = 1 - \dot{h}(t),\ \overline{\mu}_1 = |\mu_1| + \dot{h}(t),\ \gamma^T(s) = \left[x^T(s)\ \dot{x}^T(s)\right],\\
&\nu_1(t) = \int_{t-h_1}^{t} \frac{x^T(s)}{h_1} ds,\ \nu_2(t) = \int_{t-h(t)}^{t-h_1} \frac{x^T(s)}{h_{1t}} ds,\ \nu_3(t) = \int_{t-h_2}^{t-h(t)} \frac{x^T(s)}{h_{2t}} ds,\\
&\omega_1(t) = h_1\nu_1(t),\ \omega_2(t) = h_{1t}\nu_2(t),\ \omega_3(t) = h_{2t}\nu_3(t),\\
&\zeta^T(t) = \left[x^T(t)\ \ x^T(t-h_1)\ \ x^T(t-h(t))\ \ x^T(t-h_2)\ \ \omega_1(t)\ \ \omega_2(t)\ \ \omega_3(t)\right],\\
&\zeta_1^T(t) = \left[x^T(t)\ \ x^T(t-h_1)\ \ x^T(t-h(t))\ \ x^T(t-h_2)\ \ \nu_2(t)\right],\\
&\zeta_2^T(t) = \left[x^T(t)\ \ x^T(t-h_1)\ \ x^T(t-h(t))\ \ x^T(t-h_2)\ \ \nu_3(t)\right],\\
&\Delta^T(t) = \left[\omega_2^T(t)\ x^T(t-h_1) - x^T(t-h(t))\ \omega_3^T(t)\ x^T(t-h(t)) - x^T(t-h_2)\right],\\
&\xi^T(t) = [x^T(t)\ \ x^T(t-h_1)\ \ x^T(t-h(t))\ \ x^T(t-h_2)\ \ \dot{x}^T(t)\ \ \dot{x}^T(t-h_1)\ \ \dot{x}^T(t-h(t))\ \ \dot{x}^T(t-h_2)\ \ \nu_1(t)\ \ \nu_2(t)\ \ \nu_3(t)\\
&\qquad \int_{t-h_1}^{t}\int_u^t \left(x^T(s)/h_1\right) du\,ds\ \ \int_{t-h(t)}^{t-h_1}\int_u^{t-h_1}\left(x^T(s)/h_{1t}\right) du\,ds\ \ \int_{t-h_2}^{t-h(t)}\int_u^{t-h(t)}\left(x^T(s)/h_{2t}\right) du\,ds\ \ \dot{x}^T(t-\tau(t))\ \ f^T(\sigma(t))],\\
&\eta_1(t) = \begin{bmatrix} \omega_2(t) \\ x(t-h_1) - x(t-h(t)) \end{bmatrix},\ \eta_2(t) = \begin{bmatrix} \omega_3(t) \\ x(t-h(t)) - x(t-h_2) \end{bmatrix}.
\end{aligned}
$$

Box 1: Notations of several symbols and matrices in Theorems 8 and 14.

Proof. The proof is similar to lemma 2 of [42]. First, in the case of $a_2 \ge 0$, l is a convex function. So, (i) and (ii) guarantee $l(s) < 0, \forall s \in [h_1, h_2]$. Next, for $a_2 < 0$, l is a concave function. So, $l(s) \le \dot{l}(h_2)(s-h_2)+l(h_2) = (2a_2h_2+a_1)s-a_2h_2^2+a_0 := g(s)$. Then $g(h_1) = -a_2h_{12}^2 + a_2h_1^2 + a_1h_1 + a_0 = -a_2h_{12}^2 + l(h_1) < 0$ from (iii) and $g(h_2) = l(h_2) < 0$ from (ii) guarantee that $l(s) < 0$, for all $l \in [h_1, h_2]$. This completes the proof. □

Remark 5. It is interesting to note that, in Lemma 4, when $h_1 = 0$, inequalities (11) can be rewritten in those of lemma 2 in [42]. Hence the established Lemma 4 covers the lemma in [42].

Lemma 6 (see [43]). *Given matrices Γ, Ξ, and $\Omega = \Omega^T$, the following inequality*

$$\Omega + \Gamma F(\sigma)\Xi + \Xi^T F^T(\sigma)\Gamma^T < 0 \tag{12}$$

holds for any $F(\sigma)$ satisfying $F^T(\sigma)F(\sigma) \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Omega + \varepsilon^{-1}\Gamma\Gamma^T + \varepsilon\Xi^T\Xi < 0. \tag{13}$$

Remark 7. Recently, [29] improved the stability results of the uncertain neutral-type Lur'e system (1) by combining the extended double integral with Wirtinger-based inequalities technique. In practice, it is known that the range of delay with nonzero lower bound is often encountered, and such systems are referred to as interval time-delay systems. So, both the range of delay with zero lower bound and that with nonzero lower bound are considered in this paper. In addition, the lower bound of the delay derivative is also involved in this paper, which is not mentioned in [29].

3. Main Results

3.1. Absolute Stability Criteria for Nominal Form. In this section, we will investigate the robustly absolute stability problem of the system (1). First, we give an absolute stability criterion for nominal form of system (1) without uncertainties described as

$$
\begin{aligned}
\dot{x}(t) - C\dot{x}(t-\tau(t)) &= Ax(t) + A_1x(t-h(t)) \\
&\quad + Bf(\sigma(t)), \\
\sigma(t) &= H^T x(t), \quad \forall t \ge 0, \\
x(s) &= \varphi(s), \\
\dot{x}(s) &= \dot{\varphi}(s), \\
&\quad s \in [-\max(h_2,\tau), 0],\ h(t) \in \mathbf{C.1} \\
&\quad s \in [-\max(h,\tau), 0],\ h(t) \in \mathbf{C.2}.
\end{aligned}
\tag{14}
$$

For the sake of simplicity on matrix representation, the notations of several symbols and matrices are defined as Box 1 of Appendix A. The following theorem will give an absolute stability criterion for Lur'e system (14) satisfying the conditions **C. 1** and (5).

Theorem 8. *The system (14) satisfying the conditions (2) and (5) is absolutely stable for given values of $h_2 \ge h_1 \ge 0$, μ_1, $\mu_2 < 1$, $\tau_d < 1$ and $k_j > 0$ $(j = 1, 2, \ldots, m)$, if there exist symmetric matrices $P \in \mathbb{R}^{7n\times 7n}$, $(P_a, P_b \in \mathbb{R}^{5n\times 5n})$, $(Q_a, Q_b, R_{0a}, R_a, R_b \in \mathbb{R}^{n\times n})$, positive definite matrices $(Q_2 \in \mathbb{R}^{n\times n})$, $(Q_1, R_0, R_1, R_2, Q_1(t), Q_2(t) \in \mathbb{R}^{2n\times 2n})$, $S = \mathrm{diag}\{s_1, s_2, \ldots, s_m\}$, $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$ and any matrices $(U_{01}, U_{02} \in \mathbb{R}^{2n\times 2n}, \overline{U} \in \mathbb{R}^{(3n+m)\times n})$, $X_0 \in \mathbb{R}^{3n\times 2n}$, $X_i \in \mathbb{R}^{5n\times 2n}$, $Y_\lambda \in \mathbb{R}^{4n\times 2n}$ $(i = 1, \ldots, 4; \lambda = 0, \ldots, 4)$ such that the following LMIs hold for $[h(t), \dot{h}(t)] \in \{[h_1, h_2] \times [\mu_1, \mu_2]\}$:*

$$
\begin{aligned}
\overline{Q}_a &> 0, \\
\overline{Q}_b &> 0, \\
\overline{R}_{0a} &> 0,
\end{aligned}
$$

$$\begin{aligned}\overline{R}_a &> 0,\\ \overline{R}_b &> 0,\\ \Omega_{1[h(t)]} &> 0,\\ \Omega_{2[h(t)]} &> 0,\end{aligned} \tag{15}$$

$$\begin{gathered}\begin{bmatrix}\Omega_{4[h_1]} + \dfrac{1}{h_{12}} J^T \Omega_{3[h_1]} J & E_1 U_{02} \\ * & h_{12}\Omega_{2[h_1]}\end{bmatrix} > 0,\\ \begin{bmatrix}\Omega_{4[h_2]} + \dfrac{1}{h_{12}} J^T \Omega_{3[h_2]} J & E_2 U_{01}^T \\ * & h_{12}\Omega_{1[h_2]}\end{bmatrix} > 0,\end{gathered} \tag{16}$$

$$l\left(h_1, \dot{h}(t), \alpha\right) = \begin{bmatrix}\Pi_{[h_1,\dot{h}(t)]} & \Omega_{01} & \Omega_{02} & \Omega_{a[1,4]} & \overline{\mu_1}\Omega_{a[1,2]} & h_d\Omega_{b[2,1,3]}^{[h_1]} & \Omega_{b[2,1,1]}^{[h_1]} \\ * & -h_1\overline{R}_{0a} & 0 & 0 & 0 & 0 & 0 \\ * & * & -3h_1\overline{R}_{0a} & 0 & 0 & 0 & 0 \\ * & * & * & -h_{12}\overline{R}_b & 0 & 0 & 0 \\ * & * & * & * & -\overline{\mu_1}h_{12}\overline{Q}_b & 0 & 0 \\ * & * & * & * & * & -3h_dh_{12}\overline{R}_a & 0 \\ * & * & * & * & * & * & -3h_{12}\overline{Q}_a\end{bmatrix} < 0, \tag{17}$$

$$l\left(h_1, \dot{h}(t), 1-\alpha\right) = \begin{bmatrix}\Pi_{[h_1,\dot{h}(t)]} & \Omega_{01} & \Omega_{02} & \Omega_{a[1,4]} & \overline{\mu_1}\Omega_{a[1,2]} & \Omega_{b[3,2,4]}^{[h_1]} & \overline{\mu_1}\Omega_{b[3,2,2]}^{[h_1]} \\ * & -h_1\overline{R}_{0a} & 0 & 0 & 0 & 0 & 0 \\ * & * & -3h_1\overline{R}_{0a} & 0 & 0 & 0 & 0 \\ * & * & * & -h_{12}\overline{R}_b & 0 & 0 & 0 \\ * & * & * & * & -\overline{\mu_1}h_{12}\overline{Q}_b & 0 & 0 \\ * & * & * & * & * & -3h_{12}\overline{R}_b & 0 \\ * & * & * & * & * & * & -3\overline{\mu_1}h_{12}\overline{Q}_b\end{bmatrix} < 0, \tag{18}$$

$$l\left(h_2, \dot{h}(t), \alpha\right) = \begin{bmatrix}\Pi_{[h_2,\dot{h}(t)]} & \Omega_{01} & \Omega_{02} & h_d\Omega_{a[1,3]} & \Omega_{a[1,1]} & h_d\Omega_{b[2,1,3]}^{[h_2]} & \Omega_{b[2,1,1]}^{[h_2]} \\ * & -h_1\overline{R}_{0a} & 0 & 0 & 0 & 0 & 0 \\ * & * & -3h_1\overline{R}_{0a} & 0 & 0 & 0 & 0 \\ * & * & * & -h_dh_{12}\overline{R}_a & 0 & 0 & 0 \\ * & * & * & * & -h_{12}\overline{Q}_a & 0 & 0 \\ * & * & * & * & * & -3h_dh_{12}\overline{R}_a & 0 \\ * & * & * & * & * & * & -3h_{12}\overline{Q}_a\end{bmatrix} < 0, \tag{19}$$

$$l\left(h_2, \dot{h}(t), 1-\alpha\right) = \begin{bmatrix}\Pi_{[h_2,\dot{h}(t)]} & \Omega_{01} & \Omega_{02} & h_d\Omega_{a[1,3]} & \Omega_{a[1,1]} & \Omega_{b[3,2,4]}^{[h_2]} & \overline{\mu_1}\Omega_{b[3,2,2]}^{[h_2]} \\ * & -h_1\overline{R}_{0a} & 0 & 0 & 0 & 0 & 0 \\ * & * & -3h_1\overline{R}_{0a} & 0 & 0 & 0 & 0 \\ * & * & * & -h_dh_{12}\overline{R}_a & 0 & 0 & 0 \\ * & * & * & * & -h_{12}\overline{Q}_a & 0 & 0 \\ * & * & * & * & * & -3h_{12}\overline{R}_b & 0 \\ * & * & * & * & * & * & -3\overline{\mu_1}h_{12}\overline{Q}_b\end{bmatrix} < 0, \tag{20}$$

$$-a_2h_{12}^2 + l\left(h_1, \dot{h}(t), \alpha\right) < 0,$$
$$-a_2h_{12}^2 + l\left(h_1, \dot{h}(t), 1-\alpha\right) < 0, \tag{21}$$

where the related notations are defined in Box 3 of Appendix B.

