# Dialogues

**May 2019**

# The importance of language analysis to mathematical, quantitative and computational sciences

by Paul Glaister

*In this article, Paul Glaister, professor of mathematics and mathematics education at the University of Reading, description of article ...*

## Why mathematics is important

Mathematics – whether long-established, current, or yet to be developed – is essential for making progress in much of science, technology, and engineering. Both computational science and data science, as well as quantitative science more generally, also rely heavily on mathematics.

Mathematical and quantitative skills at all levels are essential for the workforce of tomorrow, in all fields. Ultimately, mathematics plays a key role in our lives and work, society, our future and the future of the planet.

As Lefkowitz writes in his article 'Why is math so important?':

> "A solid foundation in mathematics and science develops and enhances the skills of posing hypotheses, designing experiments and controls, analysing data, recognizing patterns, seeking evidence, conclusions and proof, solving problems and seeking absolutes, while being open to new information.

Please cite: Glaister, P. (2019). The importance of language analysis to mathematical, quantitative and computational sciences. *Languages, Society & Policy*. https://doi.org/XXX

> "Mathematical and quantitative skills at all levels are therefore essential for the workforce of tomorrow, in all fields. Studying mathematics not only will develop more engineers and scientists, but also produce more citizens who can learn and think creatively and critically, no matter their career fields."

Given that all these disciplines demand strong language and communication skills, any initiative that seeks to introduce and improve language analysis in schools will bring significant benefit to us all.

# LASER Manifesto

The recently launched Language Analysis in Schools: Education and Research (LASER) initiative is therefore as timely as it is welcome. As outlined in the LASER Manifesto, language analysis (LA) is "the analysis of patterns and is just like analysis in any other school area, from literary analysis to chemical analysis".

Clearly *mathematical and quantitative analysis* should be included here. The Manifesto continues:

> "The most important requirements for LA are the ability to spot patterns in language and to identify variables, and the ultimate point of LA is the activity of analysis, as a route to a deeper understanding of language."

The same is true of mathematics, and it is ultimately the deeper understanding of mathematics that often leads to the deeper understanding of the science that the mathematics seeks to underpin, model, understand and benefit.

The Manifesto concludes with:

> "All mathematical formulae ultimately rest on explanations provided in language, but more advanced maths works with symbols that are further and further from their linguistic sources. This is a serious intellectual barrier for some students, who decide that they're 'no good at maths'; and it's also a serious communicational barrier for expert mathematicians when asked to present their reasoning to non-experts. Arguably, LA might be able to help with both these difficulties: by demystifying the symbols and terminology of more advanced maths, and by reconnecting these symbols with their linguistic foundations.
>
> "The thinking skills required by LA, and presumably developed by doing it, are similar to those required in maths, so it may be the case that practice in LA improves mathematical skills.
>
> "The latest specifications in mathematics A level require students to model real-world situations, interpret real world data, identify and comment on underlying assumptions, and solve problems. None of these things can be achieved without well-developed language skills linked to the analytical skills of LA – in contrast with the lamentable current tendency in

> mathematics examining to oversimplify language on the grounds that language should not be allowed to get in the way of the mathematics."

While this is all true, it is not just the symbols that are "further and further from their linguistic sources". Advanced mathematics comprises symbols, equations, inequalities, other mathematical statements and expressions, along with logical connectors, and much exposition, all of which require analytical tools and skills, as well as good communication skills. All of mathematics would benefit from the skills developed in LA.

Turning to the last statement quoted from the Manifesto, it is clear that A-level Mathematics students in particular would benefit from LA, as a consequence of the recent reforms in these qualifications. But it is not just A-level Mathematics that falls into this category – the whole of the mathematical sciences curriculum, from primary to post-16 and beyond, would benefit too, as we illustrate by looking at the high-level requirements in each of the stages of the curriculum.

