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Three-dimensional viscoelastic instabilities in microchannels

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Whereas the flow of simple single-phase Newtonian fluids tend to become more complex as the characteristic length scale in the problem (and hence the *Reynolds number*) increases, for complex elastic fluids such as dilute polymer solutions the opposite holds true. Thus small-scale so-called "microfluidic" flows of complex fluids can exhibit rich dynamics in situations where the "equivalent" flow of Newtonian fluids remain linear and predictable. In the recent study of Qin et al.(J. Fluid Mech., vol. 864, 2019, R2) of the flow of a dilute polymeric fluid past a 50 µm cylinder (in a 100 x 60 µm channel), a novel 3D holographic particle velocimetry technique reveals the underlying complexity of the flow including inherent three-dimensionality, symmetry breaking and bi-stability as well as strong upstream propagation effects via elastic waves.

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1. Introduction

Purely-elastic instabilities (Shaqfeh 1996) driven by elastic normal stresses have been widely observed in the absence of significant inertial effects, in both viscometric flows such as the Taylor-Couette geometry (Larson et al. 1990) that imparts steady shearing motion on each fluid particle, and more general non-viscometric flow geometries (e.g. Groisman and Quake (2004); Arratia et al. (2006); Pakdel and McKinley (1996)). In addition to the usual material properties of density ρ and dynamic viscosity η which

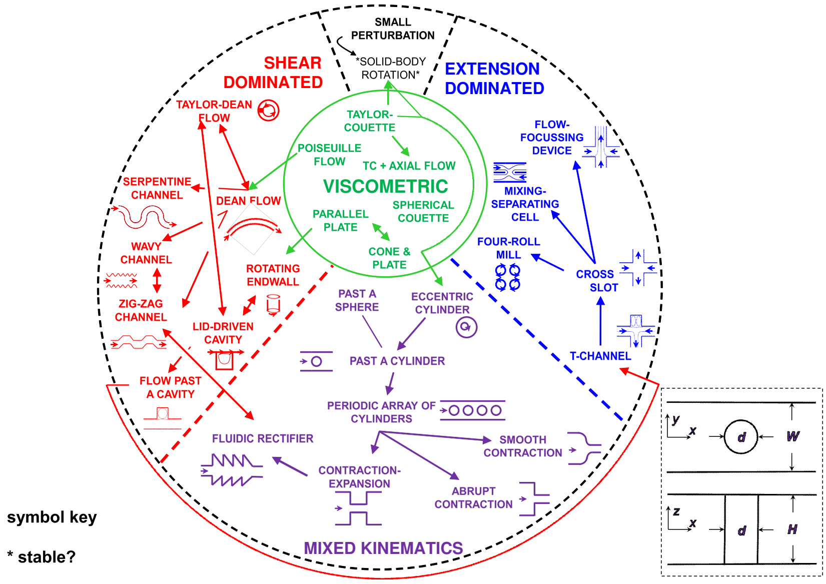

FIGURE 1. Purely-Elastic Flow Instability Map ("PEFIM"): A taxonomy for purely-elastic instabilities based on flow-type, including potential relationships between different prototype geometries. Inset in bottom right hand corner defines axes for the situation studied by Qin et al. (2019).

characterise single-phase Newtonian flows, dilute polymeric solutions, like those used by Qin et al. (2019) and most other studies in this area, are also characterised by a "relaxation" time (Bird et al. 1987). The *Deborah number* De, which is a measure of this relaxation time relative to a characteristic residence time of the flow, must become unimportant in fully developed, steady viscometric flows as the characteristic residence time tends to infinity. The second parameter governing the flow of single-phase viscoelastic fluids in the inertialess limit is the *Weissenberg number* Wi, the ratio of elastic to viscous stresses. These two parameters are often thought of as essentially interchangeable although in general, they represent subtly different properties of the flow one quantifying inherently Lagrangian unsteady effects (De) and the other the "strength" of the flow (Wi). In the problem studied by Qin et al. (2019) of flow past a cylinder of diameter d in an approximately square channel of height H, the characteristic timescale of the flow can be estimated as the time taken for a fluid particle to circumvent half the cylinder, i.e. $\pi d/2U \sim d/U \sim H/U$ as $d \sim H$. The Deborah number is then $\mathrm{De} \sim \lambda U/H$. The Weissenberg number in Qin et al. (2019) is $\mathrm{Wi} = N_1/2\tau = N_1/(2\dot{\gamma}\eta)$ where N_1 is the first-normal stress difference (the "elastic" stress), τ the shear stress and $\dot{\gamma}$ the shear rate. Only for an upper-convected Maxwell model (Bird et al. 1987) where $N_1 = 2\lambda\eta\dot{\gamma}^2$ and $\tau = \eta\dot{\gamma}$ does this yield $\mathrm{Wi} = \lambda\dot{\gamma} \sim \lambda U/H$ and hence $\mathrm{Wi} = \mathrm{De}$. However, De and Wi both increase with decreasing characteristic length scale(s). Hence, the microfluidic environment, which typically minimises inertial effects due to its small length scales, also enhances elastic effects so they become significant even for relatively dilute polymeric solutions which may appear essentially Newtonian-like on the macroscopic scale. As a result, microfluidic experiments have been widely exploited to probe purely-elastic instabilities in viscoelastic fluids, which usually arise from the combination of elastic stresses and streamline curvature. This mechanism is captured in the phenomenological criterion due to Pakdel and McKinley (1995), which states that when the product of a "local" De based on a length scale involving the local streamline curvature ($\mathfrak{R}$) and a "local" Wi based on local shear rate and local tensile (normal) stress along the streamline exceeds a critical parameter (M_{CR}^2) instability may arise i.e. $(N_1/\tau)(\lambda U/\mathfrak{R}) > M_{CR}^2$.