Proof. Construct an *LKF* candidate as

$$V(t) = \sum_{i=1}^{5} V_i(t) \tag{22}$$

with

$$\begin{aligned}
V_1(t) &= \zeta^T(t) P\zeta(t) + h_{1t}\zeta_1^T(t) P_a\zeta_1(t) \\
&\quad + h_{2t}\zeta_2^T(t) P_b\zeta_2(t), \\
V_2(t) &= \int_{t-h(t)}^{t-h_1} \gamma^T(s) Q_1(t)\gamma(s)\,ds \\
&\quad + \int_{t-h_2}^{t-h(t)} \gamma^T(s) Q_2(t)\gamma(s)\,ds, \\
V_3(t) &= \int_{t-h_1}^{t} \gamma^T(s) Q_1\gamma(s)\,ds \\
&\quad + \int_{t-\tau(t)}^{t} \dot{x}^T(s) Q_2\dot{x}(s)\,ds \\
&\quad + 2\sum_{i=1}^{m}\lambda_i \int_0^{\sigma_i} f_i(\sigma_i)\,d\sigma_i, \\
V_4(t) &= \int_{t-h(t)}^{t-h_1}\int_{\theta}^{t-h_1} \gamma^T(s)\overline{Q}_a\gamma(s)\,ds\,d\theta \\
&\quad + |\mu_1| \int_{t-h_2}^{t-h(t)}\int_{\theta}^{t-h(t)} \gamma^T(s)\overline{Q}_b\gamma(s)\,ds\,d\theta \\
V_5(t) &= \int_{t-h_1}^{t}\int_{\theta}^{t} \gamma^T(s) R_0\gamma(s)\,ds\,d\theta \\
&\quad + \int_{t-h(t)}^{t-h_1}\int_{\theta}^{t-h_1} \gamma^T(s) R_1\gamma(s)\,ds\,d\theta \\
&\quad + \int_{t-h_2}^{t-h(t)}\int_{\theta}^{t-h(t)} \gamma^T(s) R_2\gamma(s)\,ds\,d\theta,
\end{aligned} \tag{23}$$

where notations of several symbols and matrices can be found in Boxes 1 and 3 of Appendixes A and B.

First step, because the positive definiteness of the Lyapunov matrices P, P_a, and P_b is not required, the positive definiteness of the LKF (22) should be proved. The P_a- and P_b- dependent terms can be rewritten as

$$\begin{aligned}
&h_{1t}\zeta_1^T(t) P_a\zeta_1(t) + h_{2t}\zeta_2^T(t) P_b\zeta_2(t) \\
&= \begin{bmatrix} x(t) \\ x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ 0 \end{bmatrix}^T \left[h_{1t}P_a + h_{2t}P_b\right] \begin{bmatrix} x(t) \\ x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ 0 \end{bmatrix} \\
&\quad + 2\begin{bmatrix} x(t) \\ x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ 0 \end{bmatrix}^T \\
&\quad \cdot \left\{P_a\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_2(t) \end{bmatrix} + P_b\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_3(t) \end{bmatrix}\right\} \\
&\quad + \frac{\omega_2^T(t) EP_aE^T\omega_2(t)}{h_{1t}} + \frac{\omega_3^T(t) EP_bE^T\omega_3(t)}{h_{2t}},
\end{aligned} \tag{24}$$

where $E = [0\ \ 0\ \ 0\ \ 0\ \ I]$.

Based on $Q_1(t) > 0$, $Q_2(t) > 0$ and Jensen's inequality, the $V_2(t)$ term can be estimated as

$$V_2(t) \geq \frac{\eta_1^T(t) Q_1(t)\eta_1(t)}{h_{1t}} + \frac{\eta_2^T(t) Q_2(t)\eta_2(t)}{h_{2t}}. \tag{25}$$

According to $\Omega_{i[h(t)]} > 0$ $(i = 1, 2)$ and Lemma 2, we can obtain the following inequality from (24) and (25)

$$\begin{aligned}
&V_2(t) + \frac{\omega_2^T(t) EP_aE^T\omega_2(t)}{h_{1t}} + \frac{\omega_3^T(t) EP_bE^T\omega_3(t)}{h_{2t}} \\
&\geq \frac{\eta_1^T(t)\left(\begin{bmatrix} EP_aE^T & 0 \\ 0 & 0 \end{bmatrix} + Q_1(t)\right)\eta_1(t)}{h_{1t}} \\
&\quad + \frac{\eta_2^T(t)\left(\begin{bmatrix} EP_bE^T & 0 \\ 0 & 0 \end{bmatrix} + Q_2(t)\right)\eta_2(t)}{h_{2t}}
\end{aligned}$$

$$\geq \Delta^T(t)\frac{\Omega_{3[h(t)]}}{h_{12}}\Delta(t) - \frac{(1-\alpha)}{h_{12}}\eta_1^T(t)U_{02}\Omega_{2[h(t)]}^{-1}U_{02}^T\eta_1(t) - \frac{\alpha}{h_{12}}\eta_2^T(t)U_{01}^T\Omega_{1[h(t)]}^{-1}U_{01}\eta_2(t). \quad (26)$$

It follows from (15)-(16), (22), (24), (25), and (26) and $Q_1 > 0$, $Q_2 > 0$, $R_i > 0$ $(i = 0, 1, 2)$ that

$$V(t) > 0. \quad (27)$$

Thus, the LKF (22) is positive definite.

Second step, the time derivative of $V(t)$ with respect to time along the trajectory of the system (14) is as follows:

$$\begin{aligned}\dot{V}_1(t) &= \dot{h}(t)\zeta_1^T(t)P_a\zeta_1(t) - \dot{h}(t)\zeta_2^T(t)P_b\zeta_2(t) + 2\zeta^T(t)P\dot{\zeta}(t) + 2h_{1t}\zeta_1^T(t)P_a\dot{\zeta}_1(t) + 2h_{2t}\zeta_2^T(t) \\ &\cdot P_b\dot{\zeta}_2(t) = \dot{h}(t)\zeta_1^T(t)P_a\zeta_1(t) - \dot{h}(t)\zeta_2^T(t)P_b\zeta_2(t) \\ &+ 2\zeta^T(t)P\begin{bmatrix}\dot{x}(t)\\ \dot{x}(t-h_1)\\ h_d\dot{x}(t-h(t))\\ \dot{x}(t-h_2)\\ x(t)-x(t-h_1)\\ x(t-h_1)-h_dx(t-h(t))\\ h_dx(t-h(t))-x(t-h_2)\end{bmatrix} \\ &+ 2h_{1t}\zeta_1^T(t) \cdot P_a\begin{bmatrix}\dot{x}(t)\\ \dot{x}(t-h_1)\\ h_d\dot{x}(t-h(t))\\ \dot{x}(t-h_2)\\ \dfrac{x(t-h_1)-h_dx(t-h(t))-\dot{h}(t)v_2(t)}{h_{1t}}\end{bmatrix} \\ &+ 2h_{2t}\zeta_2^T(t) \cdot P_b\begin{bmatrix}\dot{x}(t)\\ \dot{x}(t-h_1)\\ h_d\dot{x}(t-h(t))\\ \dot{x}(t-h_2)\\ \dfrac{h_dx(t-h(t))-x(t-h_2)+\dot{h}(t)v_3(t)}{h_{2t}}\end{bmatrix},\end{aligned} \quad (28)$$

$$\begin{aligned}\dot{V}_2(t) &= \gamma^T(t-h_1)Q_1(t)\gamma(t-h_1) + h_d\gamma^T(t-h(t)) \cdot [Q_2(t)-Q_1(t)]\gamma(t-h(t)) - \gamma^T(t-h_2)Q_2(t) \\ &\cdot \gamma(t-h_2) - \dot{h}(t)\int_{t-h(t)}^{t-h_1}\gamma^T(s)Q_{11}\gamma(s)\,ds - \dot{h}(t) \cdot \int_{t-h_2}^{t-h(t)}\gamma^T(s)Q_{21}\gamma(s)\,ds,\end{aligned} \quad (29)$$

$$\begin{aligned}\dot{V}_3(t) &\leq \gamma^T(t)Q_1\gamma(t) - \gamma^T(t-h_1)Q_1\gamma(t-h_1) + \dot{x}^T(t)Q_2\dot{x}(t) - (1-\tau_d)\dot{x}^T(t-\tau(t)) \\ &\cdot Q_2\dot{x}(t-\tau(t)) + 2f^T(\sigma(t))\Lambda H^T\dot{x}(t),\end{aligned} \quad (30)$$

$$\begin{aligned}\dot{V}_4(t) &= h_{1t}\gamma^T(t-h_1)\overline{Q}_a\gamma(t-h_1) + |\mu_1| \cdot h_dh_{2t}\gamma^T(t-h(t))\overline{Q}_b\gamma(t-h(t)) \\ &- h_d\int_{t-h(t)}^{t-h_1}\gamma^T(s)\overline{Q}_a\gamma(s)\,ds - |\mu_1| \cdot \int_{t-h_2}^{t-h(t)}\gamma^T(s)\overline{Q}_b\gamma(s)\,ds,\end{aligned} \quad (31)$$

$$\begin{aligned}\dot{V}_5(t) &= h_1\gamma^T(t)R_0\gamma(t) + h_{1t}\gamma^T(t-h_1)R_1\gamma(t-h_1) + h_dh_{2t}\gamma^T(t-h(t))R_2\gamma(t-h(t)) \\ &- \int_{t-h_1}^{t}\gamma^T(s)R_0\gamma(s)\,ds - h_d\int_{t-h(t)}^{t-h_1}\gamma^T(s)R_1\gamma(s)\,ds - \int_{t-h_2}^{t-h(t)}\gamma^T(s)R_2\gamma(s)\,ds.\end{aligned} \quad (32)$$

For additional symmetric matrices Q_a, Q_b, R_{0a}, R_a, and R_b the following zero equations are satisfied

$$\begin{aligned}0 = \dot{h}(t)\Big[&x^T(t-h_1)Q_ax(t-h_1) - x^T(t-h(t))Q_ax(t-h(t)) - 2\int_{t-h(t)}^{t-h_1}x^T(s)Q_a\dot{x}(s)\,ds \\ &+ x^T(t-h(t))Q_bx(t-h(t)) - x^T(t-h_2)Q_bx(t-h_2) - 2\int_{t-h_2}^{t-h(t)}x^T(s)Q_b\dot{x}(s)\,ds\Big],\end{aligned} \quad (33)$$

$$0 = x^T(t)R_{0a}x(t) - x^T(t-h_1)R_{0a}x(t-h_1) - 2\int_{t-h_1}^{t} x^T(s)R_{0a}\dot{x}(s)\,ds, \quad (34)$$

$$0 = h_d\Big[x^T(t-h_1)R_ax(t-h_1) - x^T(t-h(t))R_ax(t-h(t)) - 2\int_{t-h(t)}^{t-h_1} x^T(s)R_a\dot{x}(s)\,ds\Big] + x^T(t-h(t))R_bx(t - h(t)) - x^T(t-h_2)R_bx(t-h_2) - 2\int_{t-h_2}^{t-h(t)} x^T(s)R_b\dot{x}(s)\,ds. \quad (35)$$