# How is mathematics developed pre-18?

Mathematical knowledge, understanding and skills are developed in schools and colleges (in England) through study of a mathematics curriculum which develops from Key Stage 1 through to Key Stage 4 (GCSE), and then advanced mathematics post-16: A-levels and Core Maths. Mathematical and quantitative skills are clearly essential to many other areas of study pre-18, e.g. science and social science, and thus the mathematics curriculum supports many other subjects. The same is true post-18 – both in employment and further study – with knowledge, understanding and skills being developed further in more advanced areas, or in context, or both.

Given that mathematics is 'mission-critical', what is it about mathematics in the school curriculum that would support a case for introducing language analysis in schools?

The evidence for this is provided by the descriptors in the pre-18 school mathematics curriculum, all of which apply to, and are essential for, all of mathematical study. In particular, language analysis would support problem-solving, understanding, communication, reasoning, and analysis in mathematics, as is clear from the key statements identified in **bold** in the following statements for each Key Stage or qualification.

# Key Stages 1–4/GCSE Mathematics

The following key statements are to be found in the statutory programmes of study and attainment targets for mathematics at Key Stages 1 to 4 within the national curriculum in England; items in **bold** would benefit from skills developed in LA:

> "A mathematics education provides the ability to **reason** mathematically and **solve problems**:
>
> - **reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language,**
>
> - **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including **breaking down problems into a series of simpler steps** and persevering in seeking solutions.
>
> **Language:**
>
> "The national curriculum for mathematics reflects the **importance of spoken language** in pupils' development across the whole curriculum – **cognitively**, socially and **linguistically**. The **quality** and **variety** of **language** that pupils hear and speak are key factors in developing their mathematical **vocabulary** and presenting a mathematical **justification, argument or proof**.
>
> "They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions."

# Key Stage 5/A-level Mathematics and Further Mathematics, and Core Maths

Across the suite of level 3 mathematics qualifications, including A-level Mathematics and Further Mathematics and Core Maths, there is clear importance placed on:

- **understanding, problem solving, and inference;**
- **transferable skills;**
- **presentation of mathematics clearly and logically**, **using mathematical terminology correctly;**
- **representation;**

- **interpretation;**
- **extracting information;**
- **analysis** of mathematical problems;
- **construction** of **related arguments** (and **proofs**, at A-level only);
- **generalisation;**
- **evaluation;**
- **critical reflection.**

## A-level

The following key statements are to be found in relevant guidance documents on A-level Mathematics and Further Mathematics (see also ALCAB 2014), with items that would benefit from skills developed in LA in **bold**:

> "A-level mathematics emphasises how mathematical ideas are interconnected and how mathematics can be applied to model situations mathematically, to help make sense of data, to understand the physical world and to solve problems in a variety of contexts, including social sciences and business.
>
> "Students are expected to:
>
> - use their mathematical knowledge to make **logical and reasoned decisions in solving problems** both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly,
> - **reason logically and recognise incorrect reasoning,**
> - **generalise** mathematically,
> - **construct** mathematical proofs,
> - **represent** situations mathematically and understand the **relationship between problems in context and mathematical models** that may be applied to solve them,
> - **make deductions and inferences and draw conclusions by using mathematical reasoning,**
> - **interpret solutions and communicate their interpretation effectively in the context of the problem,**

- **read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding,**
- **read and comprehend articles concerning applications of mathematics and communicate their understanding.**"

Overarching Theme 1 in the reformed AS-/A-levels for 2017 is 'Mathematical argument, language and proof', and Glaister and Glaister (2017) provide a commentary and examples on this theme, showing how the value that LA skill would bring to this is abundantly clear.

Two different types of proof in A-level Mathematics for which language is clearly important are provided here:

**Example of proof by exhaustion**

Prove that every integer that is a perfect cube is either a multiple of 9, or 1 less, or 1 more than a multiple of 9.

*Proof*

Each cube number is the cube of some integer $n$. Every integer is either a multiple of 3 or is one less or two less than a multiple of 3 since the maximum remainder when dividing by 3 is 2. The set of integers can therefore be divided into three non-overlapping cases which are exhaustive.