Purely-elastic instabilities and, at higher flowrates, even highly chaotic flows termed "elastic turbulence" have been observed in a wide range of different flows. One way of categorising these flows is via a schematic representation - a "map" - demarcating these flows into; viscometric, shear-dominated, extension-dominated and those of mixed kinematics as shown in Fig. 1. The map highlights the interrelation between flows of apparently differing character - between the cross slot (Arratia et al. 2006) and the mixing-separating geometry (Afonso et al. 2011) for example or the similarity between flow past a cylinder and into a contraction. The flow past a cylinder in a channel studied by Qin et al. (2019), shown schematically as an inset in Fig. 1, falls into the island of "mixed kinematics". This is because of the substantial shear around the cylinder and especially in the gap between the cylinder and the walls, combined with the extension/compression flows near the front/rear stagnation points of the cylinder.

2. Overview

Following on from earlier seminal work in the same group by Pan et al. (2013), where an array of cylinders in a microfluidic channel was used to show that a sustained elastic instability leading to elastic turbulence could also be achieved in the absence of streamline curvature far downstream of the array, the recent study of Qin et al. (2019) looks in detail at the instability around a single cylinder. It is precisely these "finite amplitude" perturbations (Morozov and van Saarloos 2007) which - when combined in an array of cylinders - were able to drive a self-sustaining turbulent motion far downstream (~ 400 channel widths) in the straight channel. Hence an understanding of the underlying instability mechanisms and complex flow fields around an isolated cylinder is potentially of broad interest. The significant advance of Qin et al. (2019) on most previous studies of purely-elastic instabilities (with limited exceptions such as the beautiful study by Afik and Steinberg (2017)) is in the use of a novel in-line 3D holographic particle tracking technique. The technique, detailed in Salipante et al. (2017), involves the recording using a high-speed camera of the flow seeded with tracer particles and illuminated by a laser mounted on an inverted microscope. In this manner, the positions of the particles are determined using back-scattering reconstruction and a fully three-dimensional (3D) velocity field can be reconstructed by differentiating Lagrangian particle trajectories. These 3D velocity fields reveal much more complex flow transitions as the flowrate (and hence Wi) is increased than could previously be inferred from two-dimensional measurements. In particular, the findings of Qin et al. (2019) of strong three-dimensionality at higher flowrates are significant in that they are likely to be systemic to most microfluidic flows studied to date because most microfluidic channels have finite depth, with an approximately square cross-section (as in Qin et al. (2019)).

Two-dimensional measurements in the xy-centreplane (see inset of Fig. 1 for axis definition) of Qin et al. (2019) reveal that an upstream vortex develops immediately in front of the cylinder as Wi is increased (Shi and Christopher 2016). Beyond a critical Wi ~ 4, the vortex begins to fluctuate weakly in time although it retains its symmetry about the midline of the xy-centreplane. This symmetry is lost beyond Wi ~ 8 and for Wi > 9 the flow enters a stronger time-dependent regime where the length of the upstream vortex frequently collapses to $\sim 2d$ and then regenerates (up to 6d).

The novel 3D measurements reveal that the "top" and "bottom" walls play a fundamental role in the dynamics as the flow at high Wi is revealed to be made up of a pair of separate recirculation zones each originating between the "corner" of the cylinder and the top/bottom walls. These zones are anticorrelated in that as one grows the other decays and vice versa. Thus symmetry-breaking first noted across the midline of the xy-centreplane is followed by symmetry-breaking across the midline of the xz-centreplane and the flow is strongly 3D, something which is only fully-revealed by these new measurements. Finally the results also suggest the presence of apparently different instability mechanisms upstream and downstream of the cylinder. The upstream propagation of disturbances suggest that perturbations may be transmitted against the primary flow direction via an "elastic wave" but with a wave speed which does not simply scale with λ. For a viscoelastic shear wave, based on a Maxwell type model with constant polymeric viscosity η_p and relaxation time λ, the wave speed is constant $c_s = \sqrt{(\eta_p/\rho\lambda)}$ and this leaves open precisely what is the physical interpretation of the wave (see also Varshney and Steinberg (2019)). Additionally, if these effects are related to viscoelastic shear waves, this suggests that inertia (i.e. non-zero density) may *still* be important in these flows.