Taking the zero inequalities in $\dot{V}_2$ and $\dot{V}_4$, we have the following integral terms.

$$\varphi = -\int_{t-h(t)}^{t-h_1}\gamma^T(s)\overline{Q}_a\gamma(s)\,ds - \left(|\mu_1| + \dot{h}(t)\right)\int_{t-h_2}^{t-h(t)}\gamma^T(s)\overline{Q}_b\gamma(s)\,ds - \int_{t-h_1}^{t}\gamma^T(s)\overline{R}_{0a}\gamma(s)\,ds - h_d\int_{t-h(t)}^{t-h_1}\gamma^T(s)\overline{R}_a\gamma(s)\,ds - \int_{t-h_2}^{t-h(t)}\gamma^T(s)\overline{R}_b\gamma(s)\,ds. \quad (36)$$

It follows from Lemma 3 with an augmented vector $\gamma(s)$ that

$$-\int_{t-h_1}^{t}\gamma^T(s)\overline{R}_{0a}\gamma(s)\,ds \le \begin{bmatrix} W_{01}^T \\ W_{02}^T \end{bmatrix}^T \begin{bmatrix} h_1X_0\overline{R}_{0a}^{-1}X_0^T & X_0H_{01} \\ * & \frac{h_1}{3}G_0Y_0\overline{R}_{0a}^{-1}Y_0^TG_0 + \mathrm{sym}\{G_0Y_0H_{02}\} \end{bmatrix}\begin{bmatrix} W_{01}^T \\ W_{02}^T \end{bmatrix}, \quad (37)$$

$$-\int_{t-h(t)}^{t-h_1}\gamma^T(s)\overline{Q}_a\gamma(s)\,ds \le \begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix}^T \begin{bmatrix} h_{1t}X_1\overline{Q}_a^{-1}X_1^T & X_1H_1 \\ * & \frac{h_{1t}}{3}G_1Y_1\overline{Q}_a^{-1}Y_1^TG_1 + \mathrm{sym}\{G_1Y_1H_3\} \end{bmatrix}\begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix}, \quad (38)$$

$$-\overline{\mu}_1\int_{t-h_2}^{t-h(t)}\gamma^T(s)\overline{Q}_b\gamma(s)\,ds \le \overline{\mu}_1\begin{bmatrix} W_1^T \\ W_3^T \end{bmatrix}^T \begin{bmatrix} h_{2t}X_2\overline{Q}_b^{-1}X_2^T & X_2H_2 \\ * & \frac{h_{2t}}{3}G_2Y_2\overline{Q}_b^{-1}Y_2^TG_2 + \mathrm{sym}\{G_2Y_2H_4\} \end{bmatrix}\begin{bmatrix} W_1^T \\ W_3^T \end{bmatrix}, \quad (39)$$

$$-h_d\int_{t-h(t)}^{t-h_1}\gamma^T(s)\overline{R}_a\gamma(s)\,ds \le h_d\begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix}^T \begin{bmatrix} h_{1t}X_3\overline{R}_a^{-1}X_3^T & X_3H_1 \\ * & \frac{h_{1t}}{3}G_1Y_3\overline{R}_a^{-1}Y_3^TG_1 + \mathrm{sym}\{G_1Y_3H_3\} \end{bmatrix}\begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix}, \quad (40)$$

$$-\int_{t-h_2}^{t-h(t)}\gamma^T(s)\overline{R}_b\gamma(s)\,ds \begin{bmatrix} W_1^T \\ W_3^T \end{bmatrix}^T \begin{bmatrix} h_{2t}X_4\overline{R}_b^{-1}X_4^T & X_4H_2 \\ * & \frac{h_{2t}}{3}G_2Y_4\overline{R}_b^{-1}Y_4^TG_2 + \mathrm{sym}\{G_2Y_4H_4\} \end{bmatrix}\begin{bmatrix} W_1^T \\ W_3^T \end{bmatrix}. \quad (41)$$

For any appropriately dimensioned matrices $\overline{U} = [U_1^T\ \ U_2^T\ \ U_3^T\ \ U_4^T]^T$, it is true that

$$0 = 2\left[x^T(t)\ \ \dot{x}^T(t)\ \ \dot{x}^T(t-\tau(t))\ \ f^T(\sigma(t))\right] \cdot \overline{U}\left[Ax(t) + A_1x(t-h(t)) + Bf(\sigma(t)) + C\dot{x}(t-\tau(t)) - \dot{x}(t)\right]. \quad (42)$$

Letting $S = \mathrm{diag}\{s_1, s_2, \ldots, s_m\} > 0$, it follows from (5) that

$$2f^T(\sigma(t))S\left[KH^Tx(t) - f^T(\sigma(t))\right] \ge 0. \quad (43)$$

Finally, from the above derivation, we have

$$\dot{V}(t) \le \xi^T(t)\Big\{\Pi_{[h(t),\dot{h}(t)]} + h_1\Big[W_{01}X_0\overline{R}_{0a}^{-1}X_0^TW_{01}^T + \frac{1}{3}W_{02}G_0Y_0\overline{R}_{0a}^{-1}Y_0^TG_0W_{02}^T\Big] + h_{1t}\Big[h_dW_1X_3\overline{R}_a^{-1}X_3^TW_1^T + W_1X_1\overline{Q}_a^{-1}X_1^TW_1^T\Big] + h_{2t}\Big[W_1X_4\overline{R}_b^{-1}X_4^TW_1^T + \overline{\mu_1}W_1X_2\overline{Q}_b^{-1}X_2^TW_1^T\Big] + h_{1t}\Big[\frac{h_d}{3}W_2G_1Y_3\overline{R}_a^{-1}Y_3^TG_1W_2^T$$

$$
\begin{aligned}
&+\frac{1}{3}W_2G_1Y_1\overline{Q}_a^{-1}Y_1^TG_1W_2^T\Big]\\
&+h_{2t}\left[\frac{1}{3}W_3G_2Y_4\overline{R}_b^{-1}Y_4^TG_2W_3^T\right.\\
&\left.\left.+\frac{\overline{\mu_1}}{3}W_3G_2Y_2\overline{Q}_b^{-1}Y_2^TG_2W_3^T\right]\right\}\xi(t)\\
&=\xi^T(t)\left\{\Pi_{[h(t),\dot{h}(t)]}+h_1\left[W_{01}X_0\overline{R}_{0a}^{-1}X_0^TW_{01}^T\right.\right.\\
&\left.+\frac{1}{3}W_{02}G_0Y_0\overline{R}_{0a}^{-1}Y_0^TG_0W_{02}^T\right]\\
&+h_{1t}\left[h_dW_1X_3\overline{R}_a^{-1}X_3^TW_1^T+W_1X_1\overline{Q}_a^{-1}X_1^TW_1^T\right]\\
&+h_{2t}\left[W_1X_4\overline{R}_b^{-1}X_4^TW_1^T+\overline{\mu_1}W_1X_2\overline{Q}_b^{-1}X_2^TW_1^T\right]\\
&+\alpha h_{12}\left[\frac{h_d}{3}W_2G_1Y_3\overline{R}_a^{-1}Y_3^TG_1W_2^T\right.\\
&\left.+\frac{1}{3}W_2G_1Y_1\overline{Q}_a^{-1}Y_1^TG_1W_2^T\right]+(1-\alpha)\\
&\cdot h_{12}\left[\frac{1}{3}W_3G_2Y_4\overline{R}_b^{-1}Y_4^TG_2W_3^T\right.\\
&\left.\left.+\frac{\overline{\mu_1}}{3}W_3G_2Y_2\overline{Q}_b^{-1}Y_2^TG_2W_3^T\right]\right\}\xi(t)
\end{aligned}
\tag{44}
$$

with $\alpha=(h(t)-h_1)/h_{12}\geq 0$, $1-\alpha=(h_2-h(t))/h_{12}\geq 0$.

Therefore, LMIs (17)–(21) hold, which together with *Schur complement equivalence*, Lemma 4 and the convex function theory imply that $\dot{V}(t)<0$. Hence, it follows from the Lyapunov stability theory that the nominal system (14) is absolutely stable for any nonlinear function $f(\sigma(t))$ satisfying (5). From Definition 1, this completes the proof. □

The following theorem will give an absolute stability criterion for the Lur'e system (14) satisfying the conditions **C. 2** and (5).

Corollary 9. *The system (14) satisfying the conditions (3) and (5) is absolutely stable for given values of $h\geq 0$, $\mu_1,\mu_2<1$, $\tau_d<1$, and $k_j>0$ ($j=1,2,\ldots,m$), if there exist symmetric matrices $P\in\mathbb{R}^{5n\times5n}$, ($P_a,P_b\in\mathbb{R}^{4n\times4n}$), ($Q_a,Q_b,R_a,R_b\in\mathbb{R}^{n\times n}$), positive definite matrices ($Q_2\in\mathbb{R}^{n\times n}$), ($R_1,R_2,Q_1(t),Q_2(t)\in\mathbb{R}^{2n\times2n}$), $S=\mathrm{diag}\{s_1,s_2,\ldots,s_m\}$, $\Lambda=\mathrm{diag}\{\lambda_1,\lambda_2,\ldots,\lambda_m\}$ and any matrices ($U_{01},U_{02}\in\mathbb{R}^{2n\times2n}$,$\overline{U}\in\mathbb{R}^{(3n+m)\times n}$), $X_i\in\mathbb{R}^{5n\times2n}$, $Y_i\in\mathbb{R}^{4n\times2n}$ ($i=1,\ldots,4$) such that LMIs (15) and (16) and the following LMIs hold for $[h(t),\dot{h}(t)]\in\{[0,h]\times[\mu_1,\mu_2]\}$:*

$$
\begin{aligned}
&l\left(0,\dot{h}(t),\alpha\right)\\
&=\begin{bmatrix}
\Pi_{[0,\dot{h}(t)]} & \Omega_{a[1,4]} & \overline{\mu_1}\Omega_{a[1,2]} & h_d\Omega_{b[2,1,3]}^{[0]} & \Omega_{b[2,1,1]}^{[0]}\\
* & -h\overline{R}_b & 0 & 0 & 0\\
* & * & -\overline{\mu_1}h\overline{Q}_b & 0 & 0\\
* & * & * & -3h_dh\overline{R}_a & 0\\
* & * & * & * & -3h\overline{Q}_a
\end{bmatrix}\\
&<0,
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
&l\left(0,\dot{h}(t),1-\alpha\right)\\
&=\begin{bmatrix}
\Pi_{[0,\dot{h}(t)]} & \Omega_{a[1,4]} & \overline{\mu_1}\Omega_{a[1,2]} & \Omega_{b[3,2,4]}^{[0]} & \overline{\mu_1}\Omega_{b[3,2,2]}^{[0]}\\
* & -h\overline{R}_b & 0 & 0 & 0\\
* & * & -\overline{\mu_1}h\overline{Q}_b & 0 & 0\\
* & * & * & -3h\overline{R}_b & 0\\
* & * & * & * & -3\overline{\mu_1}h\overline{Q}_b
\end{bmatrix}\\
&<0,
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
&l\left(h,\dot{h}(t),\alpha\right)\\
&=\begin{bmatrix}
\Pi_{[h,\dot{h}(t)]} & h_d\Omega_{a[1,3]} & \Omega_{a[1,1]} & h_d\Omega_{b[2,1,3]}^{[h]} & \Omega_{b[2,1,1]}^{[h]}\\
* & -h_dh\overline{R}_a & 0 & 0 & 0\\
* & * & -h\overline{Q}_a & 0 & 0\\
* & * & * & -3h_dh\overline{R}_a & 0\\
* & * & * & * & -3h\overline{Q}_a
\end{bmatrix}\\
&<0,
\end{aligned}
\tag{47}
$$

$$
\begin{aligned}
&l\left(h,\dot{h}(t),1-\alpha\right)\\
&=\begin{bmatrix}
\Pi_{[h,\dot{h}(t)]} & h_d\Omega_{a[1,3]} & \Omega_{a[1,1]} & \Omega_{b[3,2,4]}^{[h]} & \overline{\mu_1}\Omega_{b[3,2,2]}^{[h]}\\
* & -h_dh\overline{R}_a & 0 & 0 & 0\\
* & * & -h\overline{Q}_a & 0 & 0\\
* & * & * & -3h\overline{R}_b & 0\\
* & * & * & * & -3\overline{\mu_1}h\overline{Q}_b
\end{bmatrix}\\
&<0,
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
&-a_2h^2+l\left(0,\dot{h}(t),\alpha\right)<0,\\
&-a_2h^2+l\left(0,\dot{h}(t),1-\alpha\right)<0,
\end{aligned}
\tag{49}
$$

where the related notations are defined in Box 4 of Appendix C.