**Case 1:** If $n = 3m$ for some $m \in \mathbb{Z}$, then $n^3 = (3m)^3 = 27m^3 = 9(3m^3)$, which is a multiple of 9.

**Case 2:** If $n = 3m - 1$ for some $m \in \mathbb{Z}$, then $n^3 = (3m - 1)^3 = 27m^3 - 27m^2 + 9m - 1 = 9(3m^3 - 3m^2 + m) - 1$, which is 1 less than a multiple of 9.

**Case 3:** If $n = 3m - 2$ for some $m \in \mathbb{Z}$, then $n^3 = (3m - 2)^3 = 27m^3 - 54m^2 + 36m - 8 = 9(3m^3 - 6m^2 + 4m - 1) + 1$, which is 1 more than a multiple of 9.

Therefore the result is true for all integers $n$, as required.

**Example of proof by contradiction**

Prove that for all $n \in \mathbb{N}$, if $4^n - 1$ is prime, then $n$ is odd.

*Proof*

We can prove this by contradiction, as follows.

Suppose that this is not the case, i.e. that $4^n - 1$ is prime, where $n \in \mathbb{N}$, and $n$ is *even*.

In this case, $n = 2k$ for some $k \in \mathbb{N}$, and hence $4^n - 1 = 4^{2k} - 1 = (4^k)^2 - 1 = (4^k - 1)(4^k + 1)$.

However, since $k \in \mathbb{N}$, we have that both $4^k - 1$, $4^k + 1 \in \mathbb{N}$ and $4^k - 1$, $4^k + 1 > 1$, and hence $4^n - 1$ has at least two factors that are greater than 1, which contradicts the fact that $4^n - 1$ is prime.

Therefore our assumption that $n$ is even in incorrect, and hence $n$ must be odd, as required.

## Core Maths

The following key statements are to be found in relevant guidance documents on Core Maths, with items that would benefit from skills developed in LA in **bold**:

"Core Maths fosters the ability to think mathematically and to apply mathematical techniques to a variety of unfamiliar situations, questions and issues with confidence.

"Students are expected to:

- **represent and analyse** complex and unfamiliar situations,
- **identify and understand** quantifiable **information** and related **assumptions,**
- use mathematical and statistical **representations** and techniques appropriately,
- **derive new information to draw meaningful conclusions,**
- **evaluate the relevance** of solutions in context of the situation, and **establish how they could be used and communicate findings accurately and meaningfully,**
- **develop skills in mathematical thinking, reasoning and communication,**
- **interpret** new situations in terms of mathematical and quantitative characteristics,
- **make judgements** about strategies and methods to achieve a solution,
- take creative approaches where appropriate,
- test and **evaluate** answers and conclusions,
- solve non-routine problems where specific methods and solutions are not immediately obvious because there may be **limited, ambiguous or contradictory information**,
- **make judgements or assumptions**,
- **explain** mathematical reasoning and conclusions to others and **justify** specific approaches taken to the problem,

- **interpret** conclusions on the basis of mathematical understanding and **explain** limitations to answers and conclusions."

# Computing, coding, computer science, data analysis, data science, and quantitative science and skills

We have focussed our attention on the importance of mathematics, and particularly on the role that mathematics plays in science, technology and engineering, to make clear the benefit that LA would bring to these disciplines. But there is much more beyond these areas that would benefit from LA.

The British Academy's Count Us In and State of the Nation publications make abundantly clear the importance of data in the future:

> "The ability to understand and interpret data is an essential feature of life in the 21st century: vital for the economy, for our society and for us as individuals. The ubiquity of statistics makes it vital that citizens, scientists and policy makers are fluent with numbers. Data analysis is revolutionising both how we see the world and how we interact with it."

They offer

> "a vision of how the UK can rise to the potentially transformational challenge of becoming a data-literate nation. Count Us In calls for a cultural change across all phases of education and employment, together with a concerted, continuous national effort led by government."

In order that we become a 'data-literate' nation we must develop strong data-handling and analytical skills, using associated tools from mathematics and statistics, to make effective use of data. Given that LA would support mathematics, and therefore the use of mathematics in statistics and in quantitative sciences more generally, LA would ultimately benefit social sciences, and indeed any discipline for which data is fundamental.