3. Future

Given that the results of Qin et al. (2019) reveal the potentially strong influence of the walls on the time-dependent flow which develops, an obvious next step would be to investigate the effect of varying the depth aspect ratio of the channel to determine if this instability always arises at the same preferential location (the "corner" between the cylinder and the sidewall) or if new mechanisms arise when the channel is deeper. Novel fabrication methods which allow much higher aspect ratios (e.g. Haward et al. (2018)) than conventional PDMS microchannels may prove fruitful in this regard. Studies which can probe systematically the effect of varying the viscoelastic shear wave speed, perhaps by changing the polymeric viscosity contribution whilst holding the density and relaxation time approximately constant (using the model elastic liquids suggested by Dontula et al. (1998) for example), may shed more light on the upstream propagation mechanisms. In this vein, and given the rich dynamics observed, fully-resolved 3D time-dependent numerical simulations using viscoelastic constitutive equations would also seem like an interesting avenue to pursue. In this latter case, the simulations have the ability to fully "turn off" inertia, something that a *real* experiment can never truly do. However, the very complicated flow patterns and dynamics observed experimentally present a significant challenge for numerical simulations and a stringent test of existing viscoelastic constitutive equations.

References

REFERENCES

E. Afik and V. Steinberg. On the role of initial velocities in pair dispersion in a microfluidic chaotic flow. *Nature Communications*, 8(1):468, 2017.

A. M. Afonso, M. A. Alves, R. J. Poole, P. J. Oliveira, and F. Pinho. Viscoelastic flows in mixing-separating cells. *Journal of Engineering Mathematics*, 71(1):3–13, 2011.

P. E. Arratia, C. C. Thomas, J. Diorio, and J. P. Gollub. Elastic instabilities of polymer solutions in cross-channel flow. *Physical Review Letters*, 96(14):144502, 2006.

R. B. Bird, R. C. Armstrong, and O. Hassager. Dynamics of polymeric liquids. vol. 1: Fluid mechanics. 1987.

P. Dontula, C. W. Macosko, and L. E. Scriven. Model elastic liquids with water-soluble polymers. *AIChE journal*, 44(6):1247–1255, 1998.

A. Groisman and S. R. Quake. A microfluidic rectifier: anisotropic flow resistance at low reynolds numbers. *Physical Review Letters*, 92(9):094501, 2004.

S. J. Haward, K. Toda-Peters, and A. Q. Shen. Steady viscoelastic flow around high-aspect-ratio, low-blockage-ratio microfluidic cylinders. *Journal of Non-Newtonian Fluid Mechanics*, 254: 23–35, 2018.

R. G. Larson, E. S. G. Shaqfeh, and S. J. Muller. A purely elastic instability in Taylor-Couette flow. *Journal of Fluid Mechanics*, 218:573–600, 1990.

A. N. Morozov and W. van Saarloos. An introductory essay on subcritical instabilities and the transition to turbulence in visco-elastic parallel shear flows. *Physics Reports*, 447(3-6): 112–143, 2007.

P. Pakdel and G. H. McKinley. Elastic instability and curved streamlines. *Physical Review Letters*, 77(12):2459, 1996.

L. Pan, A. Morozov, C. Wagner, and P. E. Arratia. Nonlinear elastic instability in channel flows at low reynolds numbers. *Physical Review Letters*, 110(17):174502, 2013.

B. Qin, P. F. Salipante, S. D. Hudson, and P. E. Arratia. Upstream vortex and elastic wave in the viscoelastic flow around a confined cylinder. *Journal of Fluid Mechanics*, 864, R2, 2019.

P. F. Salipante, C. A. E. Little, and S. D. Hudson. Jetting of a shear banding fluid in rectangular ducts. *Physical Review Fluids*, 2(3):033302, 2017.

E. S. G. Shaqfeh. Purely elastic instabilities in viscometric flows. *Annual Review of Fluid Mechanics*, 28(1):129–185, 1996.

X. Shi and G. F. Christopher. Growth of viscoelastic instabilities around linear cylinder arrays. *Physics of Fluids*, 28(12):124102, 2016.

A. Varshney and V. Steinberg. Elastic Alfven waves in elastic turbulence. *Nature Communications*, 10(652), 2019.