Proof. The *LKF* (22) can be reduced to the following one by taking $h_1=0$, $h_2=h$, $Q_1=0$, and $R_0=0$:

$$
\widetilde{V}(t)=\sum_{i=1}^{4}\widetilde{V}_i(t)
\tag{50}
$$

with

$$\begin{aligned}\widetilde{V}_1(t) &= \zeta^T(t)P\zeta(t) + h(t)\zeta_1^T(t)P_a\zeta_1(t) \\ &\quad + (h-h(t))\zeta_2^T(t)P_b\zeta_2(t), \\ \widetilde{V}_2(t) &= \int_{t-h(t)}^{t}\gamma^T(s)Q_1(t)\gamma(s)\,ds \\ &\quad + \int_{t-h}^{t-h(t)}\gamma^T(s)Q_2(t)\gamma(s)\,ds, \\ \widetilde{V}_3(t) &= \int_{t-\tau(t)}^{t}\dot{x}^T(s)Q_2\dot{x}(s)\,ds \\ &\quad + 2\sum_{i=1}^{m}\lambda_i\int_0^{\sigma_i}f_i(\sigma_i)\,d\sigma_i, \\ \widetilde{V}_4(t) &= \int_{t-h(t)}^{t}\int_{\theta}^{t}\gamma^T(s)\overline{Q}_a\gamma(s)\,ds\,d\theta \\ &\quad + |\mu_1|\int_{t-h}^{t-h(t)}\int_{\theta}^{t-h(t)}\gamma^T(s)\overline{Q}_b\gamma(s)\,ds\,d\theta \\ \widetilde{V}_5(t) &= \int_{t-h(t)}^{t}\int_{\theta}^{t}\gamma^T(s)R_1\gamma(s)\,ds\,d\theta \\ &\quad + \int_{t-h}^{t-h(t)}\int_{\theta}^{t-h(t)}\gamma^T(s)R_2\gamma(s)\,ds\,d\theta,\end{aligned} \tag{51}$$

where notations of several symbols and matrices can be found in Boxes 2 and 4 of Appendixes A and C. The proof of Corollary 9 is omitted because of the similarity to Theorem 8. □

Remark 10. Theorem 8 and Corollary 9 can reduce the conservatism of stability conditions based LMI via the LKFs (22) and (50) application. For nonintegral item $V_1(t)$, the matrices P, P_a, and P_b are just symmetrical, not always positive definite, and $Q_1(t)$ and $Q_2(t)$ of the single-integral item $V_2(t)$ are delay-dependent matrices which can also further improve the utilization of time delay and its derivative information. When proving $V(t) > 0$, we calculate $V_1(t)$ and $V_2(t)$ as a whole applying Lemma 2, which may expand the feasible regions of LMIs (15) and (16). When bounded the derivative of the LKFs, three additional zero equations (33)–(35) and Lemma 3 have been used to narrow the gap between the upper bound and the true value, which may be another contribution of reducing the conservatism of stability conditions.

Remark 11. It is worth noting that the delay-product-type item was introduced firstly by C. Zhang et al. [31, 44]; another novel LKF with delay-dependent matrix was constructed by W. Kwon et al. [33], where some improved stability conditions for linear systems with time-varying delay were given. C. Zhang and W. Kwon et al. pointed that the LKFs with delay-product-type item or delay-dependent matrix may reduce the conservatism of stability conditions based on the same inequality scaling technique. The LKFs (22) and (50) constructed in this paper, which combine the advantage of the delay-product-type item and delay-dependent matrix, are more general than those given in [31, 33]. In fact, letting $Q_{11} = Q_{21} = 0$, $Q_{20} = \operatorname{diag}\{Q_2, 0\}$, $\overline{Q}_a = \overline{Q}_b = 0$, and $R_1 = R_2$, the LKF (50) can be reduced to the LKF (18) of [31]. And taking $P_a = P_b = 0$, $\overline{Q}_a = \overline{Q}_b = 0$, and $R_1 = R_2$ the LKF (50) can be reduced to the LKF (16) of [33]. However, the derivation method of [31, 33] cannot be applied directly. The positive definiteness of the LKF and negative definiteness of the LKF's derivative are proved in Theorem 8.

Remark 12. It is worth noting that in [33], to bound the integral item for $-\mu \le \dot{h}(t) \le \mu$, $(\mu > 0)$

$$-\dot{h}(t)\int_{t-h(t)}^{t}\gamma^T\overline{Q}_a\gamma\,ds - \dot{h}(t)\int_{t-h}^{t-h(t)}\gamma^T\overline{Q}_b\gamma\,ds \tag{52}$$

via the QGFMI technique, the following addition zero equation was introduced

$$\begin{aligned}0 &= \mu\int_{t-h(t)}^{t}\gamma^T(s)\overline{Q}_a\gamma(s)\,ds \\ &\quad - \mu\int_{t-h(t)}^{t}\gamma^T(s)\overline{Q}_a\gamma(s)\,ds \\ &\quad + \mu\int_{t-h}^{t-h(t)}\gamma^T(s)\overline{Q}_b\gamma(s)\,ds \\ &\quad - \mu\int_{t-h}^{t-h(t)}\gamma^T(s)\overline{Q}_b\gamma(s)\,ds.\end{aligned} \tag{53}$$

Then, the above integral item can be rewritten as the following form:

$$\begin{aligned}\widetilde{\varphi} &= -\left(\mu + \dot{h}(t)\right)\int_{t-h(t)}^{t}\gamma^T\overline{Q}_a\gamma\,ds \\ &\quad - \left(\mu + \dot{h}(t)\right)\int_{t-h}^{t-h(t)}\gamma^T\overline{Q}_b\gamma\,ds \\ &\quad + \mu\int_{t-h(t)}^{t}\gamma^T\overline{Q}_a\gamma\,ds + \mu\int_{t-h}^{t-h(t)}\gamma^T\overline{Q}_b\gamma\,ds.\end{aligned} \tag{54}$$

The first two items on the right can be bounded via the QGFMI technique like (39) of this paper; however, there are fewer proper techniques for obtaining a tight upper bound of the integral terms $\mu\int_{t-h(t)}^{t}\gamma^T(s)\overline{Q}_a\gamma(s)ds$ and $\mu\int_{t-h}^{t-h(t)}\gamma^T(s)\overline{Q}_b\gamma(s)ds$ due to their positive definiteness. Thus, to avoid introducing the two positive define integral terms, we give the modified LKF (22) with a double integral item $V_4(t)$.

Remark 13. In addition, the main result of [33] was not LMI due to the terms $h^2(t)$ even $h^3(t)$. In this paper, all matrices inequations of Theorem 8 and Corollary 9 are LMI via Lemma 4 application, which can be solved easily by using Matlab LMI-toolbox. Moreover, the double integral items $V_4(t)$ and $V_5(t)$ of the LKF (22) divide the time-delay interval $[h_1, h_2]$ into two subintervals, that is, $[h_1, h(t)]$ and $[h(t), h_2]$,

$$
\begin{aligned}
& h_d = 1 - \dot{h}(t), \ \gamma^T(s) = \left[x^T(s) \ \dot{x}^T(s)\right], \ \overline{\mu}_1 = |\mu_1| + \dot{h}(t), \\
& v_1(t) = \int_{t-h(t)}^{t} \frac{x^T(s)}{h(t)} ds, \ v_2(t) = \int_{t-h}^{t-h(t)} \frac{x^T(s)}{h - h(t)} ds, \\
& \omega_1(t) = h(t) v_1(t), \ \omega_2(t) = (h - h(t)) v_2(t), \ \zeta^T(t) = \left[x^T(t) \ x^T(t-h(t)) \ x^T(t-h) \ \omega_1(t) \ \omega_2(t)\right], \\
& \zeta_1^T(t) = \left[x^T(t) \ x^T(t-h(t)) \ x^T(t-h) \ v_1(t)\right], \ \zeta_2^T(t) = \left[x^T(t) \ x^T(t-h(t)) \ x^T(t-h) \ v_2(t)\right], \\
& \Delta^T(t) = \left[\omega_1^T(t) \ x^T(t) - x^T(t-h(t)) \ \omega_2^T(t) \ x^T(t-h(t)) - x^T(t-h)\right], \\
& \xi^T(t) = [x^T(t) \ x^T(t-h(t)) \ x^T(t-h) \ \dot{x}^T(t) \ \dot{x}^T(t-h(t)) \ \dot{x}^T(t-h) \ v_1(t) \ v_2(t), \\
& \qquad \int_{t-h(t)}^{t} \int_u^t \left(x^T(s)/h(t)\right) du\, ds \ \int_{t-h}^{t-h(t)} \int_u^{t-h(t)} \left(x^T(s)/(h-h(t))\right) du\, ds \ \dot{x}^T(t-\tau(t)) \ f^T(\sigma(t))], \\
& \eta_1(t) = \begin{bmatrix} \omega_1(t) \\ x(t) - x(t-h(t)) \end{bmatrix}, \ \eta_2(t) = \begin{bmatrix} \omega_2(t) \\ x(t-h(t)) - x(t-h) \end{bmatrix}.
\end{aligned}
$$

Box 2: Notations of several symbols and matrices in Corollaries 9 and 15.

instead of using the item $\int_{t-h}^{t} \gamma^T(s) R \gamma(s) ds$ directly, which further make full use of the information of time-varying delays $h(t) - h_1$, $h_2 - h(t)$ and their derivative $\dot{h}(t)$. Thus, the QGFMI technique can be used fully in each subinterval, which can further reduce the conservatism of the stability conditions.

3.2. Robustly Absolute Stability Criteria for Uncertain Form. Next, we extend the obtained absolute stability conditions to robustly absolute stability problem for the uncertain neutral-type Lur'e system (1) with time-varying parameter uncertainties satisfying (7) and (8).