Finally, as outlined in the Universities UK (UUK) report Making the most of data: Data skills training in English universities,

> "Data presents a significant opportunity for the UK. Businesses are increasingly collecting and analysing data to enhance their productivity. Policy makers are considering how best to use data to transform public service delivery. Researchers are using data to advance scientific research and engineering. The potential value of data is far reaching. Across a range of sectors and industries, developing capacity in data analytics can advance knowledge, improve services and drive innovation."

The UUK report places much emphasis on coding, computing, programming, computer science, data science, and data analytics, while acknowledging that "computer science is a mathematical subject" and that "data analysis requires a number of mathematical and computational skills". The report includes support for current initiatives in schools:

> "The new Computing curriculum in English schools is a significant development. Similarly, the coding curriculum is a positive step forwards, enabling young people to develop more advanced computer skills and become aware of the possibilities of technology. The next step will be to embed both computer science and mathematical skills across subjects taught at primary and secondary level."

The following key statements are to be found in relevant guidance documents on GCSE Computer Science, with items that would benefit from skills developed in LA in **bold**:

"GCSE specifications must require students to develop the following skills:

- take a **systematic** approach to **problem solving** including the use of **decomposition** and **abstraction**, and make use of **conventions** including pseudo code and flowcharts,
- **design, write, test and refine** programs, using one or more high-level **programming language** with a textual program definition, either to a specification or to solve a problem,
- use appropriate security techniques, including **validation** and authentication,
- **evaluate** the fitness for purpose of algorithms in meeting requirements efficiently using **logical reasoning** and test data,
- use **abstraction** effectively to appropriately **structure** programs into **modular** parts with **clear**, **well-documented** interfaces."

The Government has also made clear the importance of coding and computer programming (DfE 2018).

These initiatives are critical to realising making the most of data through data analysis, as described in the UUK report:

> "A number of stakeholders suggest that the UK currently produces great computer scientists and great mathematicians, but not a combination of both. This is problematic because advanced computer science and mathematical skills are both necessary to derive value from complex data. Although it may be that within data teams individuals have a particular area of expertise, working knowledge of statistics and basic computational and programme skills is necessary to critically engage in data analysis and make sense of data challenges, opportunities and outputs."

We have already seen that mathematics is a discipline that can require great precision. The same is clearly true of coding and programming: a code or a program must be error-free to work as intended, so precision is key here too. Moreover, coding and programming are built on a sequence of logical steps, so these require a good understanding of logic, whether in mathematical or linguistic terms. Thus, LA is also highly likely to be of benefit to the disciplines encompassed here.

# Conclusion

The UUK report concludes with:

> "We believe that if our recommendations are acted upon as a group, they will make the UK a stronger analytic nation, best placed to assume a leading role in the data economy."

It is therefore clear that any improvement in analysis skills through LA will support the UK becoming a stronger analytic nation. Given the importance of mathematics to this, and the benefit LA would bring to mathematics, LA has much to offer in support of UK plc.

# Further reading

ALCAB [A Level Content Advisory Board] (2014). *Report of the ALCAB panel on mathematics and further mathematics*.

DfE [Department for Education] (2018). *Schools Minister announces boost to computer science teaching*.

Glaister, P. & Glaister, E. M. (2017). Mathematical argument, language and proof: AS-/A-level 2017. *Mathematics in School*, 46, 16–19.

## About the author

Paul Glaister is Professor of Mathematics and Mathematics Education in the Department of Mathematics and Statistics at the University of Reading. As well as 39 years' teaching and leadership experience, his 444 publications span research in numerical analysis, mathematics/science education, and teaching and learning. He has taught students in HE on a wide range of applied mathematics courses for mathematics and mathematical sciences programmes, and on programmes in other disciplines, and at all levels, for 37 years. He has considerable experience of education policy and practice in the pre-HE sector, particularly post-16 Mathematics, and on the school-university

transition. He has reviewed A-level Mathematics and Further Mathematics as a member of ALCAB, and reviewed Core Maths qualifications for the Department for Education, and A-levels for Ofqual.