Theorem 14. *The system (1) satisfying the conditions (2), (5), (7), and (8) is robustly absolutely stable for given values of $h_2 \geq h_1 \geq 0$, $\mu_1, \mu_2 < 1$, $\tau_d < 1$, and $k_j > 0$ ($j = 1, 2, \ldots, m$), if there exist symmetric matrices $P \in \mathbb{R}^{7n\times 7n}$, ($P_a, P_b \in \mathbb{R}^{5n\times 5n}$), ($Q_a, Q_b, R_{0a}, R_a, R_b \in \mathbb{R}^{n\times n}$), positive definite matrices ($Q_2 \in \mathbb{R}^{n\times n}$), ($Q_1, R_0, R_1, R_2, Q_1(t), Q_2(t) \in \mathbb{R}^{2n\times 2n}$), $S = \operatorname{diag}\{s_1, s_2, \ldots, s_m\}$, $\Lambda = \operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$, any matrices ($U_{01}, U_{02} \in \mathbb{R}^{2n\times 2n}, \overline{U} \in \mathbb{R}^{(3n+m)\times n}$), $X_0 \in \mathbb{R}^{3n\times 2n}$, $X_i \in \mathbb{R}^{5n\times 2n}$, $Y_\lambda \in \mathbb{R}^{4n\times 2n}$ ($i = 1, \ldots, 4$; $\lambda = 0, \ldots, 4$) and $\varepsilon > 0$ such that LMIs (15)-(16) and the following LMIs hold for $[h(t), \dot{h}(t)] \in \{[h_1, h_2] \times [\mu_1, \mu_2]\}$:*

$$
\begin{aligned}
& J_1 l\left(h_1, \dot{h}(t), \alpha\right) J_1^T + \varepsilon J_2 \Phi_1^T \Phi_1 J_2^T + \operatorname{sym}\left\{J_2 \Phi_2^T J_3^T\right\} \\
& \quad - \varepsilon J_3 J_3^T < 0,
\end{aligned} \tag{55}
$$

$$
\begin{aligned}
& J_1 l\left(h_1, \dot{h}(t), 1-\alpha\right) J_1^T + \varepsilon J_2 \Phi_1^T \Phi_1 J_2^T \\
& \quad + \operatorname{sym}\left\{J_2 \Phi_2^T J_3^T\right\} - \varepsilon J_3 J_3^T < 0,
\end{aligned} \tag{56}
$$

$$
\begin{aligned}
& J_1 l\left(h_2, \dot{h}(t), \alpha\right) J_1^T + \varepsilon J_2 \Phi_1^T \Phi_1 J_2^T + \operatorname{sym}\left\{J_2 \Phi_2^T J_3^T\right\} \\
& \quad - \varepsilon J_3 J_3^T < 0,
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
& J_1 l\left(h_2, \dot{h}(t), 1-\alpha\right) J_1^T + \varepsilon J_2 \Phi_1^T \Phi_1 J_2^T \\
& \quad + \operatorname{sym}\left\{J_2 \Phi_2^T J_3^T\right\} - \varepsilon J_3 J_3^T < 0,
\end{aligned} \tag{58}
$$

$$
\begin{aligned}
& - a_2 h_{12}^2 + J_1 l\left(h_1, \dot{h}(t), \alpha\right) J_1^T + \varepsilon J_2 \Phi_1^T \Phi_1 J_2^T \\
& \quad + \operatorname{sym}\left\{J_2 \Phi_2^T J_3^T\right\} - \varepsilon J_3 J_3^T < 0,
\end{aligned} \tag{59}
$$

$$
\begin{aligned}
& - a_2 h_{12}^2 + J_1 l\left(h_1, \dot{h}(t), 1-\alpha\right) J_1^T + \varepsilon J_2 \Phi_1^T \Phi_1 J_2^T \\
& \quad + \operatorname{sym}\left\{J_2 \Phi_2^T J_3^T\right\} - \varepsilon J_3 J_3^T < 0,
\end{aligned} \tag{60}
$$

where

$$
\begin{aligned}
\Phi_1 &= \left[e_1 E_a^T + e_2 E_{a1}^T + e_{16} E_b^T\right]^T, \\
\Phi_2 &= \left[e_1 U_1 D + e_5 U_2 D + e_{15} U_3 D + e_{16} U_4 D\right]^T, \\
J_1^T &= [I \ I \ I \ I \ I \ I \ I \ 0], \\
J_2^T &= [I \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0], \\
J_3^T &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ I].
\end{aligned} \tag{61}
$$

Proof. If we replace A, A_1, and B in LMIs (17)–(21) with $A + DF(t)E_a$, $A_1 + DF(t)E_{a1}$, $B + DF(t)E_b$, respectively, Theorem 14 can be proved based on Lemma 6 easily. □

The following corollary will give the robustly absolute stability criterion for the Lur'e system (1) satisfying the condition **C. 2**.

Corollary 15. *System (1) satisfying the conditions (3) and (5), (7), and (8) is robustly absolutely stable for given values of $h \geq 0$, $\mu_1, \mu_2 < 1$, $\tau_d < 1$, and $k_j > 0$ ($j = 1, 2, \ldots, m$), if there exist symmetric matrices $P \in \mathbb{R}^{5n\times 5n}$, ($P_a, P_b \in \mathbb{R}^{4n\times 4n}$), ($Q_a, Q_b, R_a, R_b \in \mathbb{R}^{n\times n}$), positive definite matrices ($Q_2 \in \mathbb{R}^{n\times n}$), ($R_1, R_2, Q_1(t), Q_2(t) \in \mathbb{R}^{2n\times 2n}$), $S = \operatorname{diag}\{s_1, s_2, \ldots, s_m\}$, $\Lambda = \operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$, any matrices ($U_{01}, U_{02} \in \mathbb{R}^{2n\times 2n}, \overline{U} \in \mathbb{R}^{(3n+m)\times n}$), $X_i \in \mathbb{R}^{5n\times 2n}$, $Y_i \in \mathbb{R}^{4n\times 2n}$ ($i = 1, \ldots, 4$) and $\varepsilon > 0$ such that LMIs (15)-(16) and the following LMIs hold for $[h(t), \dot{h}(t)] \in \{[h_1, h_2] \times [\mu_1, \mu_2]\}$:*

$$
\begin{aligned}
& \widetilde{J}_1 l\left(0, \dot{h}(t), \alpha\right) \widetilde{J}_1^T + \varepsilon \widetilde{J}_2 \widetilde{\Phi}_1^T \widetilde{\Phi}_1 \widetilde{J}_2^T + \operatorname{sym}\left\{\widetilde{J}_2 \widetilde{\Phi}_2^T \widetilde{J}_3^T\right\} \\
& \quad - \varepsilon \widetilde{J}_3 \widetilde{J}_3^T < 0,
\end{aligned} \tag{62}
$$

$$\begin{aligned} &\widetilde{J}_1 l\left(0, \dot{h}(t), 1-\alpha\right) \widetilde{J}_1^T + \varepsilon \widetilde{J}_2 \widetilde{\Phi}_1^T \widetilde{\Phi}_1 \widetilde{J}_2^T + \operatorname{sym}\left\{\widetilde{J}_2 \widetilde{\Phi}_2^T \widetilde{J}_3^T\right\} \\ &\quad - \varepsilon \widetilde{J}_3 \widetilde{J}_3^T < 0, \end{aligned} \tag{63}$$

$$\begin{aligned} &\widetilde{J}_1 l\left(h, \dot{h}(t), \alpha\right) \widetilde{J}_1^T + \varepsilon \widetilde{J}_2 \widetilde{\Phi}_1^T \widetilde{\Phi}_1 \widetilde{J}_2^T + \operatorname{sym}\left\{\widetilde{J}_2 \widetilde{\Phi}_2^T \widetilde{J}_3^T\right\} \\ &\quad - \varepsilon \widetilde{J}_3 \widetilde{J}_3^T < 0, \end{aligned} \tag{64}$$

$$\begin{aligned} &\widetilde{J}_1 l\left(h, \dot{h}(t), 1-\alpha\right) \widetilde{J}_1^T + \varepsilon \widetilde{J}_2 \widetilde{\Phi}_1^T \widetilde{\Phi}_1 \widetilde{J}_2^T + \operatorname{sym}\left\{\widetilde{J}_2 \widetilde{\Phi}_2^T \widetilde{J}_3^T\right\} \\ &\quad - \varepsilon \widetilde{J}_3 \widetilde{J}_3^T < 0, \end{aligned} \tag{65}$$

$$\begin{aligned} &- a_2 h^2 + \widetilde{J}_1 l\left(0, \dot{h}(t), \alpha\right) \widetilde{J}_1^T + \varepsilon \widetilde{J}_2 \widetilde{\Phi}_1^T \widetilde{\Phi}_1 \widetilde{J}_2^T \\ &\quad + \operatorname{sym}\left\{\widetilde{J}_2 \widetilde{\Phi}_2^T \widetilde{J}_3^T\right\} - \varepsilon \widetilde{J}_3 \widetilde{J}_3^T < 0, \end{aligned} \tag{66}$$

$$\begin{aligned} &- a_2 h^2 + \widetilde{J}_1 l\left(0, \dot{h}(t), 1-\alpha\right) \widetilde{J}_1^T + \varepsilon \widetilde{J}_2 \widetilde{\Phi}_1^T \widetilde{\Phi}_1 \widetilde{J}_2^T \\ &\quad + \operatorname{sym}\left\{\widetilde{J}_2 \widetilde{\Phi}_2^T \widetilde{J}_3^T\right\} - \varepsilon \widetilde{J}_3 \widetilde{J}_3^T < 0, \end{aligned} \tag{67}$$

where

$$\begin{aligned} \widetilde{\Phi}_1 &= \left[e_1 E_a^T + e_2 E_{a1}^T + e_{12} E_b^T\right]^T, \\ \widetilde{\Phi}_2 &= \left[e_1 U_1 D + e_4 U_2 D + e_{11} U_3 D + e_{12} U_4 D\right]^T, \\ \widetilde{J}_1^T &= \left[I \;\; I \;\; I \;\; I \;\; I \;\; 0\right], \\ \widetilde{J}_2^T &= \left[I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right], \\ \widetilde{J}_3^T &= \left[0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; I\right]. \end{aligned} \tag{68}$$

Remark 16. If the nonlinear function $f(\sigma)$ in the feedback path satisfies the infinite sector conditions (6), for any $s_i \geq 0$, $i = 1, 2, \ldots, m$, it follows from (6) that

$$s_i f_i(\sigma_i) h_i^T x(t) \geq 0, \tag{69}$$

which is equivalent to

$$2x^T(t) HSf(\sigma) \geq 0, \tag{70}$$

where $S = \operatorname{diag}\{s_1, s_2, \ldots, s_m\}$.

Therefore, the corresponding absolute and robustly absolute stability criteria can be obtained by replacing the matrix Θ_1 of Theorems 8 and 14 and Corollaries 9 and 15 with $\Theta_2 = e_1 HSe_{16}^T$ and $\widetilde{\Theta}_2 = e_1 HSe_{12}^T$, respectively.

Remark 17. For one special case, in the absence of the nonlinear function, that is, in case of $B = 0$, the system (1) is simply written as the following linear neutral system

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t - \tau(t)) &= (A + \Delta A) x(t) \\ &\quad + \left(A_1 + \Delta A_1\right) x(t - h(t)), \\ x(s) &= \varphi(s), \\ \dot{x}(s) &= \dot{\varphi}(s), \\ &\qquad s \in \left[-\max\left(h_2, \tau\right), 0\right], \; h(t) \in \mathbf{C.1} \end{aligned} \tag{71}$$

$$s \in \left[-\max(h, \tau), 0\right], \; h(t) \in \mathbf{C.2}, \tag{72}$$

and for another special case, in the absence of the nonlinear function and neutral-type item, that is, in case of $B = 0$ and $C = 0$, the system (1) is simply written as the following linear time-delayed system:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A) x(t) + \left(A_1 + \Delta A_1\right) x(t - h(t)), \\ x(s) &= \varphi(s), \\ &\qquad s \in \left[-\max\left(h_2, \tau\right), 0\right], \; h(t) \in \mathbf{C.1} \\ &\qquad s \in \left[-\max(h, \tau), 0\right], \; h(t) \in \mathbf{C.2}. \end{aligned} \tag{73}$$

Obviously, take Theorem 8 as an example. Only letting $\Theta_1 = 0, \Pi_4 = [e_1U_1 + e_5U_2 + e_{15}U_3]$ and $\Pi_5 = [Ae_1^T + A_1e_2^T + Ce_{15}^T - e_5^T]$, the stability criteria proposed in this paper are also applied to this linear neutral system (71) with time-varying delays; taking $\Theta_1 = 0$, $Q_2 = 0$, $\Pi_4 = [e_1U_1 + e_5U_2]$ and $\Pi_5 = [Ae_1^T + A_1e_2^T]$, the stability criteria proposed in this paper are also applied to this linear system (73) with time-varying delays. We will not elaborate here due to the limited space available. However, some results of detailed comparisons will be given in the next section directly.

Remark 18. It is worth pointing out that in this paper, the upper and lower bounds constraints of the time-delay derivative are $\mu_1 \leq \dot{h}(t) \leq \mu_2$, where $\tau_d < 1, \mu_2 < 1$ due to the diagonal entry $-(1 - \dot{\tau}(t))Q_2$, $-(1 - \dot{h}(t))\dot{x}^T(t - h(t))(Q_1(t) - Q_2(t))\dot{x}(t - h(t))$ in $\Pi_{[h(t),\dot{h}(t)]}$, and $-(1 - \dot{h}(t)) \int_{t-h(t)}^{t} \gamma^T(s) R_1 \gamma(s) ds$ in $\dot{V}_5(t)$. Similar to Remark 5 of [45], one can establish a stability criterion for system (1) in the case $\tau_d \geq 1$ and $\mu_2 \geq 1$ if $\xi(t), \zeta(t), \zeta_1(t), \zeta_2(t), \gamma(s)$ and $V_5(t)$ are replaced with

$$\begin{aligned} &\bar{\xi}^T(t) \\ &= \left[x^T(t) \;\; x^T(t-h_1) \;\; x^T(t-h(t)) \;\; x^T(t-h_2) \;\; \dot{x}^T(t) \;\; \dot{x}^T(t-h_1) \;\; \dot{x}^T(t-h_2) \;\; v_1(t) \;\; v_2(t) \;\; v_3(t) \;\; \int_{t-h_1}^{t}\int_{u}^{t} \frac{x^T(s)}{h_1} du\, ds \;\; \int_{t-h(t)}^{t-h_1}\int_{u}^{t-h_1} \frac{x^T(s)}{h_{1t}} du\, ds \;\; \int_{t-h_2}^{t-h(t)}\int_{u}^{t-h(t)} \frac{x^T(s)}{h_{2t}} du\, ds \;\; f^T(\sigma(t))\right], \\ &\bar{\zeta}^T(t) = \left[x^T(t) \;\; x^T(t-h_1) \;\; x^T(t-h_2) \;\; \omega_1(t) \;\; \omega_2(t) \;\; \omega_3(t)\right], \\ &\bar{\zeta}_1^T(t) = \left[x^T(t) \;\; x^T(t-h_1) \;\; x^T(t-h_2) \;\; v_2(t)\right], \\ &\bar{\zeta}_2^T(t) = \left[x^T(t) \;\; x^T(t-h_1) \;\; x^T(t-h_2) \;\; v_3(t)\right], \\ &\bar{\gamma}^T(s) = x^T(s), \\ &Q_2 = 0, \\ &\overline{V}_5(t) = \int_{t-h_1}^{t}\int_{\theta}^{t-h_1} x^T(s) R_0 x(s)\, ds\, d\theta + \int_{t-h_2}^{t-h_1}\int_{\theta}^{t-h_1} x^T(s) R_1 x(s)\, ds\, d\theta. \end{aligned} \tag{74}$$

Due to page limitation, this result is omitted.

4. Numerical Examples

In this section, we give three types of examples, including the Lur'e system, the linear neutral system, and two linear systems, to show the effectiveness of the criteria proposed in this paper. Moreover, the conservatism of the criteria is checked based on the calculated maximal admissible delay upper bounds (MADUBs). And the index of the number of decision variables (NoVs) is applied to show the complexity of criteria. The stability criteria proposed in this paper are just dependent on τ_d, h, μ_1, and μ_2, however, independent on τ and $\tau(t)$. So, only τ_d of the information of the neutral-type time delay $\tau(t)$ is given for Examples 1 and 3. For the sake of simplicity, let $-\mu_1 = \mu_2 = \mu$.

Example 1. Consider the nominal neutral-type Lur'e system (14) with the time-varying delays satisfying **C.2** and the nonlinearity satisfying (6), and the system parameters are described as

$$A = \begin{bmatrix} -2 & 0.5 \\ 0 & -1 \end{bmatrix},$$
$$A_1 = \begin{pmatrix} 1 & 0.4 \\ 0.4 & -1 \end{pmatrix},$$
$$B = \begin{pmatrix} -0.5 \\ -0.75 \end{pmatrix}, \tag{75}$$
$$C = \begin{pmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{pmatrix},$$
$$H = \begin{pmatrix} 0.2 \\ 0.6 \end{pmatrix}.$$

Let the nonlinearity $f(\sigma(t)) = \sigma^3(t)$, where $\sigma(t) = H^T x(t) = 0.2x_1(t)+0.6x_2(t)$. Then, it follows from the infinite sector condition (6) and $f_i(\sigma_i(t))/\sigma_i(t) = f(\sigma_i(t))/\sigma(t) = \sigma^2(t) > 0$, that is $f_i(\sigma_i(t)) \in K_{[0,\infty)}$. Under the condition **C. 2**, in Table 1, the MADUBs h of the Lur'e system (14) for $\tau_d = 0.1$ and different μ by using Remark 16 and recent methods in [26, 27, 29, 34] are compared. From the table, notwithstanding the NoVs of our criteria are bigger than those of the criteria [26, 27, 29, 34], only [29] is less conservative than Remark 16 under $\mu = 0.2$. However our results become better and better with the increasing of μ. Figure 1 displays the responses of system states $x(t)$ for $h(t) = 2.8490/2+(2.8490/2)\sin(0.8t/2.8490)$, $\tau(t) = 0.5+|\sin(0.1t)|$ and initial condition $x(0) = [0.2 \quad -0.2]^T$. It is possible to see that, for this realization, the trajectory converges to the origin, as expected.

Remark 2. The MADUBs h of the Lur'e system (14) for $\tau_d = 0.1$ and different μ by using our results are less than those of [21] because the slope restrictions for nonlinearity and the lower bound of the derivative of the neutral-type delay $\dot{\tau}(t)$ were considered in [21], where these restrictions are more strict than those of this paper. Thus, the related results of [21] were not compared with the ones of this paper.

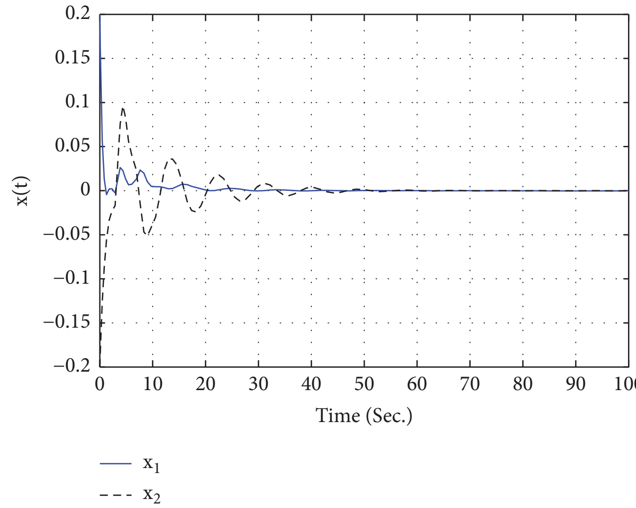

Figure 1: The state responses for Example 1.

In addition, the construction of the LKF with delay-dependent matrices is the main reason to reduce the conservatism of the stability criterion in [29], and another one is that the lower bound of the delay derivative is also involved in this paper, which is not mentioned in [29].

Example 2. Consider the following Chua's circuit

$$\begin{aligned} \dot{x} &= \alpha\left(y - g(x)\right) \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y, \end{aligned} \tag{76}$$

where $m_0 = -1/7$, $m_1 = 2/7$, $\alpha = 9$, $\beta = 14.28$, and $c = 1$, and the nonlinear function $g(x)$ is given by $g(x) = m_1\theta_1 + 0.5(m_0 - m_1)(|x+c| - |x-c|)$. This Chua's circuit can be expressed as a Lur'e-type system.

In [19], a master-slave synchronization scheme through a time-delayed state error feedback control is devised for the Chua's circuit (76), which is given as

$$\begin{aligned} &\text{Master}: \begin{cases} \dot{x}(t) = Ax(t) + Bf(Cx(t)) \\ p(t) = Wx(t), \end{cases} \\ &\text{Slave}: \begin{cases} \dot{y}(t) = Ay(t) + Bf(Cy(t)) + u(t) \\ q(t) = Wy(t), \end{cases} \\ &\text{Controller}: \ u(t) = -K_a\left(x(t) - y(t)\right) \\ &\qquad + K_b\left(p(t - h(t)) - q(t - h(t))\right), \end{aligned} \tag{77}$$

where $f(\theta) = 0.5(|\theta+1| - |\theta-1|) \in [0,1]$, $C = W = [1 \quad 0 \quad 0]$ and

Table 1: MADUBs h for different μ (Example 1).

τ_d	Methods\\μ	0.2	0.4	0.6	0.8	NoVs
	[26]	2.9962	2.1316	1.7138	1.3204	75
	[27]	3.1743	2.1789	1.7467	1.7153	96
0.1	[34]	3.4880	2.3787	1.8062	1.4625	93
	[29]	3.8557	2.8490	2.4014	2.2130	117
	Remark 16	3.6076	2.8490	2.5501	2.4526	705

Table 2: MADUBs h for different μ (Example 2).

Methods\\μ	0	0.3	0.6	0.9	NoVs
[19]	0.1622	0.1591	0.1566	0.1527	10
[23]	0.1745	0.1698	0.1698	0.1698	160
[24]	0.1747	0.1710	0.1703	0.1703	140
[27]	0.1771	0.1721	0.1715	0.1715	194
[21]	0.1894	0.1894	0.1894	0.1893	627
[30]	0.2638	0.2578	0.2540	0.2510	809
Corollary 9	0.2707	0.2700	0.2545	0.2544	1548

$$A = \begin{bmatrix} -\alpha m_1 & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} -\alpha\left(m_0 - m_1\right) \\ 0 \\ 0 \end{bmatrix}. \tag{78}$$

Letting $e = x - y$, then the resultant error system is given by

$$\begin{aligned} \dot{e}(t) = {} & \left(A + K_a\right) e(t) - K_b W e(t - h(t)) \\ & + B\varphi(Ce(t)). \end{aligned} \tag{79}$$

Suppose the synchronization controller gains are designed by [18]

$$K_a = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$
$$K_b = \begin{bmatrix} 6.0029 \\ 1.3367 \\ -2.1264 \end{bmatrix}. \tag{80}$$

In Table 2, we calculate the MADUBs h of the error system (79) for different μ and the condition **C. 2** by using our results and methods in [18, 19, 21, 23, 27, 30] are compared. From the table, it is found that notwithstanding the NoVs of Corollary 9 are bigger than those of the criteria [19, 23, 24, 27], the MADUBs computed by Corollary 9 are larger. Compared with the criteria in [21, 30], Corollary 9 obtains less conservative MADUBs but requires less computation complexity. Moreover, for $h(t) = 0.2707$ and initial condition $e(0) = [-0.1 \ \ 0.4 \ \ -0.3]^T$, Figure 2 depicts the error state responses for the error system (79) under control of the synchronization controller. Thus, one can see that the error system is asymptotically stable for the time-varying delay $h(t)$ less than 0.2707.

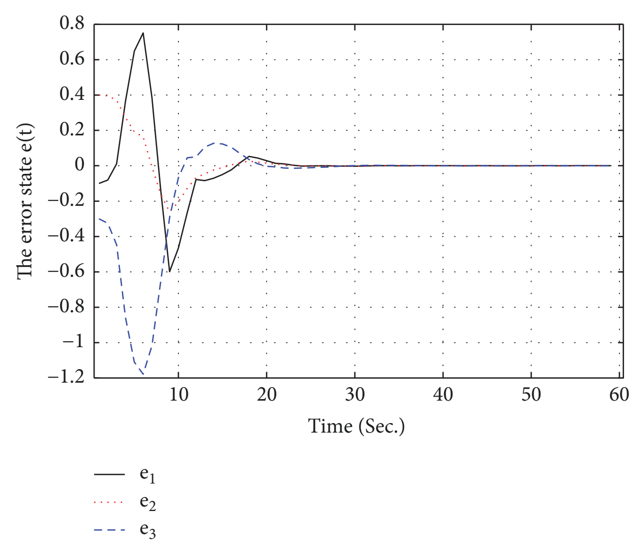

Figure 2: The error state responses for Example 2.

Remark 3. In Example 2, the purpose of this paper is to enlarge the MADUBs under the same controller to [19]. At this point, the error system (79) can be seen as a new Lure system within the given gain matrices $A + K_a$, $-K_bW$ and B proposed by [19]. The MADUBs of are calculated by solving the LMIs in Theorem 8. For an example, an MADUB $h = 0.2700$ is obtained for $\mu = 0.3$ by Theorem 8, then the stability of the error system (79) must be guaranteed by Theorem 8 under the same controller to [19] for $0 \le h(t) \le h$ and $|\mu| \le 0.3$. In other words, this paper does not design any controllers, but analyzes the stability. The same controller gains of [19] are seen as the known system matrices of the error system

Table 3: MADUBs h for $\tau_d = 0.6$ and different μ_1 and μ_2 (Example 3).

μ_1	Methods\μ_2	0.5	0.9	NoVs
Ignore μ_1	[25]	1.5572	1.5572	10
	[27]	1.6635	1.5742	160
	[35]	1.5812	1.5745	140
-0.5	[28]	1.8763	1.7352	120
	Remark 17	2.0555	1.9357	518
-0.2	[28]	1.8780	1.7453	120
	Remark 17	2.0671	1.9558	518

Table 4: MADUBs h for different μ (Example 4).

Methods\μ	0.1	0.2	0.5	0.8	NoVs
[36]	6.6103	4.0034	1.6875	1.0287	$23n^2 + 4n$
[37]	7.1672	4.5179	2.4158	1.8384	$142n^2 + 18n$
[38]	7.1765	4.5438	2.4963	1.9225	$114n^2 + 18n$
[39]	7.2030	4.5126	2.3860	1.8476	$203n^2 + 9n$
[40]	7.1905	4.5275	2.4473	1.8562	$70n^2 + 12n$
[30]	7.2734	4.6213	2.6505	2.0612	$185.5n^2 + 21.5n$
[3]	7.4001	4.7954	2.7175	2.0894	$108n^2 + 12n$
[33]	8.6565	5.8907	3.1754	2.3953	$91.5n^2 + 4.5n$
Remark 17	8.9647	7.1866	4.9390	3.8477	$165.5n^2 + 19.5n$

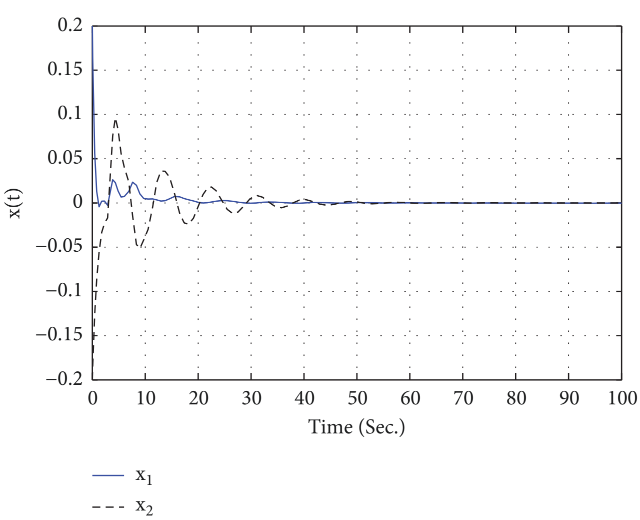

Figure 3: The state responses for Example 3.

(79). Therefore, the controller gains obtained in [19] will also stabilize the system considered in Example 2 under a bounds of $h(t)$ obtained by Theorem 8.

Example 3. Consider the linear neutral-type system (71) with the time-varying delays satisfying ***C.2***, and the system parameters are described as

$$A = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix},$$

$$\begin{aligned} A_1 &= \begin{pmatrix} 0.5 & 0 \\ 0.5 & -0.5 \end{pmatrix}, \\ C &= \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}, \\ D &= \operatorname{diag}\{1, 1\}, \\ E_a &= \operatorname{diag}\{0.05, 0.05\}, \\ E_{a1} &= \operatorname{diag}\{0.1, 0.1\}. \end{aligned} \tag{81}$$

Under the condition **C. 2**, in Table 3, we calculate the MADUBs h of the linear neutral-type system (71) for $\tau_d = 0.6$ and different μ_1, μ_2 by using Remark 16 and methods in [25, 27, 28, 35] are compared. From the table, it is found that notwithstanding the NoVs of our criteria are bigger than those of the criteria [25, 27, 28, 35], the MADUBs computed by our criteria are larger. Figure 3 displays the responses of system states $x(t)$ for $h(t) = 2.0555/2 + (2.0555/2)\sin(t/2.0555)$, $\tau(t) = 1.2|\sin(0.5t)|$ and initial condition $x(0) = [-0.2 \;\; 0.2]^T$. It is possible to see that, for this realization, the trajectory converges to the origin, as expected.

Example 4. Consider the linear systems (73) with time-varying delays, where the system parameters are described as

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}. \end{aligned} \tag{82}$$

As a special case pointed out by Remark 16, Corollary 9 removed some redundant terms which can be used to check the stability of the linear systems with time-varying delays. In order to make a comparison with some existing stability criteria, the MADUBs under the condition **C. 2** are listed in Table 4, which shows that notwithstanding the NoVs of Corollary 9 are bigger than those of the criteria [3, 33, 36, 38, 40], the MADUBs computed by Corollary 9 are larger. Compared with the criteria in [30, 37, 39], Corollary 9 obtains less conservative MADUBs but requires less computation complexity.

5. Conclusion

In this paper, some improved absolute and robustly absolute stability criteria are proposed for the uncertain neutral-type Lur'e systems with mixed time-varying delays and sector-bounded nonlinearities via a novel LKF combining the delay-product-type function and the delay-dependent matrix. An effective technique, QGFMI, is applied to further reduce the conservatism of the proposed criteria from some existing results. Finally, some numerical examples are used to illustrate the effectiveness of the proposed approaches.

Appendix

A. Boxes 1 and 2

See Boxes 1 and 2.

B. Box 3

$$\Omega_{1[h(t)]} = \begin{bmatrix} EP_aE^T & 0 \\ 0 & 0 \end{bmatrix} + Q_1(t),$$

$$\Omega_{2[h(t)]} = \begin{bmatrix} EP_bE^T & 0 \\ 0 & 0 \end{bmatrix} + Q_2(t),$$

$$\Omega_{3[h(t)]} = \begin{bmatrix} (2-\alpha)\Omega_{1[h(t)]} & (1-\alpha)U_{01} + \alpha U_{02} \\ * & (1+\alpha)\Omega_{2[h(t)]} \end{bmatrix},$$

$$\Omega_{4[h(t)]} = P + \begin{bmatrix} \tilde{e}_1^T \\ \tilde{e}_2^T \\ \tilde{e}_3^T \\ \tilde{e}_4^T \\ 0 \end{bmatrix}^T [h_{1t}P_a + h_{2t}P_b] \begin{bmatrix} \tilde{e}_1^T \\ \tilde{e}_2^T \\ \tilde{e}_3^T \\ \tilde{e}_4^T \\ 0 \end{bmatrix}$$

$$+ \operatorname{sym}\left\{ \begin{bmatrix} \tilde{e}_1^T \\ \tilde{e}_2^T \\ \tilde{e}_3^T \\ \tilde{e}_4^T \\ 0 \end{bmatrix}^T \cdot \left(P_a \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tilde{e}_6^T \end{bmatrix} + P_b \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tilde{e}_7^T \end{bmatrix} \right) \right\},$$

$$\alpha = \frac{h(t) - h_1}{h_{12}},$$

$$J^T = [E_1 \ \ E_2],$$

$$E_1 = [\tilde{e}_6 \ \tilde{e}_2 - \tilde{e}_3]$$

$$E_2 = [\tilde{e}_7 \ \tilde{e}_3 - \tilde{e}_4],$$

$$\Pi_{[h(t),\dot{h}(t)]} = \operatorname{sym}\left\{\Pi_{1[h(t),\dot{h}(t)]} + \Pi_{3[h(t),\dot{h}(t)]}\right\} + \Pi_{2[h(t),\dot{h}(t)]},$$

$$\Pi_{1[h(t),\dot{h}(t)]} = \Gamma_{[h(t)]} P \Psi^T_{[\dot{h}(t)]}$$
$$+ [e_1 \ \ e_2 \ \ e_3 \ \ e_4 \ \ e_{10}] P_a \left[h_{1t}e_5 \ h_{1t}e_6 \ h_{1t}h_d e_7 \ h_{1t}e_8 \ e_2 - h_d e_3 - \dot{h}(t) e_{10}\right]^T$$
$$+ [e_1 \ \ e_2 \ \ e_3 \ \ e_4 \ \ e_{11}] P_b \left[h_{2t}e_5 \ h_{2t}e_6 \ h_{2t}h_d e_7 \ h_{2t}e_8 \ h_d e_3 - e_4 + \dot{h}(t) e_{11}\right]^T + \Theta_1 + \widetilde{\Theta}_1 + \Pi_4\Pi_5,$$

$$\Pi_{2[h(t),\dot{h}(t)]} = [e_1 \ \ e_5][Q_1 + h_1R_0][e_1 \ \ e_5]^T$$
$$+ [e_2 \ \ e_6][Q_1(t) - Q_1][e_2 \ \ e_6]^T$$
$$+ h_d[e_3 \ \ e_7][Q_2(t) - Q_1(t)][e_3 \ \ e_7]^T - [e_4 \ \ e_8]Q_2(t)[e_4 \ \ e_8]^T$$
$$+ e_1R_{0a}e_1^T + e_2\left[h_dR_a - R_{0a} + \dot{h}(t)Q_a\right]e_2^T$$
$$+ e_3\left[R_b - h_dR_a + \dot{h}(t) \cdot (Q_b - Q_a)\right]e_3^T - e_4\left[R_b + \dot{h}(t)Q_b\right]e_4^T + e_5Q_2e_5^T$$
$$- (1-\tau_d)e_{15}Q_2e_{15}^T$$
$$+ h_{1t}[e_2 \ \ e_6]\left(\overline{Q}_a + R_1\right)[e_2 \ \ e_6]^T$$
$$+ |\mu_1| \cdot h_dh_{2t}[e_3 \ \ e_7]\overline{Q}_b[e_3 \ \ e_7]^T$$
$$+ h_dh_{2t}[e_3 \ \ e_7]R_2[e_3 \ \ e_7]^T + \dot{h}(t)\Pi_6P_a\Pi_6^T$$
$$- \dot{h}(t)\Pi_7P_b\Pi_7^T,$$

$$\Pi_{3[h(t),\dot{h}(t)]} = W_{01}X_0H_{01}W_{02}^T + W_{02}G_0Y_0H_{02}W_{02}^T$$
$$+ h_dW_1X_3H_1W_2^T + h_dW_2G_1Y_3H_3W_2^T$$
$$+ W_1X_4H_2W_3^T + W_3G_2Y_4H_4W_3^T + W_1X_1H_1W_2^T$$
$$+ W_2G_1Y_1H_3W_2^T + \overline{\mu}_1W_1X_2H_2W_3^T$$

$$
\begin{aligned}
&+\overline{\mu}_1 W_3 G_2 Y_2 H_4 W_3^T,\\
&\Omega_{01} = h_1 W_{01} X_0,\\
&\Omega_{02} = h_1 W_{02} G_0 Y_0,\\
&\Omega_{a[k,j]} = h_{12} W_k X_j,\\
&\Omega_{b[k,r,j]}^{[h(t)]} = h_{12} W_k G_r Y_j,\\
&\qquad k \in [1,2,3],\ j \in [1,2,3,4],\ r \in [1,2],\\
&\Gamma_{[h(t)]} = \left[e_1 \quad e_2 \quad e_3 \quad e_4 \quad h_1 e_9 \quad h_{1t} e_{10} \quad h_{2t} e_{11}\right],\\
&\Psi_{[\dot{h}(t)]} = \left[e_5\ e_6\ h_d e_7\ e_8\ e_1 - e_2\ e_2 - h_d e_3\ h_d e_3 - e_4\right],\\
&W_{01} = \left[e_1 \quad e_2 \quad e_9\right],\\
&W_{02} = \left[h_1 e_9\ e_1 - e_2\ e_{12}\ e_1 - e_9\right],\\
&W_1 = \left[e_2 \quad e_3 \quad e_4 \quad e_{10} \quad e_{11}\right],\\
&G_0 = \operatorname{diag}\left\{h_1,\ 0,\ 0,\ 0\right\},\\
&G_1 = \operatorname{diag}\left\{h(t) - h_1,\ 0,\ 0,\ 0\right\},\\
&G_2 = \operatorname{diag}\left\{h_2 - h(t),\ 0,\ 0,\ 0\right\},\\
&W_2 = \left[e_{10}\ e_2 - e_3\ e_{13}\ e_2 - e_{10}\right],\\
&W_3 = \left[e_{11}\ e_3 - e_4\ e_{14}\ e_3 - e_{11}\right],\\
&a_2\\
&\quad = \operatorname{sym}\Big\{e_{10}\overline{E}^T (Y_1\\
&\quad + h_d Y_3) F e_{10}^T + e_{11}\overline{E}^T (\overline{\mu}_1 Y_2 + Y_4) F e_{11}^T\Big\},\\
&\overline{E}^T = \left[I \ \ 0 \ \ 0 \ \ 0\right],\\
&F^T = \left[I \quad 0\right],\\
&Q_1(t) = Q_{10} - h(t) Q_{11},\\
&Q_2(t) = Q_{20} + (h - h(t)) Q_{21},\\
&\overline{Q}_a = Q_{11} + \begin{bmatrix} 0 & Q_a \\ Q_a & 0 \end{bmatrix},\\
&\overline{Q}_b = Q_{21} + \begin{bmatrix} 0 & Q_b \\ Q_b & 0 \end{bmatrix},\\
&\overline{R}_{0a} = R_0 + \begin{bmatrix} 0 & R_{0a} \\ R_{0a} & 0 \end{bmatrix},\\
&\overline{R}_a = R_1 + \begin{bmatrix} 0 & R_a \\ R_a & 0 \end{bmatrix},\\
&\overline{R}_b = R_2 + \begin{bmatrix} 0 & R_b \\ R_b & 0 \end{bmatrix},\\
&H_{01} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix},\\
&H_{02} = \begin{bmatrix} -I & 0 & 2I & 0 \\ 0 & -I & 0 & 2I \end{bmatrix},\\
&H_1 = \begin{bmatrix} h_{1t} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix},\\
&H_2 = \begin{bmatrix} h_{2t} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix},\\
&H_3 = \begin{bmatrix} -h_{1t} I & 0 & 2I & 0 \\ 0 & -I & 0 & 2I \end{bmatrix},\\
&H_4 = \begin{bmatrix} -h_{2t} I & 0 & 2I & 0 \\ 0 & -I & 0 & 2I \end{bmatrix},\\
&\Theta_1 = e_1 HKS e_{16}^T - e_{16} S e_{16}^T,\\
&\widetilde{\Theta}_1 = 2 e_5 H \Lambda e_{16}^T,\\
&\Pi_4 = \left[e_1 U_1 + e_5 U_2 + e_{15} U_3 + e_{16} U_4\right],\\
&\Pi_5 = \left[A e_1^T + A_1 e_3^T + C e_{15}^T + B e_{16}^T - e_5^T\right],\\
&\Pi_6 = \left[e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_{10}\right],\\
&\Pi_7 = \left[e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_{11}\right].
\end{aligned}
\tag{B.1}
$$

C. Box 4

$$
\begin{aligned}
\Omega_{4[h(t)]} &= P + \begin{bmatrix} \tilde{e}_1^T \\ \tilde{e}_2^T \\ \tilde{e}_3^T \\ 0 \end{bmatrix}^T \left[h(t) P_a + (h - h(t)) P_b\right] \begin{bmatrix} \tilde{e}_1^T \\ \tilde{e}_2^T \\ \tilde{e}_3^T \\ 0 \end{bmatrix}\\
&+ \operatorname{sym}\left\{\begin{bmatrix} \tilde{e}_1^T \\ \tilde{e}_2^T \\ \tilde{e}_3^T \\ 0 \end{bmatrix}^T \left(P_a \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{e}_4^T \end{bmatrix} + P_b \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{e}_5^T \end{bmatrix} \right)\right\},\\
\alpha &= \frac{h(t)}{h},\\
J^T &= \left[E_1 \quad E_2\right],\\
E_1 &= \left[\tilde{e}_4\ \tilde{e}_1 - \tilde{e}_2\right]\\
E_2 &= \left[\tilde{e}_5\ \tilde{e}_2 - \tilde{e}_3\right],\\
\Pi_{[h(t),\dot{h}(t)]} &= \operatorname{sym}\left\{\Pi_{1[h(t),\dot{h}(t)]} + \Pi_{3[h(t),\dot{h}(t)]}\right\}\\
&+ \Pi_{2[h(t),\dot{h}(t)]},\\
\Pi_{1[h(t),\dot{h}(t)]} &= \Gamma_{[h(t)]} P \Psi_{[\dot{h}(t)]}^T + \left[e_1 \quad e_2 \quad e_3 \quad e_7\right] P_a \Big[h(t)\\
&\cdot e_4\ h(t) h_d e_5\ h(t) e_6\ e_1 - h_d e_2 - \dot{h}(t) e_7\Big]^T\\
&+ \left[e_1 \quad e_2 \quad e_3 \quad e_8\right] P_b \Big[(h - h(t)) e_4 \quad (h - h(t))\\
&\cdot h_d e_5 \quad (h - h(t)) e_6\ h_d e_2 - e_3 + \dot{h}(t) e_8\Big]^T + \Theta_2\\
&+ \widetilde{\Theta}_2 + \Pi_4 \Pi_5,\\
\Pi_{2[h(t),\dot{h}(t)]} &= \left[e_1 \quad e_4\right] Q_1(t) \left[e_1 \quad e_4\right]^T\\
&+ h_d \left[e_2 \quad e_5\right] \left[Q_2(t) - Q_1(t)\right] \left[e_2 \quad e_5\right]^T
\end{aligned}
$$

$$
\begin{aligned}
&- [e_3 \ \ e_6] Q_2(t) [e_3 \ \ e_6]^T + e_1 \left[h_d R_a + \dot{h}(t) Q_a\right] e_1^T \\
&+ e_2 \left[R_b - h_d R_a + \dot{h}(t)(Q_b - Q_a)\right] e_2^T - e_3 \left[R_b\right. \\
&\left.+ \dot{h}(t) Q_b\right] e_3^T + e_4 Q_2 e_4^T - (1-\tau_d) e_{11} Q_2 e_{11}^T \\
&+ h(t) [e_1 \ \ e_4] \left(\overline{Q}_a + R_1\right) [e_1 \ \ e_4]^T + |\mu_1| \\
&\cdot h_d h [e_2 \ \ e_5] \overline{Q}_b [e_2 \ \ e_5]^T \\
&+ h_d h [e_2 \ \ e_5] R_2 [e_2 \ \ e_5]^T + \dot{h}(t) \Pi_6 P_a \Pi_6^T \\
&- \dot{h}(t) \Pi_7 P_b \Pi_7^T, \\
\Pi_{3[h(t),\dot{h}(t)]} &= h_d W_1 X_3 H_1 W_2^T + h_d W_2 G_1 Y_3 H_3 W_2^T \\
&+ W_1 X_4 H_2 W_3^T + W_3 G_2 Y_4 H_4 W_3^T + W_1 X_1 H_1 W_2^T \\
&+ W_2 G_1 Y_1 H_3 W_2^T + \overline{\mu}_1 W_1 X_2 H_2 W_3^T \\
&+ \overline{\mu}_1 W_3 G_2 Y_2 H_4 W_3^T, \\
\Omega_{a[k,j]} &= h W_k X_j, \\
\Omega_{b[k,r,j]}^{[h(t)]} &= h W_k G_r Y_j, \\
&\qquad k \in [1,2,3], \ j \in [1,2,3,4], \ r \in [1,2], \\
\Gamma_{[h(t)]} &= \left[e_1 \ \ e_2 \ \ e_3 \ \ h(t) e_7 \ \ (h - h(t)) e_8\right], \\
\Psi_{[\dot{h}(t)]} &= \left[e_4 \ h_d e_5 \ e_6 \ e_1 - h_d e_2 \ h_d e_2 - e_3\right], \\
W_1 &= [e_1 \ \ e_2 \ \ e_3 \ \ e_7 \ \ e_8], \\
W_2 &= [e_7 \ e_1 - e_2 \ e_9 \ e_1 - e_7], \\
W_3 &= [e_8 \ e_2 - e_3 \ e_{10} \ e_2 - e_8], \\
G_1 &= \operatorname{diag}\{h(t), \ 0, \ 0, \ 0\}, \\
G_2 &= \operatorname{diag}\{h - h(t), \ 0, \ 0, \ 0\}, \\
H_1 &= \begin{bmatrix} h(t) I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix}, \\
H_2 &= \begin{bmatrix} (h - h(t)) I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix}, \\
H_3 &= \begin{bmatrix} -h(t) I & 0 & 2I & 0 \\ 0 & -I & 0 & 2I \end{bmatrix}, \\
H_4 &= \begin{bmatrix} -(h - h(t)) I & 0 & 2I & 0 \\ 0 & -I & 0 & 2I \end{bmatrix}, \\
\Theta_2 &= e_1 HKS e_{12}^T - e_{12} S e_{12}^T, \\
\widetilde{\Theta}_2 &= 2 e_4 H \Lambda e_{12}^T \\
\Pi_4 &= \left[e_1 U_1 + e_4 U_2 + e_{11} U_3 + e_{12} U_4\right], \\
\Pi_5 &= \left[A e_1^T + A_1 e_2^T + C e_{11}^T + B e_{12}^T - e_4^T\right], \\
\Pi_6 &= [e_1 \ \ e_2 \ \ e_3 \ \ e_7], \\
\Pi_7 &= [e_1 \ \ e_2 \ \ e_3 \ \ e_8]. \\
a_2 &= \operatorname{sym}\left\{e_7 \overline{E}^T (Y_1 + h_d Y_3) F e_7^T + e_8 \overline{E}^T (\overline{\mu}_1 Y_2 + Y_4)\right. \\
&\left.\cdot F e_8^T\right\}
\end{aligned}
\tag{C.1}
$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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